



Sensor placement optimization applied to laminated composite plates under vibration

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Abstract

The location optimization of sensors is an essential problem in structural health monitoring systems. Taking the cost of sensors into account, it is uneconomical to install sensors on every part of a structure and moreover in aeronautical industry, the weight is a crucial factor. In this paper, an optimal placement optimization of sensor locations for structural health monitoring systems is studied. Several techniques of optimization of sensors are approached and applied in a shell structure. The structure, a laminate of carbon fiber, was modeled by the finite element method (FEM) and then subject to free vibration. Genetic algorithms (GAs) are then employed to locate the best sensor distribution to cover a specific number of low frequency modes. Numerical results have demonstrated the overall efficiency of sensor delivery methods. Specific problems occurred, especially regarding the method of effective independence, being less efficient and discrepant in relation to the other methods employed. In summary, the results obtained in this paper provide an optimal position for sensors in real SHM systems and experiments.

Keywords Sensor placement optimization · Structural health monitoring · Genetic algorithm · Modal data · Composite plates

1 Introduction

Structural Health Monitoring (SHM) is an interdisciplinary field in engineering that addresses innovative methods of structural monitoring, integrity and performance without affecting the structure itself or harming its operation (Gomes et al. 2018). The SHM methodology uses several types of sensors to detect the presence, location, and severity of structural damage. Such technology integrates non-destructive evaluation (NDE) techniques using sensing and intelligent materials to create self-monitoring mechanisms characterized by greater reliability and long structural life. The method is applied mainly to systems with critical requirements regarding structural performance, where the classical evaluation of localized inspection is related to high

costs, difficult or even impossible in terms of operation (Stepinski et al. 2013).

According to Bandara et al. (2014), the methods that can find the changes in structural characteristics due to damage or degradation can be defined as damage detection methods. Damage detection and identification methods are defined as SHM strategies. According to Zaher (2003), a damage can be considered as an undesirable weakening of a structure that has a negative effect on its performance and affects the safety of the structural system. Damage can also be defined as any change in geometric characteristics or material properties of the structure in question, which may cause undesirable stresses, displacements or vibrations. The effects of damage on a structure can be classified as linear or non-linear. According to Farrar and Doebling (1997), a linear damage situation is defined as the case when the initially linear-elastic structure remains linear-elastic after damage. Bakhary et al. (2007) defined that non-linear damage is the case where the initially linear-elastic structure behaves non-linearly after the damage.

Many SHM techniques developed over the years are based on detecting changes in the dynamic behavior of monitored components. Valuable state-of-the-art reviews

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on SHM-based dynamics can be found in Doebling et al. (1996), Sohn et al. (2003), Staszewski et al. (2004) and Gopalakrishnan et al. (2011). Inspection techniques based on structural dynamics are typically classified as vibration-based methods and wave propagation methods. Vibration-based damage detection techniques usually monitor changes in natural frequencies, modes of vibration, and their derivatives.

Modal analysis is one of the few non-destructive methods that are technically mature enough to be used as an integrated damage detection system (Boller 2000). Local damage identification techniques, such as ultrasonic methods and X-ray methods, require, according to Fan and Qiao (2011), that the vicinity of damage is known by assumption and readily accessible for testing, which cannot be guaranteed for most cases in civil or aerospace engineering. Thus, the vibration-based harm identification method as a global harm identification technique addressed in this study is developed to overcome these difficulties (Fan and Qiao 2011).

The basic problem of damage detection comes from responses obtained through distributed sensors in structures. The quality of these responses and therefore the quality of the damage detection depends heavily on the location and number of sensors used in the structure in question. Relevant issues at cost and practicality preclude instrumentation at all positions of interest and require that there be a smaller alternative of sensor positions. For this, an ideal positioning must be found for these sensors, which is a complex optimization problem. Particularly, for a SHM system based on vibration metrics, acquiring data at a high number of points is not feasible. For this, knowing the locations that can obtain the largest amount of modal information is fundamental.

Effectively, Guo et al. (2004) presented an index of performance of optimization of the positioning of sensors based on the detection of damages using GAs to determine the location of sensors in the structures. For this, the authors fixed the number of sensors. However, GAs may result in non-viable solutions to the problem, so some improved strategies have been presented by the authors, such as code-based crossover, mutation based on two gene bits, and improved convergence. To attack the problem itself, the authors chose to maximize the Fisher Information Matrix (FIM), since this matrix is the sum of the contribution of each degree of freedom of the sensor to the structure mode shape. Results showed that the proposed improved algorithm was effective in determining optimal sensor positions based on modal information criteria.

An extremely important part of experimental dynamics testing is the placement of sensors in structures, usually in the form of accelerometers. Accelerometers must be placed in such a way that all relevant dynamic information

is obtained during the test. Kammer and Tinker (2004) studied the optimal positioning of triaxial accelerometers, presenting a new technique based on effective independence (Efi). The technique is applied and compared to standard approaches using the X-33 spacecraft.

Another aspect that draws attention is the development of a monitoring system for large scale engineering structures, since these require a large number of sensors and their positioning to be of great relevance to such structures. In this idea, Rao and Anandakumar (2007) have suggested an optimal positioning algorithm for sensors using Particle Swarm Optimization (PSO) to solve those problems in large scale structures. The proposed algorithm aimed precisely at achieving the best identification of frequencies and modal forms in those large structures. Numerical experiments were performed considering civil engineering structures to evaluate the performance of the algorithm based on proposed swarm intelligence (PSO). Numerical studies have indicated that the proposed hybrid PSO algorithm generates sensor configurations superior to conventional iterative approaches based on information that has been popularly used for large structures. In addition, the proposed hybrid PSO algorithm exhibited superior convergence characteristics when compared to other PSO counterparts.

Padula and Kincaid (1999) give an excellent survey of sensor and actuator placement in various engineering problems and the application of optimization to improve their performance. According to Ganguli et al. (2016), it is important to develop approximations (surrogate models) of the performance index and constraints and employ these computationally efficient approximations to determine the optimal sensor locations. Zhang et al. (2010) presented a method for simultaneous identification of structural damage and support excitation that is modeled using a finite series of Chebyshev polynomials; their amplitudes are treated as the optimization variables along with the stiffness modification coefficients. In the current years, Gu et al. (2016) have proposed an algorithm for the sensor layout to solve the problem that arbitrary sensor positioning affects the efficiency of data collection and reduces the accuracy of the monitoring system, on a wind turbine blade, in his studies. This algorithm integrated the advantages of the kinetic energy method, effective independence method and the modal assurance criterion (MAC). In order to avoid redundancy of information caused by partial concentration sensors, the Euclidean distance was introduced as the iteration condition of the algorithm. The simulation of finite elements and the experimental results show that the arrangement obtained by the proposed method was beneficial to improve the signal-to-noise ratio.

Zhou et al. (2017) presented a generic structure of sensor positioning, in which four methodologies were developed and implemented through the integration with an

optimization tool based on genetic algorithms. To validate the effectiveness of the developed *layout*, the authors conduct a comprehensive case study of sensor positioning for the Factor Building at the University of California, Los Angeles. The results obtained show a good improvement in the optimized positioning of sensors when compared with those initially instrumented.

Equally important, Chisari et al. (2017) proposed a novel method for optimal sensor placement based on the definition of the representatives of the data with respect to the global displacement field. The method apply an optimization procedure based on GA and allows for the assessment of any sensor layout independently from the real inverse problem solution.

The Table 1 presents a brief synthesis of some of the main work done on the methodology of optimal positioning of sensors. Specific details about the nomenclatures displayed in the Table will be given in the next section.

Finally, this bibliographic review aimed to review some of the works present in the literature related to the subject, in order to substantiate the relevance of this work that aims to study the optimally distributed sensor for signal acquisition in experimental modal analysis. It is clear that the need to seek and design efficient structural monitoring is a scientific challenge to be overcome. Vibration metrics as damage criteria are strongly employed as parameters to be diagnosed. Evolutionary computational techniques and computational intelligence are powerful tools used to aid the diagnosis of structural responses. In addition, obtaining answers in a small number of sensors without significant loss of quantity and quality of structural information is a

dilemma to be solved that demands a significant amount of work. In addition, it is observed that related research on the subject is recent and of high industrial and scientific interest across the globe.

In this study, an optimal strategy for data acquisition applied in Structural Health Monitoring system is studied. Accurate data acquisition in SHM systems depends of measurement of structural response in sensitivity points. Because of sensor errors or lack of sensitivity, the measurements may lead to erroneous estimates of the parameters. These errors can be ameliorated if the sensors are placed at points of maximum sensitivity.

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This manuscript is organized as follows: Section 2 a general bibliographic review and background about sensor placement optimization is presented, addressing the scientific interest about the subject. Section 3 displays the backgrounds about optimization applied heuristic. Section 4 the methodological procedure is presented. Section 5 presents the main results and discussion about

Table 1 Review of the parameters used in sensor positioning optimization studies (organized in chronological order)

Author	Structure Type	Algorithm	Number of Sensors	Metric
Penny et al. (1994)	Plate	–	6 and 12	Efi, MAC and SVD
Fadale et al. (1995)	Plate	–	–	FIM
Papadopoulos and Garcia (1998)	Beam and Plate	SVD	–	FIM, KE and ADPR
Shi et al. (2000)	Truss	–	20	MDLAC e FIM
Worden and Burrows (2001)	Plate	GA e SA	10 and 4	MAC, Efi e FIM
Staszewski and Worden (2001)	Plate	GA	25	FIM
Kammer and Tinker (2004)	Spacecraft X-33	–	389	Efi and FIM
Coote et al. (2005)	Helicopter Fuselage	–	20	Efi
Meo and Zumpano (2008)	Plate	GLO	10	Efi
Barthorpe and Worden (2009)	Plate	GA	17	KE, Efi, EVP, ADPR
Borissova et al. (2012)	Beam	Combination	6 a 10	Error function
Rao et al. (2014)	Beam	–	10	PCA and Efi
Fendzi et al. (2014)	Plates	GA	4 and 5	PoD
Jung et al. (2015)	Cylindrical shell	GA	10	MAC
Zhu et al. (2015)	Mechanical Mill Model	–	12	Efi
Gomes (2017)	Shell	GA	4, 6 and 9	ADPR, KE, Efi, FIM, EVP
Gu et al. (2016)	Shell	–	7	MAC

the SPO in a laminated plate. Finally Section 6 draws the conclusions.

