Continuum Mech. Thermodyn. (2017) 29:1197–1205
DOI 10.1007/s00161-017-0566-x

ORIGINAL ARTICLE

Artur V. Dmitrenko

Analytical determination of the heat transfer coefficient for gas, liquid and liquid metal flows in the tube based on stochastic equations and equivalence of measures for continuum

Received: 8 October 2016 / Accepted: 11 April 2017 / Published online: 6 May 2017

Abstract The stochastic equations of continuum are used for determining the heat transfer coefficients. As a result, the formulas for Nusselt (Nu) number dependent on the turbulence intensity and scale instead of only on the Reynolds (Peclet) number are proposed for the classic flows of a nonisothermal fluid in a round smooth tube. It is shown that the new expressions for the classical heat transfer coefficient Nu, which depend only on the Reynolds number, should be obtained from these new general formulas if to use the well-known experimental data for the initial turbulence. It is found that the limitations of classical empirical and semiempirical formulas for heat transfer coefficients and their deviation from the experimental data depend on different parameters of initial fluctuations in the flow for different experiments in a wide range of Reynolds or Peclet numbers. Based on these new dependences, it is possible to explain that the differences between the experimental results for the fixed Reynolds or Peclet numbers are caused by the difference in values of flow fluctuations for each experiment instead of only due to the systematic error in the experiment processing. Accordingly, the obtained general dependences of Nu for a smooth round tube can serve as the basis for clarifying the experimental results and empirical formulas used for continuum flows in various power devices. Obtained results show that both for isothermal and for nonisothermal flows, the reason for the process of transition from a deterministic state into a turbulent one is determined by the physical law of equivalence of measures between them. Also the theory of stochastic equations and the law of equivalence of measures could determine mechanics which is basis in different phenomena of self-organization and chaos theory.

Keywords Stochastic equations · Equivalence of measures · Turbulence · The heat transfer coefficient

1 Introduction

Based on the analysis of publications [1–23], stochastic equations and theoretical regularity of equivalence of measures between deterministic and stochastic processes were obtained in articles [24–33]. Using these results, the basic characteristics for isothermal turbulent flow depending on initial turbulence of flow were determined. Among them are as follows: (1) the first and second critical Reynolds numbers; (2) the expression for the critical point of the transition beginning from the deterministic to turbulent state [24,30]; (3) the indicators of velocity and temperature profiles [25–27,30]; (4) using the new fractal equations by Landau [25–27,30], expressions

Communicated by Andreas Öchsner.

A. V. Dmitrenko (✉)
Department of Thermal Physics, National Research Nuclear University MEPhI, 31 Kashirskoe Shosse, Moscow, Russia 115409
E-mail: avdmitrenko@mephi.ru; ammsv@yandex.ru

A. V. Dmitrenko
Department of Power Engineering, Moscow State University of Railway Engineering (MIIT), 9 Obraztsov St., Moscow, Russia 127994

for the correlation moments of the second order and for the power spectrum of turbulence depending on initial turbulence were written. As a result, the formulas for the hydraulic drag coefficients dependent on the turbulence intensity and scale instead of only on the Reynolds number were proposed [28] for the classic flows of an incompressible fluid along a smooth flat plate and a round smooth tube. It was shown that the new expressions for the classical drag coefficients, which depend only on the Reynolds number, should be obtained from these new general formulas if to use the well-known experimental data for the initial turbulence. For the forced flows of the nonisothermal medium, analytical expressions were presented in [26,32–34] for calculating the critical point and critical Reynolds number depending on the initial flow turbulence in a pipe and on a flat plate. Analytical formulas for the indices of velocity and temperature profiles as functions of the parameters of initial turbulence and the Eckert and Prandtl numbers were obtained on the basis of the stochastic system of equations for energy, momentum, and mass [26]. It was also found [28] that the limitations of classical empirical and semiempirical formulas for the heat transfer coefficients and their deviation from the experimental data depend on various parameters of initial fluctuations in the flow for various experiments in a wide range of Reynolds numbers. So here, the formulas for the heat transfer coefficients dependent on the turbulence intensity and scale instead of only on Reynolds or Peclet numbers are proposed for the classic flows of a nonisothermal fluid in the round smooth tube. It is shown that the new expressions for the classical heat transfer coefficient Nusselt (Nu) number, which depend only on the Reynolds number or Peclet number, should be obtained from these new general formulas if we use the well-known experimental data for the initial turbulence. Thus, based on these new dependencies for Nu and St numbers, it is possible to explain that the differences between the experimental results for the fixed Reynolds or Peclet numbers are caused by the difference in values of flow fluctuations for each experiment instead of only due to the systematic error in the experiment processing in various power devices.

