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Optimization of acoustic response — a numerical and experimental comparison

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Abstract Acoustic optimization within structural dynamics involves automatic changes of structural design variables such as geometric dimensions, shell thickness, material parameters, fiber density and orientation angles, and others to obtain minimum noise or a specified sound quality in specified regions inside or outside the structure. The objective of the present paper is to compare numerical optimization results with experimental ones. The analysed structure is geometrically simple; a closed cylinder. The objective function is the sound intensity at specified points outside the structure. The variable used is the shell thickness. The structural dynamic behaviour is analysed with the finite element method and the acoustic analysis is performed with the boundary element method.

1 Introduction

To reduce the development time and cost and to make simulations for new products possible, it is today necessary to use a number of numerical tools in the design process. Computer aided engineering, CAE has become a natural part of the design process which uses numerical and computer based tools/methods. Natural tools/methods in CAE are, among others, CAD (computed aided design), CAM (computed aided manufacturing), and FEM (finite element method). The use of modern optimization tools in conjunction with the above has increased in recent years. A very simple and straightforward problem formulation for optimization together with stress analysis could be: minimize the structural weights such that the maximum stresses in the structure do not exceed some given value. Or the other way around: minimize the maximum stress in the structure such that the weight does not exceed some given value. The present paper deals with optimization in relation to acoustic response. The necessity to determine and control the sound field generated by vibrating mechanical structures has gained in importance. As machines, especially in the vehicle industry, become lighter in weight, i.e. as the ratio between stiffness and weight increases, the problem with noise generation increases. Low noise has also become a competitive factor for products on the market. By optimization of acoustic response, that is, the optimization of the sound field generated by vibrating structures, a minimization of noise is usually meant. However, not all vibrating structures produce noise; a musical instrument, for example, produces a sound field called music and a great effort is therefore made to optimize the sound, i.e. to get the right sound out of the structure. Also in industrial machines it is sometimes desirable to obtain a certain quality of the sound produced, for example a Ferrari should sound like a Ferrari and not like a sewing machine. A typical formulation of a problem involving acoustic response could be: minimize the sound intensity in a certain domain in the surroundings of the vibrating structure such that the structural weight does not exceed some given value. If we also apply stress analysis to the problem the resultant multidisciplinary optimization task could be formulated as: minimize the structural weight such that the sound intensity in certain domains and the maximum stress in the structure do not exceed some given values. A more thorough discussion of the formulation of optimization problems involving acoustic response is given by Christensen *et al.* (1998a,b). In order to calculate acoustic-related quantities, such as sound intensity in an open or closed domain, a code based on the boundary element method (BEM) has been developed (see Tinnsten 1994; Tinnsten *et al.* 1998). The BEM code is linked together with a modified version of the finite element code FEMP (see Nilsson and Oldenburg 1983) and the optimization code MMA (see Svanberg 1987, 1993) to achieve an acoustic optimization program

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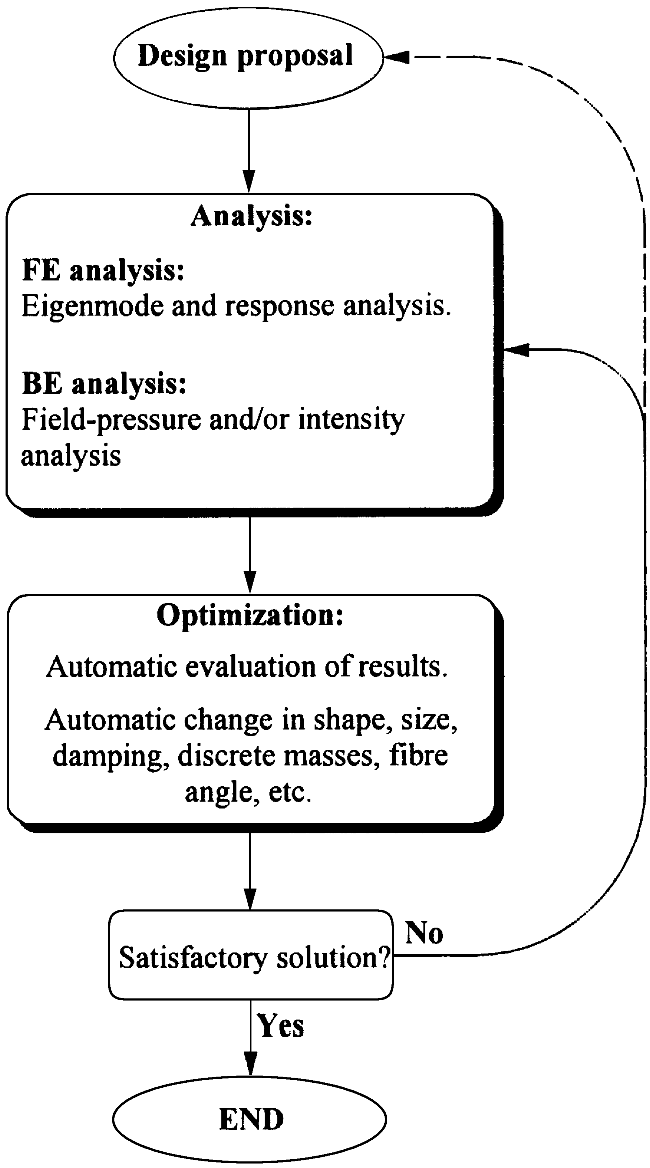

