Brief note

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On the validity of ESO type methods in topology optimization

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Abstract It is shown on a simple test example that ESO's rejection criteria may result in a highly nonoptimal design. Reasons for this failure are also discussed.

Key words "hard-kill" methods, evolutionary structural optimization (ESO), adaptive biological growth, compliance design, stress design, rejection criteria, inclusion criteria

1 Introduction

So-called "hard-kill" methods generate structural topologies by eliminating in each iteration elements having a low value of a "criterion function", such as von Mises stress, compliance, or some other response parameter. Moreover, new elements are inserted around existing elements with a high value of the criterion function.

The above procedure was used in the "Adaptive Biological Growth" method of Mattheck *et al.* (1991). A similar approach termed ESO (Evolutionary Structural Optimization) was introduced later (Xie and Steven 1992) with extensive literature consisting of over 80 research papers, half a dozen doctoral theses and a book (Xie and Steven 1997). Whilst ESO uses only element rejections, its modified version BESO ("Bidirectional" ESO, Querin 1997; Querin *et al.* 1998) also admits new elements.

ESO has the advantage of conceptual simplicity but it is an intuitive method without proof of optimality for given objective function and constraints.

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It is shown in this note on a simple example that (a) ESO's rejection criteria may result in an extremely nonoptimal design and (b) BESO is unable to reverse this failure.

2 Break-down of ESO in a test example: compliance or stress design

Figure 1a shows a simple topology optimization problem in which the total compliance is to be minimized for various volume fractions. There is a fixed support at the left and a roller support at the top. Young's modulus is taken as unity and Poisson's ratio as zero. The element thickness may only be unity or zero. The *single* load condition consists of a horizontal load of intensity 2.0 and a vertical load of intensity 1.0.

An FE model with 100 four-node quadrilateral elements (Fig. 1a) has given the following results.

(a) In the original structure (Fig. 1a), the lowest von Mises stress (0.9983) and compliance (0.9974) are in element "a" at the rollers. The obvious reason for this is the fact that this element of the vertical "tie" receives the least diffusion from the horizontal stresses in the "beam". As expected, the horizontal stress level in the "beam" was found around 2.0 and the vertical stress in the "tie" around 1.0. ESO's rejection criteria for stress or compliance will therefore first take out element "a" from the structure. The total compliance given by the FE model for the original structure in Fig. 1 a is 388.5.

(b) After rejecting element "a", ESO yields the structure in Fig. 1b, for which the total compliance by the FE model is 4371. Note that this is *1125 percent* of the compliance in the prior cycle. This large increase in compliance value is caused by the fact that the vertical load is now transmitted by flexural action.

(c) If we reject element "c" in Fig. 1a, the resulting total compliance becomes 396, which is only marginally higher than the compliance in the prior iteration. In fact, ESO's rejection criterion gives *11 times higher compliance* than this intuitively selected change.

(d) A clearly optimal solution for a volume fraction (V_f) of 40% (weight reduction of 60%!) is shown in Fig. 1c. This gives a compliance value of 1121, which is about *one quarter of the compliance of the almost 2.5 times heavier ESO design* in Fig. 1b.
(e) For higher values of the "rejection ratio" (RR), ESO may remove 2, 3 and 4 elements from the vertical "tie", with similarly large increase in the total compliance value.
(f) Even BESO fails to rectify the highly nonoptimal solution in Fig. 1b because the highest stress value (14.57) occurs in element "*b*" in Fig. 1a. BESO, therefore, would insert an element in that location (see dotted element in Fig. 1b). This would not reduce the large compliance value significantly since the addition of this element will improve only marginally the flexural stiffness of the beam.
(g) The modified design generated by ESO's rejection criteria would also be extremely nonoptimal for stress design owing to very high flexural stresses at the left support (e.g. in element "*b*" the Mises stress increases from 2.01 to 14.57).
(h) The considered type of failure is *not* restricted to relatively coarse meshes, as used in Fig. 1a. If we replaced, for example, each element with $10 \times 10 = 100$ elements (having 10000 elements in all), for certain values of the "rejection ratio" (RR) entire horizontal rows (consisting of 10 elements) or even the entire "vertical tie" at the top would be taken out, having the same effect. But even if in previous iterations 9 elements were to be rejected out of each horizontal row of 10 in the vertical tie, we would get the same effect at the end if we changed the vertical load in Fig. 1a from 1.0 to 0.1. Also note that the demonstrated effect could be made arbitrarily large by increasing the span of the beam.

