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Sequential approximate optimization using variable fidelity response surface approximations*

J.F. Rodríguez, V.M. Pérez, D. Padmanabhan and J.E. Renaud

Abstract The dimensionality and complexity of typical multidisciplinary systems hinders the use of formal optimization techniques in application to this class of problems. The use of approximations to represent the system design metrics and constraints has become vital for achieving good performance in many multidisciplinary design optimization (MDO) algorithms. This paper reports recent research efforts on the use of variable fidelity response surface approximations (RSA) to drive the convergence of MDO problems using a trust region model management algorithm. The present study focuses on a comparative study of different response sampling strategies based on design of experiment (DOE) approaches within the disciplines to generate the zero order data to build the RSAs. Two MDO test problems that have complex coupling between disciplines are used to benchmark the performance of each sampling strategy. The results show that these types of variable fidelity RSAs can be effectively managed by the trust region model management strategy to drive convergence of MDO problems. It is observed that the efficiency of the optimization algorithm depends on the sampling strategy used. A comparison of the DOE approaches with those obtained using a optimization based sampling strategy (i.e. concurrent subspace optimization – CSSO) shows the DOE methodologies to be competitive with the CSSO based sampling methodology in some cases. However, the CSSO based sampling strategy was found to be, in general, more efficient in driving the optimization.

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1 Introduction

The complex systems defining typical MDO problems are often characterized by a large number of design variables and constraints. The data that is shared is very large resulting in severe communication requirements for the designers. Hence, the application of formal optimization techniques to the design of these systems is presently hindered because the number of design variables and constraints is so large that the optimization is both intractable and costly and can easily saturate even the most advanced computers available today. Therefore, the use of approximations to represent the design space is essential to the efficiency of MDO algorithms.

Approximations provide information about the system necessary for the optimization process without the cost of executing CPU-intensive analysis tools. Moreover, the use of approximations allows for the temporary decoupling of disciplines which avoids the constant transfer of information among disciplines required during an iterative system analysis. Consequently, most MDO algorithms couple, in an iterative fashion, a traditional optimization code to lower-cost computational models of the objective function and constraints. In a traditional MDO algorithm, a solution to the approximate problem is found, a full analysis is executed at this new design, the approximate model is updated and the process repeated until convergence to a solution of the original problem is achieved.

Lower-cost computational models can be categorized in: lower complexity models which are less physically faithful representations of the actual physical problem (Barthelemy and Haftka 1993); and model approximations which are algebraic representations obtained from design sites at which objective and constraints are known (Guinta and Watson 1998; Lewis 1998). Most MDO algorithms differ in how the approximate models are built and managed in order to drive convergence to a solution of the original problem.

In recent years there has been a growing interest in the use of response surface approximations in model management frameworks to optimize large multidisciplinary systems (Rodríguez *et al.* 2000). Rodríguez *et al.* (1998b) proposed a trust region approach for lower fidelity approximate model management for the class of constrained approximate optimization problems to manage convergence. The framework is used to manage the convergence of low fidelity response surface approximations in an augmented Lagrangian optimization. A cumulative response surface approximation of the augmented Lagrangian is sequentially optimized subject to a trust region constraint which insures convergence of the strategy. Rodríguez *et al.* (1998b) conducted the application studies on a suite of single level optimization problems. High fidelity system models were queried directly using a central composite design (CCD) sampling strategy to build response surface approximations over the design space during each iteration of the algorithm. Querying the high fidelity model during each iteration can itself become cost prohibitive as the size of the design space grows. In a later study Rodríguez *et al.* (1998a) the scope of application studies was extended to include the class of multidisciplinary design optimization (MDO) test problems with the high fidelity model no longer being queried directly to generate response surface approximations. Rodríguez *et al.* (1998a) used the design sampling strategy in the CSSO approach of Wujek *et al.* (1997). Instead of querying the high fidelity model directly using a statistical sampling strategy, the Wujek approach queries approximation models available at the discipline design level. These models are of variable fidelity and will be discussed in detail in a later section. The sampling strategy in the Wujek *et al.* (1997) approach is optimization based. The design sites visited during the subspace optimizations are the response sampling sites. The resulting response surface is therefore constructed from data of variable fidelity.

The concurrent subspace optimization (CSSO) approach has the attribute of providing for the temporary decoupling of the disciplinary analysis during subspace optimizations facilitating interdisciplinary communications. However, the number of design sites visited during a subspace optimization could be much larger than the minimum required for building the RSA leading to unnecessary computational expense. Another potential problem is the generation of ribbons of data during the CSSO process which is a result of the line searches performed by the optimizer during the subspace optimizations. In other words, the data generated might not be "space filling" leading to inaccurate response surface approximations which could slow the convergence of the optimization, reducing the efficiency of the algorithm.

In this research, the decoupled system used in the CSSO methodology is sampled using design of experiments (DOE) methodologies (Box *et al.* 1978; Ross 1988) and this data used to build a second order RSA of the system function and of each of the constraints to build the augmented Lagrangian in the algorithm of Rodríguez *et al.* (1998a). The design of experiments methodologies provide an efficient alternative for sampling the design space which avoids data clustering and reduces the number of unnecessary sampling sites. In this investigation three different experimental designs are investigated: two level factorial experiments; central composite designs experiments; and orthogonal arrays. The subspace samplings are executed following a random schedule in order to reduce, the possibility of neighbouring sites being sampled at the same subspace. Note that since the sampling is executed at the subspaces, the data is of variable fidelity.

2 Design of Experiments

When searching for improved designs, the designer typically runs a number of tests at certain design sites and, observes the system's design performance and makes a decision of accepting or rejecting each new design. Design of Experiments (DOE) is a statistical based methodology for systematically and efficiently designing and analyzing experiments for product evaluation.

In the language of DOE, one has to distinguish between the inputs to an experiment called *independent variables*, and the system outputs or *responses*. A typical experiment consists of a combination of runs, where each run defines a particular setting of independent variables (i.e. a *site*). The particular combination of runs is called experimental design and the possible settings for each independent design variable are called levels.