Fig. 1 The acoustic optimization process

which involves structural design changes in an automatic fashion. The idea of this process is shown schematically in Fig. 1.

MMA has been used with great success for a variety of problems (see Esping 1995). There are many optional ways with which to change the sound field emanating from a vibrating structure. Discrete masses, with respect to weight and location, can equally well be used as optimization variables (Constants *et al.* 1998; St. Pierre and Koopmann 1995; Christensen *et al.* 1998a,b). In shell structures the thickness can serve as variable (Belegundu *et al.* 1994; Lamancusa and Eschenauer 1994). Optimization involving fiber-reinforced material offers many possible choices of variables. The fiber direction can be fixed and the variable can be the volume fraction between fiber and matrix (Lamancusa and Eschenauer 1994). Or the volume fraction can be constant and the fiber direction chosen as variable (Tinnsten *et al.* 1998).

The BEM code for acoustic calculations has, when compared with analytical and experimental results, proven to be satisfactory (see Tinnsten *et al.* 1998). The process of acoustic optimization has been implemented and tested in an earlier study (see Tinnsten *et al.* 1998). The test cases there in had the geometry of a rectangular box with the dimensions: length $= 0.4$ m, width $= 0.2$ m, and height $= 0.1$ m, according to Fig. 2.

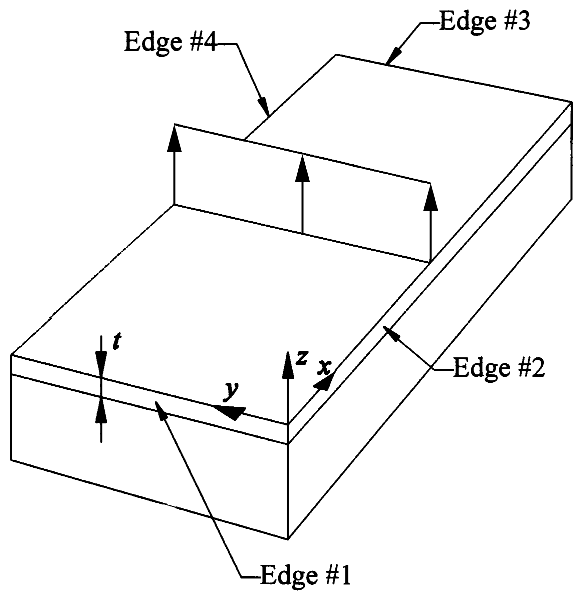

Fig. 2 Test structure for earlier study

The material in the structure was orthotropic with Young's modulus: $E_1 = 14554$ and $E_2 = 1022$ MPa (white spruce). The excitation forces were perpendicular to the top surface ($z = 0$), harmonic and in phase with each other, and applied from the middle of edge 2 to the middle of edge 4 according to Fig. 2. In the response analysis of the top surface (the only moving part in the model) edges 1 and 3 were simply supported and edges 2 and 4 were free. This simulates a very soft connection of edges 2 and 4 to the walls on the model and that the walls are very weak in bending. The optimization problem was formulated as: minimize the sound intensity such that the structural weight does not exceed a given value. The objective function (sound intensity) was calculated 0.1 m directly above the centre of the box ($x, y, z = 0.2, 0.1, 0.1$ m). The variables used were plate thickness at nodes, and the fiber orientation angle, of the top surface. The four different analyses performed gave satisfactory results. In one of the analyses, the frequency of the excitation forces was constant and below the lowest eigenfrequency and the variables were the plate thickness at nodes on the top surface. The fiber orientation was fixed so that E_1 coincided with the x-axis. The initial thickness was $t = 6$ mm and the lower and upper limit for the thickness was $5 \leq t \leq 15$ mm. The analysis converged rather quickly, according to Fig. 3.

