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ORIGINAL ARTICLE

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# Euromech 579 Arpino 3–8 April 2017: *Generalized and microstructured continua: new ideas in modeling and/or applications to structures with (nearly)inextensible fibers*—a review of presentations and discussions

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**Abstract** In the present paper, a rational report on Euromech 579, *Generalized and Microstructured Continua: New ideas in modeling and/or Applications to Structures with (nearly)inextensible fibers* (Arpino 3–8 April 2017), is provided. The main aim of the colloquium was to provide a forum for experts in generalized and microstructured continua with inextensible fibers to exchange ideas and get informed about the latest research trends in the domain. The interested reader will find more details about the colloquium at the dedicated web page http://www.memocsevents.eu/euromech579/.

**Keywords** Euromech 579 · Microstructured continua · Modeling · Inextensible fibers

## 1 Aims and main topics of the colloquium

Actual complexity of open problems in scientific research demands nowadays an orchestral effort of different research groups working in different branches of science. Needs and questions in some research area are often linked to (apparently) completely different investigations by the deep nature of the addressed problems. The recent activity on generalized continua with inextensible fibers is of course a perfect example of these interdisciplinary aspect. Research activity in this area spreads a wide set of applications, like design of special fabrics and biological tissue engineering, and methods, like mathematical modeling, numerical analysis and experimental investigations.

Euromech Colloquium 579 is exactly intended for providing precious help in direction of increasing the interaction between researchers and promoting a larger and deeper coordination of their efforts. The present work is aimed to present to the interested scientific community the most recent advances in the field of generalized continua and their use in the design of composites and metamaterials, with particular attention to inextensible fibers.

Communicated by Francesco dell’Isola.

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The underlying question behind the whole field is how to develop methods and results from generalized continuum mechanics to increase the prediction power of the theory when advanced architectured materials are concerned. In order to give an answer, both theoretical and experimental efforts are needed. One of the main challenge of this topic is to prescribe the right constitutive characteristic microscale in order to get a certain behavior at the macroscale. From this point of view, key tools are homogenization techniques which allow for the proper micro–macro-identification. Of great importance is of course a deep knowledge of functional analysis and modeling techniques in order to develop continuous models able to allow for an accurate description of reality but being simple enough to be concretely useful. The resulting integro-differential equations may admit the presence of non-local operators, and the corresponding solutions can possibly have jump discontinuities. Such difficulties require an exceptional effort in developing numerical methods. This approach is crucial also because of the youth of this research line, and it will be helpful to fill the rather large gap between theoretical prediction and experimental results. Experimental methods needed for the rational investigation of the considered structure are far from trivial too. Indeed, often the system is very sensitive to the precision reached in approximating ideal conditions about constraints and fibers and moreover the data collecting process requires accurate measurements of several kinematic and dynamic variables at the same time. Finally, one has also to remark that in the recent past the development of new technical manufacturing possibilities, like 3D printing, were crucial to push the development of this research area.

The colloquium has addressed mainly the following topics:

- to find the correct micro–macro-identification procedure which allows the characterization of suitable macro-models for continua inextensible fibers;
- to determine the characteristics of fibers and constraints to require in order to obtain the desired macroscopic behavior;
- to identify new mathematical problems and numerical methods useful for the conceived applications and new independent research lines;
- to contribute to the establishment of suitable experimental protocols for continua with inextensible fibers;
- to find the most interesting technological end engineering applications for continua with inextensible fibers.

We conclude this introductory section by remarking that among the objectives of the colloquium there is the promotion of researches which are at the moment motivated by the problems set in the given context, but that may entail a much more general scope, thus originating new independent lines of investigation. This is indeed, in our opinion, one of the highest task of a scientific workshop.

## 2 Thematic sessions

The aim of the this section is to provide a brief overview of the topics discussed in the colloquium. The following subsections, in spite of their list-like form, could help the reader to grasp the general investigation lines in the field. Of course, the different areas of research are not completely independent and different contributions could be placed in several of them, as it is evident from the fact that the arguments treated in the contributions can be often linked to each other.

### 2.1 Phenomenology and dynamics of materials reinforced with nearly inextensible fibers

In the communication of Loïc Le Marrec (‘Three-dimensional vibrations of a helically wound cable modeled as a Timoshenko model’), an analytical model for vibration of a rod containing helical fibers is proposed for both three-dimensional bending and torsion compression. The model is supported by numerical simulations, and the nonlinear contribution of the lay angle is provided analytically [1].

Francesco D’Annibale, in his talk (‘A mixed numerical-perturbative algorithm for static analysis of non-linear planar fabrics’), has illustrated a numerical perturbation method for analyzing statics and buckling problems of general geometrically nonlinear planar fabrics made of inextensible and shear-undeformable fibers. The method is based on a local perturbation analysis in which the displacement fields are not interpolated between the nodes, but rather obtained by asymptotic integration of the field equations. A third-order relation between end-forces and end-displacements is found, and numerical examples were provided to study the effectiveness of the model [2,3].

In Vladimir Sadovskii's contribution ('Constitutive relationships of fiber-reinforced composites having different resistance to tension and compression'), it was developed a general approach for constructing constitutive relationships of unidirectional fiber composites, resisting differently to compression and tension. In particular, in order to justify the necessity of taking into account different resistance to tension and compression of the composite materials reinforced with rigid fibers, the problem on bending of a hinged beam under the action of concentrated force and concentrated moment was analyzed. Finally, using this approach the constitutive relationships of fiber-reinforced composites are constructed for the case of isotropic matrix as well as for the case of heteromodular elastic matrix [4].

Sergey Lurie, in his talk ('Prediction of effective properties of multifunctional composites with CNTs "fuzzy" fiber across the length scale'), has shown a method to estimate the effective storage and loss moduli of bristled fiber composite materials, where the surface of the fibers is coated with nanostructures such as carbon nanotubes. In particular, he discussed the effects of nanofiber length, density of nanostructure near the surface of fibers, and changing of density of nanostructure near surface of fibers on the effective mechanical and dynamical properties of a multifunctional fiber composites by employing a self-consistent Eshelby method of a multi-phase model using a viscoelastic corresponding principle [5,6].

In the communication of Salvatore Di Stefano ('Plasticity and remodelling in fiber-reinforced tissues'), the description of fiber-reinforced, soft, hydrated biological tissues featuring a statistical orientation of reinforcing fibers was addressed. The main aim is to study the mechanical behavior of a tissue of this type by highlighting the coupling among deformation, plastic distortions, reorganization of the fiber pattern, and evolution of the interstitial fluid. Whereas the tissue's deformation and the flow of the interstitial fluid are described by equations characterizing the standard biphasic model of hydrated tissue, the equations determining the anaelastic distortions and the fiber reorientation are obtained within the theory of two-layer dynamics [7].

The talk by Alfio Grillo ('A theory of remodelling in fiber-reinforced anisotropic media based on the Ginzburg-Landau free energy density') is in the same spirit. In this communication, the role played by the reorganization of the fiber pattern in the structural evolution of fiber-reinforced biological tissue is described. The main assumption of the considered model is that the collagen fibers are oriented statistically according to a probability density distribution compatible with experimental data. In particular, the active reorientation is modeled by introducing a suitable remodeling energy that accounts for the interaction with the deformation and is assumed to comprise a term of the Ginzburg-Landau type and a term depending on the gradient of the mean angle of the probability density [8,9].

### 2.2 General concepts for microstructured continuum modeling

Francesco dell'Isola, in his contribution ('Description versus design: Describing metamaterials and fabrics with continuum and/or discrete models'), has presented the mathematical models which have been introduced in order to describe a family of composite fiber reinforcements used in aeronautical engineering and a novel class of metamaterials, the so-called pantographic sheets [10–13]. Fiber reinforcements are governed, if modeled with continuum mechanical models, with a second gradient deformation energy [14,15]. Unfortunately, large computations are sometimes required when using continuous models for complex mechanical systems and the order reduction made possible by higher gradient models is not always sufficient to get the desired reduction of computation time. Therefore, together with continuous models, also intrinsically discrete ones getting more easily computable equations were introduced [16,17]. The performance of continuous and discrete models are, in the most significant situation, comparable [18–22]. A careful correspondence between experimental evidence and theoretical and numerical predictions is obtained [23]. The talk was concluded with some interesting considerations about future immediate research perspectives and some major possible modeling improvements are delineated [24,25].