2 Sensor placement optimization - SPO

The basic problem of fault and damage detection is to deduce the existence of damage to a structure from measurements made on distributed sensors. It is known that the quality of these measurements, that is, the quality of the structural monitoring, is largely dependent on where the sensors are located in the structure. Cost and practicality issues prevent the instrumentation of all points of interest in the structure and lead to the selection of a smaller set of measurement sites (Barthorpe and Worden 2009). The objective of this study is to indicate the problem of sensor placement optimization (SPO) and to describe some methods that have been investigated for its solution besides proposing an alternative optimization of the optimal positioning of these sensors. The following discussion focuses on sensor optimization techniques based on the dynamic structure.

Traditionally, a successful sensor distribution has been heavily dependent on the knowledge and experience of those conducting experimental tests. Practical methods, for example, by choosing sites close to anti-knots of low-frequency vibration modes, are combined to create coherent sensor distributions (Barthorpe and Worden 2009). However, for a lot of the times, a single mode of vibration does not have enough information on a damaged structural state, being necessary the use of a set of modes. Therefore, a distribution in the anti-nodes would not be feasible for a predefined number of sensors.

According to the theory discussed in Barthorpe and Worden (2009), the objective of sensor positioning can be stated as the need to select a subset of measurement locations from a large finite set of locations, so as to represent the system with the highest possible accuracy using a limited number of degrees of freedom accessible. This can be seen as a three-step decision process:

- Number of sensors - How many sensors need to be placed in the structure to allow a satisfactory dynamic test?
- Sensor positioning optimization - Where should these sensors be located to obtain more accurate data?
- Evaluation - How can the performance of different sensor configurations be measured?

In general, on the first aspect, the minimum requirement for the system to be observable is that the number of sensors required can not be less than the number of modes to be identified, with an upper limit usually imposed by the cost or availability of the equipment. The second aspect is the area that has attracted the most interest, and object of study

in this work. For the limited number of available sensors, the problem is the development of a suitable sensor positioning performance measurement to be optimized and the selection of an appropriate method. Some approaches require a single calculation to be performed, some are iterative, and many others take the form of an objective function to which an optimization technique must be applied. The third and last aspect includes several possibilities for evaluating the performance of chosen sensor sets. In this work, preferably the positioning item will be approached, where a predefined number of sensors will be fixed.

In addition, the sensor placement issue attracts a lot of attention from academia and industry, especially due to the growing number of large instrumented monitoring structures over the last decade. This is due in part to economic reasons, to the high cost of data acquisition systems (sensors and their supporting instruments), partly because of the limitations of structural accessibility (Rao et al. 2014).

The set of degrees of freedom measured in most large structures, usually the shifts of the low frequency modes, provide enough information to describe the dynamic behavior of a structural system with sufficient accuracy to allow its structural state and/or modifications determined in an effective manner. Thus, the fundamental problem is how many and which degrees of freedom must be considered in the process of structural identification. To solve this problem, economic factors that require a limited number of sensors to be placed in accessible locations in the actual structure (Rao and Anandakumar 2007) should be duly taken into account. Still, according to Rao and Anandakumar (2007), it is crucial that the sensors are located in the most advantageous locations. Otherwise incomplete modal properties will be measured and an accurate assessment of structural health monitoring will be impossible.

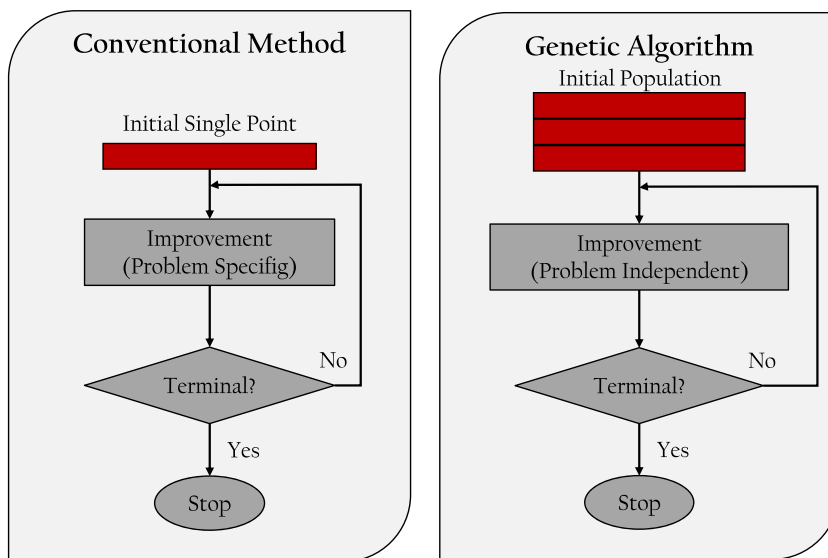
Some performance indices have been developed for the sensor distribution problem, but it is only comparatively recent that the problem was considered in the SHM perspective, according to Rao and Anandakumar (2007). In this work, some strategies were adopted to find and solve the optimal positioning of the sensors in square laminated plates. The next section discuss the main strategies used in this manuscript.

2.1 Fisher information matrix - FIM

As shown by Kammer (1991), the array of sensors can be given in the form of a estimation problem with a corresponding Fisher Information Matrix (FIM) given by (1).

$$Q = \phi_s^T W \phi_s \quad (1)$$

Fig. 1 Comparison between conventional search methods and genetic algorithms (adapted from Gopalakrishnan et al. 2011)



(a) Conventional optimization

(b) Genetic Algorithm

where W is a weighting matrix. The modal response is estimated based on the data measured by the sensors. The maximization of Q results in the minimization of the corresponding error covariance matrix, which results in the best estimate. The sensors must be placed in such a way that Q is maximized in an appropriate matrix norm. The maximization of the FIM determinant is a commonly used criterion for the estimation of optimal parameters.

Maximizing the determinant of the information matrix will maximize a combination of the spatial independence of the target modal partitions and their signal strength in the sensor data (Kammer and Tinker 2004). It has been shown that the FIM can be decomposed into the contributions of each candidate sensor location in the form shown in (2).

$$Q = \sum_{i=1}^{n_c} \phi_{si}^T \phi_{si} = \sum_{i=1}^{n_c} Q_i \tag{2}$$

being ϕ_{si} the i -th line of the mode partition array associated with the i -th candidate sensor location, n_c is the number of candidate sensors.

Then, the sensors must be placed so as to provide the best estimate of the target modal response. The maximization of the determinant of the information matrix is chosen as the criterion of positioning of the sensor, since it results in the maximization of the signal intensity and the independence of the main directions (Rao et al. 2015).

2.2 Information entropy

A potential method in the category of optimal sensor distribution, the so-called information entropy (or Shannon

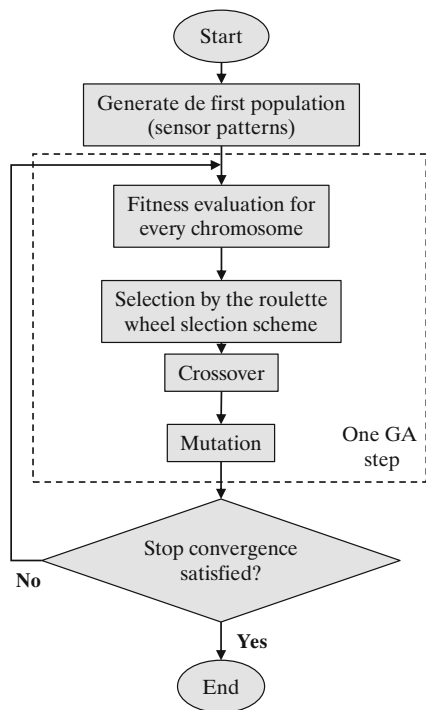


Fig. 2 Basic flowchart of an GA and its genetic operators (adapted from Beygzadeh et al. 2014)

Table 2 Properties of the material considered in this study

Property	Value
E_1	83.02 GPa
E_2	5.13 GPa
G_{12}	8.37 GPa
ν_{12}	0.32
ρ	1408.10 kg/m ³

Table 3 Applied criteria for sensor placement optimization (minimization or maximization procedure)

Method	Objective Function
Fisher Information Matrix - FIM	$J_1 = \max \det(Q) $
Information Entropy	$J_2 = \min(H(D))$
Effective Independence - Efl	$J_3 = \max(E_D)$
Kinetic Energy - KE	$J_4 = \max(KE)$
Average Driving-Point Residue - ADPR	$J_5 = \max(ADPR)$
Eigenvector Product - EVP	$J_6 = \max(EVP)$

entropy), is widely adopted to address measurement uncertainty, finding the best combination of structural tests that can minimize the negative consequence of uncertainty (Zhou et al. 2017). The importance of entropy is that it is a measure of uncertainty in parameter values, since it evaluates the disorder in predictions (Papadopoulou et al. 2014).

The optimal distribution of the sensor is obtained by minimizing the change in the information entropy $H(D)$, given by (3).

$$H(D) = E_{\theta}[-\ln p(\theta|D)] = - \int p(\theta|D) \ln p(\theta|D) d\theta \tag{3}$$

where θ is the set of uncertain parameters (for example, stiffness, modal parameters, etc.), D is the data of the dynamic tests, and E_{θ} is the mathematical expectation with respect to θ .