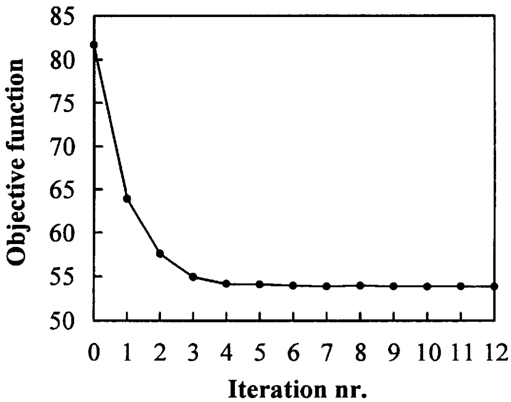

Fig. 3 Iteration history. Sound intensity in [dB] (objective function) versus no. of iterations

The objective function decreased from 81.7 dB to 53.8 dB at optimum. The structural weight increased by 80%, reaching its upper limit, and the plate stiffness increased by approximately 265%. The plate thickness at optimum is shown schematically in Fig. 4.

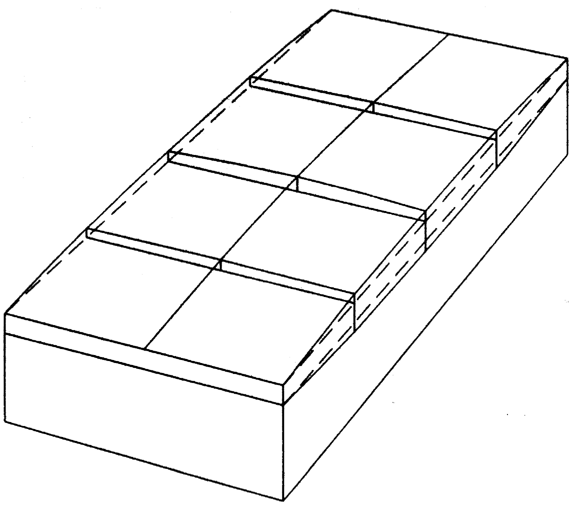

Fig. 4 Plate thickness at optimum

It seems reasonable that the analyses try to push the eigenfrequencies as far as possible from the frequency of the excitation forces. The plate thickness at optimum indicates that the allowed structural weight is distributed in such a way that it affects the intensity calculation at most. These analyses were not compared with analytical results or experimental measurements. The present investigation is mainly concerned with the comparison of the results from numerical acoustic optimization and from experimental measurements.

2 Problem definition

Earlier results, discussed above, indicated that the idea of the automatic acoustic optimization process seems to hold. In order to, as far as possible, eliminate problems associated with geometrical complexity, the chosen geometry for the comparison between numerical and experimental results is simple. The optimization analysis is performed on a structure that has the form of a cylinder with top and bottom plates as in Fig. 5.

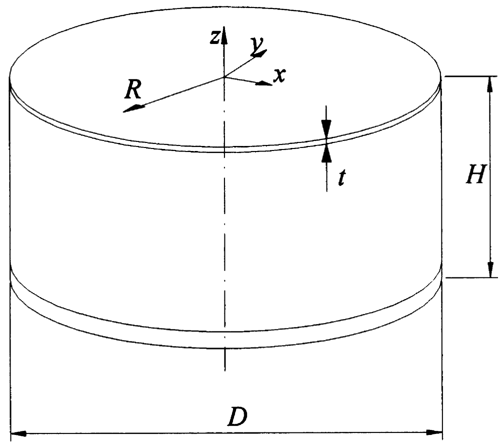

Fig. 5 Structure used in the numerical calculations

For the numerical calculations a cylindrical structure according to Fig. 5 is used, with diameter $D = 200$ mm and height $H = 100$ mm. The initial thickness t is constant over the surface. The structure consists of two materials, steel and aluminum; the cylindrical wall is of steel, the top and bottom plates of aluminum with Young's modulus $E = 70$ GPa, Poisson's ratio $\nu = 0.3$, and density $\rho = 2750\,\mathrm{kg/m^3}$. The cylindrical wall and the bottom plate are much stiffer than the top plate so they are modelled as rigid in the numerical analysis. The top plate is excited with a harmonic force applied perpendicular to the surface at its centre. The optimization problem is formulated as

$$\min_{x} I(x)\,,$$

such that

$$w(x) \leq \overline{w}\,, \quad \underline{x}_j \leq x_j \leq \overline{x}_j\,; \quad j = 1, j\,.$$

