Structural and Multidisciplinary Optimization (2018) 58:785–796
https://doi.org/10.1007/s00158-018-1908-4

INDUSTRIAL APPLICATION

Role of multi-response principal component analysis in reliability-based robust design optimization: an application to commercial vehicle design

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Received: 2 October 2017 / Revised: 22 November 2017 / Accepted: 8 January 2018 / Published online: 30 January 2018

Abstract

The Taguchi method is a widely used conventional approach for robust design that combines experimental design with quality loss functions. However, this method can be only used in a single-response problem. In this study, we propose the use of principal component analysis (PCA) to consider multi-response problems in the Taguchi method and to investigate the influence factor of a cab suspension system. We compute the normalized quality loss for each response and perform PCA to calculate the multi-response performance index. In this study, control factors with three level combinations and noise factors with random sampling from each normal distribution are considered. Additionally, we applied multi-objective reliability based robust design optimization (RBRDO) to accommodate design uncertainties and its data scattering based on rational probabilistic approaches. This is used to develop the reliability assessment and reliability based design optimization and corresponds to an integrated method that accounts for the design robustness in the objective function and reliability in the constraints.

Keywords Uncertainty · Principal Component Analysis (PCA) · Multi-response Performance Index (MPI) · Reliability Based Design Optimization (RBDO) · Reliability Based Robust Design Optimization (RBRDO)

1 Introduction

In order to evaluate and improve the performance of any system, it is necessary to analyze the experimental or observation data associated with the system. Generally, various types of errors including observation errors, experimental errors, and modeling errors are inherent in experimental and observation data. Therefore, it is important to eliminate these errors while analyzing the data or to develop robust statistical models. In this study, we developed a scatter analysis technique to control the performance uncertainty due to the variation in topology by dimension, shape, position, and the scattering in the material. In the front-loading stage, we propose an optimal design method by considering the uncertainty to satisfy various objectives such as NVH, durability, and collision safety performance. Specifically, we derive a vehicle performance design solution that analyzes and evaluates the characteristics of performance scattering with respect to various body structures and parts of a commercial vehicle cab suspension system. Therefore, it is expected that this will contribute to increase in the work efficiency and shorten the development period by developing a parameter analysis technique that can predict the performance without repeated analysis for any given design change. Specifically, with respect to commercial vehicles that are similar to the subject of the study, the weight applied to the cabin suspension varies based on the number and weight of passengers as well as the characteristics of the component material with the operating environment, and the center of gravity also exhibits uncertainty. Several uncertainties are also involved in the performance test of the general system. Most CAE models conduct a deterministic performance analysis although this approach reduces the prediction accuracy of the model. Therefore, various uncertainties and scattering that exist in real engineering problems are considered to control the scattering and to reduce the uncertainties (Tu et al. 1999).

The aim of this study involves increasing efficiency of a reliability based robust design optimization via principal component analysis by considering the uncertainty as shown in Fig. 1. First, we analyze the physical and epistemic uncertainty for uncertainty

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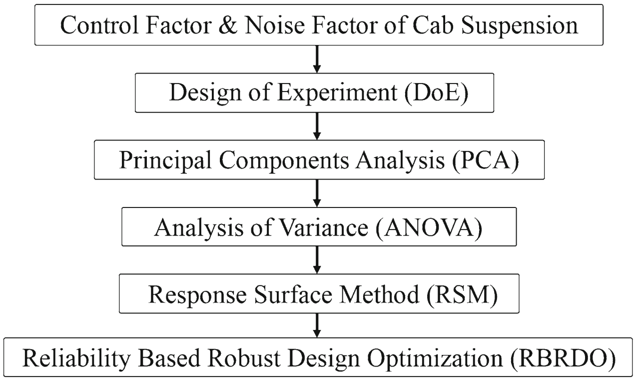


Fig. 1 Flowchart of research

quantification in cab suspension components. Here, physical uncertainty implies a knowledge random factor that is expressed as a probability distribution by knowing accurate statistical characteristics, and it requires a significant amount of numerical cost and time in advance. On the contrary, epistemic uncertainty comes from a lack of knowledge, and it can be reducible if more data are collected. In the present study of a cap suspension design, the weight and center of gravity of a driver are assumed to be noise factors that are classified as physical uncertainty.

For this purpose, the main input variables that significantly affect the performance of the cab suspension system among the multiple random input parameters are selected as control factors. The design of experiment (DOE) is performed by considering the noise factors of random sampling, and a meta-model is created for approximate optimization. The Taguchi method that combines the experimental design method and quality loss is applied to a single-response. However, it is difficult to apply the same to multi-response problems in which correlation exists between different values such as the scenario considered in the present study. To solve these types of complicated problems, we apply PCA to determine the importance of various design variables and to reduce the time required for subsequent design optimization.

Deterministic design optimization (DDO) is a commonly used as an approximate optimization technique, and it uses only a single fixed value for the meta-model and does not consider the uncertainty and tolerance of design variables (Sim et al. 2012). Thus, there is a tendency that the reliability with respect to the optimum value is slightly low. In the case of robust design optimization (RDO) that is a representative probabilistic approach, we first determine the optimal conditions for the factors that reduce dispersion and then minimize the quality loss by moving the average of the quality characteristics to the target. Therefore, we set the conditions for the controllable factors that reduce the dispersion by determining the factor that affects the dispersion of the quality characteristics and make the quality characteristic insensitive to changes in the uncontrollable factors. Another approach, namely reliability based design optimization (RBDO), is formulated as an optimal design problem that minimizes the objective function with constraints for reliability requirements. Therefore, the user can give a desired index of reliability to the constraint.

