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Topology optimization of continuum structures subjected to pressure loading

V.B. Hammer and N. Olhoff

Abstract This paper presents a generalization of topology optimization of linearly elastic continuum structures to problems involving loadings that depend on the design. Minimum compliance is chosen as the design objective, assuming the boundary conditions and the total volume within the admissible design domain to be given. The topology optimization is based on the usage of a SIMP material model.

The type of loading considered in this paper occurs if free structural surface domains are subjected to static pressure, in which case both the direction and location of the loading change with the structural design.

The presentation of the material is given in a 2D context, but an extension to 3D is straightforward. The robustness of the optimization method is illustrated by some numerical examples in the end of the paper.

Key words Design dependent loads, pressure loading, topology optimization

1 Introduction

The field of structural topology optimization has developed extensively in the last two decades and has successfully addressed a variety of problems within a wide range of applications, see e.g. Bendsøe (1995), Rozvany *et al.* (1995), Olhoff and Rozvany (1995), and Gutkowksi and Mróz (1997). However, although being very common, the class of problems where applied surface loads on a continuum structure depend on the design of the structure itself in terms of both location and direction, seems to be considered for the first time in the present paper (see also Hammer and Olhoff 2000).

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The first optimum topology (layout) solutions for loads of design dependent locations were the so-called "Prager structures" (Rozvany and Prager 1979), see also Rozvany (1989, pp. 330–341), and Rozvany *et al.* (1995) and references cited therein. A Prager structure is a (3D or 2D) Michell structure in which the permissible stress for either compression or for tension tends to zero and the loads are movable along their line of action (but not in terms of direction as well, as in the present paper).

Design dependent loads manifest themselves in many everyday structural design problems where the location, size, and direction of the loading is directly coupled to the shape or topology of the structure. As a few examples we may mention wind and snow loading on civil engineering structures; internal and external pressure loading in pump housings and pressure containers, and on underwater storage constructions, respectively; and loading due to internal and external fluid flows in ducts, diffusers, and turbomachinery, and around aircraft wings and fuselages, respectively. For simplicity, this paper primarily deals with the problem of optimizing a structure exposed to static pressure loading in which case both the direction and the location of the loading change with the design.

The outline of the paper is as follows. The basic problem formulation of minimum compliance subject to a constraint on the total structural volume is stated in Sect. 2. For the problem under study, the principal extensions relative to the usual topology design methodology comprise an enhanced design model that encounters one or more parameterized, smooth surface domains associated with a prescribed iso-volumetric density of material. The(se) surface domain(s) serve to define the action of the design dependent loading on the structure, and is (are) determined by the design variables, i.e. the distribution of the volumetric densities of material in the finite elements of the analysis model. How these surfaces are defined and represented is described in Sect. 3. In Sect. 4 the pressure applied to the iso-volumetric density curves in the design model is related to the finite element analysis model by derivation of the consistent nodal forces. The optimization itself is performed by application of a new, simple and effective fixed-point type optimality criterion. This criterion is derived in Sect. 5 from the Lagrange function of the problem, and the following Sect. 6 briefly presents the sensitivity analysis of the design dependent loads. Finally, in Sect. 7 some numerical examples illustrating the topology optimization are shown.

2 Problem formulation

The problem considered in the following is the basically classical one of distributing a limited amount of material in the most favourable way. Figure 1 shows the design domain, $\Omega \in I\!R^2$ (or $I\!R^3$), with parts being void, Ω_v, and parts containing material, $\Omega_m \equiv \Omega \setminus \Omega_v$. The description is standard except for the pressure loadings $\mathbf{p}$ acting on parts Γ_p of the surface of the material domain, Ω_m. Therefore, if the domain Ω_m changes, the pressure loadings $\mathbf{p}$ change accordingly.

