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Robust shape control of beams with load uncertainties by optimally placed piezo actuators

S. Adali*, J.C. Bruch, Jr., I.S. Sadek, and J.M. Sloss

Abstract The shape of a laminated beam is controlled by an optimally placed piezo actuator so as to minimize its maximum deflection. The locations and magnitudes of the external loads are not known a priori and belong to a specified load uncertainty domain. The optimal actuator location is obtained for any load combinations and locations which are determined so as to produce the maximum deflection corresponding to the worst case of loading. In this sense, loading uncertainties lead to an anti-optimization problem which is coupled to the optimization problem via the design parameter and loading. The resulting coupled problems are solved simultaneously to compute the optimal actuator location and the least favourable loading condition. Multiple load cases are also considered. Numerical results are given to assess the effect of load uncertainty and actuator length on the actuator location and the design efficiency which is defined with respect to the corresponding uncontrolled beam.

Key words uncertainty, control, piezoelectric, optimal, robust

1 Introduction

With the advent of smart structures technology, it is now possible to control the shape and vibrations of structural components using a number of different techniques and materials. A widely employed technique is to exercise control using piezoelectric actuators which can be integrated into the structure to provide an actuation force via converse piezo effect. The focus of the present study is the shape control of a laminated beam subject to load uncertainties with the control force provided by a piezo patch actuator the placement of which has to be determined optimally under all load combinations within a given domain.

Load uncertainties occur quite often in practice as the precise locations and magnitudes of loads acting on a structure may not be known a priori. In such cases, the location of the actuator has to be selected on the basis of the worst-case loading in terms of load locations and magnitudes. Even though uncertainty in loading is a fairly common occurance, shape control has been mostly studied for structures under deterministic loads. Haftka and Adelman (1985) proposed the use of thermal actuators which are controlled by heating or cooling. Koconis *et al.* (1994a,b) studied the shape control of plates and shells with embedded actuators for given voltage and desired shape. Experimental and analytical results were given by Austin *et al.* (1994) on the shape control of adaptive wings using internal translational actuators. Nitinol strips were used for the shape control of composite beams by Baz *et al.* (1995). Studies on the deflection control using piezoelectric actuators include Lin *et al.* (1996) for plates, and Drozdov and Kalamkarov (1996); Donthireddy and Chandrashekhara (1996); Bruch *et al.* (1997); Eisenberger and Abramovich (1997), and Kalamkarov and Drozdov (1997) for beams. Placement of actuators in adaptive structures was studied for trusses by Hakim and Fuchs (1996) and Weber *et al.* (1998) where further references on this topic are given. Optimal placement of piezoelectric actuators were studied for plates and shells by Koconis *et al.* (1994a,b), and for beams by Main *et al.* (1994); Drozdov and Kalamkarov (1996); Bruch *et al.* (1997), and Kalamkarov and Drozdov (1997). In the above studies, the loading conditions were taken as deterministic. Closed loop shape control formulation was given by Chandrashekhara and Varadarajan (1997) for composite beams for the case when the external loads are not known precisely. In this case a static feedback control

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approach was used to determine the voltages of actuators, the locations of which are fixed along the beam.

Examples of design optimization of structural elements under uncertain loading conditions were given by Adali (1993); Adali *et al.* (1995) and by Lombardi and Haftka (1998). In these studies the design variables of the problem were ply angles and thicknesses of the composite structures.

Minimization of the maximum deflection of composite beams using an optimally placed piezo actuator is the subject of the present study. The magnitudes and locations of the external loads acting on the beam are not known a priori. As such the applied loads belong to an uncertainty domain which determines the maximum values of loads. The uncertain load domains can be specified in different ways and may be, for example, in the form of a cube or ellipsoid in the load space or in some other geometric form. The case of a cubic domain is specified in the present study. The loads could act at any point on the beam and in this sense the load locations are also uncertain. The uncertainty variables (load magnitudes and locations) are determined so as to produce the maximum deflection and this constitutes the anti-optimization problem of the robust design procedure.

The optimization problem consists of determining the location of the piezo actuator optimally under any load combinations. The optimal value of the design variable (actuator location) and the anti-optimal values of the uncertainty variables depend on each other leading to a nested anti-optimization/optimization problem. The simultaneous solution of this problem leads to a robust actuator design producing the min-max deflection beam under any loadings within the uncertain load domain. The case of multiple uncertain loads is also studied.

Numerical results are given for various load combinations and actuator lengths for simply supported beams and for beams with an extended section. The efficiency of the design is assessed by comparing the deflections of controlled beams with uncontrolled ones. It is observed that the efficiency increases with increasing actuator length. It also depends on the load combinations and the boundary conditions of the beam.