In a study by Choi et al. (2009), Monte Carlo Sampling (MCS) is used to assess the reliability of constraints. An approximate function of the cumulative distribution function (CDF) is used for the constraints to acquire a probability of failure and its analytic sensitivities. Lee et al. (2008a) also developed a reliability based robust design optimization (RBRDO) method with the dimension reduction method and compared it to the performance moment integration (PMI) method and percentile difference method (PDM) in terms of accuracy and efficiency.

In this study, deterministic optimization is first performed, and the result is used as the initial value of RBRDO (Shahrak and Noorossana 2014; Motta and Afonso 2016; Youn and Xi 2009) to optimize robustness as well as reliability. The RBRDO can be used as an integrated method to obtain optimal design by providing design robustness to the objective function and simultaneously providing the desired index of reliability to the constraint condition.

2 Multiple performance characteristics of a cab suspension system

2.1 Problem definition

Recently, there is an increase in the demand for ride comfort in addition to the functions and performance of commercial vehicles such as mounting an air suspension that is shocked by air instead of a spring Lee et al. (2008b). Therefore, studies on cab suspension system that directly affects vibration reduction are actively performed such that a driver can work comfortably when given the vibrations and shocks generated during a ride (Cole 2001). Specifically, the cab suspension system of a commercial vehicle absorbs even minute shocks that are transmitted to the driver's seat in the cabin through the wheels and the frame, and thus, ride comfort is excellent and the stability is maintained especially on the expressway. Therefore, the cab suspension includes a shock absorber for supporting a hydraulic mechanism and a cabin mounted on a frame of the vehicle, an air spring installed between the frame and the cabin to support the cabin with an elastic force, and a leveling valve that adjusts the height of the cabin by controlling the internal pressure of the air spring.

In the front-loading stage of new car development, studies evaluated the construction of a performance integrated vehicle development process by considering the geometry as well as the ride and handling (R&H) by predicting and controlling the scatter factors. In this process, the necessity of a technique to reduce the error between the test results under the uncertainty and the predicted value is increased by applying probability statistics techniques.

In this study, we establish the design of an experimental table by considering the control factor and noise factor for multi-response problem and determine influential design factors through principal component analysis and analysis of variance. This ensures that the robust design and reliability-based design of cab suspension are performed easily and quickly in the front-loading stage. Generally, traditional cab suspension systems must typically compromise ride comfort and stability. Therefore, the cabin weight (weight) and the center of gravity position (cogx) are considered as the noise factors with uncertainty, and thus, design parameters of cab suspension system including the bush and stabilizer bar subsystem as shown in Fig. 2 are optimized to improve ride comfort while ensuring the stability of the system. The descriptions of control factors and noise factors are summarized in Table 1. All control factors were considered by adjusting the scale to [−1, 1].

2.2 Design of experiments

The presence of various uncertainty and variability causes the deterministic model correction to degrade the predictive ability of the model. Therefore, it is necessary to create a statistical model with the random input variables through uncertainty modeling. Subsequently, several input variables that significantly affect the system performance are selected as control factors among the multiple random input variables. We apply the statistical optimization technique by using methods, such as design of experiment (DOE) and Taguchi method, to establish the random sampling meta-model. Furthermore, we calculate the optimal condition of the control factors that minimize the standard deviation while satisfying the mean of the response values to the target value.