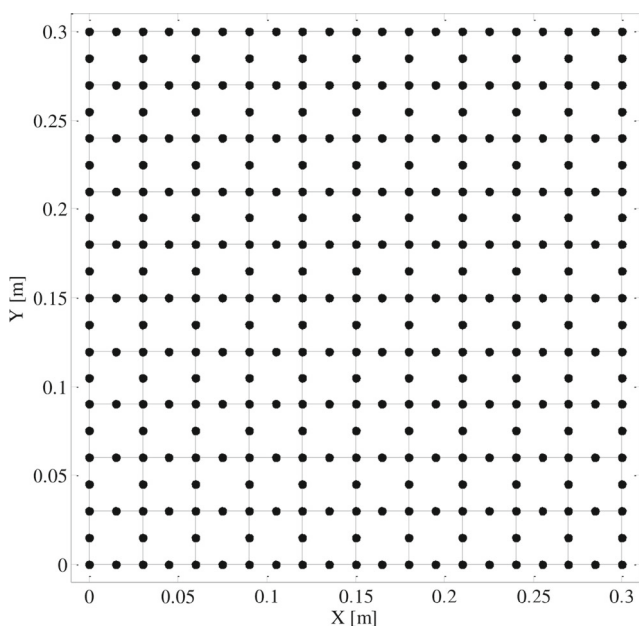


Fig. 3 Configuration of possible sensor positions

Table 4 Genetic operators used for the parameter identification problem

Genetic operator	Value
Population	$10 \times N_{var} \times N_{modes}$
Crossover	60%
Mutation	1%
Elitism	1
Generations	1000

2.3 Effective independence - Efl

The objective of the Effective Independence (Efl) method is to select the positions of the measurements that make the models of interest as linearly independent as possible, providing sufficient information about the target modal responses in the measurements. According to Zhu et al. (2015), the measured structural response can be expressed as shown in (4).

$$u = \Phi q + w = \sum_{i=1}^N q_i \phi_i + w \tag{4}$$

where Φ is the matrix containing the target vibration modes obtained by MEF, q is the response vector coefficient and also is modal coordinate, and w is a sensor noise vector, assuming the random process stationary with a mean value of zero. Assuming that this process is an unbiased estimate, the covariance matrix J is obtained in (5).

$$J = E \left[(q - \hat{q}) - (q - \hat{q})^T \right] - \left[\sigma^{-2} \Phi^T \Phi \right]^{-1} = Q^{-1} \tag{5}$$

where Q is the Fisher Information Matrix assuming that the measured noise is independent and has the same statistical properties. The Q matrix can be written as shown in (6).

$$Q = \sigma^{-2} \Phi^T \Phi = \sigma^{-2} A \tag{6}$$

Then, the maximization of Q is equivalent to the maximization of A , and thus A can be used to simplify FIM. Constructing the matrix E_D , as shown in (7).

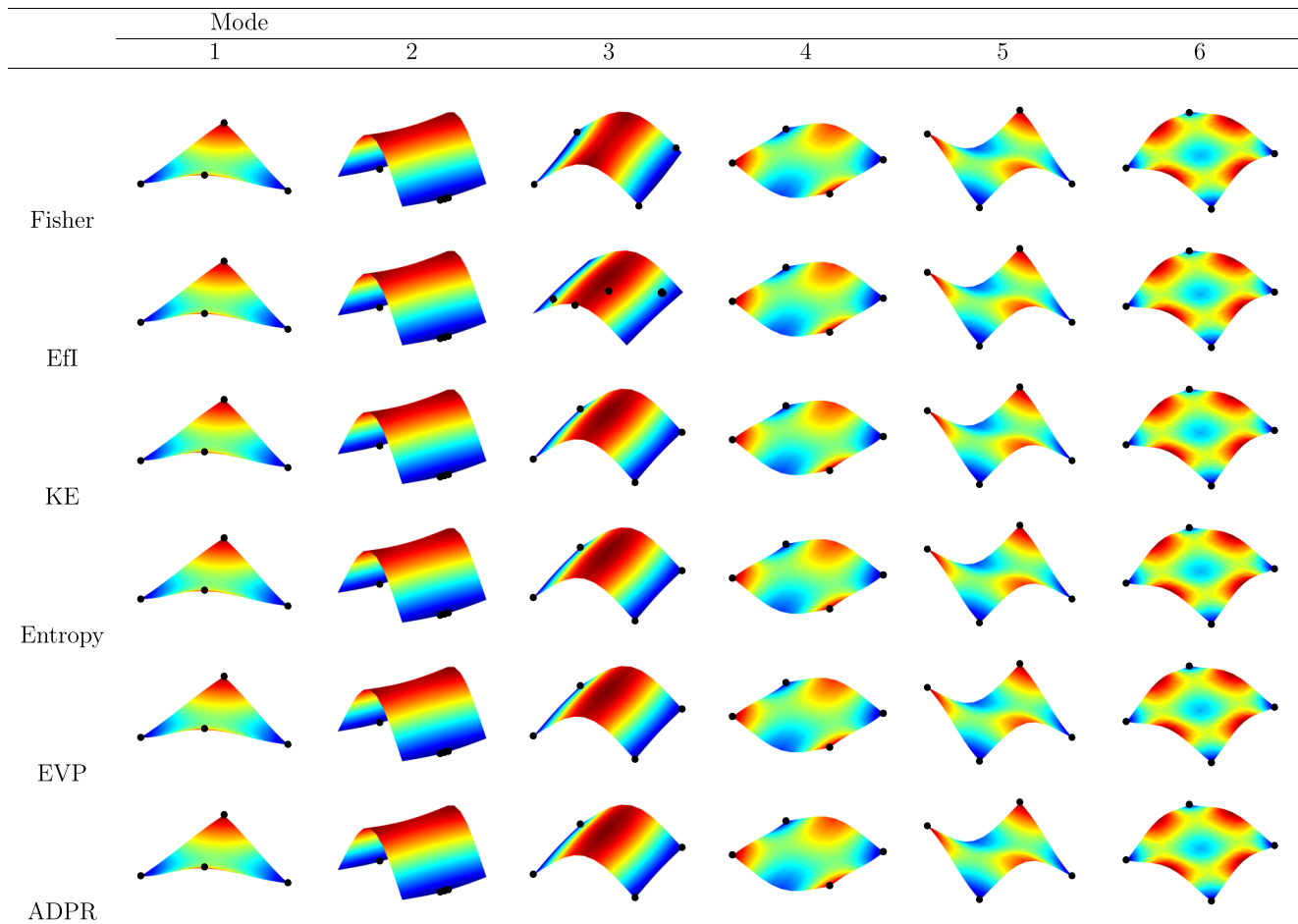
$$E_D = \Phi \left[\Phi^T \Phi \right]^{-1} \Phi^T \tag{7}$$

in which E_D is the effective independence matrix. The diagonal elements of the matrix E_D represent the contribution of the candidate points to the linearly independent modal matrix.

2.4 Kinetic energy - KE

The Kinetic Energy Method (KE) assumes that the sensors will have maximum observability of the modes of interest if the sensors are placed at maximum KE points. A procedure

Table 5 Results of optimal positioning considering 4 sensors



similar to that used in the Efi method follows. Kinetic energy is defined as shown in (8) (Barthorpe and Worden 2009).

$$KE_{ij} = \Phi_{ij} \sum_s M_{is} \Phi_{sj} \omega_j^2 \tag{8}$$

where M is the mass matrix. The locations of the sensors that offer the highest KE indexes are selected as the measurement points. As the method selects sensor locations with the highest amplitude signal, signal-to-noise ratios tend to be high, making the method attractive for use in noisy conditions. However, in contrast to Efi, the KE method does not consider the linear independence of target modes, an important consideration for modal identification.

2.5 Average driving-point residue - ADPR

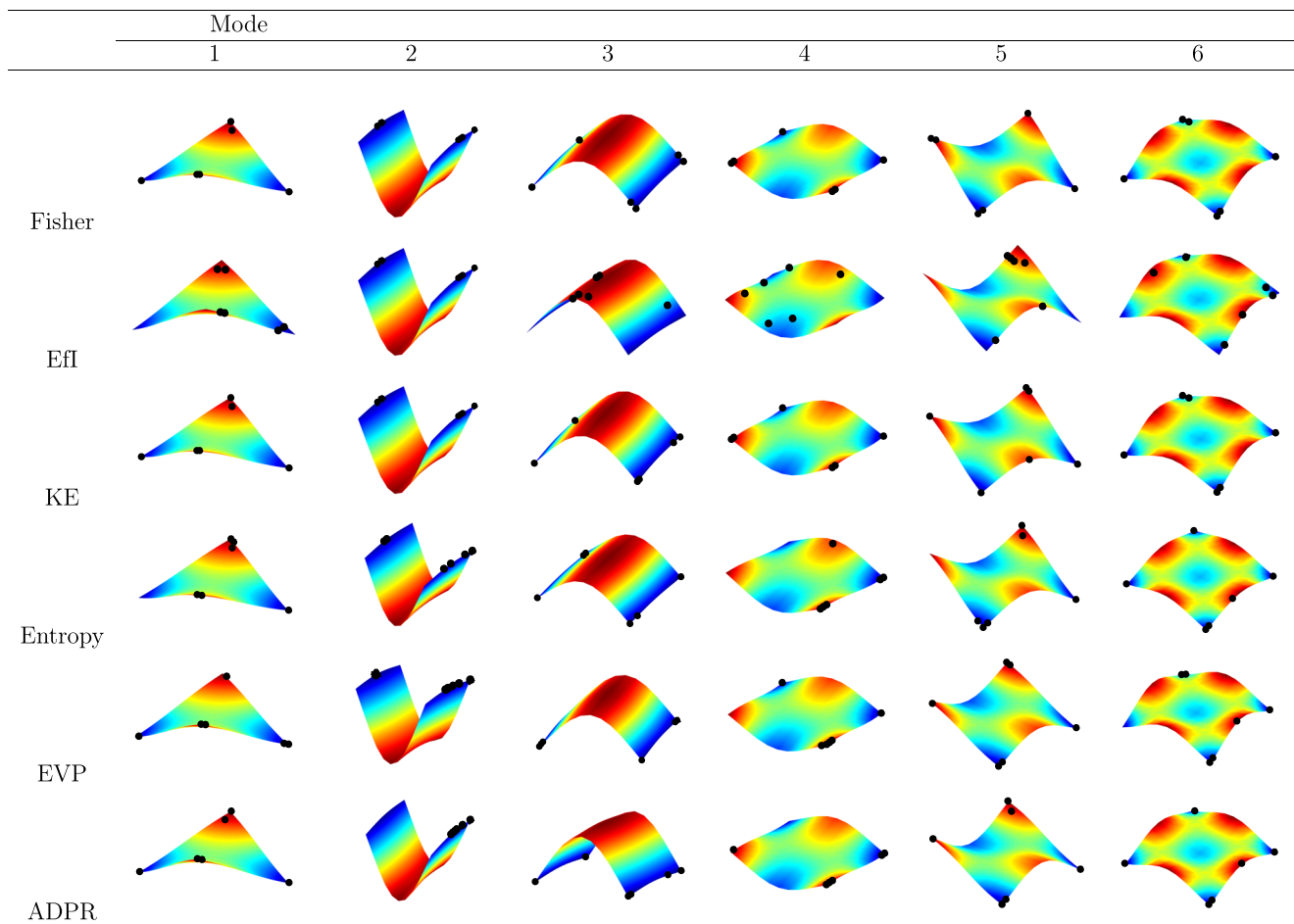
A disadvantage of the Efi approach is that the algorithm can select sensor locations that exhibit low signal strength,

