RESEARCH PAPER

An adaptive polynomial chaos expansion for high-dimensional reliability analysis

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Abstract

Efficiency is greatly concerned in reliability analysis community, especially for the problems with high-dimensional input random variables, because the computation cost of common reliability analysis methods may increase sharply with respect to the dimension of the problem. This paper proposes a novel meta-model based on the concepts of polynomial chaos expansion (PCE), dimension-reduction method (DRM), and information-theoretic entropy. Firstly, a PCE method based on DRM is developed to approximate the original function by a series of PCEs of univariate components. Compared with the PCE of the original function, the DRM-based PCE can reduce the computational cost. Before constructing the meta-model, a prior of the degree of the PCE is required, which determines the accuracy and efficiency of the PCE. However, the prior is usually determined by experience. According to the maximum entropy principle, this paper proposes an adaptive method for the selection of the polynomial chaos basis efficiently. With the adaptive PCE method based on DRM, a novel meta-model method is proposed, with which the reliability analysis can be achieved by Monte Carlo simulation efficiently. In order to verify the performance of the proposed method, three numerical examples and one structural dynamics engineering example are tested, with good accuracy and efficiency.

Keywords Polynomial chaos expansion · Dimension-reduction method · Entropy · Structural reliability analysis

1 Introduction

Due to the uncertainties frequently involved in industrial application, such as uncertainties of material, loads, and geometry, it has been well recognized that the assessment of structural safety based on probabilistic theory plays an important role in practical engineering (Du and Chen 2004; Youn et al. 2005). One of the common ways to assess structural safety is reliability analysis, which is usually modeled by the following mathematical formulation:

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$$P_f = \Pr(g(\mathbf{x}) < g_t) = \int_{-\infty}^{g_t} p(g)\mathrm{d}g = \int_{g(\mathbf{x}) < g_t} f(\mathbf{x})\mathrm{d}\mathbf{x} \qquad (1)$$

where P_f is the failure probability, $\Pr(\bullet)$ is the probability of an event, g_t is the threshold for the definition of failure, $\mathbf{x}$ is the vector of input variables with the joint probability density function (PDF) of $f(\mathbf{x})$, and $g(\mathbf{x})$ is the interested response of input variables with the PDF of $p(g)$.

Based on Eq. (1), extensive methods have been developed to achieve the structural reliability analysis with good efficiency and/or accuracy, including the sampling-based methods (Xu and Kong 2018a; Xi et al. 2014; Engelund and Rackwitz 1993), MPP-based methods (Du and Chen 2001; Meng et al. 2018), moment-based methods (Du and Sudjianto 2004; Liu et al. 2018; Wu et al. 2020; Xu et al. 2017; Xi et al. 2012; Zhang and Han 2020), and surrogate-based methods (Zhang et al. 2015; Zhang et al. 2017; Xiong et al. 2007; Zhu and Du 2016). In general, sampling-based methods are accurate and robust; therefore, many advanced sampling-based methods have been proposed recently, such as line sampling method (Lu et al. 2008), importance sampling method (Dai et al. 2015a, 2015b), and stratified

sampling method (Shields et al. 2015). However, the deficiency on efficiency still limits the application of sampling-based methods (Xu and Kong 2018). Although MPP-based methods usually have good efficiency and are widely used in reliability-based design optimization and some nonprobability reliability analysis problems (Meng and Keshtegar 2019; Meng et al. 2020b), they are not accurate enough for highly nonlinear problems. Moment-based methods can achieve the trade-off between accuracy and efficiency, but they may be confronted with the problem of numerical stability (Youn et al. 2008; Huang and Du 2006; He et al. 2019b). The basic idea of surrogate-based methods is to construct a numerical black box or an analytical model to substitute the real physical model. Then, the Monte Carlo simulation (MCS) method can be performed by the meta-model efficiently. Because the surrogate-based methods reduce the computational burden greatly with satisfactory accuracy, a large number of advanced meta-models have been proposed in the recent decades, such as neural network method (Dai et al. 2015), kriging method (Meng et al. 2020a; Zhang et al. 2019; Zhang et al. 2020) and polynomial chaos expansion (PCE) method (Wang et al. 2018; Xu and Kong 2018b; Guo et al. 2018; Zhou et al. 2019). Among these methods, the PCE method has received increasing attention, which is regarded as probably the most widely used meta-model for propagating uncertainties (Abraham et al. 2017).

The main idea of PCE is to represent a random variable, e.g., structural stochastic response, by a series of polynomial chaos basis (Soize and Ghanem 2004). It should be noted that the term chaos in PCE, which is coined by Wiener for handling Gaussian random process (Wiener 1938), is different from the concept of chaos in dynamic systems. According to Wiener's idea, Ghanem and Spanos (1991) started the study on Wiener-Hermite polynomial chaos for stochastic finite element. In order to overcome the difficulties of Hermite polynomial chaos in non-normal problems, Xiu and Karniadakis (2002a, 2002b, 2003) proposed the generalized polynomial chaos, which generates the orthogonal basis based on the probability distribution of the input random variable. On the basis of the well-chosen orthogonal basis, a recursive procedure can be used to derive the PCE for fitting the stochastic responses of mechanical systems.

To obtain the PCE meta-model, it is required to calculate the regression coefficients of the PCE accurately. The common non-intrusive methods for the calculation can be grouped into two categories: least squares approximation (LSA) method (Hadigol and Doostan 2018; Berveiller et al. 2006) and projection method (Marelli and Sudret 2015). The LSA method is a classical method for regression analysis and frequently used in PCE. Many researches have been done to enhance efficiency and convergence of LSA method (Hampton and Doostan 2015; Narayan et al. 2017). The projection method calculates the coefficients with the aid of the orthonormality of the polynomial basis. Thus, the calculation of the coefficients

is reduced to a numerical integration problem. Some algorithms have been developed to calculate the numerical integration efficiently and accurately, such as Gaussian quadrature method and Smolyak' sparse quadrature method (Gerstner and Griebel 1998). However, the applications of both LSA method and projection method are limited by the dimension of the space of original input random variables, because the difficulty of the curse of dimension always poses a great challenge in multiple- and high-dimensional problems. To overcome the tricky problem, many sparse PCE methods have been proposed, which neglect the unimportant PCE terms to reduce the number of PCE coefficients. Typically, Blatman and Sudret (Blatman and Sudret 2011) proposed an adaptive sparse PCE based on least angle regression. Cheng and Lu (2018a) proposed a sparse PCE method based on D-MORPH regression. Xu and Wang (2019) proposed a sparse PCE for efficient structural reliability analysis based on Voronoi cells. Cheng and Lu (2018b) proposed an adaptive sparse PCE for global sensitivity analysis based on support vector regression. These sparse strategies have been proven to be effective in terms of the reduction of computational cost. In addition to the sparse strategies, dimension-reduction method (DRM) is also an effective way to deal with multiple- and high-dimensional problems, but DRM receives much less attention than the sparse strategies in the researches of PCE.

Dimension-reduction method (DRM) is usually used to approximate arbitrary multivariate function by a series of lower order components. In reliability analysis community, Rahman and Xu (2004) firstly proposed a univariate DRM (UDRM) for the calculation of statistical moments of structural responses. Due to the high efficiency of UDRM, it is widely used in structural reliability analysis and reliability-based design optimization. Acar et al. (2010) employed the combination of the UDRM and the extended generalized lambda distribution for reliability analysis. Youn and Xi (2009) proposed an eigenvector dimension-reduction method for reliability-based robust design optimization. In order to calculate the fractional moments of structural responses, Zhang and Pandey (2019) proposed the multiplicative UDRM (MUDRM). Based on MUDRM and Laplace transformation, Li et al. (2019) proposed an improved fractional moment-based maximum entropy method for reliability analysis. Although widespread efforts have been made, the work about the combination of DRM and PCE is few. One of the typical representatives in this field is provided by Zhang (2013), which coupled MUDRM with PCE to calculate the PCE coefficients by projection method efficiently. In this paper, the problem of curse of dimension in PCE is solved from another angle based on the DRM and entropy.

The contribution of this paper is twofold. Firstly, different from the work of Zhang, this paper reduces a multiple- or high-dimensional function to multiple one-dimensional functions, and then fits each univariate function by PCE. Secondly, an adaptive method is proposed to select the PCE basis based

on the concept of the information-theoretic entropy. Compared with the common sparse strategies, the proposed adaptive method can construct the PCE adaptively and assess the convergence of the model, without specifying the prior of the order of the PCE and resampling for the verification of the PCE model. Organization of this paper is as follows. In Sect. 2, the fundamental theory of PCE is presented. Section 3 provides the details of DRM. Section 4 first proposes an adaptive method for selecting the polynomial chaos basis based on the concept of the information-theoretic entropy and then develops a novel surrogate-based reliability analysis method with the combination of PCE and DRM. In Sect. 5, the performance of the proposed meta-model is illustrated by three numerical examples and one engineering example. Finally, some conclusions are summarized in Sect. 6.