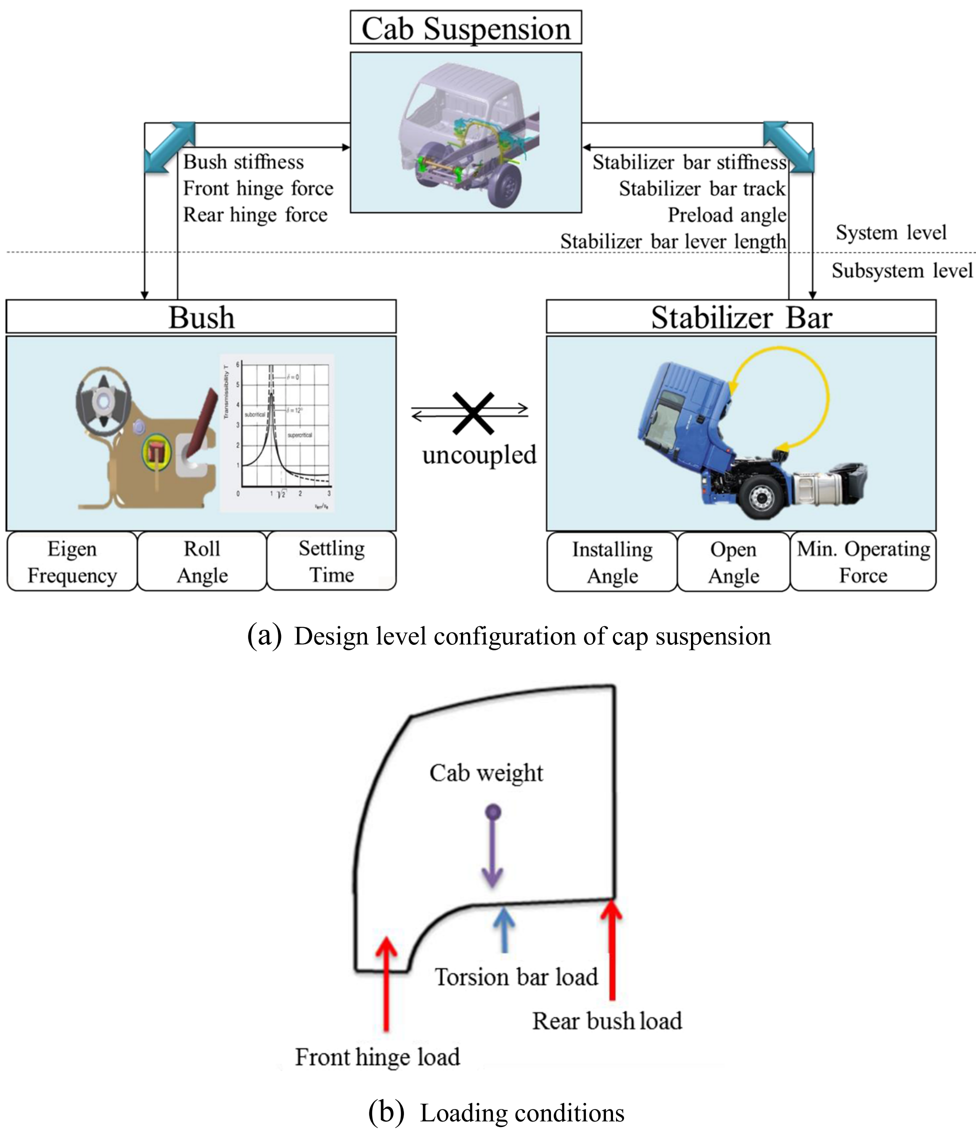


Fig. 2 Design architecture of cab suspension

Table 1 Control factor and noise factor for design optimization

Factor		Description
Control factor	x_1	Distance of hinge link
	x_2	Distance of tiliting grab handle
	x_3	Torsion bar hinge X
	x_4	Torsion bar hinge Z
	x_5	Torsion bar rate
	x_6	Preload angle of torsion bar
	x_7	Spring track FRT
	x_8	Spring track RR
	x_9	Spring track Stab. Bar
	x_{10}	Total stiffness
	x_{11}	Bush damping FRT
	x_{12}	Bush damping RR
Noise factor	Weight	Weight of cabin
	Cogx	The center of gravity X position

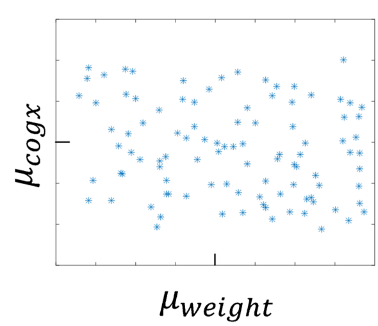
The reasons for the control factor optimization experiment are as follows. The first reason involves identifying the important control factors that reduce variability caused by noise factors. The second reason is to identify the control factors that do not significantly influence the main function such that the tolerances on these factors can be relaxed. The uncertainty of the design variables is controlled by considering the noise factors that are distributed based on the variation characteristics centering on the mean value. It is critical to focus on an engineering solution, i.e., improving the performance in a way that minimizes the cost. In the study, two cases of an experiment were performed to determine the effects of design parameters on the responses. Optimal settings are determined such that a low variability for the responses is achieved.

The experimental procedure of parameter optimization corresponds to the crossed array format as shown in Fig. 3. The use of a crossed control factor and noise factor array allows the execution of each treatment combination of the control factor at two or more treatment combinations of the noises. To verify the efficiency of PCA that will be performed in next chapter, ANOVA analysis of multi-response to all control factors was performed, and the response graph was confirmed. In the random sampling stage, the control factors of discrete sampling and noise factors of random sampling that are distributed based on the dispersion characteristics are considered. Thus, 27 trial conditions were considered, and both noise factors were considered as a normal distribution with a 15% scatter around the mean value. With respect to the two noise factors, 100 samples are randomly extracted within the range of each distribution. The loss function is calculated by considering the number of iterations in each trial condition, and the results are shown in Table 2. The F-statistic is the ratio of the mean squares. The *P*-value is the probability from the cdf of the F-distribution such that the F-statistic can assume a value that exceeds the computed test-statistic value. The preload angle of torsion bar (x_6) and total stiffness (x_{10}) correspond to the highest sum of squares value.

3 Principal component analysis

The Taguchi method is typically used for the design optimization based on the design of experiment, and it combines the quality loss and the experimental design technique. This is mostly available in the case of a single-response, and it is difficult to apply the same to multi-response problem optimization. However, most

Trial Num.	Process Parameter									Multiple Responses					
										f_1		f_2		f_3	
	x_1	x_2	x_3	x_4	...	x_9	x_{10}	x_{11}	x_{12}	Weight	Cogx	Weight	Cogx	Weight	Cogx
1	1	1	1	1	...	1	1	1	1						
2	1	1	1	2	...	3	1	2	3						
3	1	1	1	3	...	2	1	3	2						
4	1	2	3	1	...	3	2	1	3						
5	1	2	3	2	...	2	2	2	2						
6	1	2	3	3	...	1	2	3	1						
⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮						
76	3	2	1	1	...	3	3	2	1						
77	3	2	1	2	...	2	3	3	3						
78	3	2	1	3	...	1	3	1	2						
79	3	3	3	1	...	2	1	2	3						
80	3	3	3	2	...	1	1	3	2						
81	3	3	3	3	...	3	1	1	1						



100 random samples

Fig. 3 Orthogonal array of random sampling metamodel

Table 2 The analysis of variance (ANOVA) result of random sampling case

Case	Sum of squares	Mean squares	F-statistic	P-value
x_1	0.0430	0.02149	0.30	0.7663
x_2	0.0602	0.03011	0.43	0.7006
x_3	0.1054	0.05271	0.75	0.5720
x_4	0.0701	0.03504	0.50	0.6678
x_5	1.5414	0.77069	10.94	0.0838
x_6	5.5281	2.76405	39.23	0.0249
x_7	0.0117	0.00583	0.08	0.9235
x_8	0.0032	0.00159	0.02	0.9779
x_9	0.0012	0.00062	0.01	0.9912
x_{10}	5.0128	2.50641	35.58	0.0273
x_{11}	0.0137	0.00683	0.10	0.9116
x_{12}	0.0021	0.00105	0.01	0.9853

modern manufacturing processes demand the simultaneous optimization of multiple response variables, and a few of these responses are often correlated. Until now, the weight of multi-response optimization has been determined according to the engineering judgments although this approach increases uncertainty in the decision-making process. A method to solve the uncertainty problem involves determining a weight for each response although it is difficult to determine the weight. Other method involves regression techniques but this also increases the computational complexity and makes it difficult to depict the correlation between responses. Furthermore, an important factor in a single-response case may not be present in a multi-response case.