2 Set of equations

The main aspects of the stochastic theory of turbulence and the law of equivalence of measures between the deterministic and random motions were presented in [24–33]. Shortly, the general system of stochastic equations of conservation for isothermal and nonisothermal medium includes:

the equation of mass (continuity)

$$\left(\frac{\mathrm{d}(\rho)_{\mathrm{col_{st}}}}{\mathrm{d}\tau}\right) = -\left(\frac{\rho_{\mathrm{st}}}{\tau_{\mathrm{cor}}}\right) - \left(\frac{\mathrm{d}\rho_{\mathrm{st}}}{\mathrm{d}\tau}\right), \tag{1}$$

the momentum equation

$$\frac{\mathrm{d}\rho u_{i\,\mathrm{col_{st}}}}{\mathrm{d}\tau} = \mathrm{div}(\tau_{i,j})_{\mathrm{col_{st}}} + \mathrm{div}(\tau_{i,j})_{\mathrm{st}} - \frac{(\rho U)_{\mathrm{st}}}{\tau_{\mathrm{cor}}} - \frac{\mathrm{d}(\rho U)_{\mathrm{st}}}{\mathrm{d}\tau}, \tag{2}$$

the energy equation

$$\frac{\mathrm{d}E_{\mathrm{col_{st}}}}{\mathrm{d}\tau} = \mathrm{div}\left(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j}\right)_{\mathrm{col_{st}}} + \mathrm{div}\left(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j}\right)_{\mathrm{st}} - \left(\frac{E_{\mathrm{st}}}{\tau_{\mathrm{cor}}}\right) - \left(\frac{\mathrm{d}E_{\mathrm{st}}}{\mathrm{d}\tau}\right). \tag{3}$$

Here the stress tensor is $\tau_{i,j} = P + \sigma_{i,j}$ $\sigma_{i,j} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\left(\xi - \frac{2}{3}\mu\right)\frac{\partial u_l}{\partial x_l}$, and $\rho, \vec{U}, u_i, u_j, u_l, \mu, \tau, \tau_{i,j}$ are the density, the velocity vector, velocity components in directions x_i, x_j, x_l $(i, j, l = 1, 2, 3)$, the dynamic viscosity, time and stress tensor. Quantity P is the liquid or gas pressure, T is the temperature, λ is the thermal conductivity, ξ is second viscosity, ν is the kinematic viscosity, $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$. Physical essence of scientific discovery of equivalence of measures [29] is that the interaction between the deterministic and random movement (the transfer of mass, momentum and energy between them) is realized only when there is the equality not only of the time (the resonance of frequencies), but also of shifts and powers of all physically substantial values of the interacting regions of space-time continuum for both of physical states—the global resonance between the states. The concept of "equivalence of measures" reflects both the physical and mathematical essence of established physical regularity (the law). Also, the essence lies in the fact that the equivalence of measures may have a fractal nature [24–26,30] and could determine mechanics of self-organization.