2 The fundamental theory of polynomial chaos expansion

2.1 The concept of polynomial chaos basis

In theory, a random variable V can be represented by a sum of polynomial chaos basis as follows (Karagiannis and Lin 2014):

$$V = \sum_{i=0}^{\infty} c_i \phi_i(\zeta) \tag{2}$$

where c_i are the PCE coefficients, ζ is random variable with the PDF of $f(\zeta)$, and ϕ_i are one-dimensional polynomial chaos basis with the following orthogonality (Blatman and Sudret 2010):

$$\langle \phi_i(\zeta), \phi_j(\zeta) \rangle = \int_Z \phi_i(\zeta) \phi_j(\zeta) f(\zeta) \mathrm{d}\zeta = \delta_{ij} \tag{3}$$

where for arbitrary $f(\zeta)$, ϕ_i can be derived by Stieltjes procedure (Wan and Karniadakis 2006) with the following recurrence relation:

$$\sqrt{\beta_{n+1}} \phi_{n+1}(\zeta) = (\zeta - \alpha_n) \phi_n(\zeta) - \sqrt{\beta_n} \phi_{n-1}(\zeta) \qquad n = 1, \ 2, \ \dots \tag{4}$$

where α_n and β_n are given by the Christoffel-Darboux formulae as follows:

$$\begin{aligned} \alpha_n &= \frac{\langle \zeta \widehat{\phi}_n, \ \widehat{\phi}_n \rangle}{\langle \widehat{\phi}_n, \ \widehat{\phi}_n \rangle} \\ \beta_n &= \frac{\langle \widehat{\phi}_n, \ \widehat{\phi}_n \rangle}{\langle \widehat{\phi}_{n-1}, \ \widehat{\phi}_{n-1} \rangle} \\ \phi_n &= \frac{\widehat{\phi}_n}{\sqrt{\langle \widehat{\phi}_n, \ \widehat{\phi}_n \rangle}} \end{aligned} \tag{5}$$

With Eqs. (4) and (5), the polynomial chaos basis can be derived for the random variable with arbitrary PDF. Herein, five classical PCEs are listed in Table 1. If V does not follow the distributions listed in Table 1, Nataf transformation can be employed to transform arbitrary random variable into the five classical ones. However, Nataf transformation can be highly nonlinear; therefore, it can lead to a significantly detrimental effect on the accuracy and/or convergence of the final truncated PCE. Therefore, Stieltjes procedure is recommended to construct the PCE for the non-classical cases.

2.2 The truncation form of multivariate polynomial chaos expansion

By partial tensorization of the one-dimensional polynomial chaos basis, a multivariable function $Y(\mathbf{x})$ with mutually independent random variables can be approximated by a truncation expression of M-dimensional PCE as follows (Cheng et al. 2019):

$$Y(\mathbf{x}) = \sum_{i=0}^{P-1} c_i \Phi_i(\zeta) \tag{6}$$

where Φ_i are multivariable PCE basis expressed as

$$\Phi_i(\zeta) = \prod_{j=1}^{M} \phi_{i_j}(\zeta_j) \tag{7}$$

where M is the dimension of multi-dimensional random vectors, $\mathbf{x} = [x_1, x_2, \dots, x_M]$ and $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_M]$, and i_j compose the following multi-indices set:

$$I = \left\{ [i_1, \dots, i_M] \in N^M, \quad \sum_{j=1}^{M} i_j \le p \right\} \tag{8}$$

whose cardinality can be calculated by (Shao et al. 2017)

$$P = \frac{(p+M)!}{p! M!} \tag{9}$$

where p is the degree of the PCE.

2.3 The calculation of the polynomial chaos expansion coefficients

Two methods are usually used for the calculation of PCE coefficients, namely, projection method (Palar et al. 2016) and least squares approximation (LSA) method (Blatman and Sudret 2010). Because the projection method is not involved in this study, only the LSA method is presented herein.

The LSA method estimates the PCE coefficients by minimizing the residue errors between PCE and the real physical model at a set of experiment designs. Thus, the following optimization formulation can be used to calculate the

Table 1 Different forms of PCE

The distribution of V	Type of polynomial chaos basis	$f(\zeta)$	Support
Normal	Hermite	$\exp(-\zeta^2/2)$	$(-\infty, +\infty)$
Uniform	Legendre	1	$[-1, 1]$
Beta	Jacobi	$(1-\zeta)^\alpha(1+\zeta)^\beta$	$[-1, 1]$
Exponential	Laguerre	$\exp(-\zeta)$	$[0, +\infty)$
Gamma	General Laguerre	$\Gamma(\zeta, \alpha+1, 1)$	$[0, +\infty)$

coefficients (Blatman and Sudret 2010):

$$\mathbf{c} = \arg\min \sum_{j=1}^{m} \left(Y_j - \sum_{i=0}^{P-1} c_i \Phi_i(\zeta_j) \right)^2 \tag{10}$$

with the solution as

$$\mathbf{c} = \left(\mathbf{\Phi}^T \mathbf{\Phi} \right)^{-1} \mathbf{\Phi}^T \mathbf{Y} \tag{11}$$

where $\mathbf{c} = [c_1, c_2, \ldots, c_{P-1}]$, $\mathbf{Y} = [Y_1, Y_2, \ldots, Y_m]$ is the response vector of real physical model at m experiment designs, and $\mathbf{\Phi}$ is defined as

$$\mathbf{\Phi} = \begin{bmatrix} \Phi_0(\zeta_1) & \Phi_1(\zeta_1) & \cdots & \Phi_{P-1}(\zeta_1) \\ \Phi_0(\zeta_2) & \Phi_1(\zeta_2) & \cdots & \Phi_{P-1}(\zeta_2) \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_0(\zeta_m) & \Phi_1(\zeta_m) & \cdots & \Phi_{P-1}(\zeta_m) \end{bmatrix} \tag{12}$$

According to Eq. (9), the dimension of $\mathbf{c}$ increases steeply with respect to the dimension of the input random variables and the degree of PCE. Therefore, the computational cost of experiment designs is intolerable for high-dimensional problems.

It can be seen that the LSA method is confront with the problem caused by the dimension of the input random variables when calculating the PCE coefficients. Extensive efforts have been made to solve the problem, such as the popular sparse PCE methods. In this paper, the problem is solved from another angle, say dimension-reduction method.

3 Dimension-reduction method

The main idea of dimension-reduction method (DRM) is to approximate arbitrary multivariate function $Y(\mathbf{x})$ by a sum of functions of lower order in an increasing hierarchy as

$$Y(\mathbf{x}) \cong Y_s(\mathbf{x}) = \sum_{i=0}^{s} (-1)^i C_{M-s+i-1}^i \sum_{k_1 < \cdots < k_{s-i}} Y \tag{13}$$

$$(\mu_1, \cdots, \mu_{k_1-1}, x_{k_1}, \mu_{k_1+1}, \cdots, \mu_{k_{s-i}-1}, x_{k_{s-i}}, \cdots, \mu_M)$$
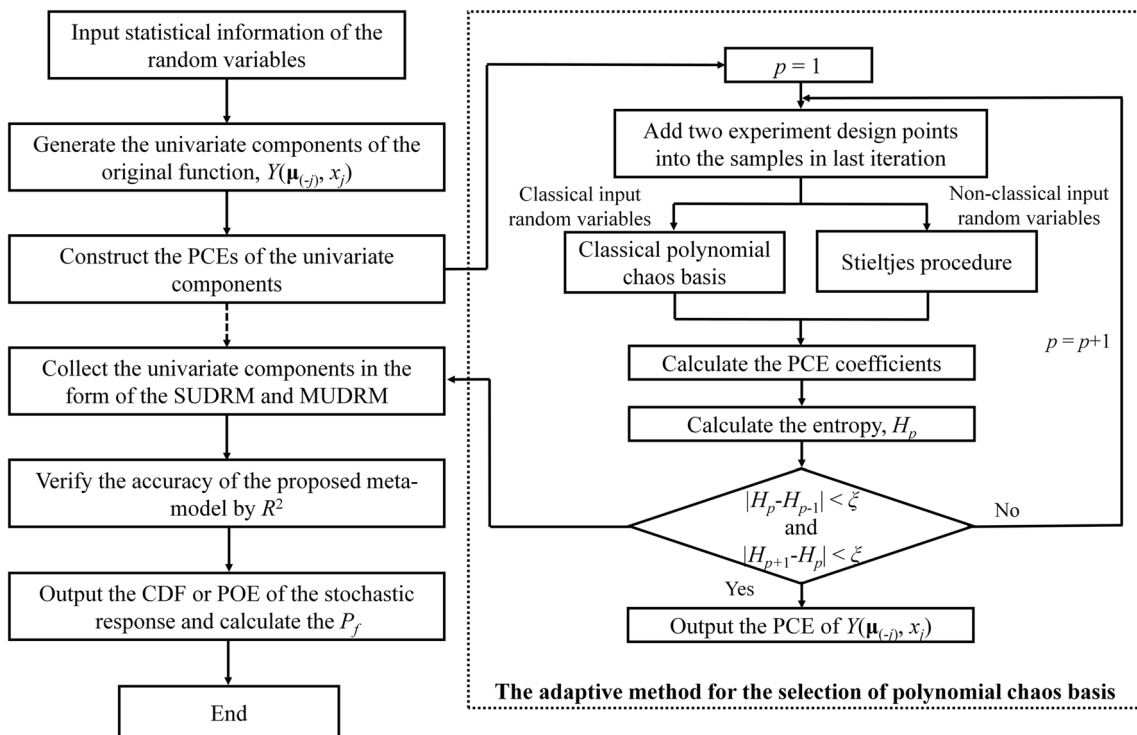

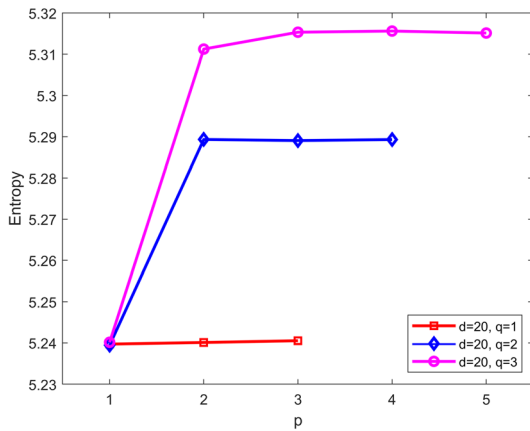**Fig. 1** The flowchart of the proposed method

Fig. 2 The convergence histories of univariate component of X_1

where $\boldsymbol{\mu} = (\mu_1, \mu_2, ..., \mu_M)$ is the vector of reference point. For a sufficiently smooth function, the higher-order components can be neglected compared with the univariate components (Zhang and Pandey 2013). Therefore, Eq. (13) can be reduced into UDRM as follows (He et al. 2019a):

$$Y(\mathbf{x}) \cong \sum_{j=1}^{M} Y\left(\boldsymbol{\mu}_{-(j)}, x_j\right) - (M-1)Y(\boldsymbol{\mu}) \tag{14}$$

where $\boldsymbol{\mu}_{-(\bullet)}$ presents the vector of reference point without the element "$\bullet$."