2 Shape control with anti-optimization

A uniform beam of length L is under the action of transverse loads $0 \le P_{in} \le P_{in,\max}$ acting at $\xi_{in} \in [0, L]$ and moments $M_{jn,\min} \le M_{jn} \le M_{jn,\max}$ acting at $\zeta_{jn} \in [0, L]$ where $i = 1, 2, \ldots, I$, $j = 1, 2, \ldots, J$, and $n = 1, 2, \ldots, N$ where $N = 1$ for a beam under a single load and $N \ge 2$ for a beam under multiple loads. The sets defined by

$$U_n = \Big\{ (P_{in}, \xi_{in}, M_{jn}) | \, 0 \le P_{in} \le P_{in,\max}\,, \\ \xi_{in} \in [0, L]\,, M_{jn,\min} \le M_{jn} \le M_{jn,\max} \Big\} \tag{1}$$

are the load uncertainty domains. The locations ξ_{in} of the loads are also unknown and they are to be determined as part of the solution of the anti-optimization problem. The locations ζ_{jn} of concentrated moments M_{jn} are specified as input parameters in the present study not to complicate the problem unduly. However, this restriction can be easily removed and the solution procedure remains the same.

The magnitudes of the loads P_{in} and moments M_{jn} can take any value within the uncertainty domain U_n and the uncertain load locations can take any value in $[0, L]$. The uncertain quantities $P_{in}, \xi_{in}, M_{jn} \in U_n$ are unknown variables of the anti-optimization problem which consists of determining the pair (P_{in}, ξ_{in}) and M_{jn} such that the maximum deflection of the beam is maximized.

Let $w_n(x; \alpha, \beta_n)$ denote the mid-plane deflection of the beam for a loading in U_n where α is the design variable to be defined later and $\beta_n = (P_{in}, \xi_{in}, M_{jn})$ is the set of uncertainty variables. The maximum deflection of the beam is given by

$$\|w_n(\bullet; \alpha, \beta_n)\| = \max_{0 \le x \le L} |w_n(x; \alpha, \beta_n)| \ . \tag{2}$$

The anti-optimization problem can be stated as

determine the uncertainty variables β_n of the n-th loading case such that

$$\|w_n(\bullet; \alpha, \beta_n^*)\| = \max_{\beta_n \in U_n} \|w_n(\bullet; \alpha, \beta_n)\| \ . \tag{3}$$

Thus, the uncertainty variables are determined for each loading case so as to maximize the maximum deflection which constitutes the solution of the anti-optimization problem which is coupled to the optimization problem since α and β_n are interdependent. Thus the solution of problem (3) yields the worst case loading on the beam in terms of load magnitudes and locations in U_n for the given values of the design variable α.

Next the optimization problem and the design variables are defined. A piezo actuator of length a is symmetrically bonded on the top and bottom surfaces of the beam. Its location is defined by b which indicates the distance between the left hand support and the actuator. A voltage V is applied on the piezoactuator the sign of which is given by $\operatorname{sgn}(V) = V/|V|$. The actuator location b and the sign of the voltage $\operatorname{sgn}(V)$ serve as the design variables of the optimization problem so that $\alpha = [b, \operatorname{sgn}(V)]$ if $V \ne 0$ and $\alpha = \alpha_u = b_u$ if $V = 0$ (uncontrolled case). Even though a single actuator is considered in the present study, the results can be easily generalized to multiple actuators. The beam deflection is to be minimized by choosing the location $0 \le b < L - a$ of the actuator and the $\operatorname{sgn}(V)$ optimally. Thus the optimization problem can be stated as

1. Single load case

$$\|w(\bullet; \alpha^*, \beta_1^*)\| = \min_{0 \le b \le L} \|w_1(\bullet; \alpha, \beta_1^*)\| \ . \tag{4}$$

2. Multiple load case with $n = 1, 2, \ldots, N$

$$\|w(\bullet;\alpha,\beta_n^*)\| = \min_{0\le b\le \overline{L}} \max_n \|w_n(\bullet;\alpha,\beta_n^*)\| \ , \tag{5}$$

where $\overline{L} = L - a$. Clearly the optimal value of α depends on the anti-optimal values of P_{in}, ξ_{in} and M_{jn} and vice versa leading to a nested optimization/anti-optimization problem. The solution of this problem gives the best location of the piezo actuator and the sign of the applied voltage so as to minimize the maximum deflection of the beam under the worst possible case of loadings which may include multiple loads. As such it provides a robust optimal piezo actuator design capable of providing min-max deflection under any loading which lies within the given uncertainty space defined as

$$S_u = U_1 \cup U_2 \cup \ldots \cup U_N \, . \tag{6}$$

3
Basic equations of beam/piezo structure

The beam is of rectangular cross-section with height H and width B. The piezo-actuator has width B, length a, and thicknesses h_p, which are activated by the application of out-of-phase electric potentials with a voltage V. The x-coordinate is taken along the midplane of the beam so that $|z| \le H/2$ indicates the beam region and $|z| \ge H/2$ the piezo layers which are denoted by S_b and S_p, respectively, viz.,

$$S_b = \{(x,z) | 0 \le x \le L \, , |z| \le H/2\} \ , \tag{7}$$

$$S_p = \Big\{(x,z) | 0 \le b \le x \le b + a \le L \, ,$$

$$-H/2 - h_p \le z \le -H/2 \, , H/2 \le z \le H/2 + h_p\Big\} \, . \tag{8}$$

Let $M_n(x)$ denote the moment at x under the loading U_n. The equations governing the flexural bending of the beam are given by

$$M_n(x) = D_1 w_n'' \, , \quad \text{for } x \in S_b \, , \tag{9}$$

$$M_n(x) = D_2 w_n'' - M_p \, , \quad \text{for } x \in S_p \, , \tag{10}$$

where D_i are the stiffness constants derived below, M_p is the moment generated by the piezo actuators, and the prime superscript stands for differentiation with respect to x.