Namely, in [24–33], for the transfer of the substantial quantity Φ (mass (density ρ), momentum ($\rho \boldsymbol{U}$), energy (E)) of the deterministic (laminar) motion into a random (turbulent) one, for domain 1 of the start of turbulence generation, the pair $(N, M) = (1, 0)$, with the equivalence of measures being written as $\left(\mathrm{d}\Phi_{\mathrm{col,st}}\right)_{1,0} = -R_{1,0}\,(\Phi_{\mathrm{st}})$ and $\left(\frac{\mathrm{d}(\Phi)_{\mathrm{col,st}}}{\mathrm{d}\tau}\right)_{1,0} = -R_{1,0}\left(\frac{\Phi_{\mathrm{st}}}{\tau_{\mathrm{cor}}}\right)$. Applying correlator $D_{N,M}(r_c; m_{ci}; \tau_c) = D_{1,1}(r_c; m_{ci}; \tau_c)$ obtained in [22–28], the equivalence relation for the pair $(N, M) = (1, 1)$ was defined as $(\mathrm{d}\Phi_{\mathrm{col,st}})_{1,1} = -R_{1,1}(\mathrm{d}\Phi_{\mathrm{st}})$, $\left(\frac{\mathrm{d}(\Phi)_{\mathrm{col,st}}}{\mathrm{d}\tau}\right)_{1,1} = -R_{1,1}\left(\frac{\mathrm{d}\Phi_{\mathrm{st}}}{\mathrm{d}\tau}\right)$ where $R_{1,0}$ and $R_{1,1}$ are fractal coefficients, $\Phi_{\mathrm{col,st}}$ is part of the field of Φ, namely its deterministic component (subscript col,st), the stochastic component of the measure of which is zero; Φ_{st} is part of Φ, namely the proper stochastic component (subscript st). The relations of the equivalence for momentum and mass (density) have been determined in the same way. For example, to obtain new analytical relations, fractal coefficients $R_{1,0}$ and $R_{1,1}$ are taken equal unity, and indices "cr" or "c" are related to critical point $r(x_{\mathrm{cr}}, \tau_{\mathrm{cr}})$ or τ_{c}. The critical point is the space–time point of the start of interaction between the deterministic and random fields. Then for nonisothermal motion of the medium, with using the definition of measures of equivalency between deterministic and random process, the set of stochastic equations of energy, momentum, and mass are determined for the next space–time areas: (1) the beginning of the generation; (2) the generation; (3) the diffusion; and (4) the dissipation of the turbulent fields. Thus, for the turbulence beginning region $r_{c0}(x_c + \Delta x_0, \tau_{\mathrm{c}} + \Delta\tau_0) - r_c$ (here $(N, M) = (1, 0)$, the "correlator" $D_{N,M}(r_c; m_{ci}; \tau_{\mathrm{c}}) = D_{1,0}(r_c; m_{ci}; \tau_{\mathrm{c}})$), the set of stochastic equations for mass, momentum and energy generation is :

$$\left(\frac{\mathrm{d}(\rho)_{\mathrm{col\,st}}}{\mathrm{d}\tau}\right)_{1,0} = -\left(\frac{\rho_{\mathrm{st}}}{\tau_{\mathrm{cor}}}\right)$$
$$\begin{cases} \left(\frac{\mathrm{d}(\rho\,\vec{U})_{\mathrm{col\,st}}}{\mathrm{d}\tau}\right)_{1,0} = -\left(\frac{(\rho\,\vec{U})_{\mathrm{st}}}{\tau_{\mathrm{cor}}}\right); \\ \mathrm{div}(\tau_{i,j})_{\mathrm{col\,st1}} = \frac{(\rho\,\vec{U})_{\mathrm{st}}}{\tau_{\mathrm{cor}}} \end{cases} \tag{4}$$
$$\begin{cases} \frac{\mathrm{d}(E_{\mathrm{col_{st}}})_{1,0}}{\mathrm{d}\tau} = -\left(\frac{E_{\mathrm{st}}}{\tau^0_{\mathrm{cor}}}\right)_{1,0}, \\ \mathrm{div}\left(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j}\right)_{\mathrm{col\,st\,1}} = \left(\frac{E_{\mathrm{st}}}{\tau^0_{\mathrm{cor}}}\right)_{1,0} \end{cases}$$

$\tau_{\mathrm{cor}} = \tau^0_{\mathrm{cor}}$ is the correlation time [24–28]. The subscript (col st1) refers to pair $(N, M) = (1, 0)$. Thus, using set (4) for flow on the flat plate and in the tube, formulas for critical points and critical Reynolds numbers were obtained [26,31–33]. For the turbulence generation region $r_{cl}(x_c+\Delta x_0+\Delta x_1, \tau_{\mathrm{c}}+\Delta\tau_0+\Delta\tau_1)-r_{c0}((N, M) = (1, 1)$, «correlator» $D_{N,M}(r_c; m_{ci}; \tau_{\mathrm{c}}) = D_{1,1}(r_c; m_{ci}; \tau_{\mathrm{c}})$), the set of stochastic equations was defined as