For the estimation of fractional statistical moments, Zhang and Pandey (2013) proposed the multiplicative UDRM to approximate multivariate functions. The method is performed as follows:
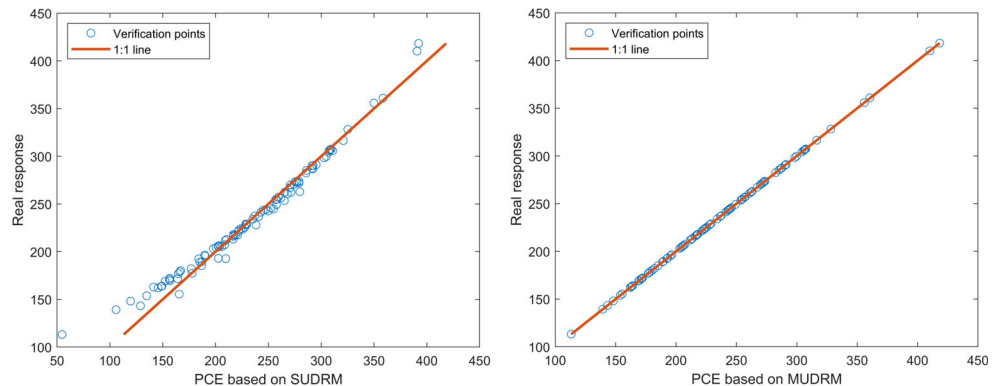
Step 1:   Transform $Y(\mathbf{x})$ into the logarithmic form:

$$T(\mathbf{x}) = \ln[Y(\mathbf{x})] \tag{15}$$

Step 2:   Approximate $T(\mathbf{x})$ by UDRM,

$$T(\mathbf{x}) \cong \sum_{j=1}^{M} T\left(\boldsymbol{\mu}_{-(j)}, x_j\right) - (M-1)T(\boldsymbol{\mu}) \tag{16}$$

Step 3:   Perform exponential transformation on Eq. (16),

$$Y(\mathbf{x}) = \exp[T(\mathbf{x})] \cong \frac{\prod_{j=1}^{M} Y\left(\boldsymbol{\mu}_{-(j)}, x_j\right)}{[Y(\boldsymbol{\mu})]^{M-1}} \tag{17}$$

Thus, the approximation of $Y(\mathbf{x})$ can be obtained by the multiplication of the univariate components. In order to avoid confusion, we name Eq. (14) as SUDRM and Eq. (17) as MUDRM and take UDRM as a general term for SUDRM and MUDRM.

4 The proposed meta-model based on polynomial chaos expansion, univariate dimension-reduction method, and entropy

In this paper, the UDRM is used to solve the problem of curse of dimension in PCE. Instead of fitting the original multivariate function, we approximate the univariate components by PCE, namely:

$$Y\left(\boldsymbol{\mu}_{-(j)}, x_j\right) = \sum_{i=0}^{P-1} c_{j,i}\phi_{j,i}(\zeta_j) \tag{18}$$

Then, the original multivariate function can be recovered by the PCEs based on SUDRM and MUDRM. Generally speaking, it is easier and more efficient to fit a univariate function than a multivariate one. Therefore, the PCE based on UDRM needs less computational cost compared with the PCE of original function. If N_i samples are used to construct PCE of the ith univariate component, the total function evaluations will be

$$\text{Function evaluations} = \sum_{i=1}^{M} N_i \tag{19}$$

Fig. 3 PCEs of SUDRM and MUDRM in case of $d = 20$ and $q = 1$ (In this paper, 1:1 line is the line with the slope of 1 and the intercept of 0 for short.). **a** PCE based on SUDRM. **b** PCE based on MUDRM

(a) PCE based on SUDRM

(b) PCE based on MUDRM

Fig. 4 PCEs of SUDRM and MUDRM in case of $d = 20$ and $q = 2$. **a** PCE based on SUDRM. **b** PCE based on MUDRM

(a) PCE based on SUDRM

(b) PCE based on MUDRM
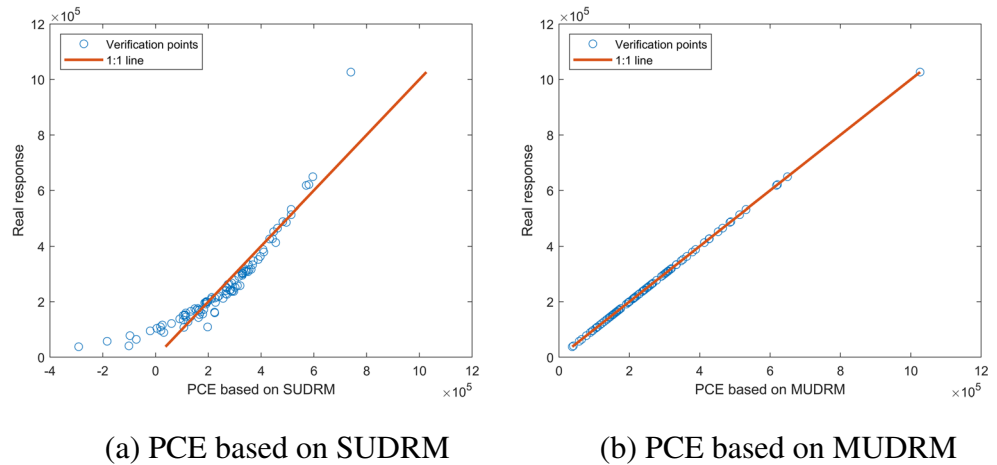
4.1 An adaptive method for the selection of polynomial chaos basis

Before the calculation of the coefficients, the degree of PCE, p, should be determined. Without the prior of complexity of the univariate function, a large p should be given for a good approximation. However, when the LSA method is used to calculate the PCE coefficients, a larger p means that more coefficients need to be calculated, that is, more experiment designs are required. Therefore, to enhance the efficiency of the meta-model, an adaptive method based on the concept of information-theoretic entropy is proposed to select the polynomial chaos basis without specifying a prior of p.

4.1.1 The concept of information-theoretic entropy

Shannon (1948) first proposed the concept of information-theoretic entropy as follows:

$$H(p_1, \ p_2, \ \ldots, \ p_d) = -h \sum_{i=1}^{d} p_i \ln p_i \qquad (20)$$

where h is a constant with positive support and p_i are the probability mass function of a discrete random variable. The

information-theoretic entropy is usually used to measure the uncertainty of a probabilistic system and widely applied in the fields of mechanic, signal process and statistical inference. On the basis of Eq. (20), Jaynes (1957) proposed the maximum entropy principle (MEP): out of all candidate distributions, one should choose the distribution that maximizes H. If no other information is available in addition to the axiom of unit measure, the distribution is $p_1 = p_2 = \ldots = p_d = 1/d$ according to MEP, which means that one can only give the most unbiased estimation for maximizing the uncertainty of the system. When some information of p_i, such as the response of real physical model at the experimental designs, is available, the formulation of MEP is expressed as

$$
\begin{aligned}
\text{find :} \quad & p_1, \ p_2, \ \ldots, \ p_d \\
\text{maximize :} \quad & H(p_1, \ p_2, \ \ldots, \ p_d) = -h \sum_{i=1}^{d} p_i \ln p_i \\
\text{s.t. :} \quad & \sum_{i=1}^{d} p_i g_k(\mathbf{x}_i) = m_k \quad k = 1, 2, \ldots
\end{aligned}
\qquad (21)
$$

where g_k are arbitrary functions of random vector $\mathbf{x}$ and m_k is the expectation of g_k. It has been proven that H is a convex function with respect to p_i, $i = 1, 2, \ldots, d$. Therefore, the unique maximum of H can be obtained if the stationary point is found.

Fig. 5 PCEs of SUDRM and MUDRM in case of $d = 20$ and $q = 3$. **a** PCE based on SUDRM. **b** PCE based on MUDRM
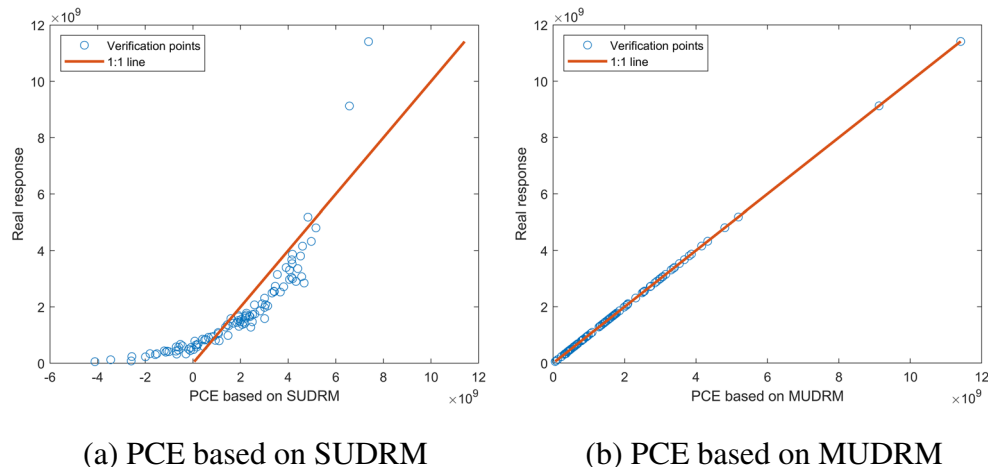
(a) PCE based on SUDRM