As mentioned earlier, one of the main concerns in designing an experiment is to define a pattern of design points (i.e sites) that best reveals the aspects of the situation of interest (i.e, response of the nonlinear objective and constraints). At this point we can distinguish between two approaches to the design of computer experiments. The parametric approach where it is assumed that f is the realization of a stochastic process, specifying a parametric family of possible processes, where the parameters are chosen such that f is approximated as accurately as possible (Torczon and Trosset 1998). This approach is usually combined with a design sampling criteria which is optimal with respect to the specified family (Osio and Amon 1995). Even though parametric experiments are appealing because of their properties, the practical difficulties of actually computing such optimal designs can be formidable, therefore limiting their applicability.

The nonparametric approach to the design of computer experiments, chooses the design sites in a manner that is perceived to be *space-filling.* Factorial experiments are part of this family of computer experiments. In this type of experiments, the independent variables or factors, are set at prescribed levels resulting in an ordered pattern in the design space. Factorial experiments can be divided in full and fractional factorial experiments. Full factorial experiments are those in which the total number of runs is given by the product of the number of levels of each independent variable. Figure 1 shows a two level full factorial experiment. On the other hand, fractional factorial experiments are those in which fewer runs than the full factorial experiment are required. The Central Composite Design (CCD) is a commonly used fractional factorial design. A CCD is a two-level factorial augmented by a centre point plus points arranged along the axis of the variables and symmetrically positioned with respect to the factorial cube (see Fig. 1).

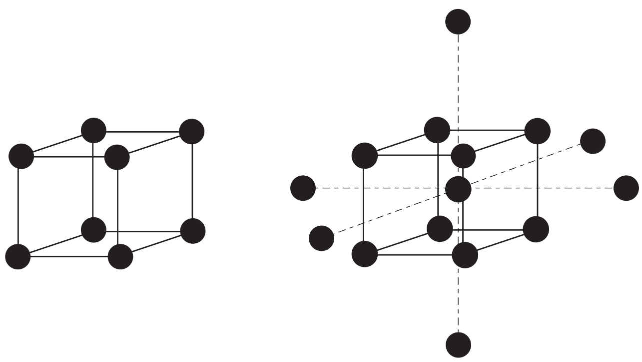

Fig. 1 Factorial designs: full factorial (left), CCD (right)

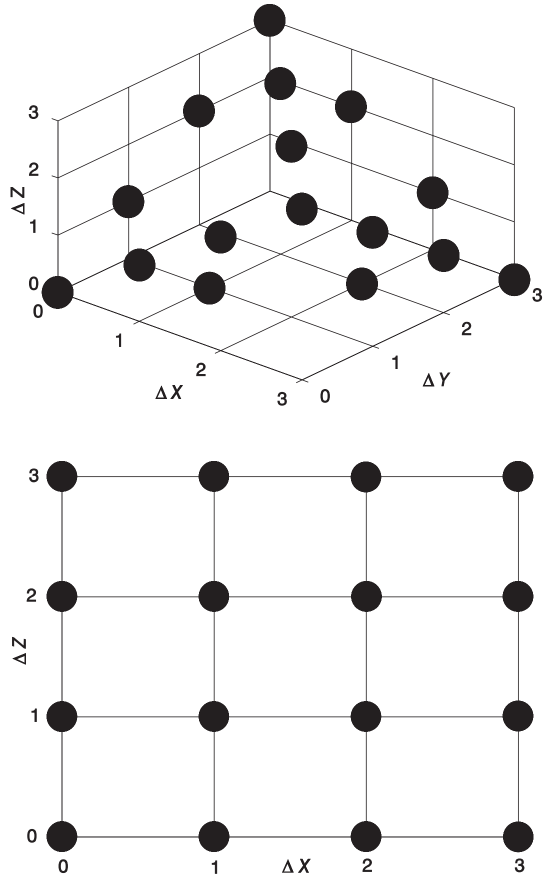

Fig. 2 Orthogonal array

Another class of experimental designs are the Orthogonal Arrays (OAs) (Owen 1992). OAs are based on fractional factorial and are constructed to minimize the number of designs required to evaluate the effects of the independent variables on the response. Orthogonal arrays possess a space-filling property called strength. An OA of strength t, for a k level experiment, represents, for every subset of t independent variables, a k^t grid. This is shown in Fig. 2, where the design sites for a four-level factorial OA of strength 2 for three independent variables are depicted. Note that even though the design sites represent a "cloud" in the whole design space, the grid pattern is observed in the projections.

3
Data base generation

In this investigation the CSSO sampling strategy is replaced by a sampling strategy based on design of experiments. In the concurrent subspace optimization (CSSO) approach of Wujek *et al.* (1997) the subspace optimizers solve the same system level optimization problem. The optimizations differ in that each discipline (i.e. subspace) optimizes the system level problem with respect to a local set of design variables allocated to the subspace. *The variable fidelity data is obtained from each design site visited during a given subspace optimization.* The subspace optimizers therefore provide the approximate response sampling strategy. *The CSSO sampling strategy is optimization based and provides for design sampling in descent directions, where the sampling sites are feasible as measured by the approximations available within the subspaces (see Fig. 3).*

In this research we elect to use the *principles* of the concurrent subspace optimization (CSSO) of Wujek *et al.* (1997) to generate variable fidelity design data for multidisciplinary systems. Evaluations of the system states (i.e. performance metrics) at the subspace level does not involve an iterative solution strategy. This is accomplished by providing the disciplinary designers with linear response representations of the other discipline states based on sensitivities generated from the global sensitivity equations (GSE) of Sobieszczanski-Sobieski (1990). Therefore nonlocal states are linearly approximated and local states are evaluated using the high fidelity local analysis tools available in the discipline. *The design data generated by a given subspace is therefore of variable fidelity.* Some of the data is generated by the high fidelity disciplinary analysis tools but based on linearized inputs of the other disciplines. Other data is simply the state prediction provided by the linearized response representations of nonlocal disciplinary states.