$$\left(\frac{\mathrm{d}(\rho)_{\mathrm{col,st}}}{\mathrm{d}\tau}\right)_{1,1} = -\left(\frac{\mathrm{d}(\rho)_{\mathrm{st}}}{\mathrm{d}\tau}\right)$$
$$\begin{cases} \left(\frac{\mathrm{d}(\rho\,\vec{U})_{\mathrm{col,st}}}{\mathrm{d}\tau}\right)_{1,1} = -\left(\frac{\mathrm{d}(\rho\,\vec{U})_{\mathrm{st}}}{\mathrm{d}\tau}\right); \\ \mathrm{div}(\tau_{i,j})_{\mathrm{col,st2}} = \frac{\mathrm{d}(\rho\,\vec{U})_{\mathrm{st}}}{\mathrm{d}\tau} \end{cases} \tag{5}$$
$$\begin{cases} \frac{d(E_{\mathrm{col_{st}}})_{1,1}}{\mathrm{d}\tau} = -\left(\frac{\mathrm{d}E_{\mathrm{st}}}{\mathrm{d}\tau}\right)_{1,1}, \\ \mathrm{div}\left(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j}\right)_{\mathrm{col\,st\,2}} = \left(\frac{\mathrm{d}E_{\mathrm{st}}}{\mathrm{d}\tau}\right)_{1,1} \end{cases}$$

The subscript (col st2) refers to pair $(N, M) = (1, 1)$. Then using mentioned above sets, the velocity fields of the turbulent flow in the tube and along the flat plate were determined. As a result, the formulas for the exponent n_T of the velocity and temperature n_T profiles were obtained as a function of the turbulence scale and the intensity of the velocity and temperature fluctuations [26]. The critical point $r(\boldsymbol{x}_{\mathrm{cr}}, \tau_{\mathrm{cr}})$, is the space–time point of the turbulence onset as the interaction between the deterministic and random fields. It should be mentioned that the idea about the critical point (the critical sublayer, the critical point $(x_2)_{\mathrm{cr}} = y_{\mathrm{cr}}$) is not new. It was for the first time proposed by Tollmien and Rayleigh for the well-known Orr–Sommerfeld's equation [37], which was used for the calculation of the laminar-turbulent transition. Later in the theory of attractors, the concept of the critical point was determined as the attraction point of trajectories [11]. Further, in the statistical semiempirical theory of Taylor, the possibility was shown for obtaining the equation for the critical Reynolds number [37] in the flow around a cylinder as a function of turbulence intensity and the scale of turbulence $L = L_{(U,P)} = L_U$ (integral length), D is the cylinder diameter: $(Re_d)_{\mathrm{cr}} = F\left\{\left(\left(\frac{U_0}{\sqrt{E_{\mathrm{st}}/\rho}}\right)\left(\frac{L_U}{D}\right)^{1/5}\right)^{-1}\right\}$, U_0 - the velocity of the main flow. So, here the stochastic theory and equivalence of measures are used for predicating the heat transfer coefficient Nu with taking into account formulas for the critical point, the critical Reynolds number and exponent for velocity and temperature profiles [24–32].