(b) PCE based on MUDRM

Table 2 R^2 of PCEs based on SUDRM and MUDRM in the three cases

d	q	PCE based on SUDRM	PCE based on MUDRM
20	1	0.9012	0.9999
20	2	0.8479	0.9999
20	3	0.4938	0.9999

4.1.2 An adaptive method for the selection of the polynomial chaos basis

PCE consists of the unknown expansion coefficients and the given polynomial chaos basis; therefore, characterizing the uncertainty of the response (the PDF of the response) is equivalent to evaluating the PCE coefficients (Cheng et al. 2019), that is, the PCE coefficients determine the probabilistic information of the predicted response to some extent. From Eq. (2), it can be concluded that the PCE converges to the interested random variable when p is large enough, namely, $p \geq t$. As we know, different values of p correspond to different expansion coefficients. Before the PCE converges, these different expansion coefficients result in different probabilistic information of the predicted response. While after the PCE converges, these coefficients provide the same probabilistic information of the predicted response because the convergent PCEs represent the information of real response. If the information-theoretic entropy is used to characterize the probabilistic information of the prediction response, it can be concluded that the entropy changes obviously with respect to p before the PCE converges but almost keeps constant after convergence. In this section, an adaptive method is proposed for selecting the polynomial chaos basis for the univariate components using the entropy as the convergence criterion, without a prior of the degree of PCE. Thus, we can obtain t and select the polynomial chaos basis for the univariate components adaptively.
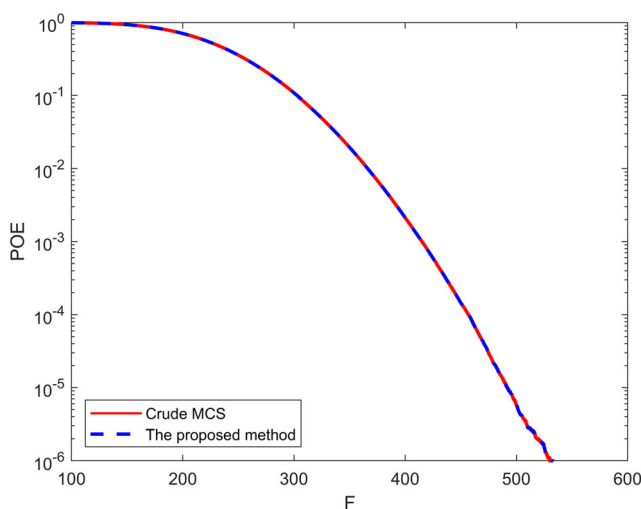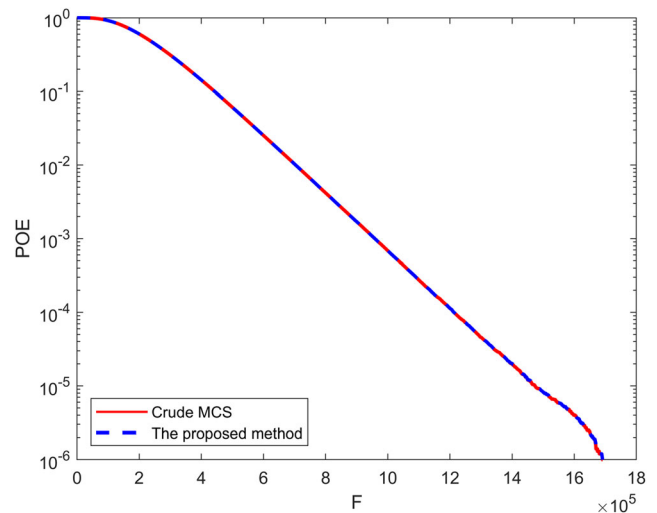

Fig. 7 Comparison of the POEs ($d = 20$, $q = 2$)

Firstly, assume p as a small integer, such as $p = 1$, for constructing the PCE of the univariate component. The expansion coefficients, $\mathbf{c}$, can be calculated by projection method or LSA method. Then, the realizations of the univariate component estimated by the PCE can be obtained by the samples of ζ; therefore, the entropy can be calculated via Eq. (20), noted as H_1. Secondly, assume $p = 2$, then H_2 can be obtained. Similarly, perform the procedure step by step until the differences of the entropy of successive three p, $p = t - 1$, $p = t$, and $p = t + 1$, are smaller than ξ, namely,

$$\begin{aligned} H_t - H_{t-1} &< \xi \\ H_{t+1} - H_t &< \xi \end{aligned} \tag{22}$$

from which it can be concluded that the PCE converges the real model when $p \geq t$.

It should be pointed out that when Eq. (22) holds, the stationary point of $H(p_1, p_2, \ldots, p_d)$ is then found and $H_{\max} = H_t$ can be obtained according to the property of convex function.
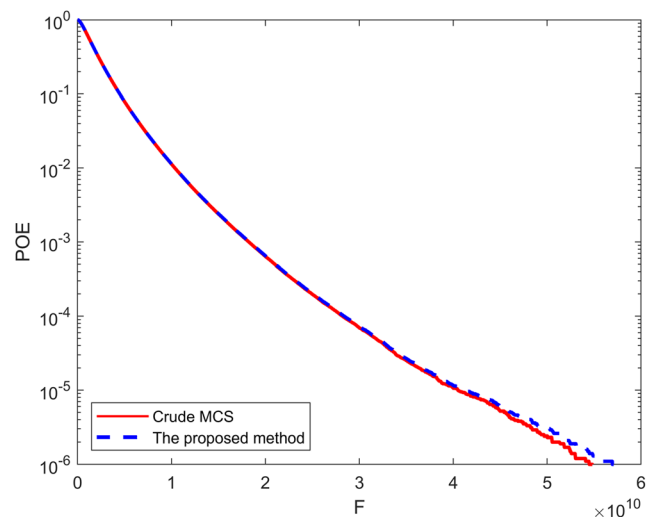

Fig. 6 Comparison of the POEs ($d = 20$, $q = 1$)

Fig. 8 Comparison of the POEs ($d = 20$, $q = 3$)

Table 3 Results of reliability analysis from different methods

d	q	Methods	P_f	Relative error	Function evaluations	Failure region
20	1	Crude MCS	1.45×10^{-4}	–	10^7	
		Full PCE	1.38×10^{-4}	4.83%	1800	$F > 450$
		Sparse PCE	1.39×10^{-4}	4.14%	1700	
		Proposed method	1.42×10^{-4}	2.07%	160	
20	2	Crude MCS	6.87×10^{-4}	–	10^7	
		Full PCE	–	–	–	$F > 10^6$
		Sparse PCE	–	–	–	
		Proposed method	6.89×10^{-4}	0.29%	200	
20	3	Crude MCS	7.03×10^{-5}	–	10^7	
		Full PCE	–	–	–	$F > 3 \times 10^{10}$
		Sparse PCE	–	–	–	
		Proposed method	7.17×10^{-5}	2.00%	240	

It means that after the PCE converges, the final result that we obtained in all possible candidates maximizes the entropy. Correspondingly, univariate component can be approximated by the following truncated PCE:

$$Y\left(\mathbf{\mu}_{-(j)}, x_j\right) = \sum_{i=0}^{t} c_{j,i}\phi_{j,i}(\zeta_j) \tag{23}$$

Thus, the proposed meta-model is finally constructed as follows:

$$Y(\mathbf{x}) \cong \widehat{Y} = \sum_{j=1}^{M} \sum_{i=0}^{t} c_{j,i}\phi_{j,i}(\zeta_j) - (M-1)Y(\mathbf{\mu}) \tag{24}$$

and

$$Y(\mathbf{x}) \cong \widehat{Y} = \frac{\prod_{j=1}^{M} \sum_{i=0}^{t} c_{j,i}\phi_{j,i}(\zeta_j)}{[Y(\mathbf{\mu})]^{M-1}} \tag{25}$$

from which the structural reliability analysis can be achieved by MCS efficiently. If $Y(\mathbf{x})$ is dominantly additive, e.g., $Y = x_1 + x_2$, Eq. (24) (the PCE based on SUDRM) can approximate the original function with good accuracy. If $Y(\mathbf{x})$ is dominantly multiplicative, e.g., $Y = x_1 x_2$, Eq. (25) (the PCE based on MUDRM) should be employed. This paper selects SUDRM or MUDRM based on the coefficient of determination with the following expression:

$$R^2 = 1 - \frac{\sum_i \left(Y_i - \widehat{Y}_i\right)^2}{\sum_i \left(Y_i - \overline{Y}\right)^2} \tag{26}$$

where Y_i is real response at the ith verification sample point, $\overline{Y}$ is the mean value of the real responses at all verification sample points, and $\widehat{Y}_i$ is the prediction of PCE at the ith verification sample point. The flowchart of the proposed method is shown in Fig. 1.