Jarkko Niiranen in his talk ('Modelling of lattice structures by strain gradient elasticity theories') has discussed the first and second gradient elasticity theories in the framework of homogenization of lattice structures. In particular, he firstly considered triangular longish planar truss structures. By means of a classical homogenization procedure, effective material parameters are found and the truss is modeled following first gradient elasticity beam model. Another approach is presented, where planar spring lattice structure is modeled by using a continuum model following the second strain gradient elasticity theory. Model comparisons and numerical simulations show that the generalized continuum models are able to capture the essential features of the lattice structures, including size effects [26,27].

In the contribution of Victor Eremeyev ('On characterization of an elastic network of orthogonal inextensible fibers with stiff knots using the six-parameter shell theory'), the deformations of an elastic network made of two orthogonal families of inextensible fibers rigidly connected to each other at contact points have been discussed in the framework of the six-parameter nonlinear shell theory. The main idea is to replace the analysis of deformations of such discrete system by homogeneous shell model using the six-parameter shell theory. The comparison between this model and a discrete model of an elastic network was discussed and the relations between material parameters in the two models were analyzed [28–30].

Another interesting point of view was given in the talk by Pierre Seppecher ('Explicit second gradient effect in periodic lattices'). In this contribution, it was addressed the problem of determining the least energetic configuration at the microscopic level in a cell once the macroscopic strain is prescribed. In the case of lattices with degenerate homogenized rigidity, it was shown that the homogenized behavior is determined by a second local minimization problem. In particular, this resulting problem is similar to the original one but with a different source term which is a function of the gradient of the macroscopic strain tensor, yielding in turn to a second gradient homogenized material [31,32].

In the contribution of Alessandro Della Corte ('Equilibria of a clamped Euler beam with distributed load: large deformations'), some rigorous results on the absolute minimizer for a clamped Euler beam under distributed load were discussed. In particular, the existence of local minimizer was proven for high enough values of the load and sufficient conditions for linear stability and instability of particular classes of solutions were provided. Finally, numerical evidence of many other stationary configurations has been shown and further rigorous results on them have to be provided [33,34].

A nonlinear generalization of extensible Euler and Timoshenko beam model was proposed in the talk by Antonio Battista ('Large deformation of extensible Euler and Timoshenko beam'). This model can describe the homogenized deformation energy of a 1D continuum with a simple microstructure. Moreover, a generalization of the model which includes an additional rotational spring in the microstructure was discussed and some numerical simulations were presented [35].

Luca Placidi, in his talk ('Modelling "pantographic beams" by means of nonlinear 1D second gradient continuum model'), has presented the so-called pantographic beam. In particular, he has introduced a mechanical discrete spring Hencky-type micro-model which resembles the microstructure of the beam. In order to obtain a continuum fully nonlinear beam model, a heuristic homogenization procedure with proper rescaling laws for the rigidities of all the springs of the micro-model and for the length of each fiber was discussed. Connection with previous literature was traced by discussing the interesting case of microstructure made of inextensible fibers [36–38].

In the talk of Maurizio Romeo ('Electromagnetic microcontinua and Maxwell equations in matter'), an alternative approach to common microcontinuum theories of electro-elastic media has been presented. This model, based on electric multipole densities, has been extended to deal with electric conductors. In particular, polarization and magnetization were obtained from macroscopic Maxwell's equations in terms of multipoles. Charge carries were considered as a continuum superimposed to the microstructured conductor and, as an example, the case of a rigid polarized conductor was discussed [39,40].

Anne Jung, in her talk ('Experimental characterisation and parameter identification for yield surfaces of metal foams'), has discussed the results of a structural investigation of aluminum metal foams with different pore sizes by computer tomography data. Metal foams are microheterogeneous materials characterized by a tripartite hierarchical structure. These results cover the most comprehensive experimental data set in the literature ever reported for open-cell metal foams [41,42].

The main aim of the study discussed in the talk by Mikhail Kariakin ('Second order effects in the problems of equilibrium and stability of nonlinear elastic body with microstructure') was to analyze classical problems for nonlinearly elastic bodies that have solutions significantly different in the classical theory of elasticity and in the theory of the Cosserat continuum. In particular, understanding such situations can be the basis for the development of experimental techniques for the identification of parameters of constitutive relations of media with microstructure and for the verification of the model used [43].

The contribution of Rainer Glüge ('On the significance of the convexity yield limit') dealt with the fundamental mathematical property of convexity of functions and sets. In particular, despite yield limits in terms of stresses have often such a nice mathematical property, requiring positive plastic dissipation does not necessarily exclude non-convex yield limits, as also experimentally observed. It was discussed an elastic-plastic, small strain, plane stress setting in a descriptive, empirical manner. The interesting result is that, at least in this setting, no specific material property can be attributed to the convexity of the yield limit [44].

Another interesting mathematical talk was the one by Boris Desmorat ('Harmonic factorization and reconstruction of the elasticity tensor'). In his contribution, he has presented a factorization of a fourth-order harmonic tensor into second-order tensors. In particular, explicit equivariant reconstruction formulas are obtained for transverse isotropic and orthotropic harmonic fourth-order tensors, and for trigonal and tetragonal harmonic fourth-order tensors up to a cubic fourth-order covariant remainder [45,46].

Dmytro Orlov, in his talk ('Harmonic-structured materials: concept, fabrication, and first attempts of elucidating their structure-performance relationship'), has presented the concept of harmonic structure materials along with possible fabrication routes. Experimental attempts aimed to understand the structure–performance relationship in harmonic structures using tensile and nano-indentation tests were reported, along with finite element simulations [47].

Results of experimental studies were discussed in the contribution by Nicolas Auffray and Martin Poncelet ('Experimental strain gradient evidence in non-central symmetric lattice'). This kind of analysis is crucial because the behaviors of strain gradient materials are up to now nearly exclusively studied from a theoretical and numerical point of view. In particular, a non-center symmetric sample, obtained by 3D printing, has been investigated and the geometrical anisotropy of the elementary cell has been chosen in order to detect strain-gradient effect. The main results discussed in this talk reveal that the architectured material behave as an effective genuine strain-gradient material [48,49].

In the presentation of Marek Pawlikowski ('Viscoelastic approach in modelling of the pantographic structures'), a formulation of a constitutive model for the polyamide PA 2200 was shown. The polyamide is the material used in 3D printing realized by means of selective laser sintering (SLS) technology. The main aim of the study was to numerically simulate printed pantographic structures using the new constitutive model. It was assumed that the material is isotropic, incompressible, nonlinearly viscoelastic. In addition, it is characterized by deformation rate dependency. The constitutive model was based on experimental tests, i.e., stress relaxation and monotonic compression. In the second part of the talk, some results of other experimental tests were presented. The tests were conducted on the printed pantographic samples. The results were shown in the form of force–displacement curves. The graphs revealed very interesting characteristics of the pantographic structures [50,51].

Finally, a talk about 3D printing techniques devoted to the production of second gradient material specimens was presented by Aron Pfaff ('Manufacturing strategy and heat treatment of a slender, second gradient material structure in AlSi10Mg by selective laser melting'). In particular, the consequences and impact on the printing of the digital preparation, exposure strategy, exposure parameters, and heat treatment were discussed [52].

### 2.3 Micro–macro-homogenization techniques and constitutive law identification

In his contribution ('Imbricated scales in granular materials: from grains to homogeneous continuum'), Antoine Wautier has discussed the mechanical behavior of granular materials. They are governed at microscale by cohesive-frictional forces acting at contact points between elementary particles. Due to the fact that only a small fraction of elementary particles participates in the stress transmission, in order to use the micro–macro-homogenization technique there is a need to define an intermediate scale [53,54].

In the same topic of granular materials, Anil Misra, in his talk ('Granular (meta)materials with high microscale rigidity-extensibility ratio') has efficaciously recalled the main models of granular materials. In particular, he has discussed the differences between discrete methods, requiring models and description of microstructure and grain-scale interactions, and continuum methods, which is related to the macro-scale containing a large number of grains and which is profoundly affected by the grain-scale interactions [55–59].