Therefore, a more efficient solution was required to solve such a complicated problem, and principal component analysis (PCA) is accordingly applied as a solution. Fig. 4 shows the flow chart for the PCA calculation. This significantly reduces the analysis time and calculation cost required for design optimization by determining and selecting the importance of many design variables (Jollife 2002, Yang et al. 2007). In summary, PCA is a data reduction technique that is used to identify a small set of design variables that account for most of the variance in the original values (Jean and Wang 2006). The key concept of PCA is to indicate the direction of the data with the greatest change. After a series of calculations, all changes

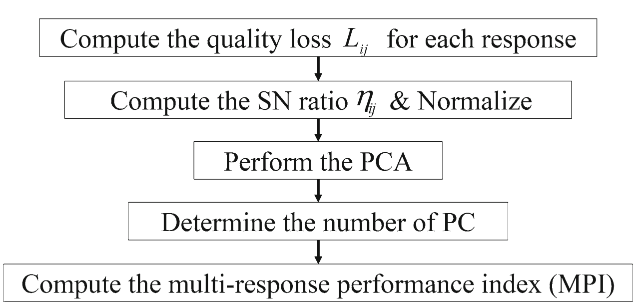


Fig. 4 Flowchart of principal component analysis

are included in the sets of coordinate axes, and the covariance corresponds to a diagonal matrix. Therefore, a set of responses is transformed into a linear combination of uncorrelated components. The ability to determine optimal factors or level combinations in a multi-response problem is considered as the greatest strength of the PCA (Antony 2000). Additionally, it is possible to reduce the uncertainty due to the engineering judgment that occurs while applying the Taguchi method.

3.1 Normalized quality loss of each quality characteristics

The input data for the principal component analysis follows the control factors, noise factors, and corresponding responses in the random sampling DOE table. Taguchi categorizes the response variables into the following three types: smaller-the-better(STB), larger-the-better (LTB), and nominal-the-best (NTB) (Phadke 1989). In this study, there are two responses with STB characteristics and one response with LTB characteristics. The formulation for quality loss is shown in (1), and it is easily obtained from Taguchi's quality loss functions as follows:

$$\begin{aligned} For\ \text{STB}, L_{ij} &= c\left(\frac{1}{n}\sum_{k=1}^{n} y_{ijk}^2\right) \\ For\ \text{LTB}, L_{ij} &= c\left(\frac{1}{n}\sum_{k=1}^{n}\frac{1}{y_{ijk}^2}\right) \end{aligned} \tag{1}$$

where, L_{ij} denotes the quality loss for the j^{th} response ($j = 1,2,\ldots,p$) in the i^{th} trial condition ($i = 1,2,\ldots,m$), and y_{ijk} denotes the experimental value of j^{th} response in the i^{th} trial condition at the k^{th} replication, c denotes the quality loss coefficient, and n denotes the number of repeated experiments (Gauri and Pal 2014). Thus, the SN ratio of each response is expressed as follows:

$$\eta_{ij} = -10{*}\log_{10} L_{ij} \tag{2}$$

In order to transform the SN ratio of each response into a scaled SN ratio, we compute the normalized quality loss (NQL, Y_{ij}). Furthermore, the maximum quality loss for the j^{th} response is η_j^+, the minimum quality loss for the j^{th} response is η_j^-, and NQL is then expressed using (3) as follows:

$$Y_{ij} = \frac{\eta_{ij} - \eta_j^-}{\eta_j^+ - \eta_j^-} \quad where, 0 \le Y_{ij} \le 1, \quad i = 1\sim 27, j = 1,2,3 \tag{3}$$

As shown in Table 3, Y_{i1}, Y_{i2}, and Y_{i3} represent the NQL values for the responses f_1, f_2, and f_3 at the i^{th} trial condition, respectively. Subsequently, we confirmed the correlation matrix of the multi-performance, and the result is summarized in Table 4. The results indicate that a high correlation exists between f_2 and f_3, while f_1 is less related to the other two values.

Table 3 Normalized quality loss values for each response

Trial(i)	Y_{i1}	Y_{i2}	Y_{i3}
1	1.0000	0.0000	0.7524
2	0.0000	0.3321	0.2640
3	0.3548	0.6349	0.5933
…	…	…	…
26	0.3548	0.4388	0.5485
27	1.0000	0.0853	0.8801

3.2 Computation of principal components

In order to solve the multi-response problem, the use of a principal component (PC) is an effective method, and it means that a small number of components (q) account for a large proportion of the variance in the original responses (p). Thus, the PCA result can be expressed as the uncorrelated linear combination of PCs, and Z_{ik} (where, $k = 1, 2, \ldots, q$ and $q \le p$) denotes a set of responses as shown in (4). Here, Z_{i1} is termed as the first principal component and it implies that it accounts for the maximum variance in the data. The expression is as follows:

$$\begin{aligned} &Z_{il} = a_{l1} Y_{i1} + a_{l2} Y_{i2} + \cdots + a_{lp} Y_{ip} \\ &where, a_{l1}^2 + a_{l2}^2 + \cdots + a_{lp}^2 = 1 \end{aligned} \tag{4}$$

Where, a_{kp} denote the elements of the eigenvector corresponding to the k th largest eigenvalue, and the components with eigenvalue larger than 1 are selected to replace the original responses (Kaiser 1960).