5 Examples

In this section, three numerical examples and one engineering example are tested to verify the performance of the proposed method. For comparison, four methods, including crude MCS, full PCE, sparse PCE based on the least angle regression, and the proposed method, are used to analyze each example. The PCE coefficients are calculated by the LSA method due to its flexibility of strategy of sample augment. According to Wang et al. (2018), the number of sample points, N_s, for building the PCE models is twice the number of coefficients. The accuracy of each method is assessed by relative error as follows:

$$\text{Relative error} = \frac{|r_e - r|}{r} \tag{27}$$

where r is the reference solution from crude MCS and r_e represents the results from other methods. Moreover, according to Marelli and Sudret (2015), the Stieltjes procedure is better than
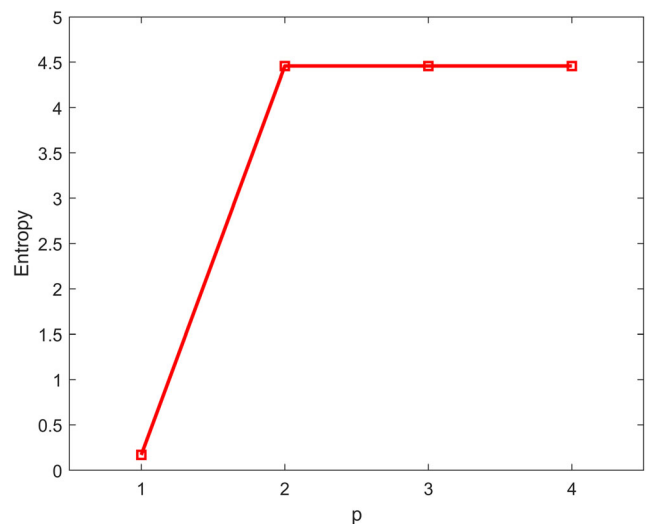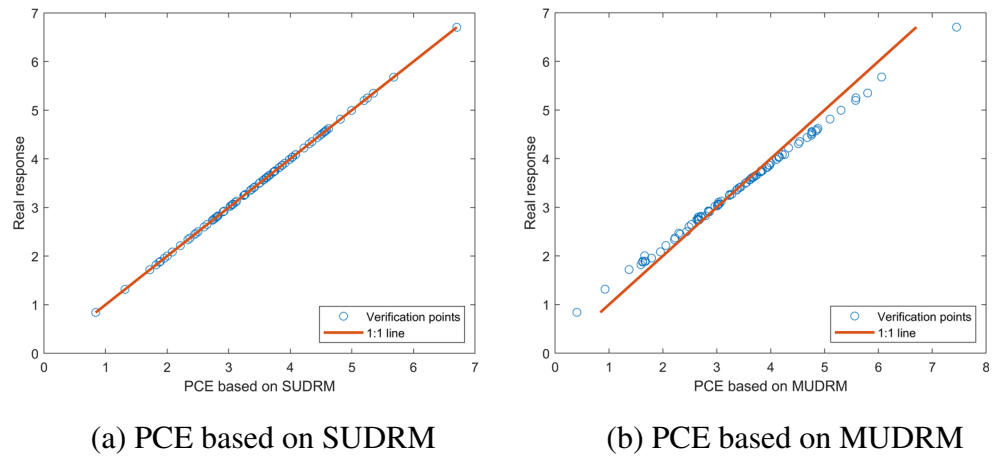

Fig. 9 The convergence history of univariate component of X_1

Fig. 10 PCEs of SUDRM and MUDRM ($d = 50$). **a** PCE-based on SUDRM. **b** PCE-based on MUDRM

(a) PCE based on SUDRM   (b) PCE based on MUDRM

Nataf transformation for the problems with non-classical random variables; therefore, we will employ the Stieltjes procedure for the non-classical cases.

5.1 Example 1: high-dimensional problem with interaction terms

A non-smooth and non-monotonous g-function (Saltelli and Sobol 1995) is first employed to verify the performance of the proposed method for the high-dimensional problem with interaction terms, whose expression is given as:

$$F(\mathbf{x}) = \prod_{i=1}^{d} \frac{|4x_i - 2|^q + a_i}{1 + a_i} \qquad (28)$$

where $a_i = 0.5(i - 2)$ and x_i's independently and identically follow Gumbel distribution with the mean value of 1 and the standard deviation of 0.05. Herein, we set $d = 20$, and examine the three cases of $q = 1, 2$, and 3. It can be seen that the effects of the interaction terms as well as the nonlinearity of the function become more significant with the increase of q. Therefore, this

function is a challenging test, due to the complex interaction terms and the presence of the absolute value which prevents the spectral convergence of the PCE (Crestaux et al. 2009).

5.1.1 The construction of the proposed meta-model

In the light of the flowchart in Fig. 1, the univariate components of the original function, namely, $F\left(\boldsymbol{\mu}_{(-i)}, x_i\right) = \frac{|4x_i - 2|^q + a_i}{1 + a_i} \prod_{j=1, j \neq i}^{d} \frac{|4\mu_j - 2|^q + a_j}{1 + a_j}$, $i = 1, 2, \ldots d$, are derived. Because the random variables in this example are not the classical cases in Table 1, the Stieltjes procedure is used to construct the polynomial chaos bases. For the three cases of $(d = 20, q = 1)$, $(d = 20, q = 2)$, and $(d = 20, q = 3)$, the convergence histories of the univariate component of X_1 are shown in Fig. 2. It can be seen that the proposed adaptive method converges after three, four, and five iterations respectively for three cases with different nonlinear degrees. Such a good convergence property facilitates the efficient reliability analysis, which will be seen in the next subsection. Repeating the
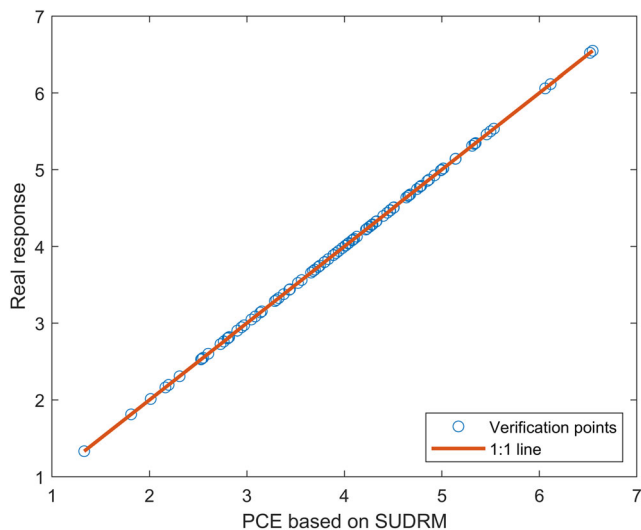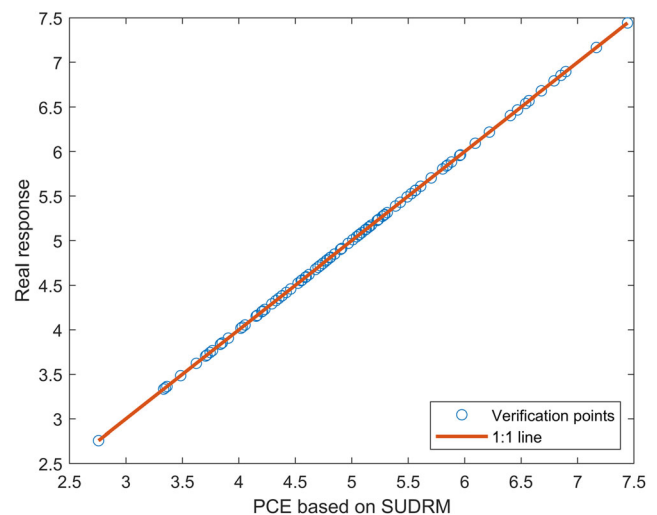

**Fig. 11** PCE based on SUDRM ($d = 100$)


Fig. 12 PCE based on SUDRM ($d = 200$)

Table 4 Results of reliability analysis from different methods

d	Methods	P_f	Relative error	Function evaluations	Failure region
50	Crude MCS	2.60×10^{-4}	–	10^7	
	Full PCE	–	–	–	$g < 0$
	Sparse PCE	2.55×10^{-4}	1.92%	450	
	Proposed method	2.66×10^{-4}	2.31%	300	
100	Crude MCS	3.96×10^{-5}	–	10^7	
	Full PCE	–	–	–	$g < 0$
	Sparse PCE	4.00×10^{-5}	1.01%	1000	
	Proposed method	4.05×10^{-5}	2.27%	600	
200	Crude MCS	4.58×10^{-5}	–	10^7	
	Full PCE	–	–	–	$g < 1$
	Sparse PCE	–	–	–	
	Proposed method	4.44×10^{-5}	3.06%	1200	

similar steps 20 times, the PCEs of all of the univariate components can be obtained. Then, collecting PCEs of the univariate components together in the forms of Eqs. (24) and (25), the proposed meta-model can be derived. To verify the performance of the proposed meta-model, we sample 100 verification points randomly and compare the results of the proposed method with the real model in Figs. 3, 4, and 5, from which it can be concluded that the MUDRM-based meta-model is superior to the SUDRM-based one. According to Eq. (26), the superiority of MUDRM-based meta-model can be quantified by R^2 listed in Table 2, from which it can be concluded that the original function is dominantly multiplicative. The results show that the proposed method can provide a good approximation for the high-dimensional problem with interaction terms. Thus, the PCE based on MUDRM will be used in the following reliability analysis.