The contribution by Xia Li ('Quantifying the micro-structure of granular materials and its impact on constitutive behavior') is again about granular materials. In this talk, it was presented a tessellation system, which serves as the mathematical description of the microstructure of granular materials. The homogenization techniques, in this framework, are able to establish the analytical connection between continuum scale stress and strain tensors with their particle-scale counterpart, contact forces and relative particle displacements [60].

In the contribution by Ranganathan Parthasarathy ('Separated temporal and spatial framework for stress calculations in atomic models') has discussed a formalism called separated temporal and spatial which obviates the necessity for canonical transforms while retaining locality at atomic scale and is applicable for continuum material points of all possible sizes. This method is a robust approach for studying mechanical behavior at high temperatures in a continuum framework and is an important add-on to granular micromechanics for applications to atomic scale [61].

Different applications of homogenization procedures, in particular biological ones, are considered in the talk by Daniel George ('3D micro-mechanical homogenization of brain tissue considering random neuronal distribution within the extracellular matrix'). In this talk, a micro-mechanical model on a representative volume element of the brain is presented. It is able to integrate anisotropic neuron distribution to simulate brain's mechanical behavior under different loading conditions [62].

Andrey Nasedkin, in his talk ('Some approaches to homogenization of porous piezoceramic materials: microscale, nanoscale and local heterogeneities near pores'), has discussed the homogenization of porous piezoceramic materials. In particular, the homogenization of porous piezoelectric material is discussed in terms of the effective moduli method and the finite element technologies by means of explicit examples [63].

Second gradient effects were considered in the talk by Abdoul Anziz Houssam ('Homogenization of periodic elastic structures leading to second gradient effects'). In particular, by means of the tool of gamma-convergence and two-scale convergence, a structure with a geometry based on a periodic planar graph is shown to yield a second gradient model.

Homogenization techniques for pantographic structures were discussed in the talk by Claude Boutin ('Homogenisation of linear pantographic sheets'). In particular, by means of a micro–macro-identification and under the hypothesis of small deformations, a continuum model associated to a linear pantographic sheet is deduced. The obtained high-order continuum model shows interesting and exotic features, related to its extreme anisotropy but also to the sub-coercivity of its deformation energy [64].

The main aim of the contribution of Ali Javili ('Micro-to-macro accounting for general imperfects interfaces') was to show how to establish a micro-to-macro transition framework to study the behavior of heterogeneous materials whereby the influence of the interface at the microscale is taken into account. In particular, general imperfect interface model was considered [65–67].

In the contribution of Lidiia Nazarenko ('A new analysis of overall thermo-elastic properties of random particulate nano-composites with various interphase models'), a new hierarchical approach to analysis of thermo-elastic properties of random composites with interphases was outlined and illustrated. The main tool is the statistical homogenization method combined with recently introduced notion of energy-equivalent inhomogeneity which can be extended to include thermal effects [68].

Finally, Frej Chaouachi, in his talk ('Microstretch beam model with enriched kinematics accounting for variable section'), has addressed the problem of construct generalized beam models accounting for Poisson's effect and the occurrence of large deformations. In particular, a beam element with eight degrees of freedom to construct a 3D isotropic microstretch continuum model of the reinforcement in composite structures was developed [69].

### 2.4 Numerical analysis for generalized and microstructured continua

Simon Eugster, in his talk ('A planar nonlinear Euler–Bernoulli beam element using B-Spline interpolation functions for the analysis of pantographic sheets'), has presented a planar Euler–Bernoulli beam element, based on B-Spline interpolation functions, which allows for large displacements and rotations as well as for large axial deformations [70,71].

In the contribution of Christian Liebold ('C1-continuous approximations using hermite finite elements for second gradient elastic problems'), hermite finite elements are used to obtain C1-continuous approximations for second gradient theories. The element stiffness matrix and the global stiffness matrix for one- and two-dimensional problems were developed and solved numerically. In particular, Euler–Bernoulli beam bending and Kirchhoff-Love plate bending were analyzed, and the results were compared to experiments [72].

Developments in the theory of finite elements calculations for micropolar solids were discussed in the talk by Andrzej Skrzat ('On FEM evaluation in linear micropolar elasticity'). In particular, the problem of capture the stress behavior in the vicinity of geometric singularities such as notches or contact areas was analyzed. An interesting analysis of the interaction between a biodegradable implant and a trabecular bone was presented [73].

The contribution of Maria Varygina ('Numerical modeling of micropolar plates and shells') was about mathematical models of micropolar plates and shells taking into account independent small rotations of particles. During the talk, numerical algorithms, based on two-cyclic splitting method with respect to the spatial variables in combination with the explicit monotone finite-difference essentially non-oscillatory scheme, were presented [74].

Philippe Boisse, in his talk ('Simulations of 3D textile composite reinforcements forming. Specificities and possible modeling of the mechanical behavior'), has discussed the mechanical behavior of 3D textile performs during the forming process. It was shown that a 3D modeling based on Cauchy hypothesis is not sufficient to describe certain aspects of the deformation of thick fibrous reinforcements. Two solutions were proposed: a second gradient, 3D orthotropic model and a finite element formulation which is able to take into account the local bending stiffness [75].

In the contribution of Aleksandr Yaroslavovich Grigorenko ('Numerical studying of the stationary dynamical process in anisotropic inhomogeneous cylindrical bodies'), the problem of natural vibrations of anisotropic inhomogeneous bodies both with circular and noncircular cross section was addressed. In particular, a numerical analytical approach based on spline approximation of model partial differential equations was employed. Finally, the influence of character of structural inhomogeneity of cylindrical bodies, of variation in geometrical and mechanical parameters, and of boundary conditions were discussed and analyzed [76].

The pantographic beam model for slender structures resembling pantographs with quasi-inextensible and inextensible fibers was discussed in the talk by Emilio Barchiesi ('Numerically tackling the solution of the (quasi-)inextensible 'pantographic beam' model by means of variational techniques'). Numerical solution of some benchmark problems was analyzed by means of finite elements minimization techniques and Hellinger–Reissner variational principle. Finally, some equilibrium shapes exhibiting highly non-standard features were shown [77–79].

Massimo Cuomo, in his talk ('Mixed variational methods with numerical applications'), has presented a homogenized model for the analysis of a 2D continuum with two straight families of inextensible fibers embedded in it. The kinematics of the continuum is analyzed and, motivated by phenomenological observations, it is assumed that the strain energy depends on the shear deformation of the fibers and on their bending curvature [80]. Several first and second gradient deformation energy models are considered, depending on the shear angle between the fibers and on its gradient, and the results obtained are compared. The proposed numerical simulations will be helpful in designing a systematic experimental campaign aimed at characterizing the internal energy for physical realizations of the ideal pantographic structure presented in this paper [81,82].

### 2.5 Strain and stress localization phenomena

In the talk of Selda Oterkus ('Peridynamic modeling of stress corrosion cracking'), the derivation of peridynamic formulation for the thermo-oxidative behavior of polymer matrix composites was presented. In particular, isothermal aging of a unidirectional composite lamina was considered, and it was shown that the model captures the effect of oxidation on damage growth and its propagation [83].

Denis Sheydakov, in his talk ('Influence of surface stresses on stability of nonlinearly elastic rod under combined loading'), has discussed the buckling analysis of nonlinearly elastic rods with surface stresses. In particular in this talk, a study on the stability of a circular rod under axial compression–tension and external pressure in the framework of Gurtin-Murdoch model has been presented. Finally, applications of the results of this study to size effect and to the influence of surface stresses on the loss of stability for circular rods was discussed [84].

### 2.6 Effects on acoustic properties of nearly inextensible reinforcement

Arkadi Berezovski, in his talk ('Internal variables associated with microstructure'), has compared different descriptions of microstructured solids on the simple example of wave propagation in the one-dimensional setting. Since in the classical continuum mechanics the existence of a microstructure is neglected, wave equations has to be modified to take into account the microstructure [85].

In the contribution of Stephan Rudykh ('Instabilities and elastic waves in fiber composites undergoing finite deformations'), the mechanical behavior of fiber composites undergoing finite deformations is discussed. To estimate the critical strains corresponding to the onset of the macroscopic or long wave instability, homogenized response for transversely isotropic fiber composites was obtained. The effect of finite deformations on small amplitude elastic waves propagating in the fiber composites was discussed [86].