The compliance W of the structure is written

$$W(\mathbf{u}) = \int_{\Omega} \mathbf{bu}\, \mathrm{d}\Omega + \int_{\Gamma_t} \mathbf{tu}\, \mathrm{d}\Gamma + \int_{\Gamma_p} \mathbf{pu}\, \mathrm{d}\Gamma_m \,, \tag{1}$$

where $\mathbf{b}$ is the vector of body forces, $\mathbf{t}$ the traction vector working on $\Gamma_t \subset \Gamma, \Gamma \equiv \partial\Omega$, and where the vector of displacements $\mathbf{u}$ are in agreement with the prescribed displacements on Γ_u. The last part in the expression of the compliance yields the effect from the pressure $\mathbf{p}$ working on the surface $\Gamma_p \subset \Gamma_m, \Gamma_m \equiv \partial\Omega_m$. The optimization problem consists in minimizing the compliance of the structure subject to a constraint on the volume V

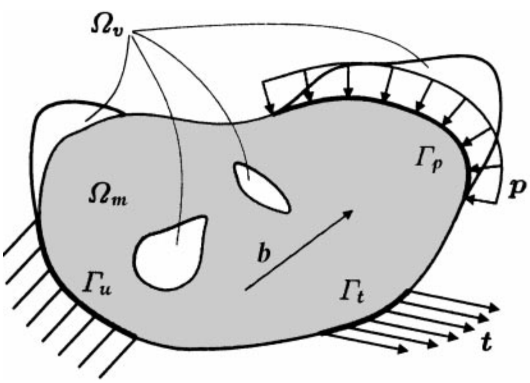

Fig. 1 The reference domain Ω parted in domains of void, Ω_v, and material, Ω_m. The pressure force $\mathbf{p}$ acts on one or more parts Γ_p of the surface of the material domain

$$\min_{\rho(x),\ x\in\Omega} W(\mathbf{u}^*) \,,$$

$$\text{s.t.} \quad V[\rho(x)] = \int_{\Omega} \rho\, \mathrm{d}\Omega \le V^* \,. \tag{2}$$

Here, x denotes the coordinates of any point belonging to the admissible design domain Ω, and $W(\mathbf{u}^*)$ is the compliance given by the displacement field $\mathbf{u}^*$ at equilibrium. W is then the work done by the external forces, which equals twice the total elastic energy at equilibrium. The design variables are the volume densities of material, $0 < \rho(x) \le 1$.

To enhance black and white (1-0) solutions, Bendsøe's (1989) approach is used, which was also derived independently and implemented extensively in 1990 by Zhou and Rozvany (Rozvany and Zhou 1991, presented in 1990) who termed it SIMP (Solid Isotropic Microstructure with Penalty, e.g. Rozvany *et al.* 1992). Intermediate densities are penalized by expressing the elasticity matrix of the SIMP material, $[E]$, as

$$[E(x)] = \rho(x)^n\, [E^*] \,. \tag{3}$$

Here $[E^*]$ denotes the elasticity matrix of the solid, isotropic material for the structure and n is a penalization power, $1 \le n \le 4$, which is increased gradually during the optimization process. For a Poisson's ratio of the solid isotropic material of e.g. $\nu = \frac{1}{3}$, (which is the value used throughout this study), a penalization factor of $n \ge 3$ assures that the SIMP material satisfies the Hashin-Shtrikman bounds for a two-phase material (Bendsøe and Sigmund 1999). Furthermore, Bendsøe and Sigmund (1999) have shown that a realization of the material can be obtained using composites.

3 Identification and representation of the surfaces subjected to pressure

The loading is applied to subdomains of the structural surface of the structure. As the design configuration is changing, the parts where the pressure loading $\mathbf{p}$ acts must still be identifiable (and of course meaningful as well).

In 2D a surface segment on which the pressure acts is defined as a line of contour connecting two points chosen a priori. These two endpoints are either defined as fixed points in the structure (for instance associated with a nodal boundary condition), or as points allowed to move according to the density distribution but only along some predefined lines, as sketched in Fig. 2. This definition implies an implicit decision on the connectivity in parts of the structure, namely that material somehow will connect the chosen endpoints. It is thus a fundamental and crucial assumption for the optimization, that the basic definition of the load carrying surfaces is meaningful.