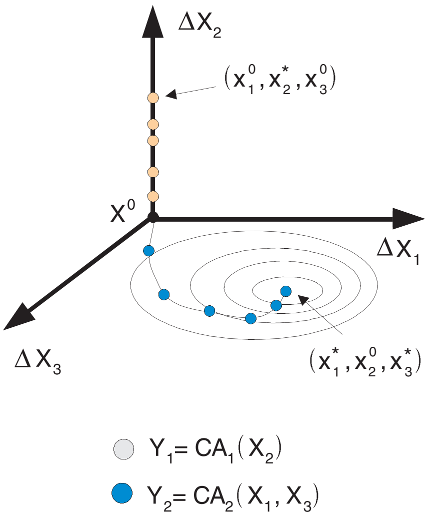

Fig. 3 CSSO: optimization based sampling

In the present paper matrices of experiments based on two level full factorial, central composite designs and orthogonal arrays are investigated. Three orthogonal arrays are considered: An OA of strength 2 and 8 levels in the design variables, an OA of strength 2 and 11 levels in the design variables, and an OA of strength 3 and 11 levels in the design variables.

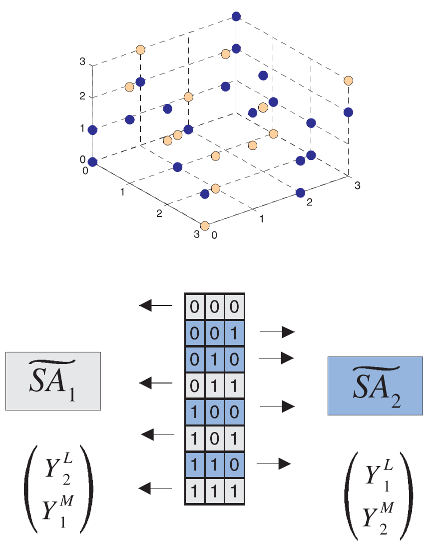

Fig. 4 Partial sampling

Two sampling methodologies are considered. In the first methodology denoted *partial sampling* methodology, the matrix of experiments is partitioned among the different disciplines or subspaces. Figure 4 shows an example of this methodology, partitioning the matrix of experiments between two subspaces named $\widetilde{SA}_1$ and $\widetilde{SA}_2$. In this way a subset of the entire matrix of experiments is sampled at each discipline. Therefore, only the required number of discipline evaluations defined by the matrix of experiments are performed in each iteration. In this methodology, three different sampling strategies are investigated. In the first strategy, the matrix of experiments as well as the assignment of the experiments to the disciplines was kept the same during the optimization process. The second strategy of sampling keeps the matrix of experiments fixed during optimization while the assignment of the experiments to the disciplines changes at each iteration. The assignment of the experiments to the disciplines is performed using a pseudo-random generator. In the third strategy, a new matrix of experiments is generated at each iteration while the assignment of the experiments to the disciplines is kept the same. In the generation of new matrices of experiments, the randomization software developed by Owen was used. A complete set of computer programs for generating OAs may be found at http://www.cmu.edu/. Strategies two and three are only applied to the orthogonal arrays since the matrix of experiments for the two level full factorial and CCD designs are unique.

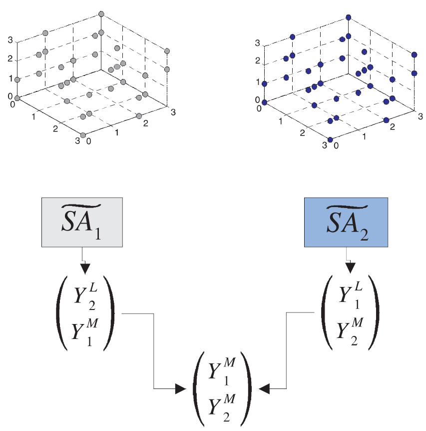

Fig. 5 Full sampling

In the second sampling methodology denoted *full sampling*, the entire experimental design was run at each discipline. Note that this sampling methodology represents a considerable increment in the cost of generating the database, however it also provides a much richer information of the system which might increase the quality of the response surface approximations and therefore the efficiency of the optimization algorithm (see Fig. 5). In this methodology, two different sampling strategies are considered. In the first strategy, the matrix of experiments is kept fixed during the optimization process. In the second strategy, which is only applied to orthogonal arrays, a new matrix of experiments is generated after each iteration. Table 1 details the characteristics of each of the sampling strategies explored.

The data generated using the procedure described above is employed to construct a cumulative response surface approximation of the augmented Lagrangian (Rodríguez *et al.* 1998b) for use in the trust region model

Table 1 Sampling strategies investigated

Strategy	Partial Sampling	Full Sampling
1	exp. matrix fixed exp. assignment fixed	exp. matrix fixed
2	exp. matrix fixed exp. assignment random	exp. matrix random
3	exp. matrix random exp. assignment fixed	–

management algorithm for constrained minimization developed by (Rodríguez *et al.* 1998a). The results presented in the next section are based on 10 independent runs corresponding to different initial matrices of experiments and/or different initial experiment assignment depending on the strategy under consideration.

4 Test problems

4.1 Autonomous hovercraft

The design of an autonomous hovercraft (AHC), shown in Fig. 6, was first presented by Sellar *et al.* (1996). This problem involves 11 design variables and the calculation of 50 states. The physical system consists of an engine, rotor, and payload. The rotor is comprised of two rectangular lifting surfaces located on opposite ends of a hollow, circular shaft. The system is to operate such that the motor speed (RPM) provides a thrust-to-weight ratio of one, imposing hover conditions.