making the system vulnerable to noisy conditions. The Average Driving-Point Residue (ADPR) method provides a measure of the contribution of any point (sensor location) to the global modal response. If $j = 1 \dots N$ modes of interest are to be measured and ω_j is the eigenvalue of j -th mode, the ADPR in the i -th degree of freedom can be calculated from (9) (Barthorpe and Worden 2009).

$$ADPR_i = \sum_{j=1}^N \frac{\Phi_{ij}^2}{\omega_j} \tag{9}$$

2.6 Eigenvalue vector product - EVP

The Eigenvalue Vector Product (EVP) method calculates the product of the components of the eigenvectors for the location of candidate sensors in the range of N modes to be measured: a maximum for this product is a dot optimum measurement candidate. The EVP of the i -th degree of

Table 6 Results of optimal positioning considering 6 sensors

freedom is calculated as shown in (10) (Barthorpe and Worden 2009).

$$EVP_i = \prod_{j=1}^N |\Phi_{ij}| \quad (10)$$

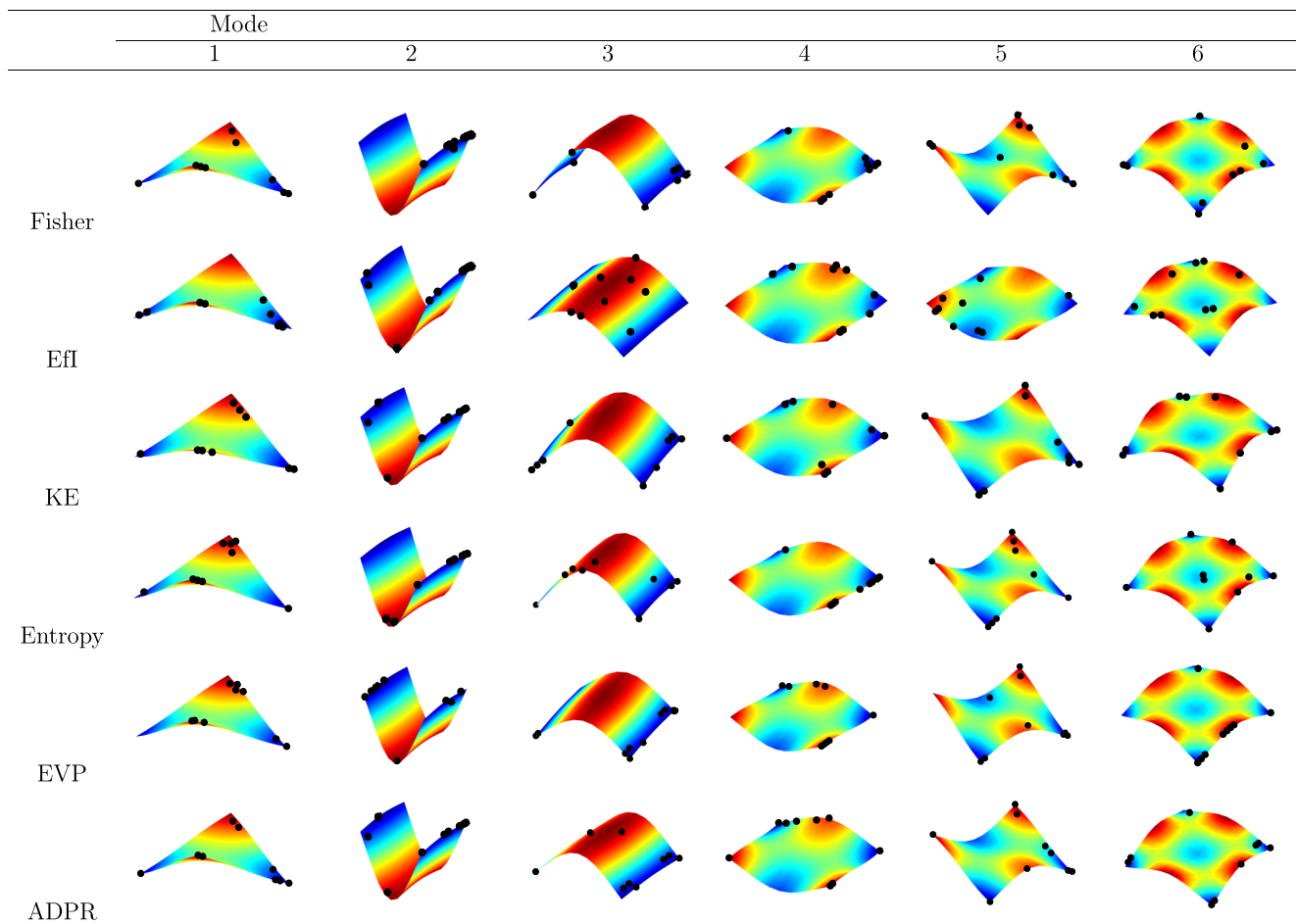
3 Genetic algorithm

According to Gopalakrishnan et al. (2011) and emphasized by Gomes et al. (2017), genetic algorithms are powerful and widely applicable methods of stochastic searches based on the principle of natural selection and natural genetics of Darwin. Conventional search and optimization techniques are generally based on derivatives, making use of the gradient of a given function to find the local minimum or maximum. These techniques depend on the problem and may work well for certain simple functions. Practical problems (such

as what we are trying to solve here) are complex in nature, and these methods can not give any satisfactory solution. The difference between the conventional search algorithms and GA is shown in Fig. 1. The main efficiency of the genetic algorithms is in their robustness, being able to find the global optimum without being stagnant in local points in multimodal functions. GAs do not require much mathematical operation for their execution and do not depend on the problem, dealing with all kinds of functions and objective constraints, linear or not. The differences between the GAs of conventional methods are summarized by Holland and Goldberg (1989).

The parental chromosomes undergo two types of genetic operations, crossover and mutation to create offspring. *Crossover* is the main genetic operation that the parent chromosomes suffer from. The performance of an GA depends to a large extent on the performance of the *crossover* operator used. It operates on one pair of chromosomes at a time and generates one or two pups retaining some characteristics of both the parent

Table 7 Results of optimal positioning considering 9 sensors



chromosomes. A crossover probability or crossover rate p_c is defined as the probability that a member of a population will be selected for such a crossover. The typical value of p_c is 0.5 to 0.8. A $p_c = 1.0$ implies that all members of a population will intersect (Gopalakrishnan et al. 2011). Higher orders of p_c lead to a better exploration of the search space of the solution and reduce the chances of obtaining a local minimum point, however, increase the computational cost. The probability of mutation p_m is defined by the probability that a single gene of the whole population will be changed. A typical value of p_m is 0.01.

The most crucial part of an GA is the selection procedure, ie the creation of a new generation from an earlier one is given by this genetic operation. Selection is the driving force for any genetic research. Selections are always based

on the fitness of a given individual. Individuals are more likely to be screened for the next generation. The fitness of an individual is usually made using the optimization function itself. The selection pressure, that is, the weight on the healthy members for selection, plays a critical role in the selection procedure. High pressure selection can lead to premature convergence. On the other hand the low selection pressure makes the convergence slow (Holland and Goldberg 1989).

The selection can be from the regular sampling space or from the expanded sampling space. Regular sampling space is usually created by replacing parents with their offspring after birth. Thus, the size of this space is always equal to the size of the population. Substitutions may also take place randomly or on the basis of their fitness values. The expanded sampling space contains the parent