The definition of the load carrying surfaces in 3D can be made in an analogous manner, only here the pres-

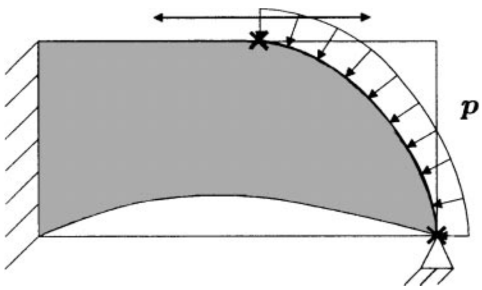

Fig. 2 A structural domain with a pressure applied to the material contour between the two points marked with crosses. One endpoint is fixed whereas the other moves along a horizontal line according to the current density distribution

sure works on contour surfaces bounded by a priori chosen curves.

The finite element method is used to perform the optimization analyses. However, as is common in topology optimization, the finite element mesh of the analysis model of the structure is kept unchanged throughout the optimization process with the drawback of yielding somewhat jagged designs. Thus a design model is needed to describe where and how the pressure acts in the current iteration.

3.1 The surface representation

A smooth surface representation is primarily needed in the domains where the surface loading acts. This is extracted from the density distribution in a way following to a large extent the work of Maute and Ramm (1994, 1995).

As the design variables (volume densities of material) are assumed to be constant within each finite element, the volume density of material is first calculated in every corner node as the average of the densities of all the elements sharing the corner node. Interpolation from these corner node values then gives a set of points $\{x_f\}$ where the volume densities equal a preselected value for a surface of iso-volumetric density, ρ_c. Along the outer boundaries of the structure the corner node densities are determined as if there is a fictitious outer layer of elements with zero density around the structure. This way and by a careful choice of the surface density ρ_c, the surface is forced to lie strictly inside the structural domain which is of great advantage when performing the sensitivity analysis, as the problem of nondifferentiability arises when the line or surface of loading coincides with a finite element boundary.

Next a set of Bézier cubic splines are used to fit these points $\{x_f\}$. The control points $\{x_c\}$ of the splines are found by solving the least square problem of minimizing the distance between the data points $\{x_f\}$ and the corresponding points on the splines $\{x_g\}$

$$\min_{\{x_c\}} \sum_i \left\| x_{f_i} - x_{g_i} \right\| . \tag{4}$$

The necessary conditions for a minimum lead to the following set of equations:

$$[B]^T [B] \{x_c\} = [B]^T \{x_f\} , \tag{5}$$

where the matrix $[B]$ contains the basis functions for the Bézier splines. Equation (5) is solved by singular value decomposition. The number of Bézier curves used is increased gradually from one until a predefined measure of the fit to the data points $\{x_f\}$ is obtained. Figure 3 illustrates for a 2D problem the procedure described above to find a curve of a given iso-volumetric density.