The multi-response performance index (MPI) for the i^{th} trial is computed based on (5) as follows:

$$\begin{aligned} &\mathrm{MPI}_i = \sum_{k=1}^{q} W_k \times Z_{ik} \\ &where, W_k = \frac{Eigenvalue\ of\ the\ l^{th}\ \mathrm{PC}}{\mathrm{Sum}\ of\ eigenvalues\ of\ \mathrm{all\ PC}}, \quad \sum_{i=1}^{q} W_k = 1 \end{aligned} \tag{5}$$

W_k denotes the proportion of overall variance of the responses and is used as the weight for the k th PC. With respect to the optimization of the MPI values, we determine the optimal parametric settings for multi-response problem (Su and Tong 1997). In the study, the variances of the PCs that are termed as eigenvalues and the coefficients that are termed as eigenvectors are computed as shown as Table 5 and Table 6. From the values, we consider the number of components that must be selected to express the original response. The first principal component accounts for more than 60.89% of the total variance in the original data as shown in Fig. 5. The eigenvector for the first principal component with an eigenvalue of 2.1009 is [0.8812, 0.4407, −0.1713]. The second principal component exhibits a variance of approximately 20.22%, an eigenvalue of 0.6978, and an eigenvector of [0.2341, −0.0919, 0.9679]. In the case of the second and third principal components, the variance is low, and the eigenvalue value is less than 1, and thus it is not considered in the MPI equation. Therefore, the multi-response performance index is calculated for all trial conditions. Subsequently, we identify the factor and interaction effects that significantly influence the multi-response performance (Antony 2000).

Table 4 Correlation matrix of the multi-response

Correlation	Eig.freq.(f_1)	Max. effort (f_2)	Min. effort (f_3)
Eig.freq.(f_1)	1.0000	0.4361	−0.1818
Max. effort (f_2)	0.4361	1.0000	−0.1369
Min. effort (f_3)	−0.1818	−0.1369	1.0000

Table 5 Eigenvalues of the correlation matrix

Principal component	Eigenvalue	% of variance	Cumulative %
1st PC	2.1009	60.8954	60.8954
2nd PC	0.6978	20.2256	81.1210
3rd PC	0.6513	18.8790	100.0000

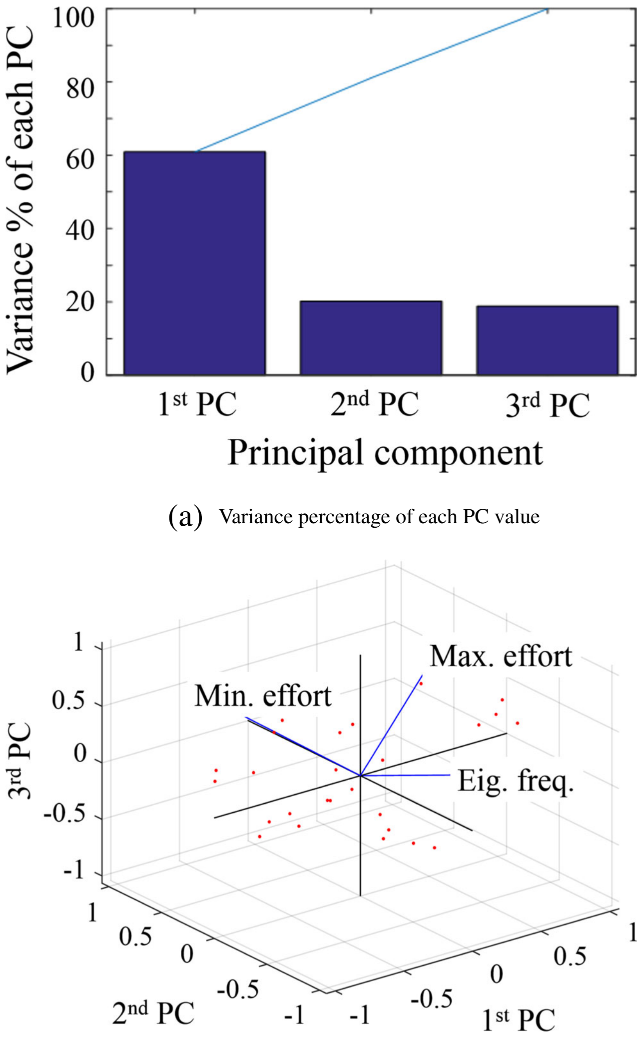
3.3 ANOVA for multi-response performance index

To compare the results, the response graphs for two cases of the random sampling meta-model, namely the weighted sum of responses case and MPI based case, are shown in Fig. 6. The weights in the case of weighted sum are assumed as the same value. From the result, the response graph shows that the influence as well as the tendency differ based on the case in which the responses are simply multiplied with arbitrary weights and the case in which the MPI values are used. Specifically, there is a difference in the effect of torsion bar rate (x_5) and front bush damping (x_{11}) when the two cases are compared.

The eigen frequency among the responses is influenced only by the total stiffness (x_{10}) as the design variable. Additionally, the tilting effort has a strong correlation with the distance between the hinge and link part as well as the rate (Nm/degree) and preload angle (degree) of the torsion bar that acts as a spring to connect the body and suspension. From this