That is, minimize the sound intensity $I(x)$ perpendicular to the top surface in such a way that the structural weight $w(x)$ does not exceed the upper limit $\overline{w}$, where x_j are the design variables with lower limit $\underline{x}_j$ and upper limit $\overline{x}_j$. The optimization process is carried out in two different cases, one where the top plate edge is free (Case 1) and one (Case 2) where the top plate edge is clamped. The objective function, the sound intesity, is in each case computed in one point above the top plate. In Case 1 the intensity is computed in the point (35, 35, 100 mm) and in Case 2 in the point (7, 7, 100 mm). The structure is discretized in a symmetric manner with constant triangular element. The discretization of the top plate is shown in Fig. 6.

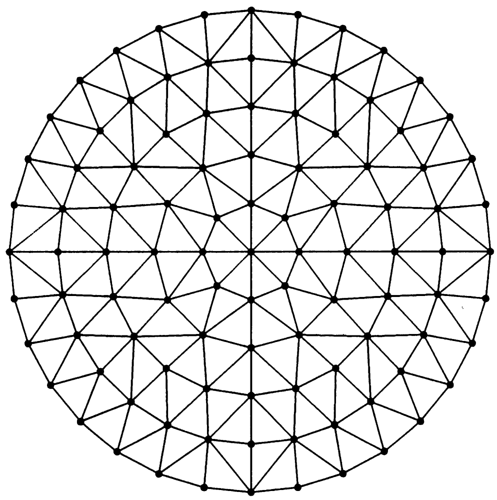

Fig. 6 Discretization of top plate

As can be seen in Fig. 6 the nodes are collected at six different radii on the top plate. These radii are: $R = 0, 20, 40, 60, 80$, and 100 mm. For manufacturing purposes the thicknesses are held constant in the circumstantial direction. The design variables are the thicknesses at the different radius, i.e. variable one is the thickness at the centre, variable two at radius $R = 20$ mm, variable three at radius $R = 40$ mm, and so on. This means that the optimization problem has six variables ($J = 6$). Figure 7 shows an axisymmetric picture of the top plate together with the design variable $x_1 - x_6$.

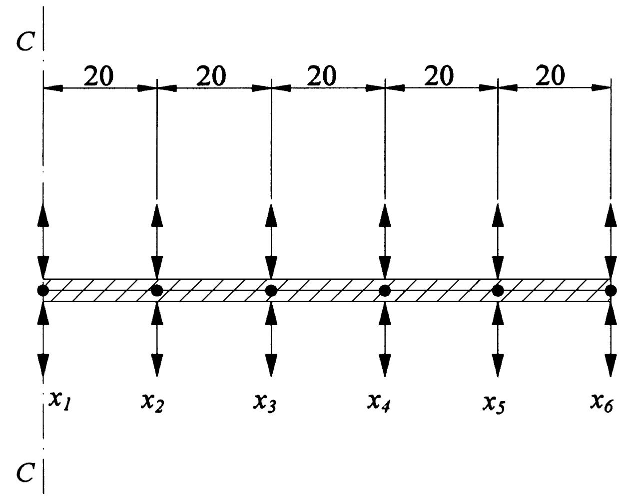

Fig. 7 Axisymmetric picture of top plate with design variables. Dimensions in [mm]

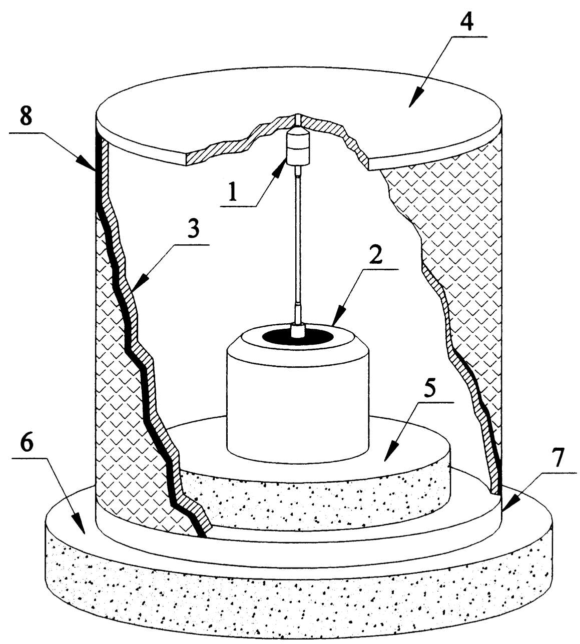

Fig. 8 Experimental setup. Pos. 1: impedance head. Pos. 2: vibrator. Pos. 3: steel cylinder. Pos. 4: aluminum top plate. Pos. 5: foam rubber. Pos. 6: soft insulating mat. Pos. 7: thick aluminum bottom plate. Pos. 8: heavy coach work mat

3
Experimental setup

For comparison with the numerical calculations experiments on the structure were performed. The experimental setup is sketched in Fig. 8.