Let ε_{nx} denote the axial strain at x and $u_n(x)$ the displacement in the x direction for loading $\beta_n \in U_n$. Then

$$\varepsilon_{nx} = u_n' + z w_n'' \, . \tag{11}$$

The expressions for the stiffnesses D_1 and D_2 will be derived for a laminated beam with the thickness of the k-th layer denoted by t_k with the total number of layers including the piezo layers denoted by K. The stress $\sigma_{nx}^{(k)}$ for the k-th layer is given by

$$\sigma_{nx}^{(k)} = E_x^{(k)} \varepsilon_{nx}^{(k)} - E_x^{(p)} d_{31} \phi_3 \, , \tag{12}$$

where

$$E_x^{(k)} = \Big[E_{11}^{-1} \cos^4\theta_k + \left(G_{12}^{-1} - 2\nu_{12} E_{11}^{-1}\right) \cos^2\theta_k \sin^2\theta_k +$$

$$E_{22}^{-1} \sin^4\theta_k\Big]^{-1} \, , \tag{13}$$

with E_{11}, E_{22}, G_{12} and ν_{12} denoting the elastic constants of the material of the k-th layer and θ_k the ply angle of the k-th layer. In (11), d_{31} is the piezoelectric constant and ϕ_3 is the electric field intensity given by $\phi_3 = V/h_p$. Let h_k denote the signed distance between the mid-plane and top of the k-th layer with $h_0 = H/2 + h_p$ and $h_K = -H/2 - h_p$. The stress resultant N_{nx} in the x-direction can be computed from

$$N_{nx} = B \sum_{k=k_0}^{K_0} \int_{h_{k-1}}^{h_k} \sigma_{nx}^{(k)} \, \mathrm{d}z \, , \tag{14}$$

using (11) and (12) where $K_0 = K - 1$, $k_0 = 2$ for $x \in S_b$ and $K_0 = K$, $k_0 = 1$ for $x \in S_p$. This computation gives

$$N_{nx} = A_x u_n' + B_x w_n'' \, , \quad \text{for } x \in S_b \, ,$$

$$N_{nx} = A_x u_n' + B_x w_n'' - N_x^{(p)} \, , \quad \text{for } x \in S_p \, , \tag{15}$$

where

$$(A_x, B_x) =$$

$$B \sum_{k=k_0}^{K_0} E_x^{(k)} \left[h_k - h_{k-1}, \frac{1}{2}\left(h_k^2 - h_{k-1}^2\right)\right] \, , \tag{16}$$

which gives different A_x and B_x values for $x \in S_b$ and for $x \in S_p$ and

$$N_x^{(p)} = B \int_{S_p} E_x^{(p)} d_{31} \phi_3 \, \mathrm{d}z \, . \tag{17}$$

Similarly, the moment resultant M_{nx} is given by

$$M_{nx} = B \sum_{k=k_0}^{K_0} \int_{h_{k-1}}^{h_k} \sigma_{nx}^{(k)} z \, \mathrm{d}z = B_x u_n' + D_x w_n'' - M_x^{(p)} \, , \tag{18}$$

so that

$$M_{nx} = B_x u_n' + D_x w_n'' \, , \quad \text{for } x \in S_b \, ,$$

$$M_{nx} = B_x u_n' + D_x w_n'' - M_x^{(p)} \, , \quad \text{for } x \in S_p \, , \tag{19}$$

where

$$D_x = \frac{1}{3} B \sum_{k=k_0}^{K_0} E_x^{(k)} \left(h_k^3 - h_{k-1}^3\right) , \tag{20}$$

$$M_x^{(p)} = B \int\limits_{S_p} E_x^{(p)} d_{31} \phi_3 z \, \mathrm{d}z . \tag{21}$$

Since there is no force in the axial direction, $N_{nx} = 0$ and from (15) it follows that

$$u_n' = -\frac{B_x}{A_x} w_n'' + \frac{N_x^{(p)}}{A_x} , \quad \text{for } 0 \le x \le L , \tag{22}$$

where $N_x^{(p)} = 0$ for $x \in S_b$. Substituting u_n' from (22) into the moment expressions (19), we obtain

$$M_{nx} = \left(D_x - \frac{B_x^2}{A_x}\right) w_n'' + \frac{B_x}{A_x} N_x^{(p)} - M_x^{(p)} ,$$

$$0 \le x \le L , \tag{23}$$

where $N_x^{(p)} = M_x^{(p)} = 0$ for $x \in S_b$. Comparing (9), (10), and (23), we obtain