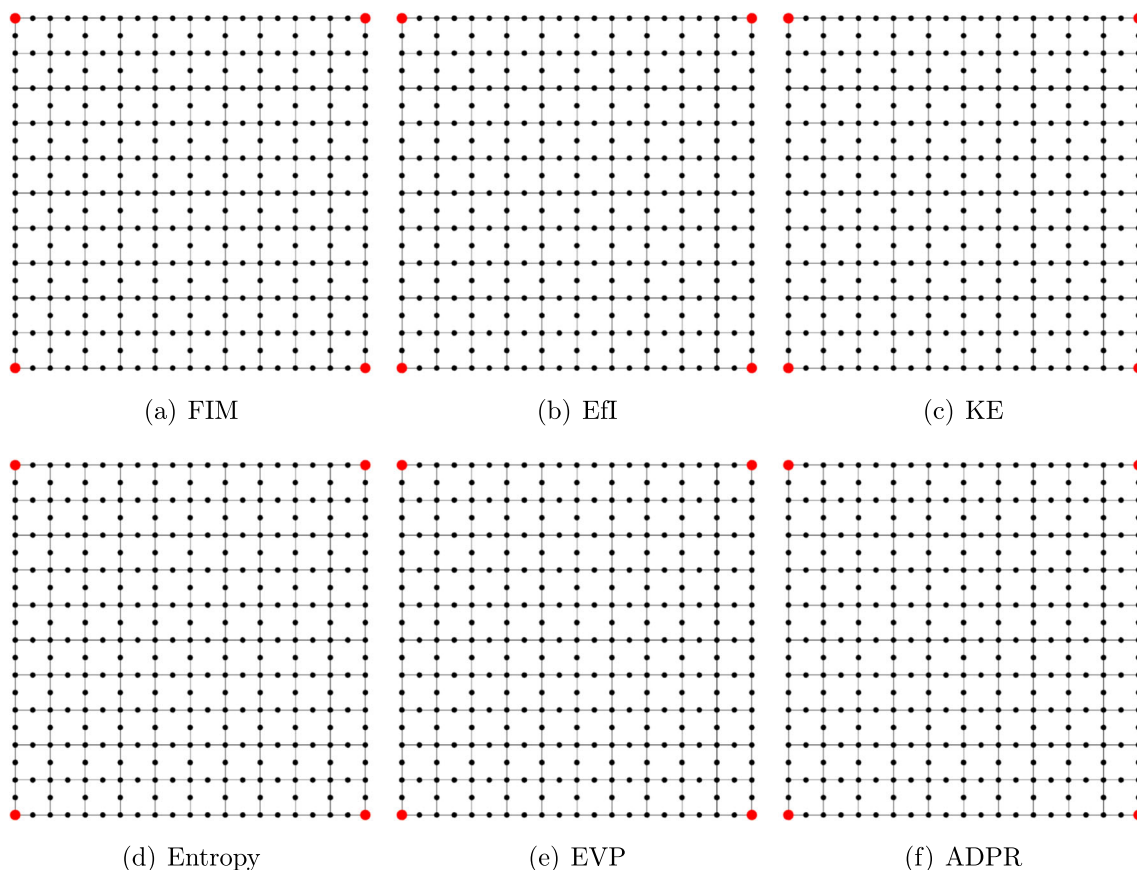


Fig. 4 General result of the positioning of 4 sensors by means of several methods (optimum location highlighted in red)

and child subspaces where they are given the same chance to compete in the selection. The size of this space will change as the number of births changes. In stochastic selection, individuals are given a certain probability of selection based on their fitness. These probabilities may be directly proportional to their fitness values or can be obtained by increasing their fitness values. In deterministic selection, the best members of the population are selected. Mixed selection is a combination of stochastic and deterministic selection (Gopalakrishnan et al. 2011). The basic flowchart on the operation of the algorithm is shown in Fig. 2.

3.1 GA - integer programming

Integer nonlinear programming refers to mathematical programming with continuous and discrete variables. The use of integer programming with genetic algorithms is a natural approach in formulating problems where it is necessary to simultaneously optimize the system structure (discrete) and (continuous) parameters.

This type of strategy has been used in a number of applications, including the process, engineering, and operational research industry. Needs in such diverse areas have motivated research and development in technology, particularly in algorithms to deal with large-scale, highly combinatorial, and highly nonlinear problems (Bussieck and Pruessner 2003).

The general formulation is given by (11).

$$\begin{aligned}
 & \text{Minimize } f(x, y) \\
 & \text{Subject to: } g(x, y) \leq 0 \\
 & \quad x \in X \\
 & \quad y \in Y \text{ integer}
 \end{aligned} \tag{11}$$

The functions $f(x, y)$ and $g(x, y)$ are non-linear functions. The variables x and y are the decision variables, where y is restricted only to integer values. X and Y are bounding constraints on variables.

No optimality criterion of the algorithm parameters was employed in this study. A trial and error was employed until an acceptable minimum and a repeatability were obtained in the different runs of the optimizer. At the beginning of the work, we used classical methods

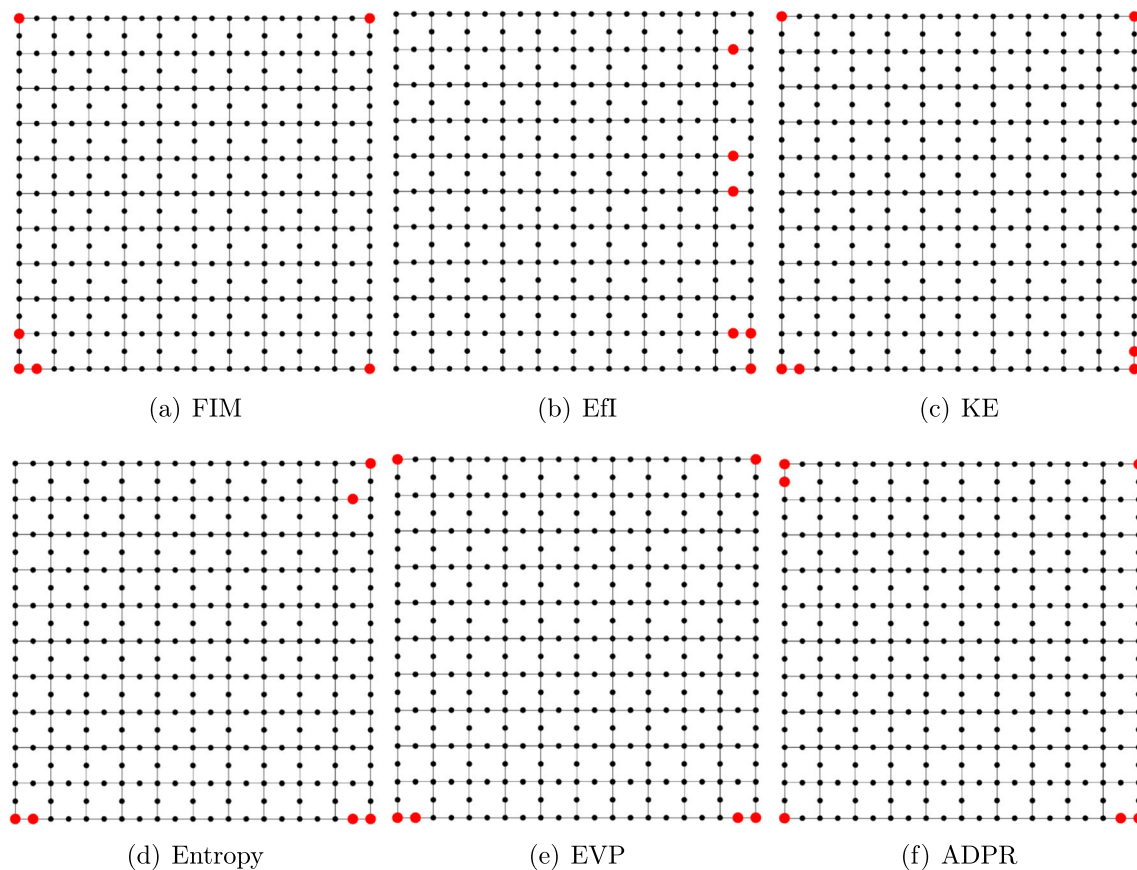


Fig. 5 General result of the positioning of 6 sensors by means of several methods (optimum location highlighted in red)

(SQP, steepest descent, etc.) and when using heuristic methods (GA), we observed that the objective function value was much lower. With GA defined, we performed several tests modifying the parameters and no significant improvement was found. Absurdly, we defined cases with a population and a very high generation, and even then, no variation to the results already obtained was verified.

In this work, the use of programming will be justified by the search for damage in structural elements that form a set of integers, not requiring numerical precision, thus avoiding a redundancy in the optimization and expense of computational effort unnecessary.

4 Optimization methodology problem for SPO

4.1 Laminated composite plates

On the model used in this study, two-dimensional studies on plate-like structures are considered. The plate in question

was modeled using shell elements (*shell*) with eight nodes and six degrees of freedom on each of its nodes. The quality of a good mesh is essential in obtaining reliable results. It was decided to use a structured mesh, because the structure in question is uniform, two-dimensional and it is added the fact of obtaining a computational economy by opting for such a configuration. The amount of elements was also chosen in such a way as to obtain a sufficient convergence in the values of the studied responses.

Regarding geometry, the direct problem was modeled as a square side plate equal to 30 centimeters. The structure consists of a symmetrical laminate of composite material consisting of 12 layers of different orientations arranged in the form $[0/90]_{3S}$. It is emphasized that this work is exclusively aimed at the study of the method of detecting damages in laminates of composite material, not emphasizing the geometric parameters and specific characteristics of the laminated material in question. Table 2 displays the set of main properties that were employed in modeling the problem via Finite Element Method (FEM).