The experimental setup consisted of: Pos. 1: impedance head B&K type 8001 measuring force and acceleration; Pos. 2: vibrator model 200 (Ling Dynamic System); Pos. 3: steel cylinder with outer diameter 204 mm, height 350 mm, and thickness of material 9.5 mm; Pos. 4: aluminum top plate, with the initial thickness of 3 mm, and optimum thickness according to Cases 1 and 2 in Table 1 and Fig. 12; Pos. 5: foam rubber with thickness of 70 mm; Pos. 6: soft insulating mat, thickness approximately 50 mm; Pos. 7: aluminum bottom plate, thickness 10 mm; and Pos. 8: heavy coach work mat, thickness 4 mm, glued to the steel cylinder. The remaining empty space in the steel cylinder was filled with a lightweight, woolly damping material. The impedance head was calibrated with B&K 4291. The force and acceleration signals were amplified with B&K 2635 before entering the dual channel FFT analyzer B&K 2032. The intensity was measured perpendicular to the top plate at specified points. The intensity probe used was B&K 3519, calibrated with B&K 3541. The signal from the intensity probe was amplified with B&K 2804 before entering the 16-channel measurement system (LMS CADA-X with a HP Paragon front end).

Table 1 Objective and variable values

Values on objective function and variables				
	Case 1 (free)		Case 2 (clamped)	
Var./ obj. fun.	initial	optimal	initial	optimal
x_1 [mm]	3.0	2.0	3.0	2.0
x_2 [mm]	3.0	2.0	3.0	10.0
x_3 [mm]	3.0	2.0	3.0	5.3
x_4 [mm]	3.0	6.1	3.0	2.0
x_5 [mm]	3.0	2.4	3.0	2.0
x_6 [mm]	3.0	2.9	3.0	2.0
I [dB]	83.6	79.6	109.7	95.0
ΔI [dB]	4.0		14.7	

Two different cases were measured; Case 1 where the top plate edge (4. in Fig. 8) was free in the numerical analysis and Case 2 where the top plate edge was clamped in the numerical analysis. For both cases two measurements were performed with the same boundary condition: one with initial thickness (3 mm) and one with optimal thickness of the top plate, according to the numerical results in Table 1. In all measurements the impedance head was joined to the centre of the top plate mechanically (screwed). In order to simulate the free case (Case 1), the top plate was placed on three short, thin ($\oslash = 0.6$ mm). wires which were attached to the end face on the top of the steel cylinder with a soft adhesive polymeric paste (Plastic Padding art. no. 415) in an axisymmetric manner according to Fig. 9. To avoid sound leakage from the interior of the cylinder an elastic rubber tape (vulcanization tape) was affixed over the gap between the top plate and the steel cylinder (see Fig. 9). To simulate the clamped case (Case 2), the edge of the top plate was glued with epoxy cement to the end face on the top of the steel cylinder.