$$D_1 = D_x - \frac{B_x^2}{A_x} , \quad x \notin S_p ,$$

$$D_2 = D_x - \frac{B_x^2}{A_x} , \quad x \in S_p \cup S_p ,$$

$$M_p = M_x^{(p)} - \frac{B_x}{A_x} N_x^{(p)} , \quad x \notin S_b . \tag{24}$$

4
Method of solution

Equations (9) and (10) must be solved for a given loading and actuator location. These equations are solved by finite differences by dividing each region R_i of the beam into n_i intervals. Thus $R_1 = [0, b]$, $R_2 = [b, b+a]$ and $R_3 = [b+a, L]$, and $\Delta x_1 = b/n_1$, $\Delta x_2 = a/n_2$, $\Delta x_3 = (L-a-b)/n_3$ where Δx_i is the interval length in region R_i. The finite difference expressions for each interval are given by

$$\delta^2 w_{nj} = M_{nx}(x_j)/D_1 , \quad \text{for } x \in S_b , \tag{25}$$

$$\delta^2 w_{nj} = \left[M_{nx}(x_j) + M_p\right]/D_2 , \quad \text{for } x \in S_p , \tag{26}$$

where $\delta^2 w_j = (w_{j+1} - 2w_j + w_{j-1})/\Delta x_i^2$, $j = 1, 2, \dots, n_1$ and $i = 1$ for $x \in R_1$, $j = n_1 + 1, \dots, n_1 + n_2$, $i = 2$ for $x \in R_2$, etc. At the end points of the piezo actuator ($x = b$ and $b + a$) an average stiffness given by $D_a = (D_1 + D_2)/2$ is taken. The above formulation leads to a system of linear equations, the solutions of which give the deflection w_{nj} at node j for the n-th loading $\beta_n \in U_n$.

The maximum deflection computed by the finite difference method discussed above gives the objective of the design problem. In order to minimize the maximum deflection under load uncertainties, the anti-optimization problem (3) and the optimization problem (4) for a single load case and (5) for multiple load case have to be solved. As these problems are coupled via external loads and actuator locations, they have to be solved simultaneously. Thus for a given value of the design parameter, the n-th anti-optimization problem is solved to obtain the max-max deflection with $n = 1, 2, \dots, N$. The design parameter is optimized using a one-dimensional optimization algorithm and in the optimization procedure the usual analysis phase corresponds to the solution of the anti-optimization problems. The optimization procedure gives the optimal location of the actuator and the min-max-max value of the deflection corresponding to the worst case loading. In the multiple load case, the min-max-max value of the deflection is valid for all load cases.

5
Numerical results and discussion

In the following, b_{opt} denotes the optimal b, i.e. the optimal distance between the left hand support and the left end point of the piezo actuator. The effectiveness of piezo control can be studied by defining an efficiency index which gives the percent decrease in the uncontrolled deflection as compared to the controlled one, viz.

$$I_f = \left[\frac{w_{\max}(\alpha_u; 0)}{w_{\max}(\alpha_{\text{opt}}; V)} - 1\right] \times 100 , \tag{27}$$

where $\alpha_u = b_u$ which is taken as $b_u = (L-a)/2$, i.e. the actuator is located centrally and $w_{\max}(\alpha_u; 0) = \max_n \|w_n(\bullet; \alpha_u \beta_n^*)\|$ with $V = 0$ is the maximum deflection of the uncontrolled beam under all $\beta_n \in U_n$. For the controlled beam $w_{\max}(\alpha_{\text{opt}}; V) = \max_n \|w_n(\bullet; \alpha_{\text{opt}} \beta_n^*)\|$ with $V > 0$ and $\alpha_{\text{opt}} = (b_{\text{opt}},\ \text{sgn}\ (V)_{\text{opt}})$. Thus the comparison is made with a passive beam where the piezo actuator contributes only to the stiffness of the beam.

The results are presented in dimensionless form by using the length L of the beam to nondimensionalize various quantities,

$$w_n = \frac{w_d}{L} , \quad a_n = \frac{a_d}{L} , \quad b_n = \frac{b_d}{L} , \quad \xi_n = \frac{\xi_d}{L} , \tag{28}$$

where the subscripts n and d denote nondimensional and dimensional quantities, respectively. In the rest of the discussion, the subscript n is dropped from the notation. The results are given for a composite beam made of graphite-epoxy (T300/5280) for which the elastic constants are $E_{11} = 181.0$ GPa, $E_{22} = 10.30$ GPa, $G_{12} = 7.17$ GPa, $\nu_{12} = 0.28$.