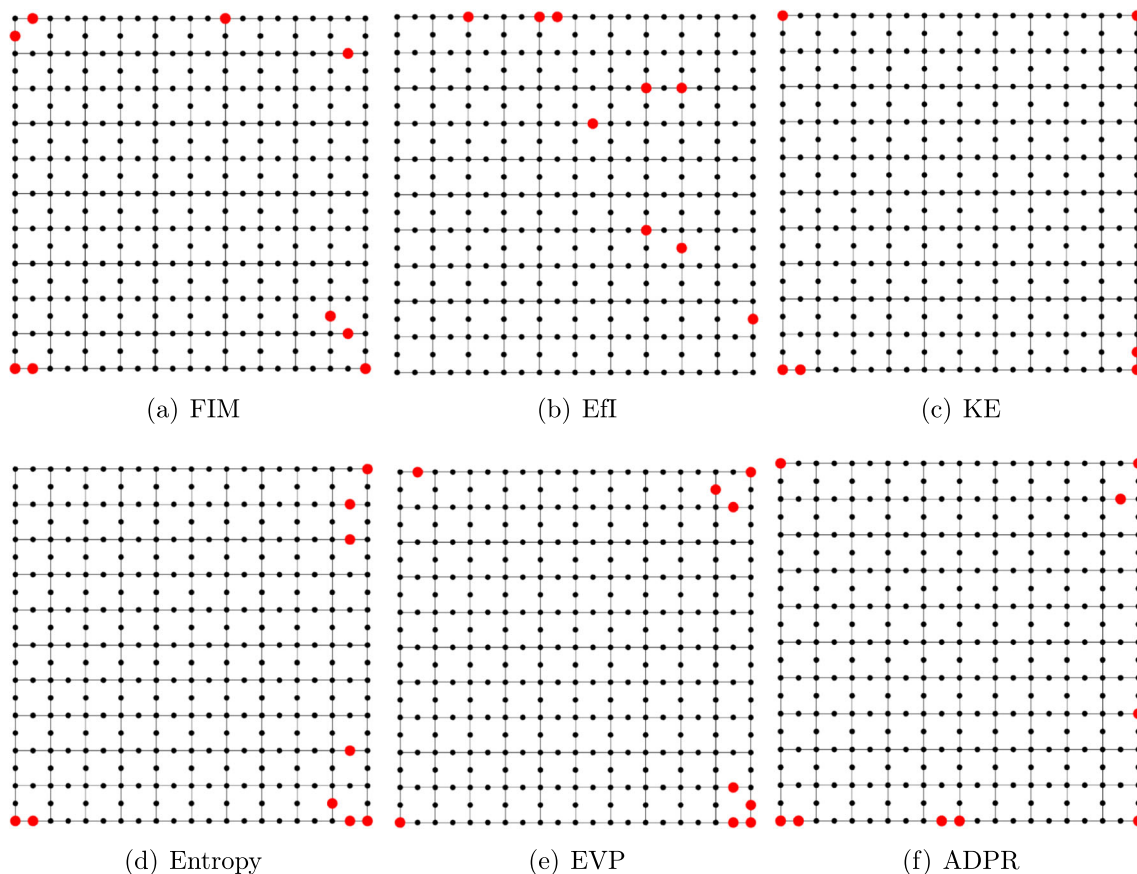


Fig. 6 General result of the positioning of 9 sensors by means of several methods (optimum location highlighted in red)

4.2 Applied sensor placement optimization

It should be remembered that the genetic algorithm becomes a useful algorithm in this study, because the problem of optimization of positioning of the sensors is characterized by the fact that the design variables are not continuous but discrete, so the use of whole programming. The objective in this part of the work is to apply the evolutionary method of optimization of positioning of sensors using genetic algorithm, for the precise modal identification in mechanical structures. A discrete-type optimization problem using genetic algorithm is formulated by defining the positions of the sensor according to the criteria quoted in the previous paragraphs. The modal parameters (i.e. modes of vibration and natural frequencies) of the real structure are obtained numerically using the finite element model.

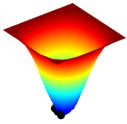
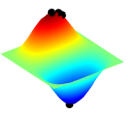
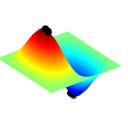
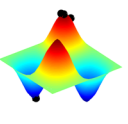
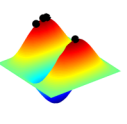
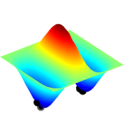
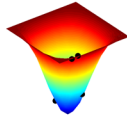
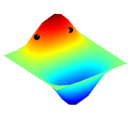
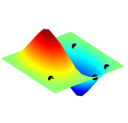
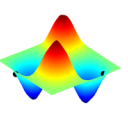
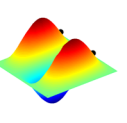
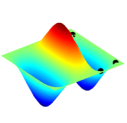
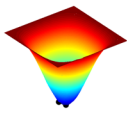
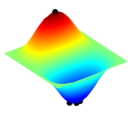
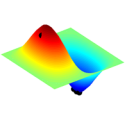
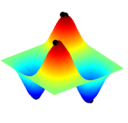
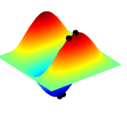
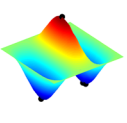
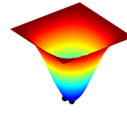
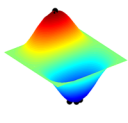
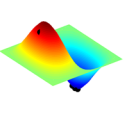
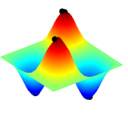
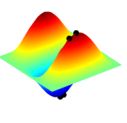
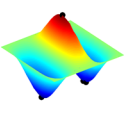
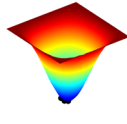
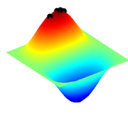
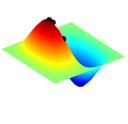
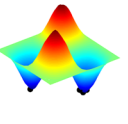
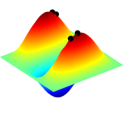
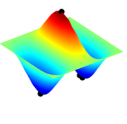
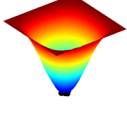
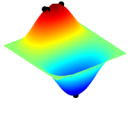
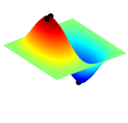
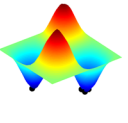
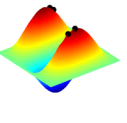
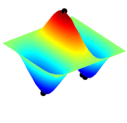
Briefly, the problem of optimal distribution of sensors is shown in Table 3, where the criteria must be maximized or minimized.

Thus, to identify the optimum location of n sensor candidates, global optimization algorithms will be used for this purpose. For example, if 9 sensors distributed are considered on the surface of the structure, taking into account a search space composed of 341 possible nodes, this would result in a combination of 1.55×10^{17} possible locations. The time of evaluation in all these possible combinations justifies the use of advanced optimization methods.

The mathematical formulation of the inverse problem can be summed up in maximizing or minimizing J_i , with $i = 1, 2, 3, 4, 5, 6, 7$. Subject to the constraints imposed of lower and upper bounds in relation to the maximum number of nodes and type $x_i - x_{i-1} \geq 1$, where x_i is the position of the candidate node. This restriction is necessary to not get multiple sensors in the same location. The search space is displayed in Fig. 3 where the possible 341 sensor positions are considered.

Another important aspect refers to the parameters used in the optimizer. In this case, genetic algorithms were used

Table 8 Results of the optimal positioning considering 4 sensors for a clamped plate

	Mode					
	1	2	3	4	5	6
FIM						
Eff						
KE						
Entropy						
EVP						
ADPR						

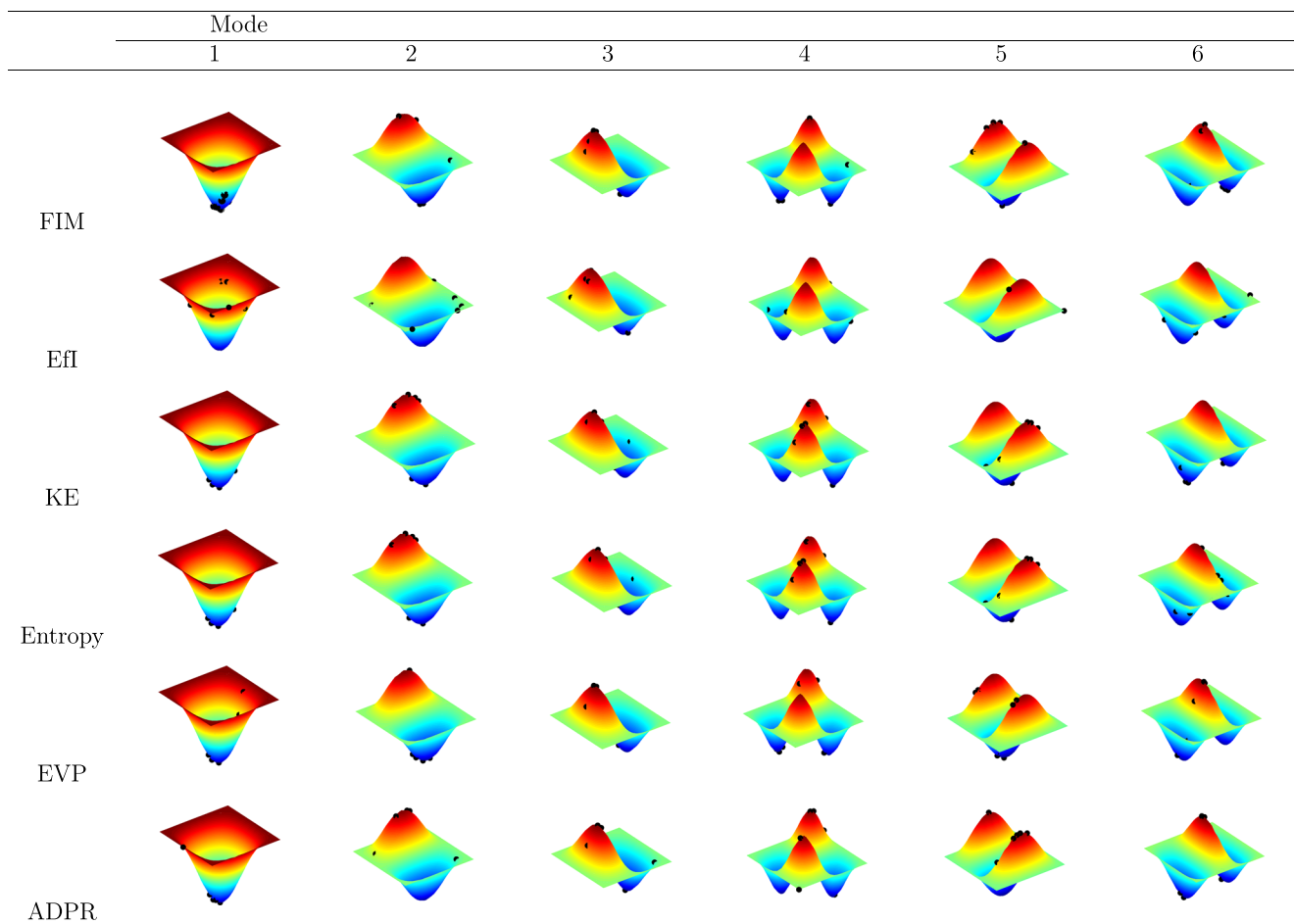
because of their proven high efficiency in similar cases. The configuration of the GA's main parameters was established after performing test cases until a satisfactory set of values were obtained. The Table 4 displays the genetic operators used in this part of the search. The population size was set at $10 \times N_{var}$ being N_{var} the total number of design variables present in the problem, i.e., in case it is desired to identify the position of the damage and its local severity, there are two project variables involved $\vec{x} = [N_e, \alpha]$, so a population of 20 individuals was sufficient to achieve convergence in the solution of the problem. The criterion for the problem was defined by the maximum number of generations of GA individuals, that is, 100.