5.1.2 Reliability analysis

With the aid of the proposed meta-model, the uncertainty of the original function can be quantified. As shown in Figs. 6, 7, and 8, the proposed method can recover the POE (probability of exceedance) curves accurately for the three cases. For comparison, the reliability analysis results from different methods are listed in Table 3. For the case of $q = 1$, each method can calculate the failure probability accurately, but the proposed method is the most efficient, whose number of function evaluations is less than one tenth of that of full PCE and sparse PCE. For the cases of $q = 2$ and 3, full PCE and sparse PCE fail to complete the construction of the meta-model because the original function has complex interaction terms and high nonlinearity. By contrast, the proposed method can accurately evaluate the failure probabilities, whose relative errors are
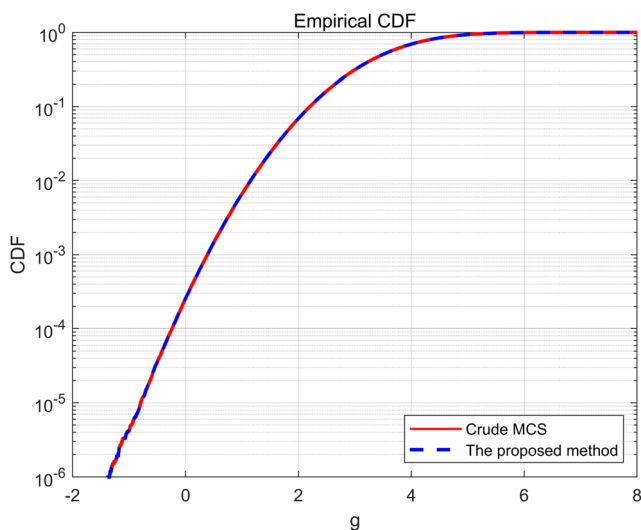
Fig. 13 Comparison of the CDFs ($d = 50$)
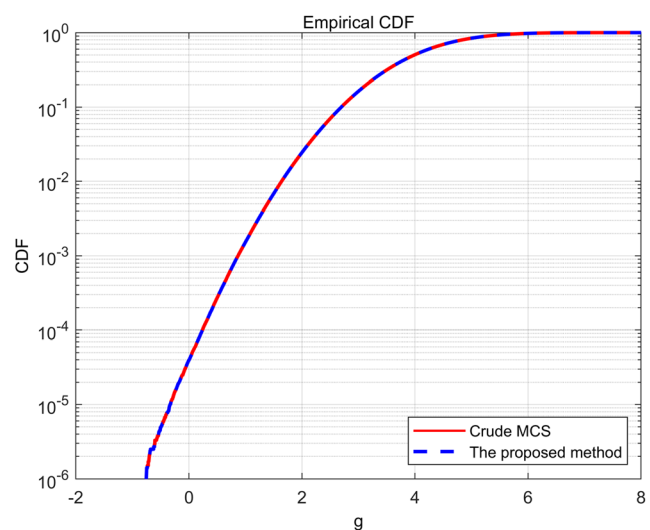
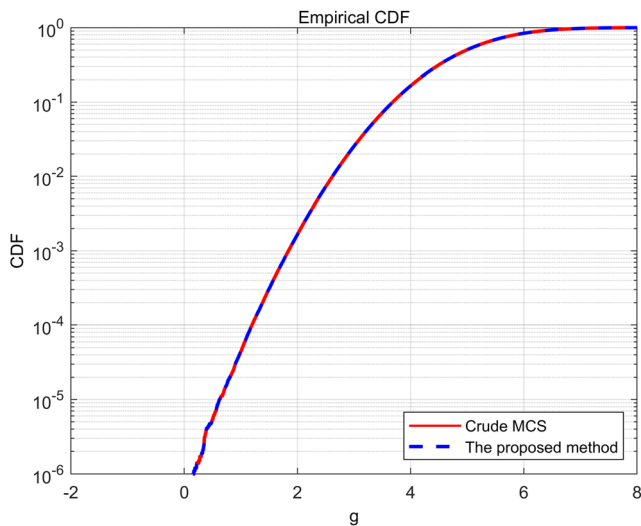**Fig. 14** Comparison of the CDFs ($d = 100$)

Fig. 15 Comparison of the CDFs ($d = 200$)

0.29% and 2.00%, respectively. According to the convergence histories, we can also calculate the number of function evaluations, namely, $5 \times 2 \times 20 = 200$ and $6 \times 2 \times 20 = 240$. It is quite efficient that the proposed method can achieve accurate reliability analysis for the complex problem with so low computational cost. Therefore, it can be concluded that the proposed method can predict the failure probability accurately and efficiently for the high-dimensional problem with interaction terms.

5.2 Example 2: high-dimensional and nonlinear case

The second example is a classic high-dimensional case for verifying the performance of reliability analysis method (Xu and Kong 2018). The response function is expressed as

$$g = 3 - X_d + 0.01 \sum_{i=1}^{d-1} X_i^2 \qquad (29)$$

where d is the dimension of the input random variables and X_i follow the independent standard normal distributions. Herein, three cases of $d = 50$, 100, and 200 are tested with the failure regions of $g < 0$, $g < 0$, and $g < 1$, respectively.

5.2.1 The construction of the proposed meta-model

Similarly, the PCEs of the univariate components are derived firstly. Limited by space, the convergence history of the univariate component of X_1 is given for illustration. As shown in Fig. 9, four iterations are enough to obtain the accurate results. For the case of $d = 50$, repeat the adaptive procedure for the selection of polynomial basis 50 times, and then all the PCEs of univariate components can be obtained step by step. Subsequently, the proposed meta-model can be derived via Eqs. (24) and (25). To verify the performance of the proposed meta-model, 100 verification points are depicted in Fig. 10,

from which it can be concluded that the SUDRM-based meta-model is superior to the MUDRM-based one. Furthermore, the performances of the two models are assessed by R^2. The R^2 of the SUDRM-based meta-model is 1 and the other one is 0.98, that is, the original function is dominantly additive, Therefore, the SUDRM-based meta-model is adopted for reliability analysis. For the cases of $d = 100$ and 200, the proposed meta-models can be built in the same way. Figures 11 and 12 show that the proposed method is accurate for high-dimensional problem and robust with respect to the dimension of the input random variables.

5.2.2 Reliability analysis

In the reliability analysis of high-dimensional problems, we focus on not only the accuracy but also the efficiency, because the curse of dimension blocks the application of many common methods in high-dimensional problems. For example, if the full PCE with the degree of three is used for the case of $d = 50$, $\frac{2(3+50)!}{3!50!} = 46852$ sample points will be required, which is computationally prohibitive. From Table 4, it can be seen that accurate results can be obtained for the cases of $d = 50$ and 100 when sparse PCE method is used for the high-dimensional problem. However, sparse PCE fails to achieve the reliability analysis in the case of $d = 200$, that is, it is still limited by the dimension of the input random variables. Compared with the sparse PCE method, the proposed method can estimate the failure probability accurately with 300, 600, and 1200 function evaluations. Obviously, it is fairly efficient that the proposed method can accurately predict such small failure probabilities, i.e., the order of 10^{-4}–10^{-6}, with so low computational cost. Again, Figs. 13, 14, and 15 show the good performance of the proposed method for high-dimensional problems, especially for capturing the tail of the CDF. Therefore,
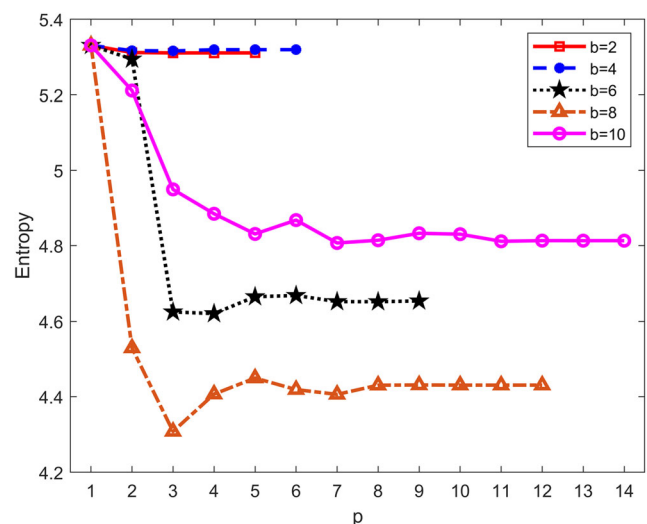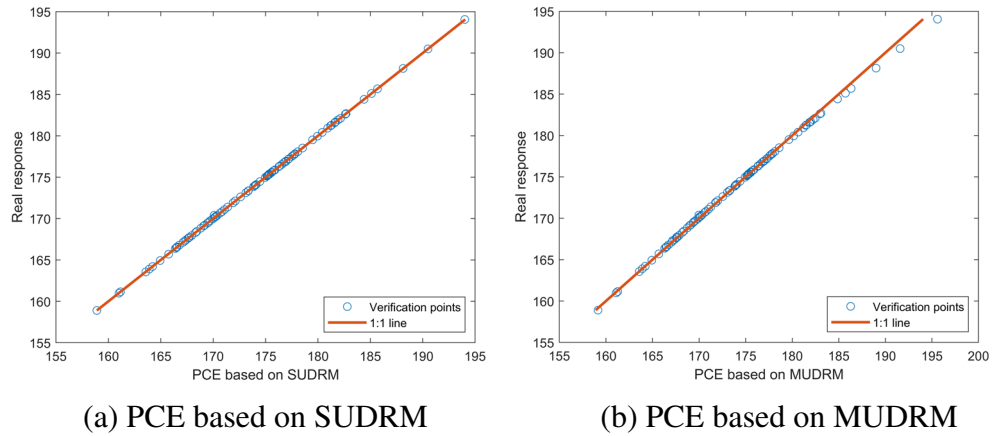

Fig. 16 The convergence histories of univariate component X_1

Fig. 17 PCEs of SUDRM and MUDRM ($b = 10$). **a** PCE based on SUDRM. **b** PCE based on MUDRM

(a) PCE based on SUDRM   (b) PCE based on MUDRM

for high-dimensional problems, the proposed method can be used to predict the failure probability with good accuracy and efficiency.

5.3 influence of nonlinearity on results of high-dimensional case

This example is employed to show the influence of the non-linearity of the original function on the reliability analysis results. The performance function is given as follows (Sadoughi et al. 2018):

$$Z = \frac{(X_1^2 + 1)(X_2 - 1)}{20} - \cos\left(\frac{bX_1}{2}\right) + \sum_{i=1}^{40} X_i^2 - 75 \qquad (30)$$

where X_i follows normal distributions with means of 1.5 and standard deviations of 0.3 and the parameter b represents the nonlinearity of Z along X_1.