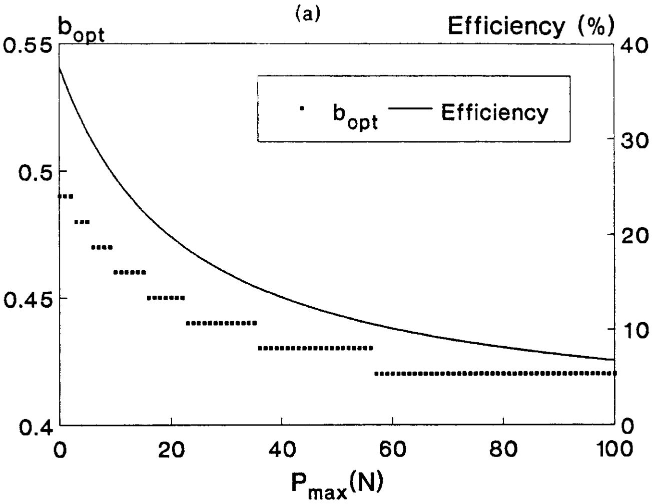
The piezo-actuators are made of PZT-5H piezoceramic material with the properties $E_p = 62$ GPa and $d_{31} = -274 \times 10^{-12}$ m/V (see Morgan Matroc, Inc. 1993). In all of the calculations the beam length is taken as $L = 1$ m, the thickness as $H = 5$ mm, the width as $B = 10$ mm, the actuator thicknesses as $h_p = 1$ mm, and the beam stacking sequence is chosen as $(0°/90°/90°/0°)$. Thus the stacking sequence is symmetrical and is given by $(\mathrm{PZT}/0°/90°/90°/0°/\mathrm{PZT})$ for $x \in S_p$. The allowable voltage for piezoceramic materials is around 500–1000 volts/mm of piezo thickness (Morgan Matroc, Inc. 1993). In the present study, the voltage is taken as $V = 500$ volts in all cases.

In the finite difference scheme $\Delta x = 0.01L$ in all regions dividing the beam into 100 intervals. The distances b_{opt} are computed as multiples of Δx. Thus the optimal locations are obtained up to an accuracy of 0.01 and these results are indicated as dots in the figures with jumps of 0.01 in the value of b_{opt} when the loading changes.

5.1 Single load cases

Figure 1 shows the curves of b_{opt} and I_f versus the maximum load $P_{\max}$ for a simply supported beam under the actions of a concentated load $0 \le P \le P_{\max}$ acting at $x = \xi_a$ and moment -10 Nm $\le M_r \le 10$ Nm applied at $x = L$. In this case the anti-optimal loading is given by $P = P_{\max}$ and $M_r = 10$ Nm with the anti-optimal load location ξ_a depending on the length of the actuator and the magnitude of $P_{\max}$ where $\operatorname{sgn}(V)_{\mathrm{opt}} = +1$. The results are given for $a = 0.2$ (Fig. 1a) and $a = 0.4$ (Fig. 1b). At low values of $P_{\max}$, the actuator is closer to the right hand support as the design is dominated by the moment loading. As $P_{\max}$ increases, the optimal actuator location moves left, i.e. b_{opt} decreases. In the limiting case of $P_{\max} \to \infty$, $\xi_a \to 0.5$ and $b_{\mathrm{opt}} \to (L - a)/2$. Efficiency of the design is highest for $P_{\max} = 0$ and decreases as $P_{\max}$ increases. Moreover, higher actuator length leads to higher design efficiency since $8\% \le I_f \le 38\%$ for $a = 0.2$ and $18\% \le I_f \le 116\%$ for $a = 0.4$.

Figure 2 shows the same curves as in Fig. 1 with $0 \le P \le P_{\max}$ and -20 Nm $\le M_r \le 10$ Nm for a simply supported beam. For this case the anti-optimal loading is given by $P = 0$ and $M_r = -20$ Nm for low values of $P_{\max}$ with the optimal $\operatorname{sgn}(V) = -1$ and by $P = P_{\max}$ and $M_r = +10$ Nm for high values of $P_{\max}$ with optimal $\operatorname{sgn}(V) = +1$. At the intermediate values of $P_{\max}$, both loadings produce the same deflection. Thus for low values of $P_{\max}$, the loading $M_r = -20$ Nm dominates the design and the actuator is closer to the right-hand support. It is noted that in both cases of loadings ($|M_r| \le 10$ Nm, Fig. 1 and the present one), the optimal actuator location is sensitive to $P_{\max}$ in certain ranges and less sensitive in other ranges of $P_{\max}$. The design efficiency is not as high as it was in the previous case due to M_r having a larger range. However, the trend as $P_{\max}$ increases remains the same.



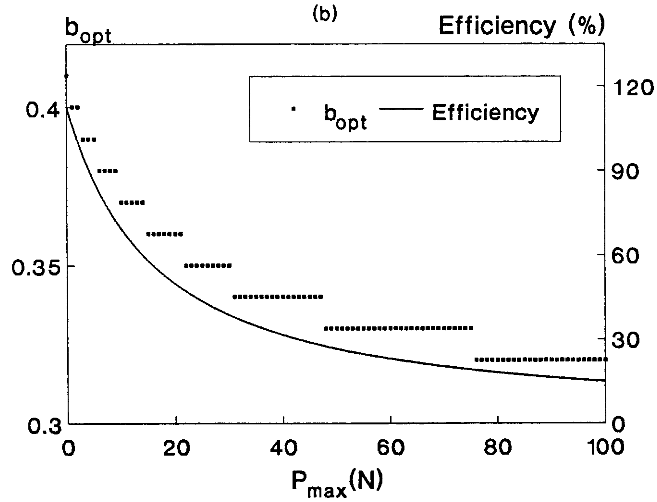


Fig. 1 Optimal actuator location and efficiency index vs $P_{\max}$ for load case $0 \le P \le P_{\max}$, $|M_r| \le 10$ Nm with (a) $a = 0.2$, (b) $a = 0.4$