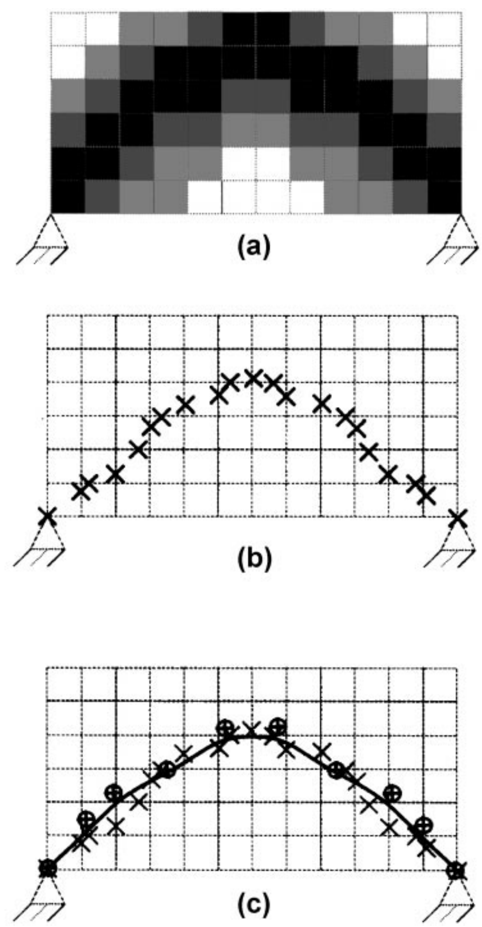

Fig. 3 A sketch of the parametrization of a load carrying surface based on nonuniform density distribution. (a) A 2D design with non-uniform and discontinous density distribution. (b) Points of equal density, $\{x_f\}$. (c) The fitted Bézier curves and their control points $\{x_c\}$

Alternatively, as a very fast and straightforward procedure, the piece-wise linear curve connecting the points of equal volume density $\{x_f\}$ can also be used as a surface representation, although this of course often leads to a somewhat more rough appearance.

4 Usage of the iso-volumetric density curves for load application

The pressure loading is related to the finite element analysis model as sketched in Fig. 4. All the finite elements

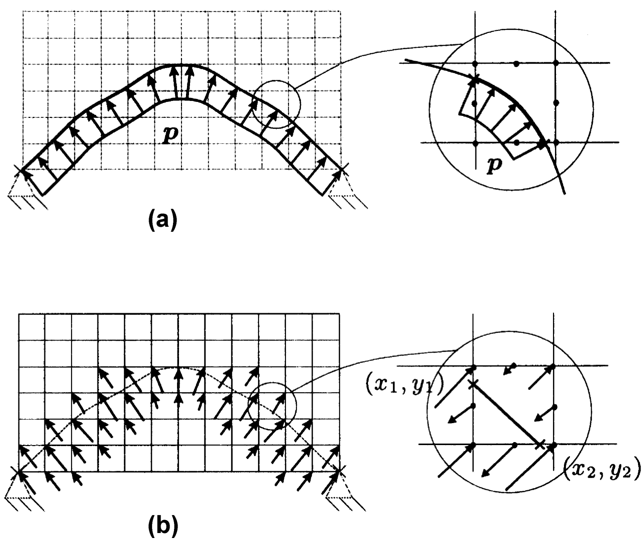

Fig. 4 The design model and the consistent nodal loads on the finite element analysis model. (a) The design model. (b) The analysis model

intersected by the curve for load application carry a part of the loading. The curve is approximated by a straight line in each finite element determined by its two intersection points on the element boundary (x_1, y_1) and (x_2, y_2). In the finite element model, the element vector of consistent nodal forces $\{A_e\}$ is determined as the integration along the line of loading L over the matrix of shape functions $[N]$ and the vector of loading $\{\Phi\}$ (see e.g. Cook *et al.* 1989),

$$\{A_e\} = \int\limits_L [N]^T \{\Phi\} \, \mathrm{d}L =$$

$$pt \int\limits_{x_1}^{x_2} [N]^T \left\{ \begin{matrix} -\frac{y_2 - y_1}{x_2 - x_1} \\ 1 \end{matrix} \right\} \mathrm{d}x \,. \tag{6}$$

In the above equation p is the size of the pressure, and t the element thickness. Hereby, the load on an element is distributed to all the nodes of the element, thus in a sense dispersing the load. This in turn implies that the situation of the curve for load application jumping past nodes is less severe than if the loads were applied to, e.g. a single row of nodes.

5 The successive optimization scheme

A simple fixed-point type of optimality criterion algorithm (Cheng and Olhoff 1982) is used to determine the density distribution that yields minimum compliance. The optimality criterion is derived from the Lagrange function for the optimization problem, see e.g. Cheng and Olhoff (1982) or Bendsøe (1995), only here the design dependent loads give rise to an additional term in the expression for the update of the design variables.