3 Determination of the equation for the Nusselt number depending on the intensity and the scale of turbulence

It is known that the classical equation for the heat transfer into the wall for recorded Fourier's law and the Newton's law [37–39] is used to determine the Nusselt number: $\left[q = \left(\lambda\frac{\partial T}{\partial x_2}\right)\right] = \alpha\,(T_0 - T_w)$ or, in the general case, $q = (\lambda\frac{\partial T}{\partial x_2}) + \mu u_1\frac{\partial u_1}{\partial x_2} = \alpha(T_0 - T_w)$, Here q, α—heat flux density, heat transfer coefficient. On the other hand, we assume the temperature and the speed profile affinity $\left[\frac{T-T_w}{T_0-T_w} = (\frac{x_2}{R})^{n_T}\right]$, $T = T_w + (T_0 - T_w)\left(\frac{x_2}{R}\right)^{n_T}$ and $u_1 = U_0\left(\frac{x_2}{R}\right)^n$, $R, U = U_0, T_0, u_1, T_w, n, n_T$—the tube radius, the velocity and the temperature at the axis, the velocity along x_1, the temperature at the wall, the exponent in the velocity profile and the exponent in the temperature profile, x_1, x_2,—the longitudinal and transverse coordinates, respectively. So we can write that $q = \left(\lambda\frac{\partial T}{\partial x_2}\right) + \mu u_1\frac{\partial u_1}{\partial x_2} = \lambda\frac{1}{n_T}(T_0 - T_w)\frac{1}{R}\left(\frac{x_2}{R}\right)^{(1/n_T)-1} + \mu U_0\left(\frac{x_2}{R}\right)^{1/n}\frac{1}{n}\left(\frac{U_0}{R}\right)\left(\frac{x_2}{R}\right)^{1/n-1} = \alpha\,(T_0 - T_w)$. So, the Nusselt number, without considering the friction, has the form $Nu_{\mathrm{d}} = 2\frac{\alpha R}{\lambda} = 2\frac{1}{n_T}\left(\frac{x_2}{R}\right)^{(1/n_T)-1}$. Taking into account friction, we have $Nu_d = \frac{\alpha 2R}{\lambda} = 2\frac{1}{n_T}\left(\frac{x_2}{R}\right)^{(1/n_T)-1}\left[1 + Pr\,Ec\frac{n_T}{n}\left(\frac{x_2}{R}\right)^{(2/n)-(1/n_T)}\right]$. The expression in square brackets is written taking into account the effect of heat transfer due to friction. Now applying the relation for the critical transition point $\left(\frac{x_2}{R}\right) = \left[\frac{1}{4}\frac{(E_{\mathrm{st}})_{UP}}{U_0^2}(\frac{R}{L_U})\right]^{\frac{1}{3}}\left[\frac{Ec\cdot Pr}{1+Ec\cdot Pr}(1 + \frac{2\cdot T_T}{Tu^2\cdot Ec})\right]^{\frac{1}{3}}$ and substituting it in the last formula for Nu_{d}, for compactness recording leaving the old record in square brackets, we have the next formula

$$Nu_d = 2\frac{1}{n_T}\left(\left[\frac{1}{4}\frac{(E_{\mathrm{st}})_{UP}}{U_0^2}\left(\frac{R}{L_U}\right)\right]^{\frac{1}{3}}\left[\frac{Ec\cdot Pr}{1+Ec\cdot Pr}\left(1+\frac{2\cdot T_T}{Tu^2\cdot Ec}\right)\right]^{\frac{1}{3}}\right)^{(1/n_T)-1} \left(1+\frac{n_T}{n}Pr\,Ec\left(\frac{x_2}{R}\right)^{(2/n)-(1/n_T)}\right) \tag{6}$$

Taking into account that in according with [31–33]

$$Re_d = R_{d(TU)} = \left(6\left(\frac{1}{2}\right)^{1/3}\left(\frac{U_0}{\sqrt{E_{\mathrm{st}}/\rho}}\right)^{5/3}\left(\frac{L_U}{R}\right)^{1/3}\right)\cdot F_{tb},$$

$$F_{tb} = \left(\frac{\left[1+\frac{2T_T}{Ec\cdot Tu^2}\right]^{\frac{2}{3}}}{\left[1+\left(\frac{Pr\cdot\left(\left(u_j^2\right)_{\mathrm{st}}\right)^{1\backslash 2}}{\left(\left(u_i^2\right)_{\mathrm{st}}\right)^{1\backslash 2}}\right)\frac{2T_T}{Ec\cdot Tu^2}\right]}\left(\frac{\left(1+2\frac{1}{Pr\cdot Ec}\right)}{\left(1+\frac{1}{Pr\cdot Ec}\right)^{2/3}}\right)\right),$$

$$\frac{\cdot\left(\left(u_j^2\right)_{\mathrm{st}}\right)^{1\backslash 2}}{\left(\left(u_i^2\right)_{\mathrm{st}}\right)^{1\backslash 2}} \approx K_u = 0.3 \div 0.5$$

we can write that $\left(\frac{\sqrt{E_{\mathrm{st}}/\rho}}{U_0}\right)^{2/3}\left(\frac{R}{L_U}\right)^{1/3} = F_{tb}\left(6\left(\frac{1}{2}\right)^{1/3}\left(\frac{U_0}{\sqrt{E_{\mathrm{st}}/\rho}}\right)\right)/Re_{d(T,U)}$

Table 1 An influence of cooling on increasing critical Reynolds numbers depending on turbulence intensities