Many global search algorithms are stochastic in nature. According to Haftka (2016), this means that to demonstrate their effectiveness. One must run them multiple times so as to establish the statistical distribution of the solution as function of the design parameters. For this,

a average of several runs were made to obtain a good statistical confidence level. For the problem of optimal positioning of the sensors, where the modeling of the inverse problem has already been discussed in the previous items of this study. It was necessary to expand the number of individuals present in the population to $10 \times N_{var} \times N_{modes}$, where N_{modes} is the number of modes considered in the process of optimal positioning of the sensors. The stopping criterion was also established as the maximum number of generations, but increased to 1000 per generation to meet sufficient convergence criteria.

5 Results and discussion

Distribute sensors efficiently in a structure is crucial in an operational way, where not all points of acquisition are

Table 9 Results of the optimal positioning considering 6 sensors for a clamped plate

available, besides the factor of structural weight saving and costs related to the sensing in a structure, for example aerospace, are weight criteria in the decision-making on structural sensing.

5.1 Optimal sensor position for a free plate

The matrix criteria adopted were: *i)* FIM, *ii)* EfI, *iii)* KE, *iv)* Entropy, *v)* EVP and *vi)* ADPR. The optimal layout of the sensor distribution was obtained using the genetic algorithm optimization method. The common basic understanding of these methods is to obtain optimal sensor positions where the greatest amount of modal information is obtained. Only the position of a predefined number of sensors at 4, 6 and 9 were adopted with optimization criteria. The sensors were then distributed in the nodal positions of the plate.

It should also be remembered that the type of sensor adopted is an ideal sensor, where its physical-mechanical characteristics, such as weight and rigidity, are not taken into account, which could lead to some small structural variations.

The Table 5 displays the results of the optimization for 4 sensors. It can be seen that the optimal distribution of sensors occurred near the board edges in most modes. This is due to the fact that the structure is free in contour conditions, and its points of greatest modal deformation are precisely at the ends. By expanding the number of sensors on the board to 6, more sensor set options are available. The Table 6 displays the graphical results obtained by GA. Similarly, the results obtained by the matrix criteria generated optimum values in the vicinity of greater deformation/modal amplitude of the plate for

Table 10 Results of the optimal positioning considering 9 sensors for a clamped plate

	Mode					
	1	2	3	4	5	6
FIM						
Efi						
KE						
Entropy						
EVP						
ADPR						

most of the methods and modes studied. In addition, the figures in the Table 7 exhibit the same phenomenon in the behavior for 9 distributed sensors. Except for the third mode of vibration studied by the Efi criterion, the optimum points were obtained again in the vicinity and in the nodes of greater amplitude of vibration. In a general way, all methods obtained similar performance, generating a distribution of sensors in anti-knots of the vibration modes. A particular case is given by the method of effective Independence, which has some discrepancy in relation to the other criteria. A possible explanation for this phenomenon, as discussed by Barthorpe and Worden (2009), is that this criterion can select sensor locations that exhibit low signal strength, which can be considered as a disadvantage.

In fact, no optimality criterion of the algorithm parameters was employed in this study. A trial and error was

employed until an acceptable minimum and a repeatability were obtained in the different runs of the optimizer. In fact there are two points. First there is no mathematical proof that in practical, complex cases the GA converges to the global optimum (or *minima*). This can be proven only by tests: trying out your GA on several appropriate example problems (what we did in our manuscript). Other is that GA can only approach the global optimum with appropriate preciseness (just like the nature does). This is why GA is especially appropriate for use in engineering problems, where no absolute preciseness is needed. The stop criteria was established when the pre-defined maximum affordable number of generations $N_{generation}$ has been exhausted. In our case, $N_{generation} = 1000$. By analyzing the individual responses of each mode of vibration, it can be seen that the optimized sensors are located at the points of maximum amplitude of vibration (global best). The metrics used in

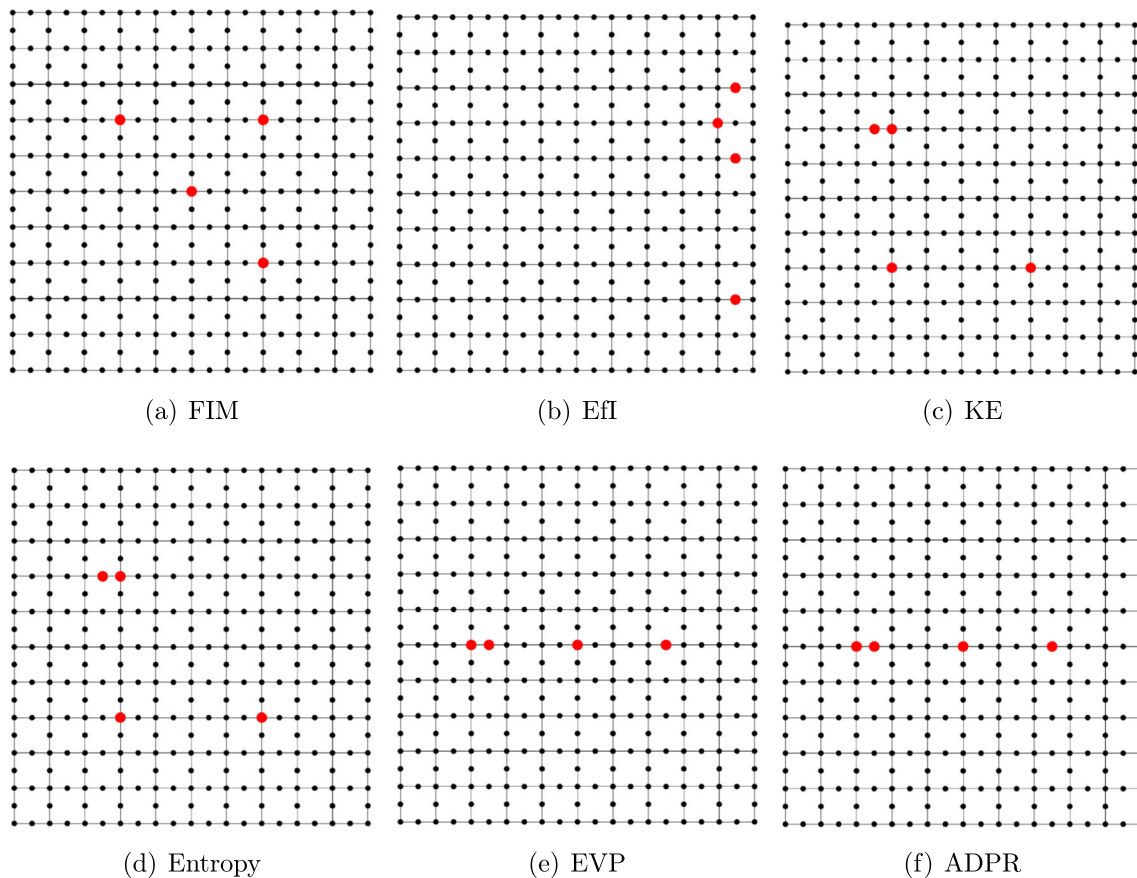


Fig. 7 General result of the positioning of 4 sensors by means of several methods considering a clamped plate

goal constructs (FIM, EI, Kinetic Energy, etc.) are actually used to achieve this result.

In order to provide more information regarding the convergence and optimality of GA, simulation using SA algorithm were performed. The results are displayed in [Appendix](#).

In a way, it may be evident that the optimum location of the sensors is ideal at points of greater amplitude of vibration, thus avoiding nodal points with zero amplitudes in certain modes. However, in the case of a geometry of greater geometric complexity, the evaluation of these maximum points is added in complexity. Although obtaining optimal sensor distributions over a vibration mode is obtained, the idea here is to obtain the sensor localization set that takes into account the first 6 modes of vibration, resulting in a complex and non-trivial problem, requiring efficient methods optimization.

The Figs. 4, 5 and 6 display the final optimization results of the $n = 6$ first vibration modes of the board in question.

It can be verified that the final result was still made up of sensor points, in general, points at the ends of the plate. The restriction employed in this study of 4, 6 and 9 sensors generated a distribution of not so symmetrical and distributed configuration. In the effort to refine the result of the optimal distribution obtained by the modal information criterion presented in this topic, we opted for the introduction of a second optimization criterion based on the modal interpolation, which will be discussed in the following item.

5.2 Optimal sensor position for a clamped plate

For the test structure of this work, all the geometric and physical characteristics were preserved, modifying only the boundary conditions. In this section, the results obtained by the optimum distribution technique of the sensors for the plate in fixed contour condition are displayed, ie all sides