Writing the Lagrange function $\mathcal{L}$ for the optimization problem, (2), in a finite element formulation gives

$$\mathcal{L} = \{D\}^T [S] \{D\} + \lambda (V - V^*) + \{\mu\}^T ([S]\{D\} - \{A\})$$

$$+ \sum_e a_e (\rho_{\min} - \rho_e) + \sum_e b_e (\rho_e - \rho_{\max}) \,, \tag{7}$$

where $\{D\}$ and $\{A\}$ are the vectors of displacements and external forces, respectively, and $[S]$ is the global stiffness matrix of the system. Thus the first term equals the compliance, the second expresses the volume constraint, and the third the equilibrium requirement. Finally λ, $\{\mu\}$, a_e, and b_e are the Lagrange multipliers for the volume and equilibrium constraints and for the lower and upper limits imposed on the design variables, respectively. Stationarity conditions on the Lagrange function $\mathcal{L}$ with respect to the displacement vector $\{D\}$ and the design variable ρ_e of the finite element e give

$$2\{D\}^T \frac{\partial \{A\}}{\partial \rho_e} - n\rho_e^{(n-1)} \{D_e\}^T [S_e^*] \{D_e\} + \lambda A_e -$$

$$a_e + b_e = 0 \,. \tag{8}$$

Here $\{D_e\}$ and $[S_e^*]$ refer to the local displacement vector and the local stiffness matrix independent of the volume density variable, and A_e to the area of the e-th element. If the side constraints on the design variable are inactive, the Lagrange multipliers a_e and b_e equal zero, whereby one upon rearranging obtains the following condition:

$$K_e =$$

$$\frac{1}{\lambda A_e} \left(n\rho_e^{(n-1)} \{D_e\}^T [S_e^*] \{D_e\} - 2\{D\}^T \frac{\partial \{A\}}{\partial \rho_e} \right) = 1 \,. \tag{9}$$

This condition can be interpreted so that the energy-like expression K_e is constant equal to one – or the design should be fully stressed – for all intermediate densities. Based on this expression a fix-point type update algorithm of the design variable in the $(i+1)$-th iteration can then be written as

$$\rho_e^{i+1} = \begin{cases} \alpha & \text{if } \rho_e^i \left(K_e\right)^{0.8} \le \alpha \\ \rho_e^i \left(K_e\right)^{0.8} & \text{if } \alpha \le \rho_e^i \left(K_e\right)^{0.8} \le \beta \,, \\ \beta & \text{if } \rho_e^i \left(K_e\right)^{0.8} \ge \beta \end{cases}$$

$$\alpha = \max\left[(1-\xi)\,\rho_e^i, \rho_{\min}\right] \,,$$

$$\beta = \min\left[(1+\xi)\,\rho_e^i, \rho_{\max}\right] \,, \tag{10}$$

where the superscripts i and $(i+1)$ refer to the iteration number. Move-limits are imposed on the design variables by the parameter ξ. As the volume V is a continuous and monotone decreasing function in the Lagrange multiplier λ, the latter can easily be found in an inner iteration loop by means of e.g. bisection or a Newton-Raphson method.

As can be seen from (9), the factor K_e can become negative due to the sensitivities of the loads. In this case K_e is taken as zero in the update algorithm, meaning that the updated value of the design variable ρ_e^{i+1} in (10) takes its minimum value of α. However, this case of $K_e < 0$ can, if at all, only occur for the elements just around the line or surface of loading, as the sensitivities of the load vector $\frac{\partial\{A\}}{\partial\rho_e}$ are zero in all other elements.

The optimization process is performed by successive iterations making use of the finite element analysis model with fixed mesh on the one hand, and the design model with the parameterized iso-volumetric density curve for the pressure loading on the other. The optimization performance can be tuned by optimizing the structure for fixed loading a couple of iterations before moving the pressure, and by the choice of ρ_c.