Long-wavelength approximation to describe propagation of longitudinal, transverse and rotational waves in a granular medium with internal stress is considered in the talk of Vladimir Erofeev and Alexey Malkhanov ('Interaction of strain solitons in granular medium with internal stresses'). In particular, effects of splitting of supersonic solitary waves were demonstrated [87].

Jean-François Ganghoffer, in his talk ('Symmetry analysis and conservation laws of fully nonlinear wave models in fiber-reinforced anisotropic incompressible hyperelastic solids'), has considered the full sets of dynamic equations for finite deformations of incompressible hyperelastic solids containing a single fiber family.

In particular, finite-amplitude wave propagation ansatz was employed for a generic fiber family orientation, and the corresponding nonlinear and linear wave equations were derived. Applications to viscoelastic deformations were considered [88,89].

In the contribution of Giuseppe Rosi ('Anisotropic wave propagation within strain gradient framework: a focus on a mixed static dynamic numerical procedure for parameter identification'), recent results in the domain of anisotropic and dispersive wave propagation within the framework of linear strain-gradient elasticity were presented. In particular, a mixed numerical static–dynamic approach that uses static measures with rich boundary conditions and dynamics measures of phase velocity based on Bloch analysis was discussed [90,91].

Oxana Sadovskaya, in her talk ('Mathematical modeling of thermomechanical and electrodynamic effects in liquid crystals'), has presented a mathematical model of a nematic liquid crystal, considered as a micropolar acoustic medium with rotating particles, under the influence of weak thermomechanical and electrodynamic perturbations. Numerical analysis of a simplified system of second-order equations for tangential stress and angular velocity in 2D case was discussed, and it was shown the agreement between numerical and exact solutions for 1D problem [92].

A Bloch wave analysis was performed to obtain the dispersion curves of a periodic structure made of piezoelectric patches shunted on optimal impedance able to generate extended phononic band gaps was discussed in the talk by Manuel Collet ('Weighted relaxed micromorphic model for modeling adaptive piezocomposite materials'). The interesting results of such an analysis suggest the possibility to design morphologically complex adaptive meta-structures which make use of the chosen piezoelectric coupling as the basic building block for controlling its ability of 'stopping' elastic wave propagation at the scale of the structure [93].

### 2.7 Applications to mechanics of living tissue, engineering fabrics and composite reinforcements

In the first part of the talk of Tomasz Lekszycki ('Some experimental and theoretical results about pantographic sheets: possible applications to metamaterials to be used in biological applications') some interesting considerations about the actual approach of the scientific community to the whole investigation line on metamaterials were discussed. In the second part of the talk, he has discussed how metamaterials and variational approach can be used in biological applications [94–97].

Ewa Bednarczyk, in her talk ('Biomechanical effects in the osteoarthritis changes'), has discussed an expanded model of bone remodeling and development of cell culture to describe the growth of osteophytes during osteoarthritis. The main features of such a model are a strong nonlinearity and a non-local structure. It was shown that it is able to describe angiogenesis process, mechanical loading and tissue microstructure [98].

In the same spirit is collocated the talk by Yanfei Lu ('Modelling of bone regeneration in bioresorbable scaffold'). The main aim of the talk was to present a new mathematical model describing bone fracture healing. Bone tissue is an intelligent material which change its mass distribution and microstructure according to current mechanical and biological conditions. Interesting results about the interaction between bone tissue and bioresorbable graft material and on the influence of microstructure, gap size and nutrients supply on healing process were presented [99,100].

In the contribution of Eleonora Crevacore ('Preliminary results on the use of the Darcy-Brinkman equation in fiber-reinforced porous media of biological interest'), a model of articular cartilage based on the Darcy–Brinkman equation aimed to give a more detailed description of material response of articular cartilage is presented. In particular, a viscous stress tensor depending on the fluid velocity gradient was introduced through an effective viscosity and the possibility to determine the fluid velocity by solving PDE was discussed [101].

A model of cartilage as a continuous two-phase medium with microstructure composed by inextensible or nearly inextensible fibers was presented in the talk by Roberto Serpieri ('Multiphase modelling of articular cartilage as continua with quasi-inextensible fibers by a purely variational macroscopic theory of poroelasticity'). In particular, general momentum balance equations and boundary conditions have been derived in the context of a general variational macroscopic continuum theory of two-phase saturated porous media [102].

### 2.8 Damage and fractures in generalized continua with inextensible fibers

Uwe Mühlich, in his talk ('Strain gradient approach for quasi-brittle failure'), has discussed a methodology for deriving a model for describing quasi-brittle failure, taking into account deterministic size effects and damage evolution. These requirements can be accomplished only by models derived from a generalized continuum theory. In particular, the methodology illustrated in the talk was for porous elastic solids [103,104].

In the contribution of Erkan Oterkus ('Peridynamic modeling of stress corrosion cracking'), stress corrosion cracking, a particular kind of corrosion mechanism which occurs only for specific materials subjected to specific loading and operating under specific environmental conditions, has been discussed. In particular, the applicability of peridynamics to the simulation of stress corrosion cracking were discussed and demonstrated [105,106].

A mathematical model for an arbitrary number of interacting hyperelastic solids undergoing large elastic–plastic deformations was derived in the talk by Sergey Gavriliuk ('Dynamic fracture and spallation via hyperbolic models of hyperelastic solids'). A numerical method, called diffuse interface method, which considers the interface cells as an artificial mixture zone through which the interface conditions must be satisfied was discussed. Finally, the model of the spallation process in metals was presented [107–109].

Mario Spagnuolo, in his talk ('Qualitative pivot damage analysis in aluminum printed pantographic sheets: numerics and experiments'), has presented a model in which the motions of two families of beams constituting a pantographic sheet, when described with two independent placement fields, allows to correctly simulate the points of fracture onset in axial torsion test. Results of experiments and numerical simulations was discussed and good agreement was found [110].

An effective implementation of boundary element multiscale method in analyzing of piezoelectric ceramics was presented in the talk by Tomasz Trzepieciński ('Modeling of fracture in piezoelectric ceramics by using boundary element method'). The main aim is to get a better understanding of damage mechanism in the ceramic in order to improve the constitutive models and to support the future design of those materials [111].

## 3 Some conclusions and perspectives

Euromech 579 was an occasion to exchange knowledge and ideas. Interesting discussions were born after the talks and several consequent informal meetings took place between people from different research areas. All participants have got the occasion to have active interaction with their research community during the forecast activities and meals. Thanks to an effective selection of the invited participants which share a common language and knowledge allowing for an effective communication, several animate discussions started. In particular, the discussions among the participants were focused on hot topic in the field, mainly related to the formulation of continuum models for complex mechanical system and multiphysics system. We would like to add that the relevant representation of promising young researchers involved in the colloquium allows us, even in the current difficult moment for scientific research, to be reasonably optimistic about future prospects in the field.

To conclude this report, we will discuss the main topics debated during the various discussions which started after the talks. Since the chairpersons during the sessions did not applied rigidly the time limits during (also if polite interruptions were allowed) and after the talks, several interesting scientific discussions started and/or were solicited. Actually we believe that the most interesting part of the meeting was represented exactly by the exchange of ideas and research perspectives which were started and imagined because of the formal presentations.

The main debates were focused on the following crucial points:

(1) The mathematical problems arising in the homogenization of systems showing a complex microstructure and in particular exhibiting high contrast in micro-physical and mechanical properties
(2) The need of the mathematical rigorous characterization of the micro-properties required to get a macro-homogenized continuous model which is not included in Cauchy postulation scheme
(3) The role of the Principle of Virtual Work and in general of Variational Principles in the mathematical modeling of complex systems and in the design of metamaterials
(4) The importance of suitably designed metamaterials and their experimentally observed behavior in the advancement of theoretical and numerical modeling of complex mechanical systems
(5) The importance of rescaling rules of physical and mechanical properties in the micro–macro-homogenization procedure
(6) m The role of modeling approaches in the choice of the most suitable numerical codes and methods to be used in order to get theoretical predictions about the behavior of considered complex systems

Concerning the point (1), it has been agreed that in the literature the available homogenization techniques, while being for sure mathematically rigorous, do not allow to get out, in the homogenized continuum models, of the postulation formulated by Cauchy [112–115]. An emerging issue in homogenization theory is therefore the one which concerns the developments of techniques allowing for the infringement of such conceptual blockage.