The system analysis is comprised of four contributing analysis, three of which interact in a complex coupled fashion as illustrated in the dependency diagram of Fig. 7. The aerodynamics CA (CA_a) calculates the aerodynamic loads on the lifting surfaces and approximates the distributed drag force along the rod while estimating the induced velocity at the lifting surfaces as a function of the thrust. CA_a requires the torsional deformation of the shaft (θ_d), the motor RPM and thrust as inputs. The shaft deformation is supplied by the structures CA (CA_s). All the quantities calculated by this CA, which also include the axial and shear stresses at the hub and the deflection of the lifting surfaces, are a function of the aerodynamic loads, thus subjecting these CAs to static aeroelastic coupling. The propulsion/performance CA (CA_p) calculates the thrust and torque necessary to spin the motor based on the loads supplied by CA_a. RPM is calculated by explicitly imposing that the total weight equals the thrust, determining the weight of the motor to achieve this total weight, and calculating the power and resulting RPM available from that size motor. At this point the formulation used here differs from that of Sellar *et al.* (1996) where the RPM is updated in

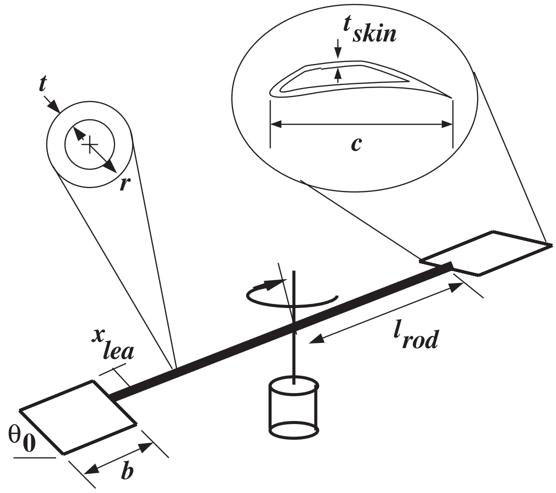

Fig. 6 Autonomous hovercraft system

a heuristic manner. The fourth CA, structural dynamics (CA_d), calculates the first natural frequencies of the rotor in bending and torsion. This CA is completely uncoupled from the other CAs as it requires no states as inputs.

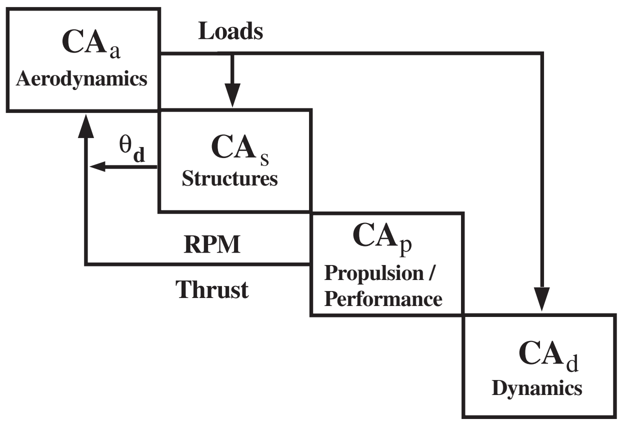

Fig. 7 AHS dependency diagram

The goal of the optimization of this system is to minimize the empty weight of the hovercraft subject to constraints on the Von Misses stress due to in-plane and normal forces in the rod, the first natural frequencies of the rod, the Mach number at the tip, and the hovercraft range. The optimization problem is posed as:

$$\min W_{\text{empty}} = W_{\text{wing}} + W_{\text{rod}} + W_{\text{fuel}} + W_{\text{motor}}\,,$$

subject to

$$g_1 = 1.0 - \frac{\sigma_N}{\sigma_{\text{all}}} \geq 0.0\,,$$

$$g_2 = 1.0 - \frac{\sigma_T}{\sigma_{\text{all}}} \geq 0.0\,,$$

$$g_3 = \frac{\omega_b}{k \cdot \mathrm{RPM}} - 1.0 \geq 0.0\,,$$

$$g_4 = \frac{\omega_t}{k \cdot \mathrm{RPM}} - 1.0 \geq 0.0\,,$$

$$g_5 = \frac{M_{\mathrm{tip}_{\mathrm{all}}}}{M_{\mathrm{tip}}} \geq 0.0\,,$$

$$g_6 = \frac{E}{E_{\mathrm{req}}} - 1.0 \geq 0.0\,.$$

where $\sigma_{\mathrm{all}} = 14\,000\,psi$, $k = 1.5$, $M_{\mathrm{tip}_{\mathrm{all}}} = 0.8$ and $E_{\mathrm{req}} = 2\,\mathrm{h}$. The global optimum for this problem was first reported by Sellar *et al.* (1996) and is depicted in Table 2. For the minimum empty weight, $W_{\mathrm{empty}} = 67.9\,\mathrm{lbs}$, 7 design variables are at their bounds and the hovercraft range constraint, g_6, is active.

Table 2 Starting and optimum designs for the AHC problem

Design Variable	Starting Design	Optimum Design
t (in)	0.5	0.15
r (in)	2.0	1.20
l_{rod} (in)	60.0	39.92
c (in)	18.0	6.00
b (in)	24.0	42.0
$\theta_\circ$ (deg)	15.0	10.66
t/c	0.2	0.05
x/c	0.0	0.1507
$\%cam$	0.0	0.035
t_{skin} (in)	0.25	0.05
W_{fuel} (lb)	50.0	19.981
W_{empty} (lb)	394.2	67.92

4.2
The cantilever beam

The structures-controls design problem shown in Fig. 8 as introduced by Sobieszczanski-Sobieski *et al.* (1991). The problem comprises a total of 11 design variables and 43 states. The physical problem consists of a cantilever beam subjected to static loads along the beam and to a dynamic excitation force applied at the free end. Two sets of actuators are placed at the free end of the beam to control both the lateral and rotational displacement.