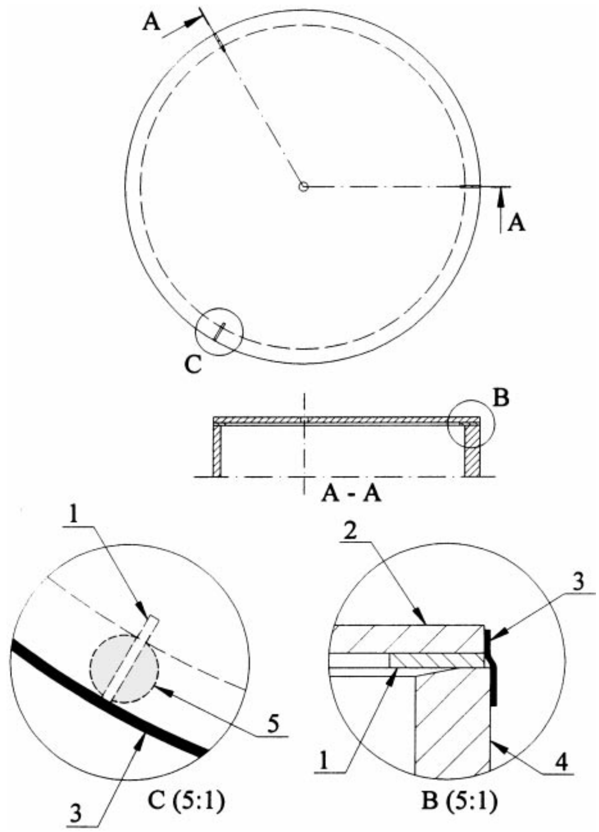

Fig. 9 Experimental setup for Case 1 (free). Pos. 1: wire of diameter 0.6 mm. Pos. 2: top plate. Pos. 3: soft rubber tape. Pos. 4: steel cylinder. Pos. 5: soft adhesive polymeric paste

4 Result

4.1 Numerical results

The optimization process was performed for two cases. In both cases the cylindrical wall and the bottom plate were modelled as rigid, i.e. the only moving part in the model was the top plate. Due to the discretization, illustrated in Fig. 6, the model of the top plate is not axisymmetric. The normal velocity for the BEM element at the top plate was determined by the response analysis in the FEM code, for all other elements in the model the normal velocity was given as zero. In both cases the excitation force was harmonic, perpendicular to the top plate with the amplitude 2.0 N, and applied at the centre of the surface. In the first case the edge of the top plate was free in the response analysis and in the second case the edge was clamped. Proportional damping is included in the analysis as $[C] = \alpha \times [K]$. In Case 1 the frequency of the exciting force was 600 Hz, the sound intensity (the objective function) was calculated at the point (35, 35, 100 mm), and the damping factor α was given the value 1.0×10^{-4}. In Case 2 the frequency of the exciting force was 700 Hz, the intensity was calculated at the point (7, 7, 100 mm), and $\alpha = 1.0 \times 10^{-5}$. In both cases a weight increase of 10% was allowed. The initial thickness t of the top plate had in both cases the constant value of 3 mm (constant distribution). The lower limit for the plate thickness was in both cases 2 mm. The iteration histories for the two cases are given in Figs. 10 and 11.

The sound intensity decreased from an initial 83.6 dB to 79.6 dB at optimum ($\Delta I_1 = 4.0$ dB) in Case 1 and from 109.7 dB to 95.0 dB ($\Delta I_2 = 14.7$ dB) in Case 2. The thickness and the value for the objective function at optimum are given in Table 1 and sketched in Fig. 12.

4.2 Experimental results

The intensity was measured perpendicular to the top plate, which in this case was axisymmetric, 100 mm above

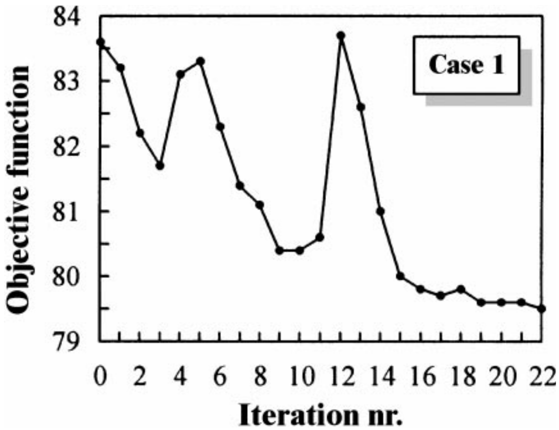

Fig. 10 Iteration history for Case 1 (free). Sound intensity in [dB] (objective function) versus no. of iterations

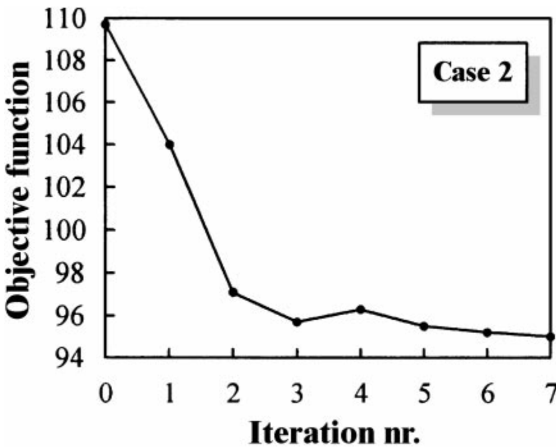

Fig. 11 Iteration history for Case 2 (clamped). Sound intensity in [dB] (objective function) versus no. of iterations