Ec	T_u	T_T	M	$(T_w - T_o)K^0$	F_{plt} (the plate)	F_{tb} (the tube)	Experiments [34–38] Increasing critical Reynolds numbers
−0.01	0.01	0.005	0.03	−10.	1.113	1.135	~1.1÷1.2
−0.1	0.03	0.01	0.1	−100.	1.769	2.179	~1.7÷2.2

$$\left(\frac{x_2}{R}\right) = \left[\frac{1}{4}\frac{(E_{st})_{UP}}{U_0^2}\left(\frac{R}{L_U}\right)\right]^{\frac{1}{3}}\left[\frac{Ec\cdot Pr}{1+Ec\cdot Pr}\left(1+\frac{2\cdot T_T}{Tu^2\cdot Ec}\right)\right]^{\frac{1}{3}}$$
$$= \left(\left(\left(\frac{1}{2}\right)^{2/3} 6\left(\frac{1}{2}\right)^{1/3}\left(\frac{U_0}{\sqrt{E_{st}/\rho}}\right)\right) F_{tb}/Re_{d(T,U)}\right)\left[\frac{Ec\cdot Pr}{1+Ec\cdot Pr}\left(1+\frac{2\cdot T_T}{Tu^2\cdot Ec}\right)\right]^{\frac{1}{3}}, \quad (7)$$

$$Nu_d = 2\frac{1}{n_T}\left[\cdot\frac{1}{3}\cdot\left(\frac{\sqrt{E_{st}/\rho}}{U_0}\right)Re_{d(TU)}/F_{tb}\right]^{(n_T-1/n_T)}\left(\left[\frac{Ec\cdot Pr}{1+Ec\cdot Pr}\left(1+\frac{2\cdot T_T}{Tu^2\cdot Ec}\right)\right]^{\frac{1}{3}}\right)^{(1/n_T)-1}$$
$$\left(1+\frac{n_T}{n}Pr\,Ec\left(\frac{x_2}{R}\right)^{(2/n)-(1/n_T)}\right). \quad (8)$$

We write this expression by determining the amount of

$$F_{tb}^{-1} = \left(\frac{\left[1+\frac{2T_T}{Ec\cdot Tu^2}\right]^{\frac{2}{3}}}{\left[1+\left(\frac{Pr\cdot\left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}{\left(\left(u_i^2\right)_{st}\right)^{1\backslash 2}}\right)\frac{2T_T}{Ec\cdot Tu^2}\right]}\left(\frac{\left(1+2\frac{1}{Pr\cdot Ec}\right)}{\left(1+\frac{1}{Pr\cdot Ec}\right)^{2/3}}\right)\right)^{-1}$$
$$= Pr\frac{\left(1+\frac{1}{Pr\cdot Ec}\right)^{2/3}}{\left(1+2\frac{1}{Pr\cdot Ec}\right)}\frac{\left[\frac{1}{Pr}+\left(\frac{\left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}{\left(\left(u_i^2\right)_{st}\right)^{1\backslash 2}}\right)\frac{2T_T}{Ec\cdot Tu^2}\right]}{\left[1+\frac{2T_T}{Ec\cdot Tu^2}\right]^{\frac{2}{3}}}, \quad (9)$$

Then taking into account values in Table 1, see [33],
the estimation of the formula for F_{tb} depending on Pr number is

$$F_{tb}^{-1} = Pr\cdot\frac{\left(1+\frac{1}{Pr\cdot Ec}\right)^{2/3}}{\left(1+2\frac{1}{Pr\cdot Ec}\right)}\frac{\left[\frac{1}{Pr}+\left(\frac{\left(\left(u_j^2\right)_{st}\right)^{1\backslash 2}}{\left(\left(u_i^2\right)_{st}\right)^{1\backslash 2}}\right)\frac{2T_T}{Ec\cdot Tu^2}\right]}{\left[1+\frac{2T_T}{Ec\cdot Tu^2}\right]^{\frac{2}{3}}} \approx 0.8\cdot Pr \quad (10)$$

Then, substituting this expression, as well as the values of Ec, Tu, T_T, and M into (8), we obtain

$$Nu_d \approx .157 - .229\left[\left(\frac{\sqrt{E_{st}/\rho}}{U_0}\right)Re_d\right]^{(n_T-1/n_T)}\left[(Pr)^{2/3(n_T-1)/n_T}\right] \quad (11)$$