The system analysis is comprised of two coupled contributing analysis as shown in Fig. 9. The structures subsystem, $\mathrm{CA_s}$ consists of a finite element model of the beam where the natural frequencies and modes of the cantilever beam are computed. $\mathrm{CA_s}$ requires, in addition to the characteristics of the beam, the weight of the control system as input. The weight of the control system is calculated at the controls CA, $\mathrm{CA_c}$. The weight of the control system is a function of the dynamic displacements and rotations of the free end of the beam. These dynamic displacements and rotations are functions of the natural frequencies and modes obtained in the structures CA, thus subjecting these CAs to coupling.

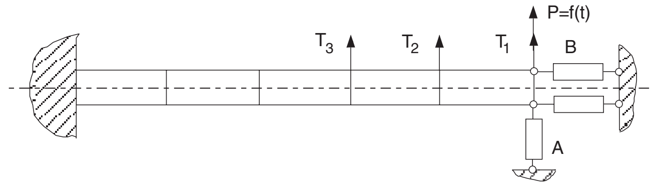

Fig. 8 Cantilever beam with actuators

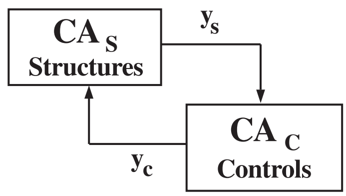

Fig. 9 Dependency diagram of the cantilever beam design problem

The objective of the optimization is to minimize the total weight of the system W_t, composed of the weight of the beam W_s plus the weight of the control system W_c. The minimization is subjected to seven constraints on the static stresses, lateral and rotational displacements, natural frequencies and dynamic lateral and rotational displacements at the free end of the beam. The problem is posed as:

$$\min\, W_t = W_s + W_c\,,$$

subject to

$$g_1 = 1 - \frac{dl}{dl_a} \geq 0\,, \quad g_2 = 1 - \frac{dr}{dr_a} \geq 0\,,$$

$$g_3 = \frac{\omega_1}{\omega_{1_a}} - 1 \geq 0\,, \quad g_4 = \frac{\omega_2}{\omega_{2_a}} - 1 \geq 0\,,$$

$$g_5 = 1 - \frac{\sigma}{\sigma_a} \geq 0\,, \quad g_6 = 1 - \frac{ddl}{ddl_a} \geq 0\,,$$

$$g_7 = 1 - \frac{ddr}{ddr_a} \geq 0\,,$$

where dl is the static lateral displacement, dr is the static rotational displacement, ddl is the dynamic lateral displacement, ddr is the dynamic rotational displacement, ω_1 is the first natural frequency, ω_2 is the second natural frequency, and σ is the static stress. The subscript a stands for the allowed value. The optimum for this problem is depicted in Table 3. The minimum weight,

$W = 1493.6$ lbs occurs where 6 design variables are at their bounds.

Table 3 Starting and optimum designs for the Cantilever Beam problem

Design Variable	Starting Design	Optimum Design
b_1 (in)	10.0	3.0
b_2 (in)	10.0	3.0
b_3 (in)	10.0	3.0
b_4 (in)	10.0	3.0
b_5 (in)	10.0	3.0
h_1 (in)	10.0	13.85
h_2 (in)	10.0	11.96
h_3 (in)	10.0	9.78
h_4 (in)	10.0	7.06
h_5 (in)	10.0	3.75
c	0.01	0.06

5 Results

The optimization algorithm described by Rodríguez *et al.* (1998a), was applied to the autonomous hovercraft and cantilever beam problems using the database generation methodologies presented in previous sections. Since both problems have a total of 11 design variables, the sampling patterns are the same in both cases. Table 4 shows the total number of design evaluations required for each of the experimental designs investigated.

Table 4 Number of evaluations for each experimental design

Experiment Design	No. of Evaluations
OA11-8-2	128
OA11-11-2	121
OA11-11-3	1331
FF	2048
CCD	2071

In Table 4 OAX-Y-Z stands for an orthogonal array of strength Z, for X design variables and Y levels per variable. Note that increasing the strength of the orthogonal array increases the number of sampling points, providing a much richer data base. However, it also increments the cost of generating the database.

For each of the sampling strategies described in Table 1, 10 different runs were performed, each with a different initial matrix of experiments. The initial design point was kept unchanged. The mean number of approximate minimizations and its standard deviation for each of the strategies are shown for each problem. The total cost of the optimization is reported in the form of the total number of subsystem evaluations performed using each of the sampling strategies.

5.1 Autonomous hovercraft

For this problem, the four contributing analysis (i.e. CA_a, CA_s, CA_p, CA_d) comprising the system analysis of the problem are taken as four independent subspaces for database generation purposes. Table 5 shows the average number of approximate minimizations for each of the sampling cases for the partial sampling methodology (i.e. the design matrix is partitioned among the subspaces).

Table 5 Mean value of the total number of approximate minimizations for the AHC problem (partial sampling)

Sampling	Approx. Minimizations		
Matrix	Strategy 1	Strategy 2	Strategy 3
OA11-8-2	31 ± 15	36 ± 14	34 ± 23
OA11-11-2	42 ± 26	40 ± 38	52 ± 33
OA11-11-3	16 ± 2	16 ± 5	18 ± 4
FF	36 ± 33	28 ± 27	–
CCD	17 ± 4	17 ± 4	–

The results show (Figs. 10–12) that increasing the strength of the orthogonal arrays reduces the total number of approximate minimizations required to solve the problem. Also, a reduction in the standard deviation of the number of approximate minimizations is observed for experimental designs where a large number of sampling points were used (i.e. OA11-11-3). Table 5 also shows the results to be almost independent of the sampling strategy used for each orthogonal array. This fact is more clearly observed in Fig. 10 where the average number of approximate minimizations for the different sampling strategies for the partial sampling methodology are depicted.