Indeed, there is a unanimous agreement about the fact that there is a definite evidence (theoretical, conceptual and experimental, see [116–118]) that when the considered complex systems exhibit a strong contrast at micro-level then the macro-models cannot be those which the followers of Cauchy consider the most general ones (i.e., first gradient continua see [119,120]). Of course, the questions to be solved to compare the most general homogenization procedures to be envisaged are relevant. All discussions converged to the acceptance of the need of developing more general mathematical results allowing for the precise characterization of the micro-properties leading to non-Cauchy, Piola-type continua (debates concerning the previous point 2). Of course, as already Piola, and later many authors among which Toupin, Germain, Mindlin and many others, a crucial role has to be assigned to Lagrangian 'metaphysics' (as Piola call the Postulation point of view championed by Lagrange). Therefore there was a general agreement, in the discussions concerning the previous point 3), about the fact that no true advancement in the theory of metamaterials and complex systems (and in general of generalized continua) can be attained without the acceptation of Lagrangian vision of mechanics. Of course nobody in the audience could be sure that the mechanical systems which were object of the discussions (among which pantographic metamaterials) can play a role in future technological applications. However their role, as academic exercises, has been generally recognized. Although the same word 'academic' is being more and more despised in many scientific milieux, all the audience agreed about the fact that Archimedean spirit has to be continued: academic examples, representing the simplest situations in which some scientific models and paradigms *cannot* be applied play a relevant role in the advancement of science. These methodological discussions were very long and interesting when debating the questions concerning the previous point 4). Very interesting has been the debate concerning the point 5). It is indeed clear that the rescaling rules in the homogenization limit are crucial in deterring the obtained microscopic models. Of course the applicability of the continuum models is always limited by the true finite value of the homogenization length scale, which in general, but not always, can be identified with the size of the micro-periodicity cell. The discussion arrived at the following conclusions: there are relevant physical situations in which the high contrast of micro-mechanical properties may introduce relevant length scales whose value can be much bigger than the cell size. These situations are those where generalized continuum models may be of importance and use. The debate concerning this point 5) was the longest one in the whole workshop and also during any social event. Finally the problem concerning the true need of using continuum models was debated (concerning the last point 6). The summary of the discussion was the following question, which was not solved and which waits deeper investigations:

Do we really need continuum models for describing micro-complexity?

Actually all continuum models at the end are to be discretized. The debate involved the correct choice of finite elements of NURBS or Isogeometric methods to be used when considering generalized continua. Some participants [121] proposed to start with *directly* discrete (Hencky–Piola-type) models. They seem to have proven that in many cases this can be the correct choice.

All participants belong to M&MOCS International Centre: they agreed to pursue a cooperative effort in trying to solve the scientific questions and problems discussed in the debate. Particular attention has been promised to attack, in a close future, the following investigation programs:

- Further investigations and generalization on modeling of fiber-based microstructured continua, like pantographic structure.
- Nonlinear generalization of the standard beam models to include large deformation regime via homogenization techniques.
- Further investigations on modeling of viscoelastic materials, by using higher gradient theories, with applications to biomechanics.
- Analyze the existing micro-mechanical models of granular materials from a thermodynamical perspective.

These investigation lines, which have been programmed during the conference, have given arise to several interesting and original papers [122–128].

In research work, clear identification of crucial problems and critical points of disagreement is as valuable as the discovery of new solutions to particular problems. This colloquium was organized duly taking into account this concept, and therefore, we can say that the main purposes which inspired the organization of Euromech 579 have been successfully accomplished.

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## References


1. Le Marrec, L., Zhang, D., Ostoja-Starzewski, M.: Three-dimensional vibrations of a helically wound cable modeled as a timoshenko rod. Acta Mech. **229**, 1–19 (2017)
2. Contento, A., Luongo, A.: Static and dynamic consistent perturbation analysis for nonlinear inextensible planar frames. Comput. Struct. **123**, 79–92 (2013)
3. Turco, E., Golaszewski, M., Giorgio, I., D'Annibale, F.: Pantographic lattices with non-orthogonal fibres: experiments and their numerical simulations. Compos. B Eng. **118**, 1–14 (2017)
4. Sadovskaya, O., Sadovskii, V.: Mathematical Modeling in Mechanics of Granular Materials, vol. 21. Springer, Berlin (2012)
5. Gusev, A.A., Lurie, S.A.: Symmetry conditions in strain gradient elasticity. Math. Mech. Solids **22**(4), 683–691 (2017)
6. Lurie, S.A., Volkov-Bogorodsky, D., Leontiev, A., Aifantis, E.: Eshelby's inclusion problem in the gradient theory of elasticity: applications to composite materials. Int. J. Eng. Sci. **49**(12), 1517–1525 (2011)
7. Tomic, A., Grillo, A., Federico, S.: Poroelastic materials reinforced by statistically oriented fibres—numerical implementation and application to articular cartilage. IMA J. Appl. Math. **79**(5), 1027–1059 (2014)
8. Grillo, A., Wittum, G., Tomic, A., Federico, S.: Remodelling in statistically oriented fibre-reinforced materials and biological tissues. Math. Mech. Solids **20**(9), 1107–1129 (2015)
9. Grillo, A., Carfagna, M., Federico, S.: An allen-cahn approach to the remodelling of fibre-reinforced anisotropic materials. J. Eng. Math. **109**, 1–34 (2017)
10. Alibert, J., Seppecher, P., dell'Isola, F.: Truss modular beams with deformation energy depending on higher displacement gradients. Math. Mech. Solids **8**(1), 51–73 (2003)
11. Giorgio, I.: Numerical identification procedure between a micro-cauchy model and a macro-second gradient model for planar pantographic structures. Zeitschrift für angewandte Mathematik und Physik **67**(4), 95 (2016)
12. dell'Isola, F., Della Corte, A., Giorgio, I., Scerrato, D.: Pantographic 2D sheets. Int. J. Nonlinear Mech. **80**, 200–208 (2016)
13. Scerrato, D., Giorgio, I., Rizzi, N.: Three-dimensional instabilities of pantographic sheets with parabolic lattices: numerical investigations. Zeitschrift für angewandte Mathematik und Physik **67**(3), 1–19 (2016)
14. Rahali, Y., Giorgio, I., Ganghoffer, J.F., dell'Isola, F.: Homogenization à la piola produces second gradient continuum models for linear pantographic lattices. Int. J. Eng. Sci. **97**, 148–172 (2015)
15. Alibert, J., Della Corte, A.: Second-gradient continua as homogenized limit of pantographic microstructured plates: a rigorous proof. Zeitschrift für angewandte Mathematik und Physik **66**(5), 2855–2870 (2015)
16. Turco, E., dell'Isola, F., Cazzani, A., Rizzi, N.L.: Hencky-type discrete model for pantographic structures: numerical comparison with second gradient continuum models. Zeitschrift für angewandte Mathematik und Physik **67**(4), 1–28 (2016)
17. dell'Isola, F., Giorgio, I., Pawlikowski, M., Rizzi, N.L.: Large deformations of planar extensible beams and pantographic lattices: heuristic homogenization, experimental and numerical examples of equilibrium. In: Proceedings of the Royal Society of London A, vol. 472, pp. 20150790. The Royal Society (2016)
18. Andreaus, U., Placidi, L., Rega, G.: Numerical simulation of the soft contact dynamics of an impacting bilinear oscillator. Commun. Nonlinear Sci. Numer. Simul. **15**(9), 2603–2616 (2010)
19. Battista, A., Rosa, L., dell'Erba, R., Greco, L.: Numerical investigation of a particle system compared with first and second gradient continua: Deformation and fracture phenomena. Mathematics and Mechanics of Solids, page 1081286516657889, (2016)
20. Battista, A., Cardillo, C., Del Vescovo, D., Rizzi, N.L., Turco, E.: Frequency shifts induced by large deformations in planar pantographic continua. Nanomech. Sci. Technol. Int. J. **6**(2), 161–178 (2015)
21. Turco, E., Golaszewski, M., Cazzani, A., Rizzi, N.L.: Large deformations induced in planar pantographic sheets by loads applied on fibers: experimental validation of a discrete lagrangian model. Mech. Res. Commun. **76**, 51–56 (2016)
22. Turco, E., Rizzi, N.L.: Pantographic structures presenting statistically distributed defects: numerical investigations of the effects on deformation fields. Mech. Res. Commun. **77**, 65–69 (2016)
23. dell'Isola, F., Lekszycki, T., Pawlikowski, M., Grygoruk, R., Greco, L.: Designing a light fabric metamaterial being highly macroscopically tough under directional extension: first experimental evidence. Zeitschrift für angewandte Mathematik und Physik **66**, 3473–3498 (2015)
24. Turco, E., dell'Isola, F., Rizzi, N.L., Grygoruk, R., Müller, W.H., Liebold, C.: Fiber rupture in sheared planar pantographic sheets: numerical and experimental evidence. Mech. Res. Commun. **76**, 86–90 (2016)
25. Giorgio, I., Della Corte, A., dell'Isola, F., Steigmann, D.J.: Buckling modes in pantographic lattices. C. R. Mec. **344**(7), 487–501 (2016)
26. Niiranen, J., Khakalo, S., Balobanov, V., Niemi, A.H.: Variational formulation and isogeometric analysis for fourth-order boundary value problems of gradient-elastic bar and plane strain/stress problems. Comput. Methods Appl. Mech. Eng. **308**, 182–211 (2016)
27. Khakalo, S., Balobanov, V., Niiranen, J.: Isogeometric static analysis of gradient-elastic plane strain/stress problems. In: Altenbach, H., Forest, S. (eds.) Generalized Continua as Models for Classical and Advanced Materials. Advanced Structured Materials, vol. 42. Springer, Cham (2016)
28. Eremeyev, V.A.: On effective properties of materials at the nano-and microscales considering surface effects. Acta Mech. **227**(1), 29–42 (2016)
29. Altenbach, H., Eremeyev, V.A.: On the variational analysis of vibrations of prestressed six-parameter shells. In: Muñoz-Rojas, P. (ed.) Computational Modeling, Optimization and Manufacturing Simulation of Advanced Engineering Materials. Advanced Structured Materials, vol. 49. Springer, Cham (2016)
30. Eremeyev, V., Altenbach, H.: Basics of mechanics of micropolar shells. In: Altenbach, H., Eremeyev, V. (eds.) Shell-like Structures. CISM International Centre for Mechanical Sciences (Courses and Lectures), vol. 572. Springer, Cham (2017)
31. Alibert, JJ., Della Corte, A., Seppecher, P.: Convergence of hencky-type discrete beam model to euler inextensible elastica in large deformation: rigorous proof. In: dell'Isola, F., Sofonea, M., Steigmann, D. (eds.) Mathematical Modelling in Solid Mechanics. Advanced Structured Materials, vol. 69. Springer, Singapore (2017)