Note. To enable the reader to check on the foregoing findings without FE calculations, very similar results are obtained in the Appendix in a few lines, by means of simple structural models and analytical calculations.

3 Reasons for the failure of ESO

In the case of compliance design, for example, the rejection criteria of ESO are based on element compliances ($[u_i]^T \cdot [K_i] \cdot [u_i]$), which correspond to sensitivities of the total compliance with respect to the density (here: thickness t_i) of the i-th element (at $t_i = 1$). Failure of the rejection criterion occurs if the sensitivity for the rejected element increases significantly as its normalized density (t_i) varies from 1 to zero. This is demonstrated by the above example, in which the sensitivities (s_a) with respect to the density (t_a) of the element "*a*" were:

$$(\text{for } t_a = 1) \quad s_a = -0.9974 \cong -1\,,$$

$$(\text{for } t_a = 10^{-6}) \quad s_a = -15889000\,. \tag{1}$$

Since the rejection given by ESO results in a change from $t_a = 1$ to $t_a = 0$, the sensitivity value for $t_a = 1$ above gives clearly an absurdly incorrect estimate of the change of the objective function (compliance).

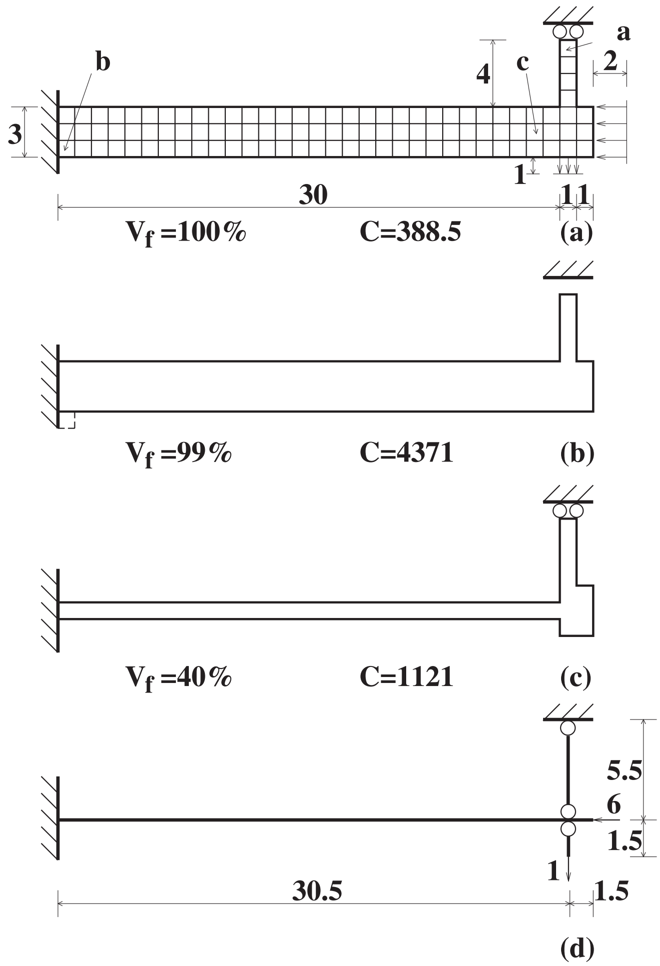

Fig. 1 Test example: (a) start design with FE mesh, (b) topology given by ESO's rejection criterion for 1% volume reduction, (c) optimal solution for 60% volume reduction (for comparison), (d) simple beam-tie model for analytical verification

4 Concluding remarks

1. It was demonstrated on a simple test example that ESO's current rejection criteria for compliance or stress design can produce an *extremely nonoptimal*

topology (with compliance about 1100% of the prior solution, Figs. 1a and b).
2. The present version of BESO (ESO with element admissions) does not rectify this highly unfavourable topology. In fact it would insert additional elements where elements should be removed for an optimal topology (Figs. 1b and c).
3. A complete explanation of these failures was also given.
4. Various shortcomings of ESO type methods and possible improvements will be discussed elsewhere.