Table 6 shows the total number of approximate minimizations obtained with the full sampling methodology

Table 6 Mean value of the total number of approximate minimizations for the AHC problem (full sampling)

Sampling	Approx. Minimizations	
Matrix	Strategy 1	Strategy 2
OA11-8-2	17 ± 3	16 ± 3
OA11-11-2	19 ± 12	15 ± 3
OA11-11-3	16 ± 1	18 ± 6
FF	13	–
CCD	18	–

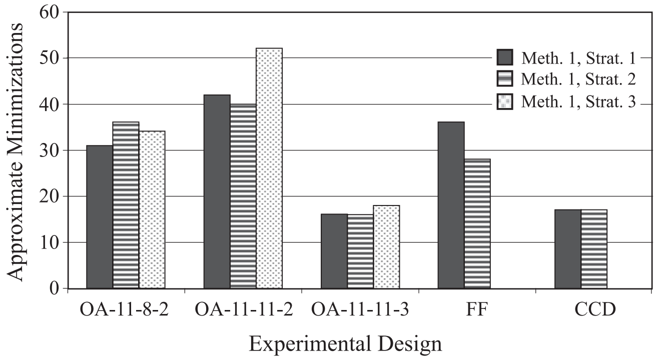

Fig. 10 Total approximate minimizations for different sampling strategies (partial sampling)

(i.e. the entire matrix of experiments is run at all subspaces).

This table shows a reduction in the number of approximate minimizations for all matrices of experiments, but also the results are obtained with a lower standard deviation. This last observation denotes a more reliable database from an optimization standpoint. Moreover, Table 6 shows that for the particular case of the AHC problem, the results obtained using full sampling seem to be also independent of the experimental design used to query the subspaces (see Fig. 11).

When the total number of CA Calls for both methodologies are compared (Fig. 12) a rather interesting result is obtained.

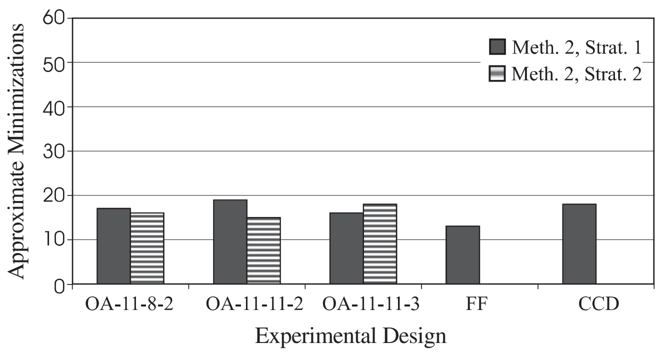

Fig. 11 Total approximate minimizations for different sampling strategies (full sampling)

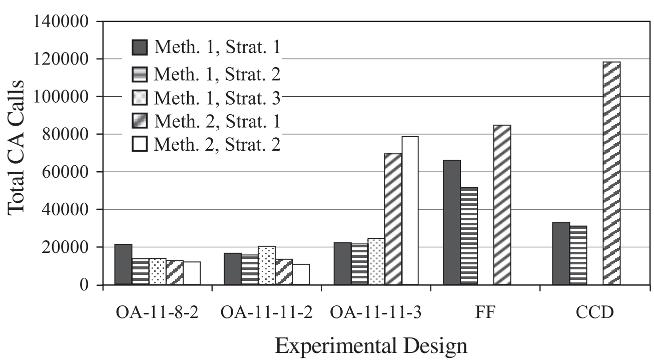

Fig. 12 Total CA calls for different sampling strategies

The figure shows that for the case of high strength orthogonal arrays, full factorial and CCD design, the number of CA Calls grows tremendously when full sampling is used. However, the same does not occur for low strength orthogonal arrays where the total number of CA Calls remains almost constant. In these two cases the system is optimized in a lower number of iterations with more repeatability of the results (i.e. lower standard deviation) at almost the same cost.

5.2 Cantilever beam

In this problem the two contributing analysis (i.e. CA_s and CA_c) comprising the system analysis are taken as two independent subspaces for database generation. Table 7 shows the average number of approximate minimizations for the partial sampling methodology.

Table 7 Mean value of the total number of approximate minimizations for the cantilever beam problem (partial sampling)

Sampling	Approx. Minimizations		
Matrix	Strategy 1	Strategy 2	Strategy 3
OA11-8-2	75 ± 39	71 ± 26	65 ± 23
OA11-11-2	88 ± 46	70 ± 24	72 ± 43
OA11-11-3	46 ± 25	30 ± 9	33 ± 10
FF	45 ± 21	37 ± 10	–
CCD	40 ± 15	37 ± 15	–

These results are in general consistent with those obtained for the AHC design problem. Increasing the size of the experimental design reduces the total number of approximate minimizations. In this case, a reduction in the standard deviation of the number of approximate minimizations for the orthogonal array of higher strength and Full Factorial and CCD designs is also observed. As found in the AHC problem the results in Table 7 appears to be indifferent to the sampling strategy used with each experimental design. Figure 13 showing the average number

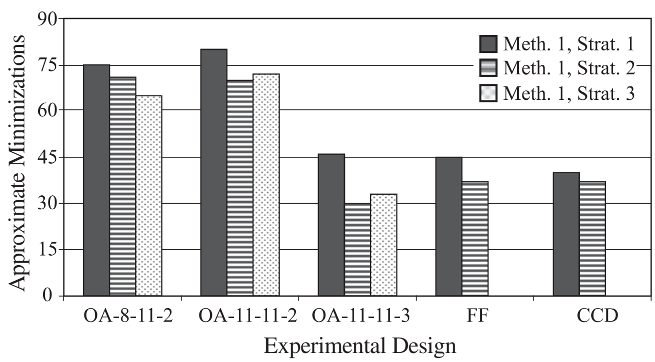

Fig. 13 Total approximate minimizations for different sampling cases (partial sampling)

Table 8 Mean value of the total number of approximate minimizations for the cantilever beam problem (full sampling)

Sampling Matrix	Approx. Minimizations Strategy 1	Strategy 2
OA11-8-2	40 ± 11	57 ± 20
OA11-11-2	55 ± 21	60 ± 18
OA11-11-3	31 ± 14	25 ± 4
FF	37	–
CCD	33	–

of approximate minimizations for the partial sampling methodology details this observation.