6
Sensitivity analysis of the design dependent loads

As explained above the load curves in the design model are controlled by the density distribution in the finite element model and in turn fully determine the global load vector of the finite element model. Thus the sensitivity analysis is based on both the analysis model and the design model. In the update algorithm also the sensitivities of the load vector $\{A\}$ with respect to a design change must be evaluated. This is determined by partial differentiation as

$$\frac{\partial\{A\}}{\partial\rho_e} = \frac{\partial\{A\}}{\partial\{x_c\}}\,\frac{\partial\{x_c\}}{\partial\{x_f\}}\,\frac{\partial\{x_f\}}{\partial\rho_e} \,, \tag{11}$$

where in turn the effect of a density variation on the points $\{x_f\}$ of fixed density, on the control points $\{x_c\}$ of the Bézier curves, and finally how this affects the nodal loads on the finite element model, must be evaluated. All parts of the sensitivity analysis are determined analytically under the provision that the density variation is considered so small that the contour curve will stay within the same set of finite elements. If this is not the case, i.e. if the pressure load acts exactly on the boundary of a finite element, then the (nonexistent) sensitivities of the finite element loads are disregarded in the optimization. However, this case is to a large extent avoided by the way the volume densities in the corner nodes along the outer boundaries of the structure are calculated, in combination with a careful choice of the density value for the iso-volumetric density line ρ_c. The sensitivities of the design dependent loads are zero for most elements except for those lying in a narrow band around the iso-volumetric density line.

7
Numerical examples

The method developed in this paper is first illustrated by the optimization of the simple short cover-like structure shown in Fig. 5a. The finite element analysis model con-

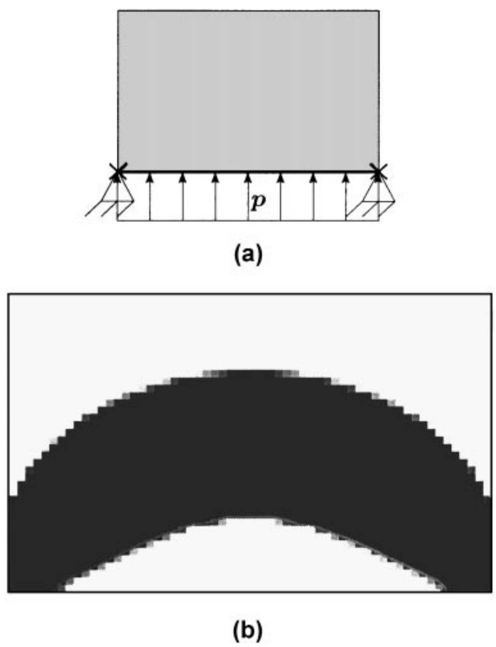

Fig. 5 Topology optimization of a cover-like structure. (a) The design domain with the initial pressure distribution from below. (b) The optimized topology for a volume fraction of 50%. In (b) the black domain corresponds to $\rho = 1$ and the grey line marks the curve for load application in the final design, $\rho_c = 0.6$

sists of 2440 elements with one design variable per element. The structure is clamped in both lower endpoints and is subjected to a constant pressure loading from below. The pressure is restricted to act on the curve of iso-volumetric density of $\rho_c = 0.6$. As this curve varies from one iteration to another, so does the loading. In the very first iterations the optimization algorithm primarily adds mass around the supports which causes the line of loading to be broken and thus meaningless in this context. This problem is avoided by keeping the curve for the load application fixed independently of the density distribution during the first few iterations. As is seen from Fig. 5b, the fixed-point type of optimality criterion algorithm used to determine the density distribution yielding minimum compliance, results in a quite distinct black ($\rho = 1$) and white ($\rho = 0$) design. The dependency of the optimization procedures on the symmetry in the initial design is insignificant, which was tested by also starting the optimization from an uneven and very asymmetric design (and thus also an asymmetric initial pressure distribution), which lead to the same optimal and symmetric design.

In the next example, Fig. 6a, the admissible design domain is a little longer and the pressure is applied from the sides and from above. Again the optimization yields a very compact structure as seen in Fig. 6b.