Taking into account that in the previous equation $n = 7$, $n_T = 8$, we can write finally

$$Nu_d = .155 - .229\left[\left(\frac{\sqrt{E_{st}/\rho}}{U_0}\right)Re_d\right]^{(7/8)}\left[(Pr)^{7\backslash 12}\right] \quad (12)$$

4 Results of calculations of Nusselt number

4.1 Air flow in the tube

The empirical formula [34–39] for the turbulent flow in a pipe, if parameters $0.6 < Pr < 200$, $10^4 < Re_d < 5\times 10^6$, is

$$Nu = 0.021 \cdot (Pr_L)^{0.43} Re_d^{0.8} (Pr_L/Pr_w)^{0.25} \tag{13}$$

Index L refers to the medium parameters on the tube axis. Index W refers to the medium parameters on the tube wall. For the air flow in the tube, equation (13) is usually written as

$$Nu = 0.018 \cdot Re_d^{0.8} \tag{14}$$

For any turbulence intensity, see Table 2, we can determine the next values.

Then formula (11)–(12) for the air flow $Pr = 0.7$, $Tu = 0.0356$ may be written as

$$Nu_d = 0.00854 \left[(Pr)^{7\backslash 12}\right][Re_d]^{(7/8)} = 0.0069[Re_d]^{(7/8)} \tag{15}$$

Let us compare the results by a new formula for the Nusselt number with experimental equation (14), see Table 3.

4.2 Water flow in the tube

We now represent calculations for water flow on the assumption that $Pr_L = 3$, $Pr_W = 4$ (the heat transfer from fluid to the wall), and then, the equation can be written as

$$Nu_d = 0.0077[(Pr)^{7\backslash 12}][Re_d]^{(7/8)} = 0.0145[Re_d]^{(7/8)} \tag{16}$$

An empirical equation for this case of the flow is

$$Nu_d = 0.021 * (Pr_L)^{0.43} Re_d^{0.8} (Pr_L/Pr_w)^{0.25} = 0.03137 Re_d^{0.8} \tag{17}$$

So, results of calculation are presented in Table 4:

In the case of the water flow with $Pr_L = 7$, $Pr_w = 6$ (the wall-to-liquid heat transfer), the equation (11)–(12) for $Tu = 0.007$–0.01 has the form :

$$Nu_d = 0.0077[(Pr)^{7\backslash 12}][Re_d]^{(7/8)} = 0.0177[Re_d]^{(7/8)} \tag{18}$$

Table 2 Changes of the intensity of turbulence and values of coefficients in formula (12)

$[\frac{\sqrt{(E_{st})_{UP/\rho}}}{U_0}]$	$A = [\frac{\sqrt{(E_{st})_{UP/\rho}}}{U_0}]^{(0.875)}$	$A1 = (0.157 - 0.229) \cdot A$
0.009	0.0162	0.0254–0.0037
0.0093	0.0167	0.0262–0.00382
0.0115	0.02	0.0316–0.0046
0.015	0.0253	0.0392–0.00579
0.02	0.0326	0.00746–0.00508
0.036	0.0545	0.00854–0.012

Table 3 Calculations using new formula (15) and the experimental formula (14) for $Pr = 0.7$

Re	$Re^{0.8}$	$Re^{7/8}$	Nu_d, Eq. (15)	Data Nu_d, Eq. (14)
10^4	1585	3163	22	28
$5*10^4$	5743	12930	91	104
10^5	10^4	23717	164	180
$5*10^5$	36239	96961	669	652
10^6	63095	177827	1233	1136

Table 4 Calculations using new formula (16) and the experimental formula (17) for $Pr_L = 3$, $Pr_W = 4$

Re	$Re^{0.8}$	$Re^{7/8}$	Nu_d, Eq. (16)	Data Nu_d, Eq. (17), $Pr_L \backslash Pr_w = 0.93$
10^4	1585	3163	46	53
$5*10^4$	5743	12930	187	180
10^5	10000	23714	344	337
$5*10^5$	36239	96961	1405	1137
10^6	63095	177827	2587	2125

Table 5 Calculations using new formula (18) and the experimental formula (19) for $Pr_L = 7Pr_w = 6$

Re	$Re^{0.8}$	$Re^{7/8}$	Nu_d Eq. (18)	Data Nu_d, Eq. (19)
10^4	1585	3163	57	77
$5*10^4$	5743	12930	228	279
10^5	10000	23717	420	485
$5*10^5$	36239	96961	1716	1757
10^6	63095	177827	3152	3063