Table 8 shows the total number of approximate minimizations for the cantilever beam problem using the full sampling methodology.

In this example a reduction in the number of approximate minimizations for all matrices of experiments is also observed. This reduction is accompanied, as in the previous problem, with a reduction in the standard deviation of the total number of approximate minimizations. Note that the results for this methodology in regard to the number of approximate minimizations, follows the same trend as in the AHC problem (see Figs. 11 and 14). However the results are not as uniform as was observed in the previous example.

If the total number of CA Calls required for each sampling strategy to solve the optimization problem are compared (Fig. 15), the result resembles that obtained for the AHC problem. As occurred in the previous example, the number of CA calls for high strength OAs, full factorial, and CCD designs grows when used with the full sampling methodology, while the number of CA calls does not manifest a significant variation when full sampling methodology is applied with low strength OAs.

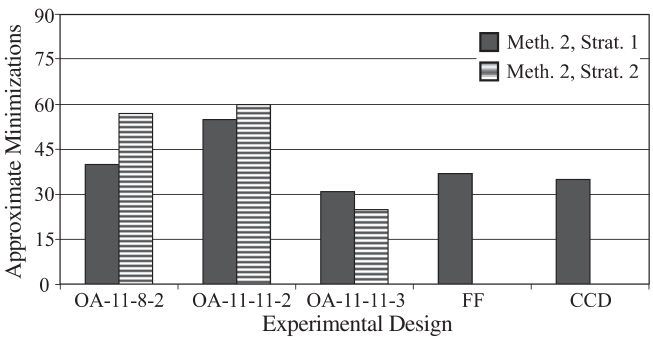

Fig. 14 Total approximate minimizations for different sampling strategies (full sampling)

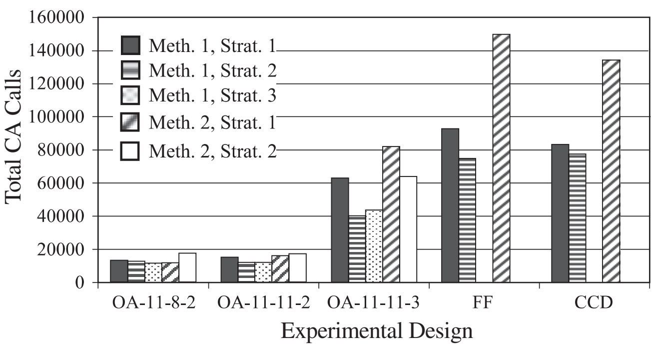

Fig. 15 Total CA calls for different sampling strategies

6 Discussion

One of the most interesting results obtained is the reduction in the number of approximate minimizations when low strength orthogonal arrays are used with the full sampling methodology. This behaviour is obtained because in the partial sampling methodology, the matrix of experiments is divided among the contributing analysis limiting the amount of information supplied for each discipline. This might prevent for some areas of the design space where the response provided by a particular CA is critical for the minimization to be sample, or queried insufficiently. Therefore, the response surfaces approximations constructed from this data will not provide a good approximation of the system reducing the rate of convergence or leading to earlier convergence of the algorithm, thus making the algorithm more sensitive to the partition of the matrix of experiments among the disciplines and the matrix of experiments itself. Note that this situation will be more critical in problems with a large number of subspaces and a relatively small number of design variables since the contribution of each discipline will be significantly reduced. This is the situation in the examples studied here. The AHC problem has four subspaces implying that each discipline contributes 30 sampling points for the OA-11-11-2 orthogonal array. On the other hand, a quadratic response surface approximation (RSA) with 11 design variables around a design point, requires 66 constants to be determined, which corresponds to the Hessian information, as first order information is already available. Even though, the total number of points in the database (122) suffices to characterize the second order approximation, the ratio of the information supplied for each discipline to the total number of sampling points required to characterize the RSA is low (i.e. 0.5). The situation changes quite abruptly for the full sampling methodology. In this scenario, the ratio of the information supplied by the discipline to the total required for characterizing the RSA is four times larger (i.e. 2) than that in the partial sampling methodology.

The considerable reduction in the number of approximate minimizations for the case of low strength OAs compensates for the extra cost involved in the generation of the database. As a consequence the total number of CA calls required to optimize the problem is kept almost constant as shown in Figs. 12 and 15. The difference is that using the full sampling methodology the algorithm gains robustness.

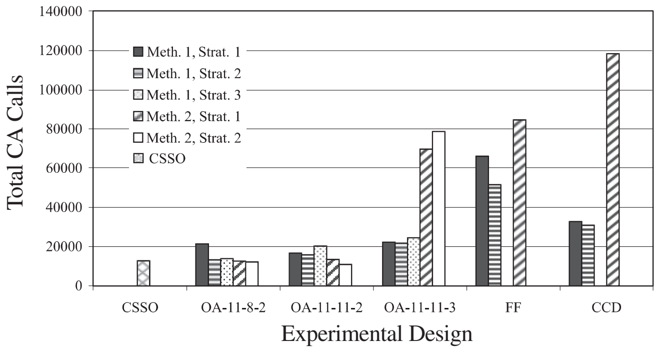

Fig. 16 Total CA calls required for CSSO, partial sampling, and full sampling methodologies for the AHC problem

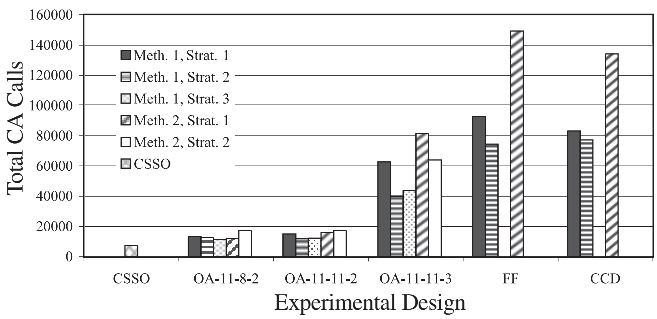

Fig. 17 Total CA calls required for CSSO, partial sampling, and full sampling methodologies for the cantilever beam problem

It is interesting to compare the cost of the optimization using a statistical based sampling methodology and an optimization based sampling methodology like CSSO. Figure 16 and Fig. 17 provide a comparison of the total CA calls required to optimize the AHC and the cantilever beam problems using both methodologies.