5.3.1 The construction of the proposed meta-model

In order to discuss influence of nonlinearity of the original function, the proposed meta-models in the cases of $b = 2$, 4, 6, 8, and 10 are tested. Based on the adaptive method for selection of the polynomial basis, the PCEs of the univariate components can be obtained. Limited by length, only the convergence histories of univariate component of X_1 are presented, as shown in Fig. 16, from which it can be seen that the iteration history will be prolonged with the increase of the nonlinearity of the original function. It should be noted that

Table 5 Results of reliability analysis from different methods

| b	Methods	P_f	Relative error	Function evaluations
2	Crude MCS	3.72×10^{-4}	–	10^7
	Full PCE	–	–	–
	Sparse PCE	3.77×10^{-4}	1.34%	600
	Proposed method	3.74×10^{-4}	0.54%	322
4	Crude MCS	1.87×10^{-4}	–	10^7
	Full PCE	–	–	–
	Sparse PCE	–	–	–
	Proposed method	1.82×10^{-4}	2.67%	324
6	Crude MCS	2.52×10^{-4}	–	10^7
	Full PCE	–	–	–
	Sparse PCE	–	–	–
	Proposed method	2.53×10^{-4}	0.40%	330
8	Crude MCS	4.37×10^{-4}	–	10^7
	Full PCE	–	–	–
	Sparse PCE	–	–	–
	Proposed method	4.38×10^{-4}	0.23%	336
10	Crude MCS	4.50×10^{-4}	–	10^7
	Full PCE	–	–	–
	Sparse PCE	–	–	–
	Proposed method	4.52×10^{-4}	0.44%	340
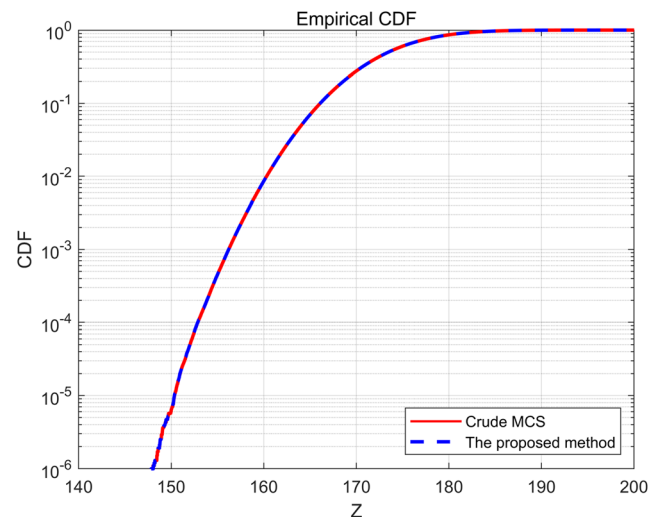
Fig. 18 Comparison of the CDFs ($b = 10$)

Fig. 19 Schematic view of simplified model
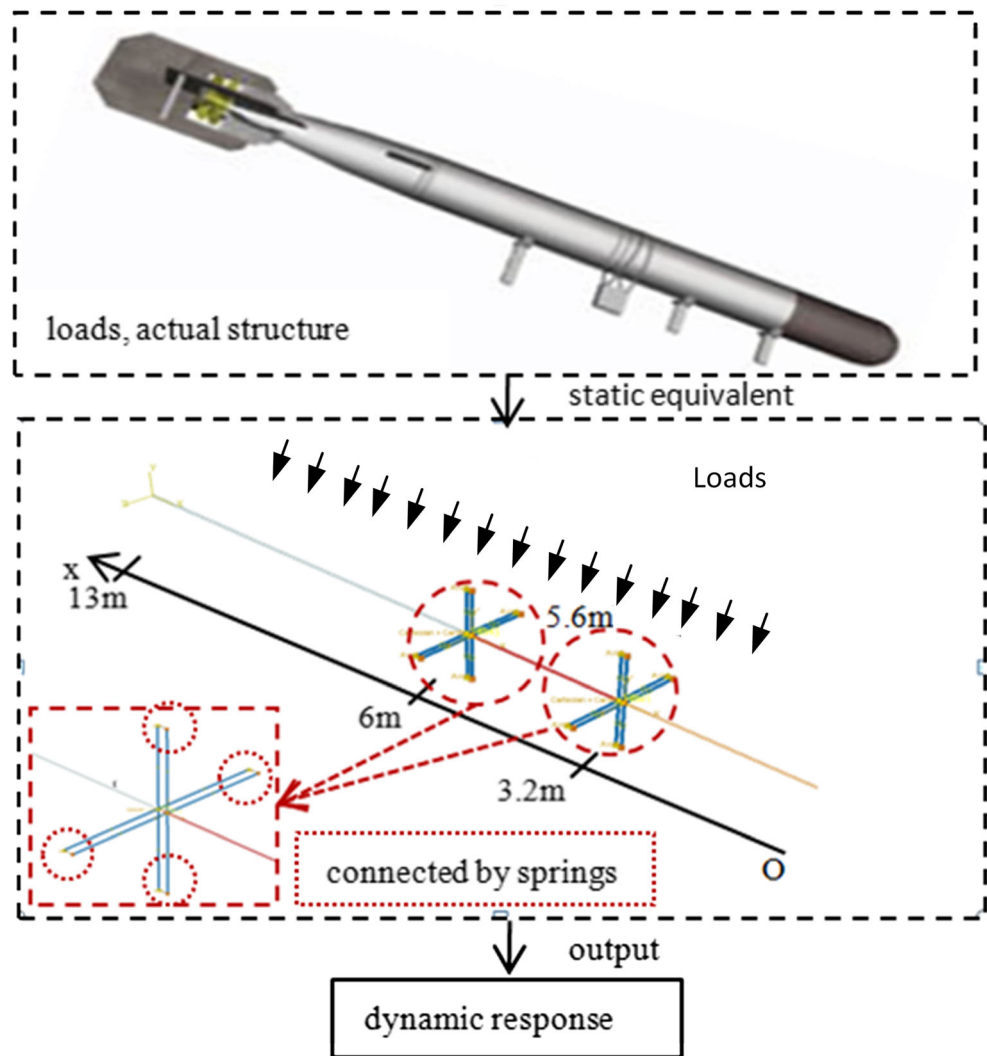
for the case of $b = 10$, $3p = 52$ experiment designs are used to fit the univariate component of X_1 due to the high nonlinearity. With the PCEs of the univariate components, the final meta-models of the five cases can be obtained. As a representative, the results of $b = 10$ are shown in Fig. 17, from which the SUDRM-based PCE approximates the original function better than the MUDRM-based one. The R^2 of the SUDRM-based PCE and the MUDRM-based PCE are also calculated by the

100 verification points, which are 0.9999 and 0.9882, respectively. Therefore, the SUDRM-based meta-model is used for the subsequent reliability analysis.

5.3.2 Reliability analysis

Table 5 lists the reliability analysis results of different methods. It can be seen that full PCE is not able to complete

Table 6 Statistical characteristics of input parameters

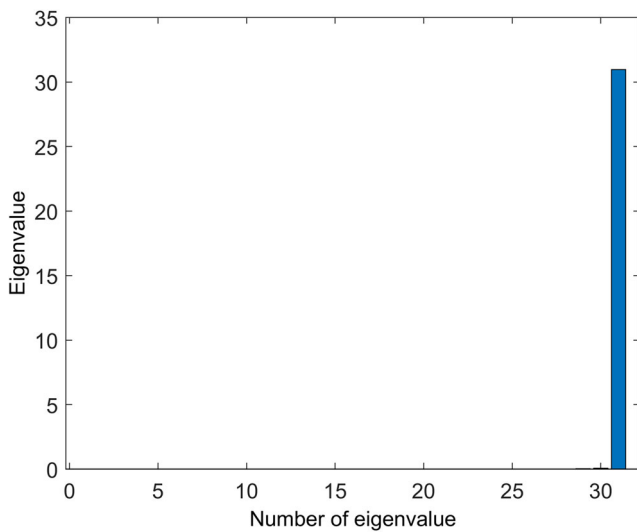| Variables	Description	Distribution	Mean	C.O.V	Autocorrelation coefficient function
K_1	Tensile stiffness at 3.2 m	Normal	3.65×10^5 N/mm	0.1	–
K_2	Tensile stiffness at 6 m	Normal	3.65×10^5 N/mm	0.1	–
K_3	Compressive stiffness at 3.2 m	Normal	3.3×10^7 N/mm	0.1	–
K_4	Compressive stiffness at 6 m	Normal	3.3×10^7 N/mm	0.1	–
L_1 - L_{15}	Random loads	Gaussian process	200 kPa	0.05	$\exp(-0.1|t_i-t_j|)$

Fig. 20 Eigen decomposition of L_i

the reliability analysis of the high-dimensional problem because of unaffordable computational cost. For the case of $b = 2$, both the sparse PCE and the proposed method can predict the failure probability accurately with the relative errors below 1%. However, the sparse PCE method pays nearly twice computational cost compared with the proposed method. For the cases of $b = 4$, 6, 8, and 10, sparse PCE method needs to select interested basis from $\frac{(p+40)}{p!40!}$ candidates, where the degree of PCE, p, increases with respect to the nonlinearity parameter, b. For example, when $b = 4$, it is rational to assume $p = 5$ based on the prior shown in Fig. 16. Thus, the total number of the candidates is 1,221,759. Therefore, the sparse PCE is excessively expensive for the high-dimensional and high-nonlinear cases. The proposed method can provide accurate predictions for the failure probabilities in different cases. It should be noted that the proposed method only employs
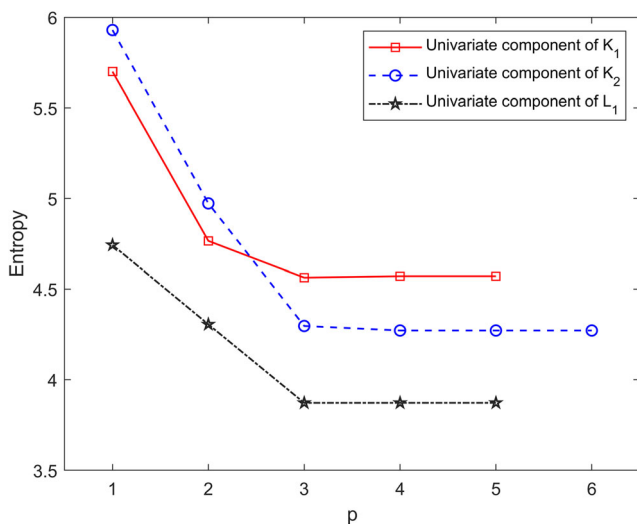

Fig. 21 The convergence histories at 0.1 s of univariate components K_1, K_2, and L_1

320–340 experiment designs to evaluate the failure probabilities with the order of 10^{-4}–10^{-5}; therefore, the proposed method is accurate and efficient for the high-dimensional and high-nonlinear cases. In order to illustrate the performance of the proposed method further, the approximated CDF is presented in Fig. 18. Because the CDFs of the five cases are similar, only the case of $b = 10$ is taken as a representative. It can be seen that the proposed method can accurately recover the probabilistic information of the stochastic response in the whole range.