Next a beam with a hanging section is studied. The beam is simply supported at the left-hand support and the other hinge support is located at $x = 0.8L$. Figure 3 shows the b_{opt} and I_f plotted against $P_{\max}$ with $0 \le P \le P_{\max}$ acting at $x = \xi_a$ with a moment $|M_\ell| \le 10$ Nm applied at the left hand support $x = 0$. In this case the anti-optimal loadings are given by $M_\ell = +10$ Nm and $P = P_{\max}$ $(0 \le \xi_a \le 0.8L)$ for low values of $P_{\max}$ [$\operatorname{sgn}(V)_{\mathrm{opt}} = +1$] and by $M_\ell = -10$ Nm and $P = P_{\max}$ at $\xi_a = L$ for higher values of $P_{\max}$ [$\operatorname{sgn}(V)_{\mathrm{opt}} = -1$]. The results are given for $a = 0.2$ (Fig. 3a) and for $a = 0.4$ (Fig. 3b). For low values of $P_{\max}$, the actuator is closer to the left-hand support as the moment loading dominates the design. As $P_{\max}$ increases, the actuator approaches the right support at $x = 0.8L$. The efficiency drops substantially as $P_{\max}$ increases, but the decrease tapers off after a certain $P_{\max}$. For $a = 0.2$, $13\% \le I_f \le 51\%$, and for $a = 0.4$, $31\% \le I_f \le 180\%$ indicating that the longer actautor leads to higher design efficiency.

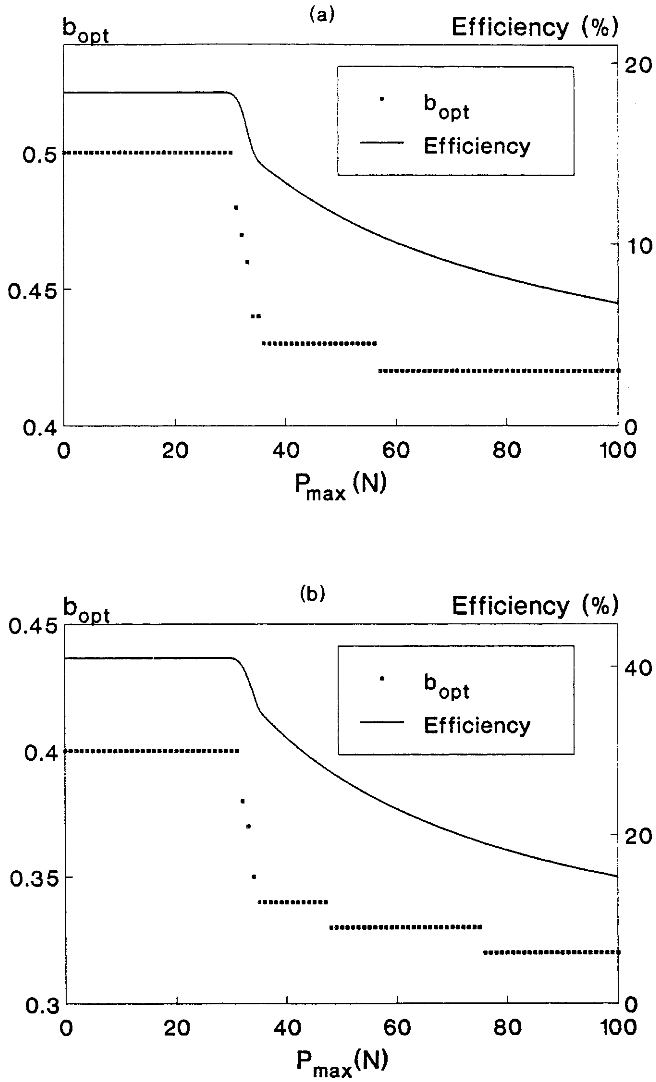


Fig. 2 Optimal actuator location and efficiency index vs $P_{\max}$ for load case $0 \le P \le P_{\max}$, $-20N_m \le M_r \le 10$ Nm with (a) $a = 0.2$ and (b) $a = 0.4$