Table 6 Eigenvectors of the correlation matrix

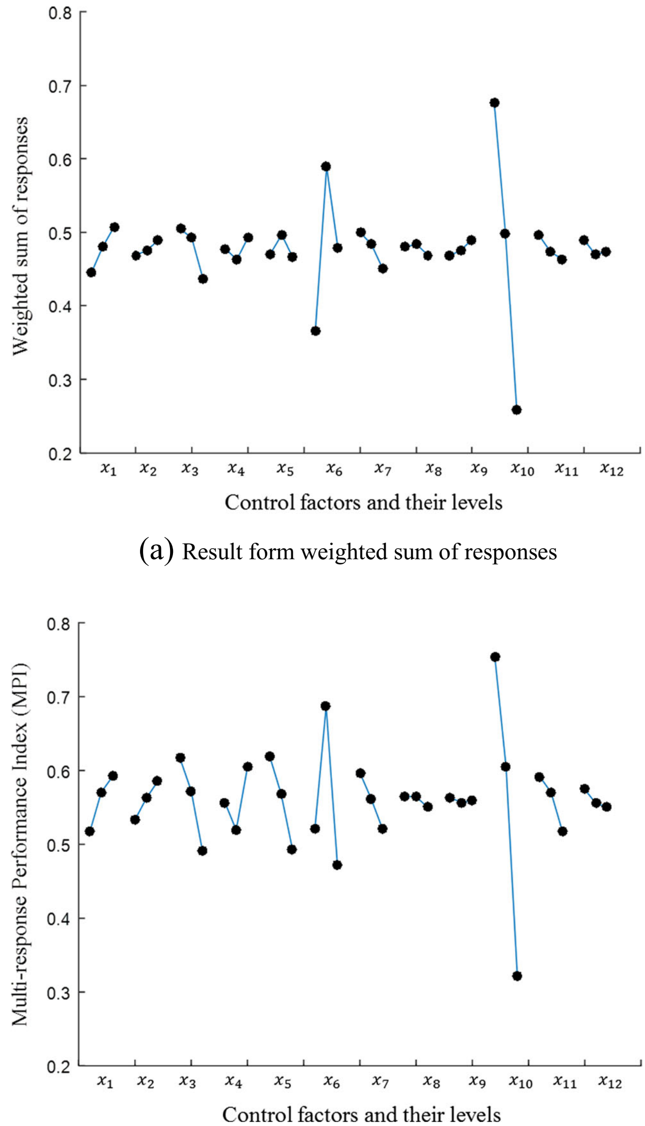
Performance characteristics	Principal component		
	1st PC	2nd PC	3rd PC
f_1	0.8812	0.2341	−0.4108
f_2	0.4407	−0.0919	0.8929
f_3	−0.1713	0.9679	0.1841



(a) Variance percentage of each PC value

(b) Three responses at PC dimension

Fig. 5 PCA result



(a) Result form weighted sum of responses

(b) Result from MPI values

Fig. 6 Response graph of random sampling metamodel

mechanism, it is desirable to consider MPI based result with linearity and the significant effect of the torsion bar rate. Therefore, the MPI concept reduces the uncertainty that arises from determining the weights based on the judgement of the engineer. From the ANOVA, the effects of factors on the MPI are summarized, and factors x_6 and x_{10} are most influential factors when compared with the other factors, and they account for more than 79% of the total variation. Furthermore, factors x_7, x_9, and x_{12} exhibit a lower influence on MPI. From the results, it is possible to determine the robust optimal design condition in the multi-response problem with a factor combination with the highest MPI value. However, in this study, the aim involves reducing the analysis time by subtracting the main factor among many factors to perform a reliability-based analysis under uncertainty as opposed to a simple approximate optimization. Based on the result, the factors in which the sum of the effect percentage on MPI exceeds 95% and which exhibit the lower rank when compared to half of the number of factors are considered in the approximate optimization process such as DDO, RBDO, and RBRDO. The ANOVA values of the six factors are summarized in Table 7.

4 Deterministic design optimization

4.1 Formulation of the DDO problem

The purpose of structural design and optimization involves satisfying any performance reference value related to safety and usability, and the performance reference value is usually formulated as a limit state. General optimization techniques in extant studies proceeded to satisfy the constraints required by the system and to optimize the objective function. This technique is known as deterministic design optimization (DDO).

Table 7 Pooled the analysis of variance (ANOVA) on MPI for low-fidelity case

Case	Sum of squares	DoF	Mean squares	F-statistic	P-value
x_1	0.0262	2	0.0131	0.57	0.6371
x_3	0.0732	2	0.0366	1.59	0.3861
x_4	0.0325	2	0.0163	0.71	0.5860
x_5	0.0709	2	0.0355	1.54	0.3937
x_6	0.2273	2	0.1136	4.94	0.1684
x_{10}	0.8690	2	0.4345	18.88	0.0503

Thus, it was systematically used to reduce costs and improve quality (Arora et al. 1995; Haftka et al. 1998). However, the deterministic design technique does not account for the uncertainties of design parameters, such as dimensions, models, materials, and loads, or indirectly consider the same by using methods such as subjective judgement and partial safety factors.

Hence, the reliability of the optimal value is generally low because it does not consider the uncertainty of the design variables, the tolerance required in the fabrication process, and the fluctuation of material properties. This is a limitation point because only one fixed value is used as a representative value (Tu et al. 1999). As a result, deterministic optimal solutions possess the potential to reduce reliability levels (Beck and de Santanna Gomes 2012). Based on the results of the deterministic optimization, we aim to provide a technique that quantitatively and reasonably considers uncertainties and fluctuations in the design parameters that occur during the initial design process.

The polynomial based response surface method (RSM) is used to create the meta-model for optimum design. The present study employs quadratic and cubic functions to establish response surface meta-models according to their nature of nonlinearity. There were a total of 81 data sample used in training response surface models. The statistical parameter R^2 for evaluating the approximation degree of the response surface must satisfy approximately 95% or more.