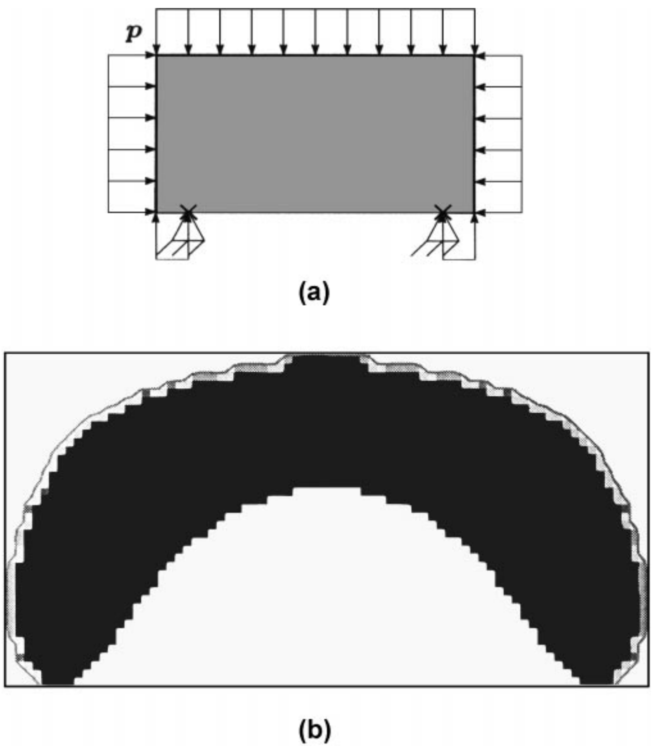

Fig. 6 The design model and the resulting optimal topology. (a) The design domain with the initial pressure distribution from the sides and from above. (b) The optimized topology for a volume fraction of 50%

The next example in Fig. 7 investigates how two levels with a gap in between should be closed. As in the previous examples an even density distribution was used first as the initial design leading to the curve of isovolumetric density following the structural boundary (Fig. 7a). The optimized design shown in Fig. 8a was however somewhat counterintuitive, so a different initial design and thus curve for the pressure application

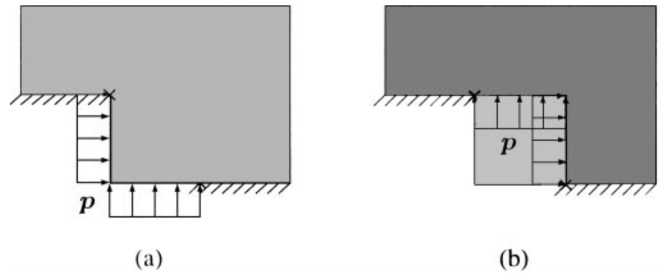

Fig. 7 The design domain with two different initial pressure distributions; (a) along the structural boundary, and (b) inside the structural domain as the consequence of an uneven distribution of mass as illustrated by the shades of grey

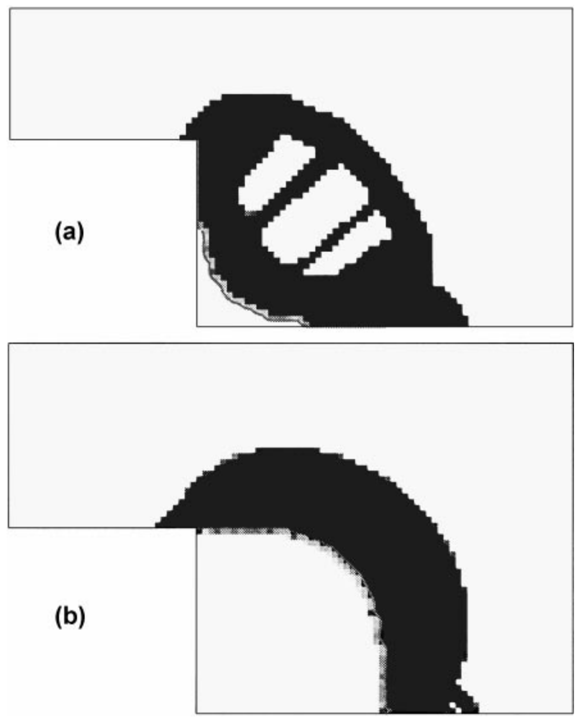

Fig. 8 The optimized topologies obtained from two different initial designs. The volume fractions are 30%. (a) Optimized from the initial design shown in Fig. 7a. Resulting total elastic energy, $U = 0.164$. (b) Optimized from the initial design shown in Fig. 7b. Resulting total elastic energy, $U = 0.104$

was tried out (see Fig. 7b). The resulting optimal design shown in Fig. 8b yields a compliance as low as 63.4% of the optimized design obtained first. This example clearly demonstrates the need for testing different initial designs to reduce the risk of ending up in local optima.