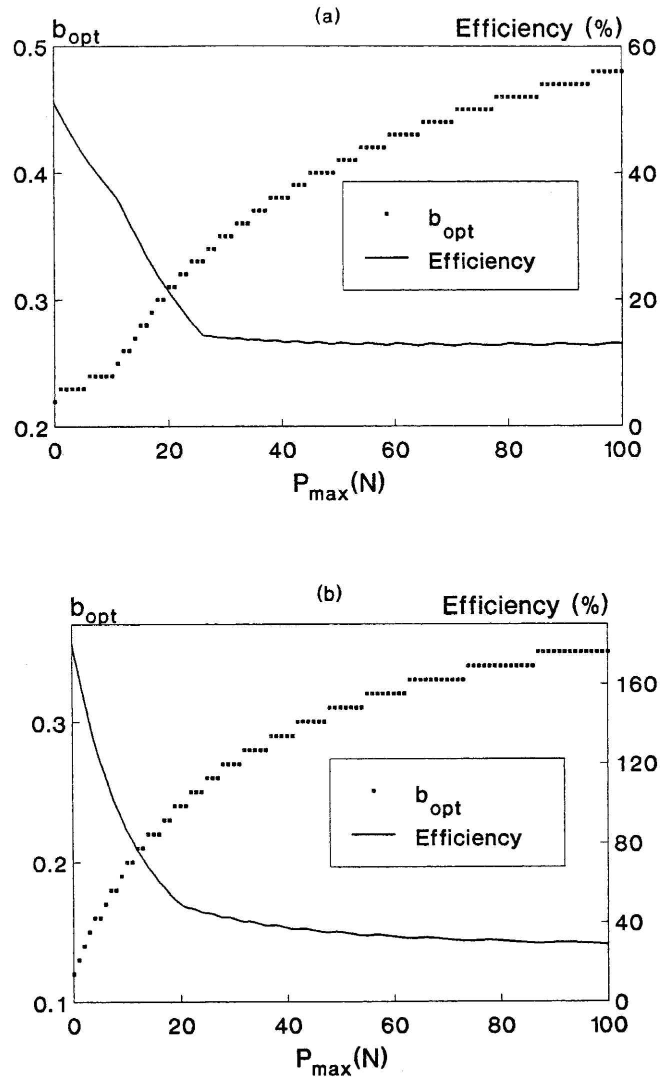


Fig. 3 Optimal actuator location and efficiency index vs $P_{\max}$ for a beam with a hanging section with $0 \le P \le P_{\max}$, $|M_\ell| \le 10$ Nm, (a) $a = 0.2$ and (b) $a = 0.4$

5.2 Multiple load cases

In the multiple load cases, the beam is subjected to different combinations of uncertain loads. As a first example, we consider a simply supported beam subjected to the following load cases denoted by LC.

LC1: Concentrated load $0 \le P \le P_{\max}$ acting at $0 \le \xi \le L$ with $\xi_a = L/2$.
LC2: Moment $|M_r| \le 20$ Nm acting at $x = L$.

Figure 4 shows b_{opt} and I_f curves plotted against $P_{\max}$ for $a = 0.2$ (Fig. 4a) and 0.4 (Fig. 4b). For $P_{\max} \le 59$ N, b_{opt} is determined by LC2 and for $P_{\max} \ge 69$ N by LC1 in which case $b_{\text{opt}} = (L - a)/2$. There exists a transition region for 59 N$\le P_{\max} \le 69$ N where both load conditions have to be taken into account in determining the optimum actuator location.

As a final example, the case of two concentrated loads with a given distance between them acting on a simply supported beam is specified as one of the load cases. Thus LC1 is given by $0 \le P_1 = P_2 \le P_{\max}$ acting at $x_1 = \xi_a$ and $x_2 = \xi_a + d$ where d is the given distance between the two loads. LC2 corresponds to $|M_r| \le 20$ Nm at $x = L$. The results are given in Fig. 5 for $a = 0.2$ (Fig. 5a) and 0.4 (Fig. 5b) which indicate a similar pattern as the previous multiple load case (Fig. 4) with each loading case determining b_{opt} at certain ranges of $P_{\max}$.

6 Conclusions

The problem of locating a piezo actuator optimally on a beam under uncertain loads was studied. The locations and magnitudes of the loads are not known a priori and have to be determined to produce the least favorable load-