The formulation of the deterministic design problem is shown in (6) below. In this case, only the 6 factors and not all the 12 factors are considered from the results of PCA and ANOVA processes, and the corresponding constraints are included. The expression is as follows:

$$\begin{array}{ll} \text{Find} & x_1, x_3, x_4, x_5, x_6, x_{10} \\ \text{Minimize} & f_{DDO} = f_1/4 + f_2/10 - f_3/10 \\ \text{Subject to} & g_{open\_a} \geq 1 \\ & 0 \leq g_{\max\_ef} \leq 20 \\ & -20 \leq g_{\min\_ef} \leq 0 \\ & g_{roll\_a} \leq 0.3 \\ & g_{settl\_t} \leq 0.4 \\ & x_i^L \leq x_i \leq x_i^U \, where, \mathrm{i} = 1, 3, 4, 5, 6, 10 \end{array} \tag{6}$$

4.2 Result of the DDO optimal solution

For each of the six control factors selected through PCA and ANOVA, the approximate optimal values of design variables and the objective function are shown as initial values in Table 8 while considering the random sampling based design of experiments. Based on the results, the following two optimization processes were performed: micro-genetic algorithm (micro-GA) of a global optimization technique and sequential two-point diagonal quadratic approximate optimization (STDQAO) of a gradient-based optimization technique (Song and Lee 2010; Kim et al. 2001; Coello and Pulido 2005). In Table 8, DDO solutions are demonstrated for two cases of PCA (i.e., with PCA) and all design variables (i.e., without PCA). In order to identify the variation of optimal solutions, the micro-GA search was conducted four times by starting with different initial populations for the case of 'with PCA'. It is detected that micro-GA solutions are similar in terms of design variables value (x_1, x_3, x_4, x_5, x_6, x_{10}) and objective function value (f_{DDO}) as shown in Table 8. From the comparison between 'with PCA' and 'without PCA' under both micro-GA and STDQAO, the latter (without PCA) produces the slightly better objective function values since 'without PCA' actually works with the larger (i.e., non-reduced) number of design variables.

The results confirmed that a sufficient objective is satisfied even if the factors with low influence are excluded from the optimum design process through the PCA and ANOVA. Therefore, we significantly reduce the time and cost for RSM calculation, and the formulation of optimization problem, such as RBDO and RBRDO, is simplified since the considered design variables are reduced. Specifically, the overall objective result value of STDQAO in both cases is the lowest, and the reliability-based optimization is performed with this result as the initial value.

5 Design optimization under uncertainty

5.1 Formulation of the RBDO problem

In the study, it is necessary to improve the limit of optimal design based on the deterministic approach as previously confirmed. Therefore, we first systematically approach the uncertainties of design variables and apply reliability-based optimal design that corresponds to a probabilistic approach that can more accurately and rationally handle the stability of structures by applying logical probability and statistical theory. The approach can be formulated as an optimal design problem that minimizes the objective function with the reliability index as a constraint (Sandgren and Cameron 2002). Thus, in contrast to the deterministic approach identified above, the probability constraint that satisfies the constraint exceeds the

Table 8 Result of DDO using PCA result (with PCA) and all design variables (without PCA)

		Design variable												Objective function			
		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	f_1	f_2	f_3	f_{DDO}
PCA result	Initial	1	–	−1	1	−1	0	–	–	–	−1	–	–	3.679	96.21	7.559	9.785
	MGA	−0.777	–	−0.163	0.962	0.144	0.465	–	–	–	−0.486	–	–	4.680	3.068	−18.64	3.341
		−0.952	–	0.531	0.908	−0.567	0.875	–	–	–	−0.669	–	–	4.345	1.201	−19.65	3.171
		−0.841	–	−0.125	0.822	−0.348	0.825	–	–	–	−0.494	–	–	4.507	0.035	−19.67	3.137
		−0.995	–	0.017	0.971	−0.309	0.455	–	–	–	−0.582	–	–	4.664	17.16	−1.388	2.982
	STDQAO	−1	–	0.352	0.856	−1	0.918	–	–	–	−0.695	–	–	4.297	18.49	0	2.924
All design var.	Initial	1	1	−1	1	−1	0	−1	−1	−1	−1	−1	−1	3.679	96.21	7.559	9.785
	MGA	0.376	1	0.160	0.735	0.407	0.376	0.036	0.590	−0.628	−0.807	1	1	4.070	16.37	−2.692	2.924
	STDQAO	0.381	1	0.160	0.736	0.407	0.377	0.036	0.590	−0.627	−0.807	1	1	4.070	16.38	−2.675	2.922

confidence probability according to the probabilistic approach. At this point, it is possible to obtain the robustness of the constraint condition by changing the design area with the reliability index that possesses the upper limit value and the lower limit value based on the distribution characteristic of the random variable. The objective function is to minimize the f_{RBDO}, and the reliability index criterion for five constraints was considered. The RBDO also accounts for two cases of all the design variables and the reduced design variables, similar to the previous DDO. The reliability-based optimization problem in the reduced case is formulated as (7) as follows:

$$
\begin{aligned}
&\text{Find} && x_1, x_3, x_4, x_5, x_6, x_{10}\\
&\text{Minimize} && f_{RBDO} = f_1/4 + f_2/10 - f_3/10\\
&\text{Subject to} && P\left(g_{open\_a} \geq 1\right) \geq R_{target} = \Phi(-\beta)\\
& && P\left(0 \leq g_{\max\_ef} \leq 20\right) \geq R_{target} = \Phi(-\beta)\\
& && P\left(-20 \leq g_{\min\_ef} \leq 0\right) \geq R_{target} = \Phi(-\beta)\\
& && P\left(g_{roll\_a} \leq 0.3\right) \geq R_{target} = \Phi(-\beta)\\
& && P\left(g_{settl\_t} \leq 0.4\right) \geq R_{target} = \Phi(-\beta)\\
& && x_i^L \leq x_i \leq x_i^U \; where, \mathrm{i} = 1, 3, 4, 5, 6, 10
\end{aligned}
\tag{7}
$$