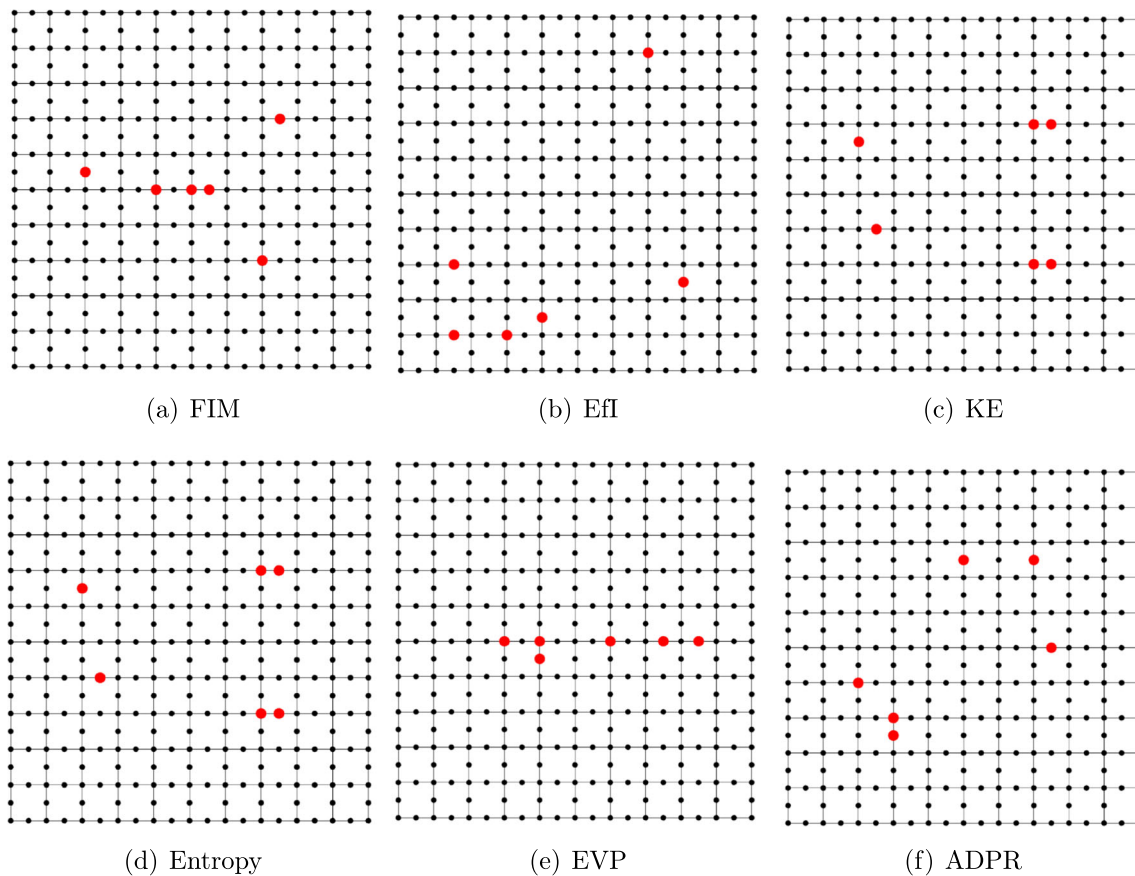


Fig. 8 General result of the positioning of 6 sensors by means of several methods considering a clamped plate

of the plate are set, having no movement of translation and rotation in the Cartesian axes. Such a boundary condition as CCCC (*clamped-clamped-clamped-clamped*) was taken as nomenclature.

Tables 8, 9 and 10 display the results of the optimal distribution of sensors considering 4, 6 and 9 sensors, respectively. It can be observed that for most methods, the “optimal” points are close to the locations of maximum vibration amplitude. Some specific methods performed more satisfactorily than others. For example, the information entropy criterion was able to distribute the sensors at points of vibration peaks (anti-knots), whereas the Effective Independence (Efi) method presented lower yield, as previously noted in the case of the free plate of condition.

Figures 7, 8 and 9 show the best sensor location results considering as the objective function the set of the $n = 6$ first vibration modes. It is observed that the optimum points are located in the vicinity of the central plate, considering

mainly 4 sensors. For 6 and 9 sensors, it can be verified that the distribution is not trivial and does not assume any specific trend.

The main efficiency of the genetic algorithms is in their robustness, being able to find the global optimum without being stagnant in local points in multimodal functions. In general, all methods investigated showed excellent results with GA. Except for the Effective Independence (Efi) method, which presented some peculiarities inherent to the method itself and not to the optimizer. Regarding the optimizer, a considerable population of individuals and a high generation (stopping criterion) was employed. In addition, the algorithm was run several times because it is a stochastic and not a deterministic. Thus, it was observed that the variance of the results was very small or even null, which makes us believe that we find the global minimum. In addition, the GA itself has internal selection to avoid being “stuck” at these local minimum points.

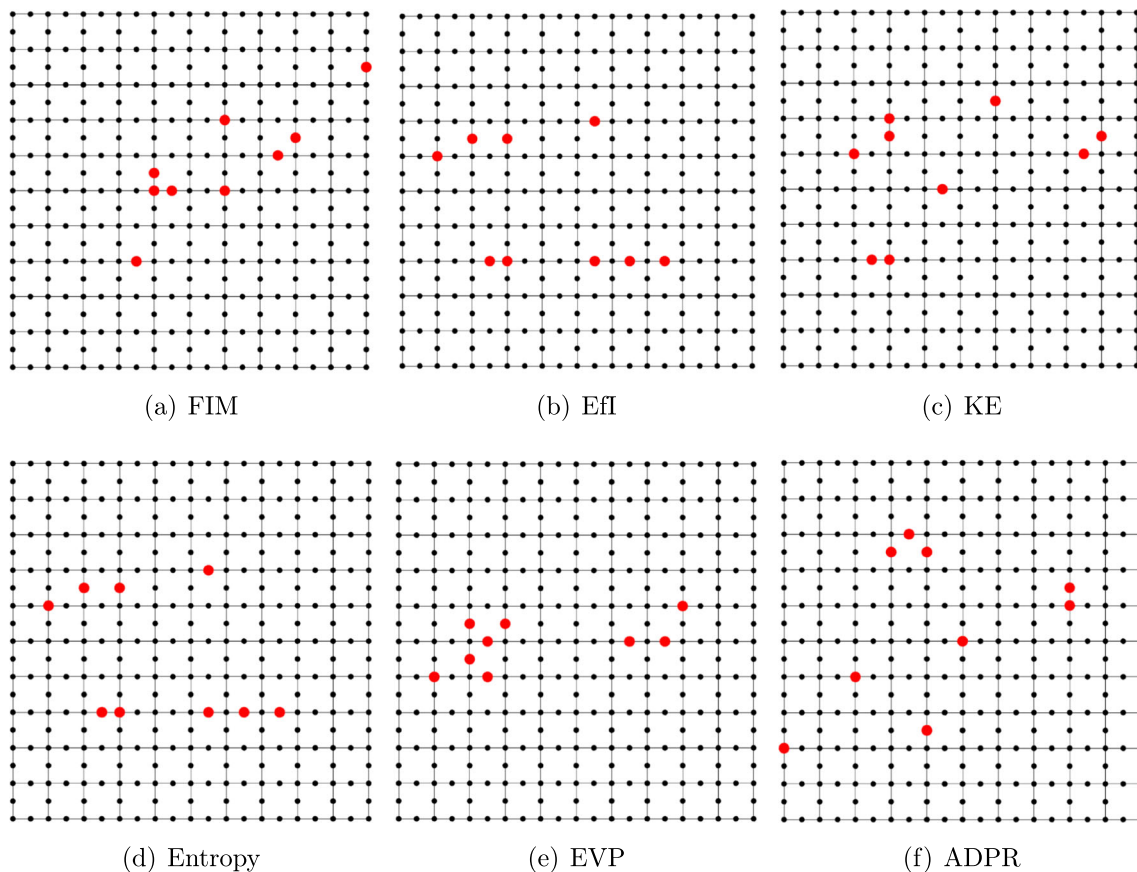


Fig. 9 General result of the positioning of 9 sensors by means of several methods considering a clamped plate

6 Conclusion

In this paper, the GA was studied and adjusted to find the optimal sensor placement based on several criterion of optimal sensor placement for modal test applied to structural health monitoring. Six optimal placement techniques were presented and objectives functions were built considering a range of all low frequency modes (first six modes) for a laminated CFRP plate.

The main objective of this study is to evaluate the different metrics of sensor optimization known in the literature. In a first approach, the case of free vibration was considered. The results obtained for this first part were discussed in the manuscript. In fact, treating only the case of free vibration is not so relevant. Therefore, the authors considered a second case, taking the plate clamped in all the four edges.

Finite element method is employed to obtain the vibration modes of a carbon fiber board. In the sequence, optimization (GA) was employed to minimize several objective functions, built according to sensor optimization metrics known in the literature.

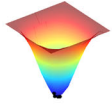
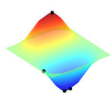
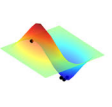
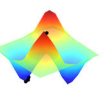
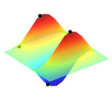
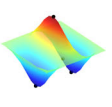
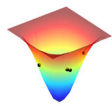
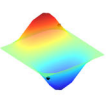
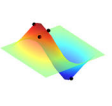
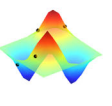
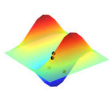
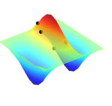
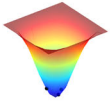
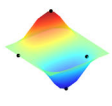
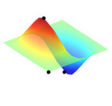
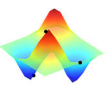
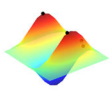
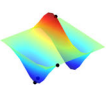
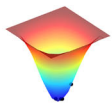
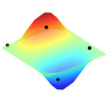
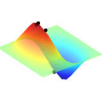
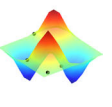
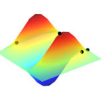
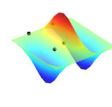
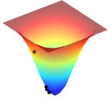
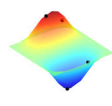
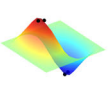
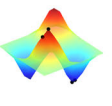
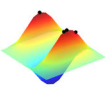
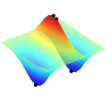
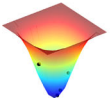
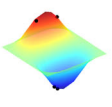
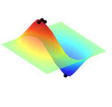
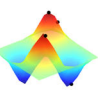
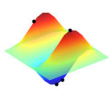
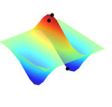
The research shows the effectiveness of GA for sensor placement optimization in a structural health monitoring system. The numerical example and analysis demonstrate that GA is able to determine the optimal sensor locations. Numerical examples indicate that GA is capable of solving problems with larger possible candidate sensor locations.

Several matrix methods were applied and in a sense all were efficient, bringing a very similar result between them. The optimum position of the sensors generated by such objectives resulted in points where the greatest amount of modal information was available. In general, all the criteria were effective in the optimal location of the sensors, except for the case of the Efi, which still has sensitive characteristics in the proposed application. Taking into account the set of several modes included in the minimization of the objective function, it is observed that the final result of sensors location is not trivial, i.e., not necessarily corresponding to vibration peaks.

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Appendix

Table 11 Results of the optimal positioning considering 4 sensors for a clamped plate using Simulated Annealing Algorithm

	Mode					
	1	2	3	4	5	6
FIM						
Efi						
KE						
Entropy						
EVP						
ADPR						

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