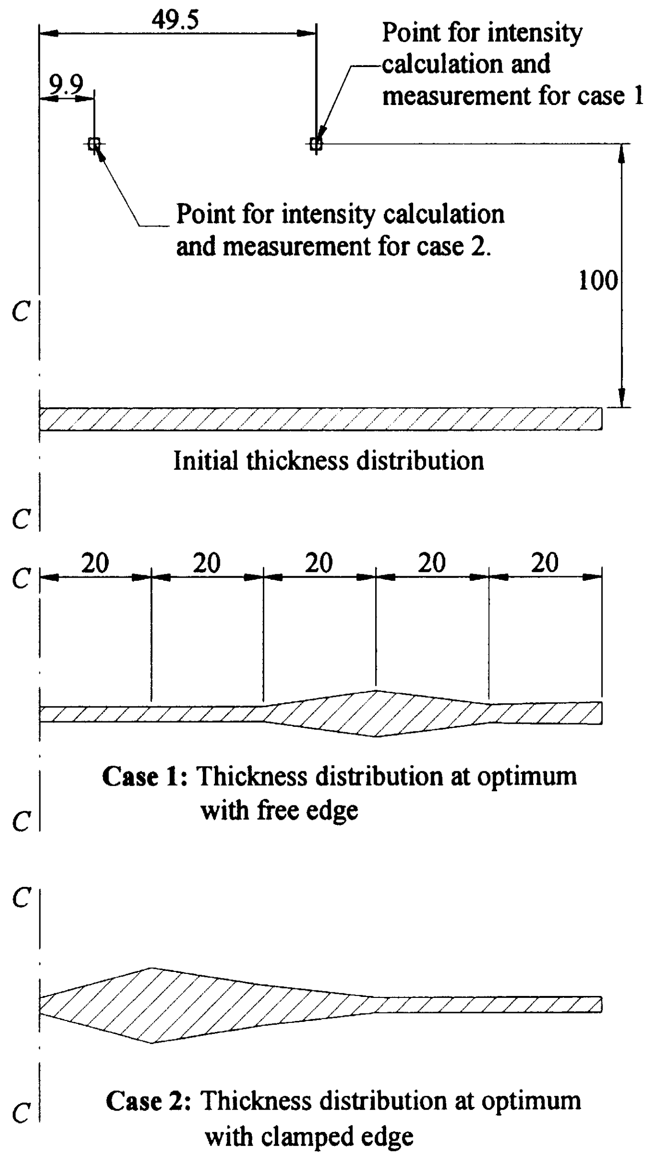

Fig. 12 Axisymmetric figure of top plate with sketched results from optimization process. Dimensions in [mm]

it ($z = 100$ mm in Fig. 5), and 49.5 mm from the centreline in Case 1 and 9.9 mm from the centreline in Case 2, according to Figs. 5 and 12. For each radii the intensity was measured at three points; the mean value was then used in the comparison with the numerical result. In both cases the excitation force was harmonic with the amplitude of 2.0 N. The frequency of the excitation force was 600 Hz in Case 1 and 700 Hz in Case 2. The results from the measurements are presented in Tables 2 and 3.

5 Discussion and conclusions

A comparison study of numerical acoustic optimization results and experimental results has been performed. To make the comparison as unambiguous as possible, the structural geometry studied was simple, a closed cylinder. The wall and the bottom plate of the cylinder were modelled as rigid, i.e. the only moving part in the model was the top plate. The top plate was excited at its centre with a constant force and at constant frequencies. The variables used were the top plate and the sound intensity (the objective function) were calculated and measured at specified points above the top plate. The optimization problem was formulated as: minimize the sound intensity at a specified point such that the structural weight of the top plate does not increase by more than 10%. A comparison of numerical and experimental results was performed for two different cases; one where the edge of the top plate was modelled as free (Case 1) and one where the edge of the top plate was modelled as clamped (Case 2). In both cases the excitation force was harmonic with the amplitude of 2.0 N. The frequency of the excitation force was in Case 1 (free) 600 Hz and in Case 2 (clamped) 700 Hz. The initial thickness of the top plate was in both cases 3 mm with uniform distribution.

In the free case (Case 1) the calculated sound intensity converged quite slowly from the initial value of 83.6 dB to the value of 79.6 dB at optimum, i.e. a decrease in intensity with 4.0 dB. The corresponding experimental result was 86.1 dB at initial geometry and 838. dB (2.3 dB decrease) at optimal geometry according to the numerical analysis.