32. Placidi, L., Giorgio, I., Della Corte, A., Scerrato, D.: Euromech 563 cisterna di latina 17–21 March 2014 generalized continua and their applications to the design of composites and metamaterials: a review of presentations and discussions. Math. Mech. Solids **22**(2), 144–157 (2017)
33. Della Corte, A., dell'Isola, F., Esposito, R., Pulvirenti, M.: Equilibria of a clamped euler beam (elastica) with distributed load: large deformations. Math. Models Methods Appl. Sci. **27**(08), 1391–1421 (2017)
34. dell'Isola, F., Cuomo, M., Greco, L., Della Corte, A.: Bias extension test for pantographic sheets: numerical simulations based on second gradient shear energies. J. Eng. Math. **103**(1), 127–157 (2017)
35. Greco, L., Giorgio, I., Battista, A.: In plane shear and bending for first gradient inextensible pantographic sheets: numerical study of deformed shapes and global constraint reactions. Math. Mech. Solids **22**(10), 1950–1975 (2017)
36. Placidi, L., Andreaus, U., Giorgio, I.: Identification of two-dimensional pantographic structure via a linear d4 orthotropic second gradient elastic model. J. Eng. Math. **103**(1), 1–21 (2017)
37. Andreaus, U., dell'Isola, F., Giorgio, I., Placidi, L., Lekszycki, T., Rizzi, N.L.: Numerical simulations of classical problems in two-dimensional (non) linear second gradient elasticity. Int. J. Eng. Sci. **108**, 34–50 (2016)
38. Barchiesi, E., dell'Isola, F., Laudato, M., Placidi, L., Seppecher, P.: A 1D continuum model for beams with pantographic microstructure: asymptotic micro-macro identification and numerical results. In: dell'Isola, F., Eremeyev, V., Porubov, A. (eds.) Advances in Mechanics of Microstructured Media and Structures. Advanced Structured Materials, vol. 87. Springer, Cham (2018)
39. Romeo, M.: A microstructure continuum approach to electromagneto-elastic conductors. Contin. Mech. Thermodyn. **28**(6), 1807–1820 (2016)
40. Romeo, M.: Electromagnetic field-current coupling in rigid polarized conductors. Math. Mech. Solids **23**(1), 85–98 (2016)
41. Jung, A., Diebels, S.: Microstructural characterisation and experimental determination of a multiaxial yield surface for open-cell aluminium foams. Mater. Des. **131**, 252–264 (2017)
42. Heinze, S., Jung, A., Diebels, S., Düster, A.: Experimental and numerical investigation of metal foams undergoing large deformations. PAMM **16**(1), 345–346 (2016)
43. Karyakin, M.I., Shubchinskaia, N.Y.: The equilibrium and stability of the nonlinearly elastic cylinder with internal stresses. Mater. Phys. Mech. **28**(1–2), 31–35 (2016)
44. Glüge, R., Bucci, S.: Does convexity of yield surfaces in plasticity have a physical significance? Math. Mech. Solids (2017). https://doi.org/10.1177/1081286517721599
45. Desmorat, B., Desmorat, R.: Tensorial polar decomposition of 2D fourth-order tensors. C. R. Méc. **343**(9), 471–475 (2015)
46. Desmorat, B., Vannucci, P.: An alternative to the kelvin decomposition for plane anisotropic elasticity. Math. Methods Appl. Sci. **38**(1), 164–175 (2015)
47. Sawangrat, C., Kato, S., Orlov, D., Ameyama, K.: Harmonic-structured copper: performance and proof of fabrication concept based on severe plastic deformation of powders. J. Mater. Sci. **49**(19), 6579–6585 (2014)
48. Poncelet, M., Auffray, N., Jailin, C., Somera, A., Morel, C.: Experimental strain gradient evidence in non-central symmetric lattice. In: EUROMECH-Colloquium 579 on Generalized and Microstructured Continua (2017)
49. Rosi, G., Placidi, L., Auffray, N.: On the validity range of strain-gradient elasticity: a mixed static-dynamic identification procedure. Eur. J. Mech. A Solids **69**, 179–191 (2017)
50. Pawlikowski, M., Barcz, K.: Nonlinear viscoelastic constitutive model for bovine cortical bone tissue. Biocybern. Biomed. Eng. **36**(3), 491–498 (2016)
51. Turco, E., Barcz, K., Pawlikowski, M., Rizzi, N.L.: Non-standard coupled extensional and bending bias tests for planar pantographic lattices. Part I: numerical simulations. Zeitschrift für angewandte Mathematik und Physik **67**(5), 122 (2016)
52. Brandl, E., Heckenberger, U., Holzinger, V., Buchbinder, D.: Additive manufactured alsi10mg samples using selective laser melting (slm): microstructure, high cycle fatigue, and fracture behavior. Mater. Des. **34**, 159–169 (2012)
53. Wautier, A., Bonelli, S., Nicot, F.: Scale separation between grain detachment and grain transport in granular media subjected to an internal flow. Granul. Matter **19**(2), 22 (2017)
54. Nicot, F., Xiong, H., Wautier, A., Lerbet, J., Darve, F.: Force chain collapse as grain column buckling in granular materials. Granul. Matter **19**(2), 18 (2017)
55. Misra, A., Poorsolhjouy, P.: Identification of higher-order elastic constants for grain assemblies based upon granular micromechanics. Math. Mech. Complex Syst. **3**(3), 285–308 (2015)
56. Misra, A., Poorsolhjouy, P.: Granular micromechanics model for damage and plasticity of cementitious materials based upon thermomechanics. Math. Mech. Solids (2015). https://doi.org/10.1177/1081286515576821
57. Misra, A., Poorsolhjouy, P.: Granular micromechanics model of anisotropic elasticity derived from gibbs potential. Acta Mech. **227**(5), 1393 (2016)
58. Misra, A., Poorsolhjouy, P.: Micro-macro scale instability in 2D regular granular assemblies. Contin. Mech. Thermodyn. **27**(1–2), 63 (2015)
59. Misra, A., Poorsolhjouy, P.: Granular micromechanics based micromorphic model predicts frequency band gaps. Contin. Mech. Thermodyn. **28**(1–2), 215 (2016)
60. Li, X.: Internal structure quantification for granular constitutive modeling. J. Eng. Mech. **143**(4), C4016001 (2016)
61. Misra, A., Chang, C.S.: Effective elastic moduli of heterogeneous granular solids. Int. J. Solids Struct. **30**(18), 2547–2566 (1993)
62. George, D., Spingarn, C., Dissaux, C., Nierenberger, M., Rahman, Ranya Abdel, Rémond, Yves: Examples of multiscale and multiphysics numerical modeling of biological tissues. Bio-Med. Mater. Eng. **28**(s1), S15–S27 (2017)
63. Nasedkin, A.V., Nasedkina, A.A, Rybyanets, A.N.: Modeling and computer design of piezoceramic materials with stochastic microporous structure and local alloying pore surfaces. In: Vandamme, M., Dangla, P., Pereira, J-M., Ghabezloo, S. (eds.) Poromechanics VI, pp. 724–731
64. Boutin, C., dell'Isola, F., Giorgio, I., Placidi, L.: Linear pantographic sheets: asymptotic micro-macro models identification. Math. Mech. Complex Syst. **5**(2), 127–162 (2017)
65. Javili, A., Steinmann, P., Mosler, J.: Micro-to-macro transition accounting for general imperfect interfaces. Comput. Methods Appl. Mech. Eng. **317**, 274–317 (2017)