References

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Appendix. Verification of the FE results using a simple structural model and analytical calculations

A close approximation of the considered structure can be obtained by considering it as a propped cantilever (Fig. 1d) consisting of a beam and a tie.

A.1 Structure in Fig. 1a

We ignore the diffusion of the horizontal stresses at the beam/tie intersection and the insignificant flexural stresses in the structure in Fig. 1a. Moreover, assuming for simplicity that the vertical stresses spread at an angle of 45° across the "beam", we get the total compliance value of

$$C = 4 + \frac{3}{2.5} + 32 \times 2 \times 2 \times 3 = 389.2\,, \tag{2}$$

where the first term is the compliance of the (vertical) stresses in the tie, the second term the compliance of the vertical stresses in the beam (spread out over an average width of 2.5) and the third term the compliance of the horizontal stresses (or forces). The compliance value in (2) differs only by 0.18% from the corresponding FE result(!)

A.2 Structure in Fig. 1b

Since the vertical tie has been disconnected, the cantilever deflection at the vertical load based on a Bernoulli-beam model becomes:

$$\Delta = \frac{L^3}{3I} = \frac{30.5^3 \times 4}{27} = 4203\,, \tag{3}$$

where L is the effective cantilever length and I the moment of inertia. The total compliance is then

$$C = 4203 + 384 = 4587\,, \tag{4}$$

where the first and second term, respectively, is the compliance of the horizontal and vertical forces. This differs by less than ten percent from the FE result (discretization error due to coarse mesh).

In calculating the horizontal stress in element "b", the moment is $M = 30.5$, the moment of inertia $I = 27/12$, the stress due to the axial load is 2, and the distance of the centre of the element "b" from the neutral axis is 1, giving a horizontal stress

$$\sigma_H = \frac{30.5 \times 12}{27} + 2 = 15.56\,, \tag{5}$$

in element "b". Since the vertical stress is negligible and the shear stress small, the value in (5) is also a good approximation of the von Mises stress in element "b". The 6.3% difference from the FE result is due to discretization error.

A.3 Structure in Fig. 1c

Assuming straight-line and 45° diffusion of the horizontal and vertical stresses, we obtain the total compliance value

$$C = 2 \times 3(6 \times 30 + 3 \times 2) + 1\left(4 + 2 \times \frac{3}{5}\right) = 1121\,. \tag{6}$$

This value agrees exactly with the FE result(!)

A.4 Sensitivity with respect to the thickness of element "a"

For the simplified model in Fig. 1d, the compatibility conditions can be expressed as

$$F\left(\frac{1}{t_a} + 3 + \frac{1.5}{1.5}\right) = (1 - F)\frac{30.5^3 \times 4}{27}\,, \tag{7}$$

where F is the force in the tie and t_a is the thickness of element "a". The factor 1.5 in the denominator allows for the diffusion of the vertical stresses. By relation (7), we have

$$F = \frac{4203}{(1+t_a)+4207}\,, \tag{8}$$

$$C_V = \frac{4203(1+4t_a)}{1+4207t_a} + 1\,, \tag{9}$$

where C_V is the compliance for the vertical load[1], the first term in (9) is the deflection at the centroid of the beam and the second term is the elongation of the "vertical bar" under the beam (Fig. 1d, length 1.5, equivalent width 1.5). The required sensitivity then becomes

$$s_a = \frac{\mathrm{d}C_V}{\mathrm{d}t_a} = \frac{4203 \times 4(1+4207t_a) - 4207 \times 4203(1+4t_a)}{(1+4207t_a)^2}\,. \tag{10}$$

The relation (10) implies

$$(\text{for } t_a = 1) \quad s_a - 0.9976 \cong -1\,, \tag{11}$$

$$(\text{for } t_a = 10^{-6}) \quad s_a = -4203^2 = -17665209\,. \tag{12}$$

The results in (11) and (12), respectively, differ by 0.02% and 10% from the corresponding FE output.

[1] The thickness t_a has a negligible effect on the compliance of the horizontal load