5.4 Example 4: high-dimensional structural dynamics problem without explicit expression

The final example presents a high-dimensional structural dynamics case without explicit expression, which is stochastic model of the dynamic response of an underwater vehicle (He et al. 2019a). The underwater vehicle is approximately modeled by Timoshenko beam, which is 13 m long and has an annular section with the diameter of 2.11 m and the thickness of 7.11 mm. The schematic view of simplified model is shown in Fig. 19. The beam is composed of three parts, which are connected by two nonlinear structures, and subjected to the time-dependent loads from water. The loads only act on the position between 0.9 and 5.6 m and are equivalent to 15 time-dependent lateral concentrated forces, L_i, on the positions of $0.3 \times (i+3)$ m, $i = 1, \ldots,15$, which are Gaussian stochastic processes. Therefore, the reliability analysis of the structural dynamics problem is essentially a time-dependent reliability analysis problem, which can be solved by extreme value-based method (Wang and Chen 2017). Because the structural failure mode is controlled by the bending moment, the limit state function is defined as

$$Q = T_M - \max(M(0, 0.3)) \tag{31}$$

where $M(0, 0.3)$ is the bending moment in 0.3 s of the interested cross-section and T_M is the threshold of failure, given as 195 kN m. Due to no explicit expression for $M(0, 0.3)$, finite element analysis is employed for the calculation. The stochastic parameters in this example are listed in Table 6.
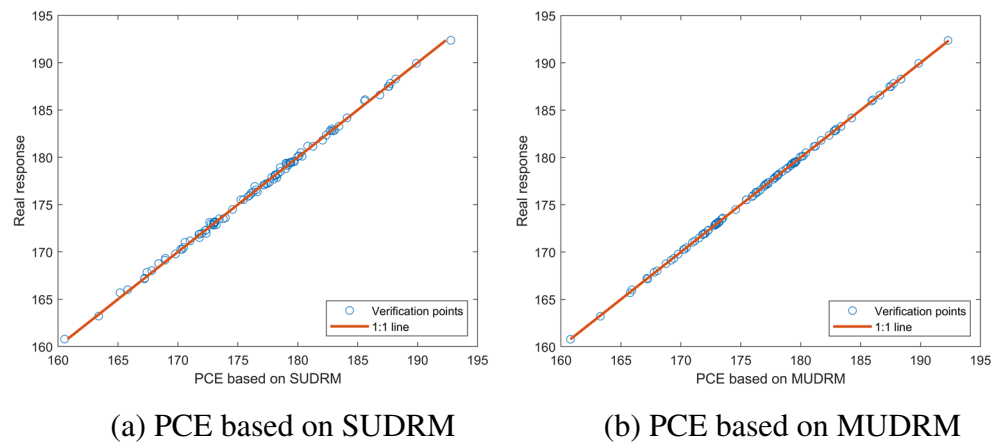
$C.O.V$ represents the coefficient of variance

5.4.1 The construction of the proposed meta-model

To solve the time-dependent problem by extreme value-based method, it is required to discretize the time interval, [0, 0.3 s], by a small time step (e.g., 0.01 s). Thus, the loads, L_i, can be approximated by the following expansion optimal linear estimation (Zhang et al. 2017):

$$L(t) \approx \mu(t) + \sigma(t) \sum_{k=1}^{d} \frac{U_k}{\sqrt{\lambda_k}} \mathbf{\Phi}_k^{\mathrm{T}} \mathbf{\Sigma} \tag{32}$$

Fig. 22 PCEs of SUDRM and MUDRM

(a) PCE based on SUDRM   (b) PCE based on MUDRM

where U_k, $k = 1, ..., d$, are the independent standard normal variables, Σ is the autocorrelation matrix, and Φ_k and λ_k are eigenvectors and eigenvalues of Σ, respectively. Figure 20 shows the result of the eigen decomposition of Σ, from which it can be concluded that each random load has only one dominated component, that is, L_i, $i = 1, 2, ..., 15$, can be simplified by 15 independent standard normal random variables. Thus, the proposed meta-model can be employed to approximate the bending moment of the interested cross-section at each time node, and then the maximum bending moment in 0.3 s can be obtained for each realization of the input random variables.

Firstly, the PCEs of the 19 univariate components at each time node are built. Figure 21 shows the convergence histories at 0.1 s of the univariate components of K_1, K_3, and L_1, from which the three univariate components can be modeled by the PCEs with $p = 4$, 5, and 4. The convergence histories of the univariate components of the K_2, K_4, and L_i, $i = 2, ..., 15$ are not presented because they are similar with the results of K_1, K_3, and L_1, respectively. Secondly, with the 19 PCEs, the meta-models at 0.1 s

can be obtained based on Eqs. (24) and (25). Repeating the above steps 300 times, the meta-models at the time nodes, $t_n = 0.01n$ ($n = 1, 2, ..., 300$) can be obtained. Finally, the proposed meta-model for the maximum bending moment can be constructed by extracting the maximum of predicted values of the 300 meta-models at each sample point. To demonstrate the accuracy of the proposed meta-models, the responses of 100 verification points are calculated, as shown in Fig. 22. Also, the coefficients of determination of the SUDRM-based meta-model and MUDRM-based meta-model are calculated by Eq. (26), say 0.9986 and 0.9999. Therefore, the MUDRM-based model is used for reliability analysis.

5.4.2 Reliability analysis

As shown in Fig. 23, the proposed method recovers the curve of POE of the maximum bending moment accurately compared with the result of crude MCS. Table 7 presents the final results of different methods, from which it can be seen that all methods can obtain the accurate prediction for the failure probability. However, considering the efficiency, the proposed method outperforms the full PCE method and the sparse PCE method. Remarkably, it can be concluded that the proposed method is a good choice for high-dimensional problems, because the proposed method employs only 194 experiment designs to estimate the failure probability accurately for the complex structural dynamics problem.
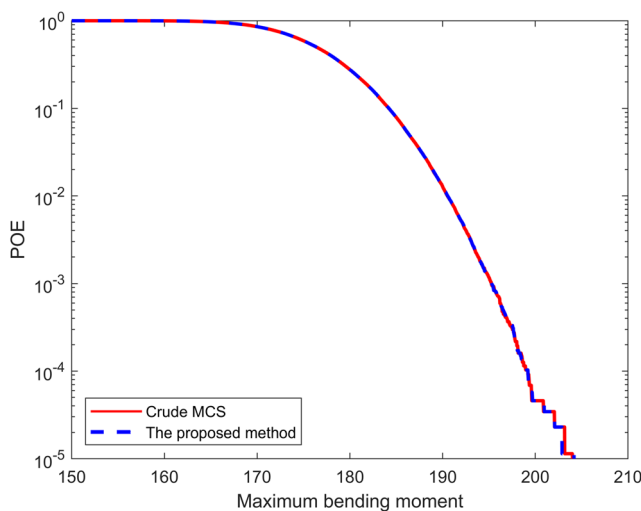

Fig. 23 Comparison of the curves of POE

Table 7 Results of reliability analysis from different methods

Methods	P_f	Relative error	Function evaluations
Crude MCS	1.18×10^{-3}	–	10^5
Full PCE	1.20×10^{-3}	1.69%	3080
Sparse PCE	1.19×10^{-3}	0.85%	350
Proposed method	1.19×10^{-3}	0.85%	194

$Q < 0$ means failure

6 Conclusion

This paper proposes a novel surrogate-based method on the basis of the concepts of polynomial chaos expansion (PCE), dimension-reduction method (DRM), and maximum entropy principle (MEP). Firstly, the univariate dimension-reduction method (UDRM) is used to decompose the original function into a series of univariate components. Then, the PCE is employed to approximate the univariate components accurately and efficiently. An adaptive method for the selection of the polynomial chaos basis is proposed to construct the PCEs of the univariate components. By collecting the PCEs of the univariate components in the form of UDRM, the proposed meta-model can be obtained. The validity of the proposed method is demonstrated by three numerical examples and one engineering example. The following conclusions can be obtained:

1. The adaptive method can select the polynomial chaos basis without the prior of the degree of the PCE. With the selected basis, the univariate components of original function can be approximated accurately by PCE.
2. With the combination of the PCE and UDRM, the proposed method can achieve the reliability analysis accurately and efficiently.
3. The common reliability analysis methods, such as full PCE and sparse PCE based on least angle regression, may be limited by the difficulty of curse of dimension, while the proposed method can solve the problem caused by high-dimensional input random variables well.

Replication of results As comprehensive implementation details are provided, we are confident that the methodology in this paper is reproducible. Therefore, no additional data and code is appended. If one is interested in the methodology and needs more help for the reproduction, please feel free to contact the corresponding author by email.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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