The last example models the inlet from a channel to a larger pressure chamber. The material around the inlet is prescribed to be solid and nonchangeable. Here, two domains of the structural surface are subjected to pressure, and the initial pressure distribution is shown along with the design domain in Fig. 9a. The value for the isovolumetric density line ρ_c was initially chosen to be 0.25, and then gradually increased to $\rho_c = 0.85$. The optimized material distribution exhibits a gradual change on the upper part and only a small stiffener in the lower part of the pressure chamber as shown in Fig. 9b. Other shapes of the initial small channel leading the pressure from the inlet to the chamber could of course have been chosen, but the final design seems unaffected hereof.

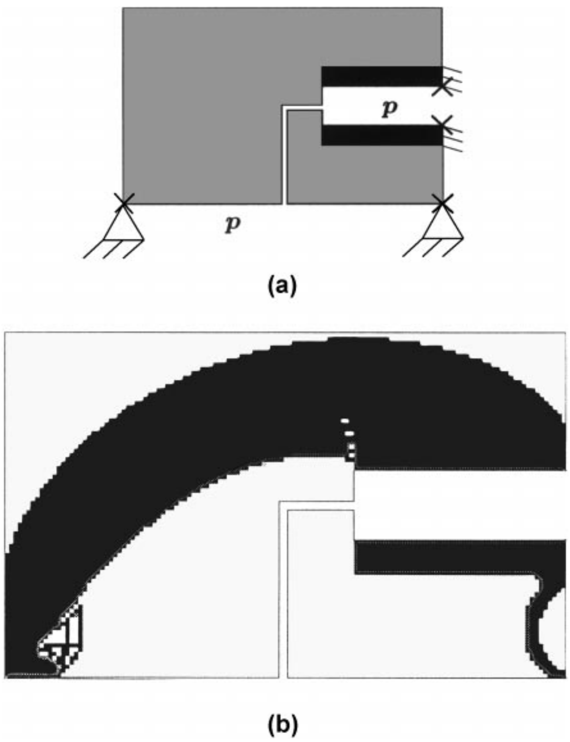

Fig. 9 Optimization of an inlet. Two separate parts of the structural surface are subjected to pressure loads. These are marked with grey lines in the optimal design. (a) The design domain with the pressure initially distributed to the parts of the structural boundary drawn with thick lines. (b) The optimized topology for a volume fraction of 40%

8 Summary

A generalization of the methodology of topology optimization of elastic continuum structures to problems involving loadings that depend on the design has been presented. While the formulation of the new topology optimization problem is based on commonplace usage of a SIMP material model and a fixed finite element mesh with the volumetric density of material within each finite element as a design variable, the extension to design dependent surface loading adds two types of complexities to the problem. First, based on the analysis model for the problem, it is necessary to develop a design model where the load carrying surfaces are defined by a given value of the volumetric density of material and represented by spline functions with control points that depend on the design variables. This smooth surface representation provides the basis for the definition and calculation of the design dependent surface loading, which upon transformation into consistent nodal forces on the finite elements intersected by the surface, constitutes the link back to the analysis model. Secondly, the sensitivity analysis becomes more involved relative to usual topology design problems due to the need to include sensitivities of the loading with respect to design. These sensitivities are established in conformity with the above dependencies and derived analytically by implicit partial differentiation. The topology optimization problem is then solved by successive iterations based on a fixed-point optimality criterion algorithm, and the applicability and efficiency of the proposed approach for solution of the new type of problem are demonstrated via example problems that have all resulted in quite distinct "black and white" topology designs.

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