66. Javili, A., Ottosen, N.S., Ristinmaa, M., Mosler, J.: Aspects of interface elasticity theory. Math. Mech Solids (2017). https://doi.org/10.1177/1081286517699041
67. Javili, A., McBride, A., Steinmann, P.: Thermomechanics of solids with lower-dimensional energetics: on the importance of surface, interface, and curve structures at the nanoscale. A unifying review. Appl. Mech. Rev. **65**(1), 010802 (2013)
68. Nazarenko, L., Stolarski, H., Altenbach, H.: Effective properties of short-fiber composites with gurtin-murdoch model of interphase. Int. J. Solids Struct. **97**, 75–88 (2016)
69. Rahali, Y., Ganghoffer, J.F., Chaouachi, F., Zghal, A.: Strain gradient continuum models for linear pantographic structures: a classification based on material symmetries. J. Geom. Symmetry Phys. **40**, 35–52 (2015)
70. Eugster, S.R.: Geometric continuum mechanics and induced beam theories. Springer (2016)
71. Eugster, S.R., Glocker, C.: Determination of the transverse shear stress in an Euler–Bernoulli beam using non-admissible virtual displacements. PAMM **14**(1), 187–188 (2014)
72. Liebold, C., Müller, W.H.: Measuring material coefficients of higher gradient elasticity by using AFM techniques and raman-spectroscopy. In: Altenbach, H., Forest, S., Krivtsov, A. (eds.) Generalized Continua as Models for Materials. Advanced Structured Materials, vol. 22. Springer, Berlin, Heidelberg (2013)
73. Eremeyev, V.A., Skrzat, A., Stachowicz, F.: On computational evaluation of stress concentration using micropolar elasticity. In: International Conference on Applied Physics, System Science and Computers, pp. 199–205. Springer (2017)
74. Varygina, M.: Numerical modeling of micropolar thin elastic plates. In: International Conference on Numerical Analysis and Its Applications, pp. 690–697. Springer (2016)
75. Boisse, P., Hamila, N., Madeo, A.: Simulations of 3D textile composite reinforcements. Specificities of the mechanical behavior. In: AIP Conference Proceedings, vol. 1896, pp. 030012. AIP Publishing (2017)
76. Grigorenko, A.Y., Vlaikov, G.G.: Investigation of the static and dynamic behaviour of anisotropic cylindrical bodies with noncircular cross-section. Int. J. Solids Struct. **41**(9), 2781–2798 (2004)
77. Placidi, L., Barchiesi, E., Battista, A.: An inverse method to get further analytical solutions for a class of metamaterials aimed to validate numerical integrations. In: dell'Isola, F., Sofonea, M., Steigmann, D. (eds.) Mathematical Modelling in Solid Mechanics. Advanced Structured Materials, vol. 69. Springer, Singapore (2017)
78. Barchiesi, E., Placidi, L.: A review on models for the 3D statics and 2D dynamics of pantographic fabrics. In: Sumbatyan, M. (eds.) Wave Dynamics and Composite Mechanics for Microstructured Materials and Metamaterials. Advanced Structured Materials, vol. 59. Springer, Singapore (2017)
79. Placidi, L., Barchiesi, E., Turco, E., Rizzi, N.L.: A review on 2D models for the description of pantographic fabrics. Zeitschrift für angewandte Mathematik und Physik **67**(5), 121 (2016)
80. Cuomo, M., Dell'ÄôIsola, F., Greco, L., Rizzi, N.L.: First versus second gradient energies for planar sheets with two families of inextensible fibres: investigation on deformation boundary layers, discontinuities and geometrical instabilities. Compos. Part B: Eng. **115**, 423–448 (2017)
81. Cuomo, M., Contrafatto, L., Greco, L.: A variational model based on isogeometric interpolation for the analysis of cracked bodies. Int. J. Eng. Sci. **80**, 173–188 (2014)
82. Contrafatto, L., Cuomo, M.: A framework of elastic-plastic damaging model for concrete under multiaxial stress states. Int. J. Plast **22**(12), 2272–2300 (2006)
83. Madenci, E., Oterkus, S.: Peridynamic modeling of thermo-oxidative damage evolution in a composite lamina. In: 58th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, pp. 0197 (2017)
84. Sheydakov, D.N., Altenbach, H.: Stability of inhomogeneous micropolar cylindrical tube subject to combined loads. Math. Mech. Solids **21**(9), 1082–1094 (2016)
85. Berezovski, A.: Internal variables associated with microstructures in solids. Mech. Res. Commun. (2017). https://doi.org/10.1016/j.mechrescom.2017.07.011
86. Galich, P., Slesarenko, V., Rudykh, S.: Elastic wave propagation in soft microstructured composites undergoing finite deformations. PAMM **16**(1), 627–628 (2016)
87. Erofeev, V.I., Kazhaev, V.V., Pavlov, I.S.: Inelastic interaction and splitting of strain solitons propagating in a one-dimensional granular medium with internal stress. In: Altenbach, H., Forest, S. (eds.) Generalized Continua as Models for Classical and Advanced Materials. Advanced Structured Materials, vol. 42. Springer, Cham (2016)
88. Cheviakov, A.F., Ganghoffer, J.F., St Jean, S.: Fully nonlinear wave models in fiber-reinforced anisotropic incompressible hyperelastic solids. Int. J. Nonlinear Mech. **71**, 8–21 (2015)
89. Cheviakov, A.F., Ganghoffer, J.F., Rahouadj, R.: Finite strain plasticity models revealed by symmetries and integrating factors: the case of dafalias spin model. Int. J. Plast **44**, 47–67 (2013)
90. Rosi, G., Auffray, N.: Anisotropic and dispersive wave propagation within strain-gradient framework. Wave Motion **63**, 120–134 (2016)
91. Rosi, G., Placidi, L., Auffray, N.: Anisotropic wave propagation within strain gradient framework: a focus on a mixed static-dynamic numerical procedure for parameter identification. In: EUROMECH-Colloquium 579 on Generalized and microstructured continua, vol. 69, pp. 195–206 (2017)
92. Sadovskii, V.M., Sadovskaya, O.V.: On the acoustic approximation of thermomechanical description of a liquid crystal. Phys. Mesomech. **16**(4), 312–318 (2013)
93. Madeo, A., Barbagallo, G., Collet, M., d'Agostino, M.V., Miniaci, M., Neff, P.: Relaxed micromorphic modeling of the interface between a homogeneous solid and a band-gap metamaterial: new perspectives towards metastructural design. Math. Mech. Solids (2017). https://doi.org/10.1177/1081286517728423
94. Ganzosch, G., dell'Isola, F., Turco, e, Lekszycki, T., Müller, W.H.: Shearing tests applied to pantographic structures. Acta Polytech. CTU Proc. **7**, 1–6 (2016)
95. Lekszycki, T.: Optimality conditions in modelling of bone adaptation phenomenon. J. Theor. Appl. Mech. **3**, 607–623 (1999)
96. Lekszycki, T.: Modelling of bone adaptation based on an optimal response hypothesis. Meccanica **37**(4–5), 343–354 (2002)
97. Lekszycki, T.: Functional adaptation of bone as an optimal control problem. J. Theor. Appl. Mech. **43**(3), 555–574 (2005)