Even though orthogonal arrays appear to be competitive with CSSO when used with the full sampling methodology, the optimization based sampling provided by CSSO seems to be, in general, more efficient in driving the optimization. However, one must extend the current study to additional MDO problems having different dimensionality and complexity in order to extend this observation to a definite conclusion.

7 Conclusions

This investigation continues research efforts on the use of data of variable fidelity to drive the convergence of MDO problems using a trust region model management algorithm. The present study focuses on the use of design of experiments at the disciplines levels for response sampling to generate the database required to build the system RSAs. Orthogonal arrays of strength 2 and 3, full factorial and CCD experiments are investigated. Two different sampling methodologies are considered, one in which the matrix of experiments is partitioned among the different disciplines (*partial sampling* methodology) and a second sampling methodology in which the entire matrix of experiments is run in each discipline (*full sampling* methodology). Two MDO problems that have complex coupling between disciplines are used to benchmark the performance of each sampling strategy. The results show that response surface approximations constructed using variable fidelity data obtained using design of experiments at the discipline level, can be effectively managed by the trust region model management strategy. The results also show that orthogonal arrays of low strength (lower dimension matrices of experiments) perform best when used with the full sampling methodology. A comparison of the current results with those obtained using CSSO showed that full sampling with low strength orthogonal arrays appears to be competitive with CSSO in some cases. However, the optimization based sampling strategy (CSSO) was found to be, in general, more efficient in driving the optimization.

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References

Alexandrov, N. 1998: On managing the use of surrogates in general nonlinear optimization and MDO. *Proc. 7-th AIAA/USAF/NASA/ISSMO Multidisciplinary Analysis & Optimization Symp.* (held in St. Louis, MO), Paper 98-4798, pp. 720–729

Barthelemy, J.F.; Haftka, R.T. 1993: Approximation concepts for optimum structural design – a review. *Struct. Optim.* **5**, 129–144

Box, G.; Hunter, W.; Hunter J. 1978: *Statistics for experiments.* New York: John Wiley and Sons

Giunta, A.; Watson, L. 1998: A comparison of approximation modeling techniques: polynomial versus interpolating models. *Proc. 7-th AIAA/USAF/NASA/ISSMO Multidisciplinary Analysis & Optimization Symp.* (held in St. Louis, MO), Paper 98-4758, pp. 392–404

Lewis, R.M. 1998: Using sensitivity information in the construction of kriging models for design optimization. *Proc. 7-th AIAA/USAF/NASA/ISSMO Multidisciplinary Analysis & Optimization Symp.* (held in St. Louis, MO), Paper 98-4799, pp. 730–737

Osio, I.G.; Amon, C.H. 1995: An engineering design methodology with multistage Bayesian surrogates and optimal sampling. *Res. Engrg. Des.* **8**, 189–206

Owen, A.B. (1992): Orthogonal arrays for computer experiments, integration and visualization. *Statistica Sinica* **2**, 439–452

Rodríguez, J.F.; Renaud, J.E.; Wujek, B.A.; Tappeta, R.V. 2000: Trust region model management in multidisciplinary design optimization. *J. Comp. Appl. Math.* (to appear)

Rodríguez, J.F.; Renaud, J.E.; Watson, L.T. 1998a: Convergence of trust region augmented Lagrangian methods using variable fidelity approximation data. *Struct. Optim.* **15**, 141–156

Rodríguez, J.F.; Renaud, J.E.; Watson, L.T. 1998b: Trust region augmented Lagrangian methods for sequential response surface approximation and optimization. *J. Mech. Des.* **120**, 58–66

Ross, 1988: *Taguchi techniques for quality engineering.* McGraw Hill

Sellar, R.S.; Batill, S.M.; Renaud, J.E. 1996: A neural network-based, concurrent subspace optimization approach to multidisciplinary design optimization. *34-th AIAA Aerospace Sciences Meeting* (held at Reno, NV), Paper 96-1383

Sobieszczanski-Sobieski, J. 1988: Optimization by decomposition: a step from hierarchic to non-hierarchic systems. *2-nd NASA/Air Force Symp. on Recent Advances in Multidisciplinary Analysis and Optimization* (held in Hampton, VA), NASA Conference Publication 3031, Part 1, pp. 28–30

Sobieszczanski-Sobieski, J. 1990: Sensitivity of complex, internally coupled systems. *AIAA J.* **28**, 153–160

Sobieszczanski-Sobieski, J.; Bloebaum, C.L. ; Hajela, P. 1991: Sensitivity of control-augmented structure obtained by a system decomposition method. *AIAA J.* **29** 264–270

Torczon, V.; Trosset, M.W. 1998: Using approximations to accelerate engineering design optimization. *Proc. 7-th AIAA/USAF/NASA/ISSMO Multidisciplinary Analysis & Optimization Symp.* (held in St. Louis, MO), Paper 98-4800, pp. 507–512

Wujek, B.A.; Renaud, J.E.; Batill, S.M. 1997: A concurrent engineering approach for multidisciplinary design in a distributed computing environment. In: Alexandrov, N.; Hussaini, M.Y. (eds) *Multidisciplinary design optimization: state-of-the-art*, Proceedings in Applied Mathematics 80, pp. 189–208. Philadelphia: SIAM