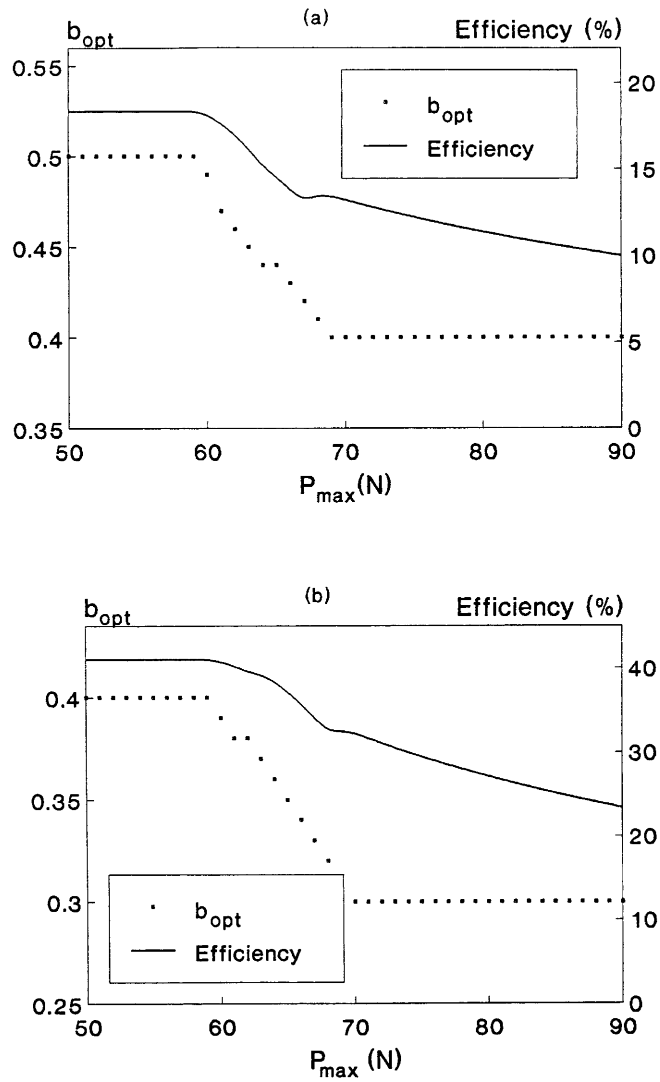


Fig. 4 Optimal actuator location and efficiency index vs $P_{\max}$ for the multiple load case (LC1: $0 \leq P \leq P_{\max}$, LC2: $|M_r| \leq 20$ Nm) with (a) $a = 0.2$ and (b) $a = 0.4$

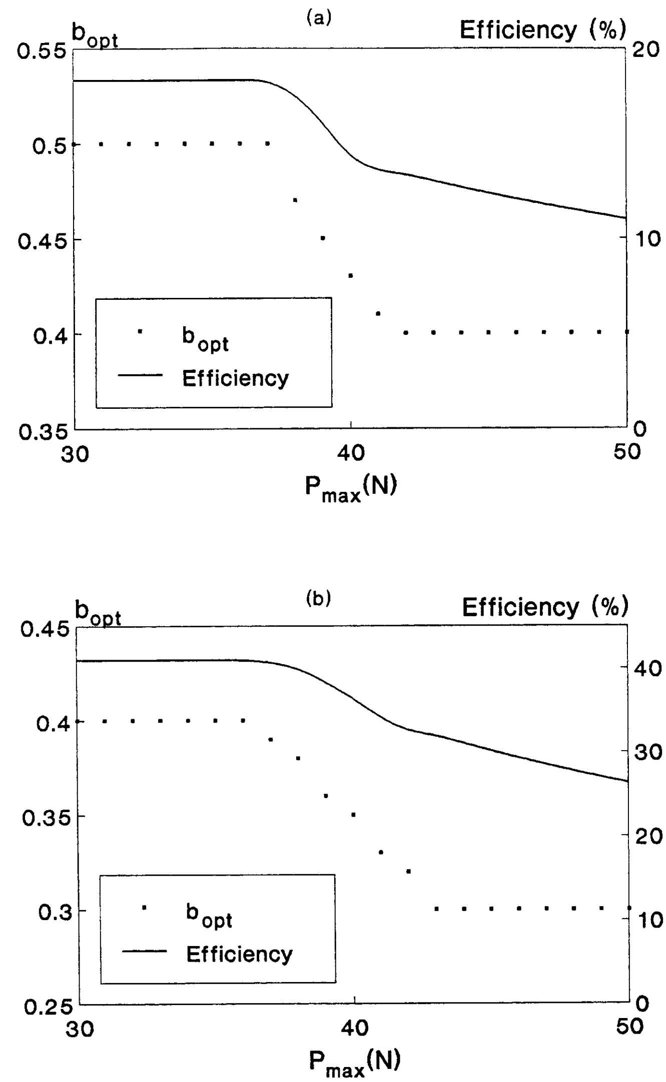


Fig. 5 Optimal actuator location and efficiency index vs $P_{\max}$ for the multiple load case (LC1: $0 \leq P_1 = P_2 \leq P_{\max}$, LC2: $|M_r| \leq 20$ Nm) with (a) $a = 0.2$ and (b) $a = 0.4$

ing condition with respect to the maximum deflection. The objective of the design is the minimization of the maximum deflection under any load combination and this is achieved by controlling the deflection using a piezo actuator of a given length.

The loads acting on the beam are defined in an uncertainty domain and maximizing the deflection over the loads within this domain constitutes the anti-optimization problem. The optimization and anti-optimization problems are coupled through the design variable (actuator location) and the loading. The solution procedure requires the solution of the anti-optimization problem for a given value of the design variable which is optimized iteratively to obtain the optimal actuator location under least favourable loading.

Results were given for beams with different boundary conditions and uncertainty domains with single and multiple load cases. The effectiveness of the piezo control is assessed by comparing the maximum deflections of controlled and uncontrolled beams. It is observed that the design efficiency depends largely on the uncertainty domain, i.e. the load configuration and the maximum values of the load magnitudes have considerable effect on design efficiencies which could be more than 100% depending on the actuator length and loading space. It was observed that the optimal location of the actuator was confined to a relatively small range for the load cases studied in the numerical examples.

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