98. Bednarczyk, E., Lekszycki, T.: A novel mathematical model for growth of capillaries and nutrient supply with application to prediction of osteophyte onset. Zeitschrift für angewandte Mathematik und Physik **67**(4), 1–14 (2016)
99. Lekszycki, Tomasz, dell'Isola, F.: A mixture model with evolving mass densities for describing synthesis and resorption phenomena in bones reconstructed with bio-resorbable materials. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik **92**(6), 426–444 (2012)
100. Lu, Y., Lekszycki, T.: Modelling of bone fracture healing: influence of gap size and angiogenesis into bioresorbable bone substitute. Math. Mech. Solids **22**(10), 1997–2010 (2016)
101. Crevacore, E., Tosco, T., Sethi, R., Boccardo, G., Marchisio, D.L.: Recirculation zones induce non-fickian transport in three-dimensional periodic porous media. Phys. Rev. E **94**(5), 053118 (2016)
102. Serpieri, R., Travascio, F.: Variational macroscopic two-phase poroelasticity. Derivation of general medium-independent equations and stress partitioning laws. In: Variational Continuum Multiphase Poroelasticity. Advanced Structured Materials, vol. 67. Springer, Singapore (2017)
103. Mühlich, U., Zybell, L., Kuna, M.: Estimation of material properties for linear elastic strain gradient effective media. Eur. J. Mech. A Solids **31**(1), 117–130 (2012)
104. Zybell, L., Mühlich, U., Kuna, M.: Constitutive equations for porous plane-strain gradient elasticity obtained by homogenization. Arch. Appl. Mech. **79**(4), 359–375 (2009)
105. De Meo, D., Diyaroglu, C., Zhu, N., Oterkus, E., Siddiq, M.A.: Modelling of stress-corrosion cracking by using peridynamics. Int. J. Hydrogen Energy **41**(15), 6593–6609 (2016)
106. De Meo, D., Russo, L., Oterkus, E.: Modeling of the onset, propagation, and interaction of multiple cracks generated from corrosion pits by using peridynamics. J. Eng. Mater. Technol. **139**(4), 041001 (2017)
107. Ndanou, S., Favrie, N., Gavrilyuk, S.: Criterion of hyperbolicity in hyperelasticity in the case of the stored energy in separable form. J. Elast. **115**(1), 1–25 (2014)
108. Ndanou, S., Favrie, N., Gavrilyuk, S.: Multi-solid and multi-fluid diffuse interface model: applications to dynamic fracture and fragmentation. J. Comput. Phys. **295**, 523–555 (2015)
109. Gavrilyuk, S., Ndanou, S., Hank, S.: An example of a one-parameter family of rank-one convex stored energies for isotropic compressible solids. J. Elast. **124**(1), 133–141 (2016)
110. Spagnuolo, M., Barcz, K., Pfaff, A., dell'Isola, F., Franciosi, P.: Qualitative pivot damage analysis in aluminum printed pantographic sheets: numerics and experiments. Mech Res. Commun. **83**, 47–52 (2017)
111. Biglar, M., Gromada, M., Stachowicz, F., Trzepieciński, T.: Synthesis of barium titanate piezoelectric ceramics for multilayer actuators (mlas). Acta Mech. Autom. **11**(4), 275–279 (2017)
112. dell'Isola, F., Madeo, A., Seppecher, P.: Cauchy tetrahedron argument applied to higher contact interactions. Arch. Ration. Mech. Anal. **219**(3), 1305–1341 (2016)
113. dell'Isola, F., Seppecher, P., Della Corte, A.: The postulations á la d'alambert and á la cauchy for higher gradient continuum theories are equivalent: a review of existing results. In: Proceedings of the Royal Society of London A, vol. 471, pp. 20150415. The Royal Society (2015)
114. dell'Isola, F., Seppecher, F., Madeo, A.: How contact interactions may depend on the shape of cauchy cuts in nth gradient continua: approach, Äúà la d'alambert, Äú. Zeitschrift für angewandte Mathematik und Physik **63**(6), 1119–1141 (2012)
115. dell'Isola, F., Seppecher, P., Madeo, A.: Beyond euler-cauchy continua: the structure of contact actions in N-th gradient generalized continua: a generalization of the Cauchy tetrahedron argument. In: dell'Isola, F., Gavrilyuk, S. (eds.) Variational Models and Methods in Solid and Fluid Mechanics. CISM Courses and Lectures, vol. 535. Springer, Vienna (2011)
116. dell'Isola, F., Della Corte, A., Giorgio, I.: Higher-gradient continua: the legacy of piola, mindlin, sedov and toupin and some future research perspectives. Math. Mech. Solids **22**(4), 852–872 (2017)
117. Carcaterra, A., dell'Isola, F., Esposito, R., Pulvirenti, M.: Macroscopic description of microscopically strongly inhomogenous systems: a mathematical basis for the synthesis of higher gradients metamaterials. Arch. Ration. Mech Anal. **1239**(218), 1239–1262 (2015)
118. dell'Isola, F., Steigmann, D., Della Corte, A.: Synthesis of fibrous complex structures: designing microstructure to deliver targeted macroscale response. Appl. Mech. Rev. **67**(6), 060804 (2015)
119. Dell'Isola, F., Andreaus, U., Placidi, L.: At the origins and in the vanguard of peridynamics, non-local and higher-gradient continuum mechanics: an underestimated and still topical contribution of gabrio piola. Math. Mech. Solids **20**(8), 887–928 (2015)
120. dell'Isola, F., Della Corte, A., Esposito, R., Russo, L.: Some cases of unrecognized transmission of scientific knowledge: from antiquity to gabrio piola's peridynamics and generalized continuum theories. In: Altenbach, H., Forest, S. (eds.) Generalized Continua as Models for Classical and Advanced Materials. Advanced Structured Materials, vol. 42. Springer, Cham (2016)
121. Turco, E., Golaszewski, M., Giorgio, I., Placidi, L.: Can a hencky-type model predict the mechanical behaviour of pantographic lattices? In: dell'Isola, F., Sofonea, M., Steigmann, D. (eds.) Mathematical Modelling in Solid Mechanics. Advanced Structured Materials, vol. 69. Springer, Singapore (2017)
122. Giorgio, I., Rizzi, N.L., Turco, E.: Continuum modelling of pantographic sheets for out-of-plane bifurcation and vibrational analysis. In Proceedings of the Royal Society of London A, vol. 473, pp. 20170636. The Royal Society (2017)
123. Turco, E., Giorgio, I., Misra, A., Dell'ÄôIsola, F.: King post truss as a motif for internal structure of (meta) material with controlled elastic properties. Open. Sci. **4**(10), 171153 (2017)
124. Abdoul-Anziz, H., Seppecher, P.: Strain gradient and generalized continua obtained by homogenizing frame lattice. hal-01672898 (2017)
125. Battista, A., Della Corte, A., Dell'Isola, F., Seppecher, P.: Large deformations of 1d microstructured systems modeled as generalized timoshenko beams. hal-01625160 (2017)
126. Lekszycki, T., Bucci, S., Del Vescovo, D., Turco, E., Rizzi, N.L.: A comparison between different approaches for modelling media with viscoelastic properties via optimization analyses. ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik **97**(5), 515–531 (2017)

127. Giorgio, I., Andreaus, U., Dell’ÄôIsola, F., Lekszycki, T.: Viscous second gradient porous materials for bones reconstructed with bio-resorbable grafts. Extreme Mech. Lett. **13**, 141–147 (2017)
128. Zhao, C.F., Yin, Z.Y., Misra, A., Hicher, P.Y.: Thermomechanical formulation for micromechanical elasto-plasticity in granular materials. Int. J. Solids Struct. **138**, 64–75 (2017)