In the clamped case (Case 2) the calculated sound intensity converged quickly from the initial 109.7 dB to 95.0 dB at optimum, i.e. a 14.7 dB decrease of the intensity. The corresponding experimental result was 112.4 dB with initial thickness and 92.9 dB (19.5 dB decrease) with optimal thickness according to the numerical analysis. In Table 4 the comparison of measured and numerical results is presented.

Proportional damping was included in the analysis as $[C] = \alpha \times [K]$. In Case 2 the top plate is lightly damped and the damping factor α was set at 1.0×10^{-5}. In Case 1 the damping is however more complex. Here there were three small spots with adhesive polymeric paste between the top plate edge and the end face on the top of the steel cylinder. There was also an elastic rubber tape (vulcanization tape) placed over the gap between the top plate edge and the end face of the steel cylinder according to Fig. 9. This case was assumed to be free which gives large deflection at the edge of the top plate. Due to the polymeric paste and the tape a damping factor α of 1.0×10^{-4} was chosen for Case 1, i.e. ten times the damping factor in Case 2. The damping factor had greatest influence on two numerical results, namely the top plate with optimal thickness in Case 1 and with initial thickness in Case 2. A reduction of the damping factor from 1.0×10^{-4} to 5.0×10^{-5} in Case 1 increased the intensity for initial thickness with insignificant 0.3 dB but decreased the intensity for optimal thickness with 2.5 dB. In Case 2, an increase of the damping factor from 1.0×10^{-5} to 2.0×10^{-5} gave a decrease of the intensity of 2.2 dB for initial geometry and of 0.2 dB for optimal geometry.

In the numerical analysis of Case 2, the edge of the top plate is clamped. To simulate this in the experiment the edge of the top plate was glued to the end face on the top of the steel cylinder with epoxy cement. This does not give an absolutely clamped boundary condition. Nevertheless, since the distribution of the epoxy cement was approximately 2–3 mm in radial direction and the top plate is less stiff the boundary condition is assumed to simulate a clamped condition quite well.

In the numerical analysis the only moving part of the model is the top plate. During the experiments the sound intensity parallel and perpendicular to the steel cylinder were measured. The levels were insignificantly low and are therefore not considered to contribute to the sound intensity measured above the top plate. Another difference between the model used for the numerical analysis and the actual structure is the hole at the centre of the top plate in the actual structure. The purpose with the hole was to join the impedance head to the top plate mechanically (screwed). This hole was not present in the numerical analysis. With a diameter of 5 mm and the estimated decrease of plate stiffness approximately 2.5%, the hole was assumed to have insignificant influence on the results. There is also a difference in the geometry of the top plate in the numerical analysis and in the experimental study. While the top plate is not axisymmetric in the numerical analysis due to the discretization into constant elements it is so in the experimental study. This difference is dependent on manufacturing demands and may influence the response.

Table 2 Measured intensity for Case 1. Radius, R, according to Figs. 5 and 12

Case 1 (free)			
		Intensity (experimental) [dB]	
Point	R [mm]	initial thickness	optimal thickness
1	49.5	86.4	83.8
2	49.5	86.4	83.9
3	49.5	85.6	83.8
I_{mean}	86.1	83.8	
ΔI_{mean}	2.3		

Table 3 Measured intensity for Case 2. Radius, R, according to Figs. 5 and 12

Case 2 (clamped)			
		Intensity (experimental) [dB]	
Point	R [mm]	initial thickness	optimal thickness
1	9.9	112.2	92.9
2	9.9	112.4	92.8
3	9.9	112.5	92.9
I_{mean}	112.4	92.9	
ΔI_{mean}	19.5		

Table 4 Comparison between numerical and experimental results. [1] Intensity calculated by numerical analysis. [2] Measured intensity in the experiments. [3] Difference between numerical and measured intensity

	Case 1 (free)		Case 2 (clamped)	
Intensity [dB]	initial	optimal	initial	optimal
[1] I_{num}	83.6	79.6	109.7	95.0
[2] I_{exp}	86.1	83.8	112.4	92.9
[3] $I_{\text{num}} - I_{\text{exp}}$	−2.5	−4.2	−2.7	+2.1

6
Future considerations

In the optimization analysis some parameters are chosen as variables and some are chosen as constants. The optimization process minimizes (or maximizes) a function without violating any constraints through changes in the variables while some parameters are held constant. In optimization of real structures it could be of value to do sensitivity analyses also on the constant parameters after the optimum is found. This to gain an idea of how sensitive the structure or design is to imperfections in different parameters.

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