Table 6 Calculations using new formula (20) and the experimental formula (21) for $Pr_L = 3Pr_w = 7$

Re	$Re^{0.8}$	$Re^{7/8}$	Nu_d, Eq. (20)	Data, Nu_d, Eq. (21)
10^4	1585	3163	32	43
$5*10^4$	5743	12930	130	156
$3*10^5$	24082	62012	620	655
10^6	63095	177827	1778	1715
$5*10^6$	228652	727107	7271	6219

Table 7 Calculations using new formula (22) and the experimental formula (23)

Re	$Re^{0.8}$	$Re^{7/8}$	Nu_d (22) $Pr = 0.05$	Nu_d, experiment Eq. (23) $Pr = 0.05$
10^4	1585	3162	$3 \div 4$	6
$5*10^4$	5743	12930	$12 \div 13$	18
10^5	10000	23717	$22 \div 23$	27
$3*10^5$	24082	62012	$58 \div 59$	59
10^6	63095	177827	$168 \div 169$	149

and the empirical equation for this case of the flow is

$$Nu_d = 0.021 * (Pr_L)^{0.43} Re_d^{0.8} (Pr_L/Pr_w)^{0.25} = 0.021 \cdot 2.308 Re_d^{0.8} = 0.0485 Re_d^{0.8} \tag{19}$$

So, the results of calculation are presented in Table 5.

We now represent the calculations for the water flow under the assumption that $Pr_L = 3Pr_w = 7$ in Table 6 (heat transfer from the fluid to the wall). When $Pr_w > Pr_L$ then turbulence intensity near the wall is lower than in the flow if $Pr_L = Pr_W = 7$, other factors being equal. Therefore, the estimated calculations of equation (13), we can imagine not to $Tu = .0356$ as in the case of isothermal flow or flow with constant thermal properties, but with the fall of the wall turbulence. Then we can write equation (11)–(12) for $Tu = .01 - 0.02$ as:

$$Nu_d = 0.01 \cdot \left[\mathrm{Re}_{d\,(TU)}\right]^{(7/8)} \tag{20}$$

and the empirical equation for this case of the flow is

$$Nu_d = 0.021 * (Pr_L)^{0.43} Re_d^{0.8} (Pr_L/Pr_w)^{0.25} = 0.0272 Re_d^{0.8} \tag{21}$$

4.3 Liquid metal flow in the tube

Finally, let us compare calculations and data for the liquid metal flow in the tube. Using new general equations (11)–(12), the new formula for the liquid metal flow ($Pr = 0.05$) has next equation

$$Nu = 0.00095 \cdot Re_x^{7/8} \tag{22}$$

Then, taking into account the equation (Seban & Shimazaki) for the liquid metal flow in the tube [39–43]

$$Nu = 5 + 0.025 * (Pr \cdot Re)^{0.8} = 5 + 0.025 * (Pe)^{0.8} \tag{23}$$

we present the comparison between the theoretical and empirical values in Table 7.

5 Conclusions

New analytical formulas are obtained for the heat transfer coefficients (Nu_d) for the gas, liquid, and liquid metal flows in the tube (Eqs. 11–12). These formulas enable us also to take into account the initial turbulence parameters (the turbulence scale and the of velocity fluctuation intensity), which are inherent for the flow, instead of the effect of the Reynolds number only. This circumstance allows us to determine the causes of the limited applicability of classical experiments and semiempirical equations of the Nu coefficient for the flow in the tube due to the different fluctuation parameters of fluid in experiments. New formulas show that the changes in the values of coefficients in the formulas for the heat coefficients are sensitive even for small changes in values of the initial turbulence. Accordingly, new formulas can serve the basis for clarifying the expressions of heat transfer in various devices in the case of different values of the initial turbulence. Obtained results show that both for isothermal and for nonisothermal flows, the reason for the process of transition from a deterministic state into a turbulent one is determined by the physical law of equivalence of measures between them. Also the theory of stochastic equations and the law of equivalence of measures could determine mechanics which is basis in different phenomena of self-organization and chaos theory.

Acknowledgements This work was supported by the program of increasing the competitive ability of National Research Nuclear University MEPhI (agreement with the Ministry of Education and Science of the Russian Federation of August 27, 2013, Project No. 02.a03.21.0005).

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