5.2 Result of the RBDO optimal solution

The STDQAO exhibits better results for the optimal value of the deterministic approach, and thus, RBDO based on a single loop single vector (SLSV) is performed based on the value. The SLSV method eliminates the iterative process to obtain the precise most probable point (MPP). This method allows MPP to approximately converge and to simultaneously optimize the design parameter. Therefore, it is possible to reduce the numerical burden due to a dual loop structure such as the reliability index approach (RIA) (Enevoldsen and Sorensen 1994; Yu et al. 1997) or the target performance approach (Jeong et al. 2012). Specifically, MPP based RBDO is more efficient when compared to the second-order reliability method (SORM) (Der Kiureghian et al. 1987) and sampling methods (Lee et al. 2013). The results of RBDO are shown in Table 9, and four cases were examined for the reliability index (β) and standard deviation of input variables (σ). The β of 2 and 3 implies that the confidence interval corresponds to 95.45% (2 sigma) and 99.7% (3 sigma), respectively. The range of σ is from zero to one, and we equally apply the case of 0.5 and 0.8 to all the design variables. It is observed that the design variables change when these values increase, and it corresponds to a conservative solution with an increased objective function. Additionally, the same procedure that considers all the design variables is performed. When the value of β and σ increases, the objective value also increases although it exhibits a lower increase in range when compared with the case of the reduced variables. The difference in the value of design variables occurs based on the number of design variables that are considered when compared with the DDO although it is confirmed that it exhibits a very similar objective value. Therefore, with respect to the optimum design that considers the reliability index, it is possible to obtain a result that is almost similar to the case in which all design variables are applied to the reduced design variables with a result of PCA for the multi-response.

5.3 Formulation of the RBRDO problem

The design optimization methods that consider uncertainty are generally classified into robust design optimization and reliability based design optimization methods. Previous studies proposed optimum design for structure by using each of these methods. In the case of the robustness design method, it is a design technique that reduces the fluctuations in performance by reducing the sensitivity as opposed to reducing the cause of fluctuation in the design process (Youn and Choi 2004). It also exhibits a feature in which the robustness of the objective function and the robustness of the constraint condition are simultaneously considered. However, a disadvantage is that

Table 9 Result of RBDO using PCA result (with PCA) and all design variables (without PCA)

		Design variables												Obj.
		x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	f_{RBDO}
PCA result	Case1	−1	–	−0.036	0.641	−0.950	0.999	–	–	–	−0.676	–	–	2.891
	Case2	−0.833	–	0.272	0.629	−0.920	1	–	–	–	−0.307	–	–	2.998
	Case3	−0.619	–	−0.207	0.998	−0.172	0.526	–	–	–	−0.777	–	–	3.163
	Case4	1	–	0.051	1	0.447	0.216	–	–	–	−0.451	–	–	3.215
All Design Var.	Case1	0.533	1	0.087	0.396	0.953	0.123	−1	0.455	0.157	−0.998	1	0	2.681
	Case2	0.515	0	−0.001	0.388	0.827	0.146	−1	0.440	0.122	−0.994	1	−1	2.843
	Case3	0.528	0	0.071	0.394	0.719	0.203	−1	0.450	0.123	−0.998	1	−1	2.808
	Case4	−0.061	1	0.103	0.558	0.930	−0.008	−1	−0.479	−1	−0.989	1	0	2.907

Case 1 ($\beta = 2$, $\sigma = 0.5$), Case 2 ($\beta = 3$, $\sigma = 0.5$), Case 3 ($\beta = 2$, $\sigma = 0.8$), and Case 4 ($\beta = 3$, $\sigma = 0.8$)

the criterion for determining the degree of reliability is not clear. Otherwise, in the case of RBDO, it is formulated as an optimal design that minimizes the objective function with the reliability index as a constraint based on the stochastic approach (Sandgren and Cameron 2002). However, an additional difficulty is that the robustness of the objective function cannot be considered while the robustness of the constraint condition is secured.

Given these reasons, the reliability-based robustness optimization (RBRDO) that is used in the present study is an integrated method to obtain robustness by providing design robustness to the objective function and the user's desired reliability index to the constraint condition. Specifically, RDO is naturally combined with RBDO for the constraint evaluation, and this corresponds to RBRDO, and the SLSV-based RBDO is used in the study. Thus, we determine a value that minimizes the critical state equation in the space of the standard normal distribution based on the given reliability index condition as follows:

$$\begin{aligned}
&\text{Minimize} \quad \phi(x) = w\left(\frac{\mu_f}{\mu_f^*}\right)^2 + (1-w)\left(\frac{\sigma_f}{\sigma_f^*}\right)^2, 0 \le w \le 1 \\
&\qquad where, \mu_f = f\left(\mu_{x_i}\right), \quad i = 1 \sim n \\
&\qquad \sigma_f \approx \sqrt{\sum_{i=1}^{n}\left(\frac{\partial f(x)}{\partial x_i}\right)^2_{\mu} \cdot \sigma_{x_i}^2} \\
&\text{Subject to} \quad P[G(X) \le 0] \le P_f
\end{aligned} \tag{8}$$

Where, μ_f^* and σ_f^* denote optimal function values that are calculated by only considering the mean and standard deviation, respectively, as in the deterministic design technique.

In this problem, it is necessary to minimize the f_{RBRDO} that correspond to a series of calculations for three responses f_1, f_2, and f_3. The robust design itself possesses the characteristics of the multi-objective for mean and variance, and thus we set their weights as a multiple objective function to solve the problem. We consider the upper and lower boundaries for each of the six design variables and the five constraints similar to the previous RBDO. The optimization problem in the study is formulated as follows:

$$\begin{aligned}
&\text{Find} \quad x_1, x_3, x_4, x_5, x_6, x_{10} \\
&\text{Minimize} \quad f_{RBRBDO} = w_1\frac{\mu_f}{\mu_f^*} + w_2\frac{\sigma_f}{\sigma_f^*} \\
&\qquad where, f = f_1/4 + f_2/10 - f_3/10 \\
&\qquad w_1 + w_2 = 1, 0 \le w_1 \le 1, 0 \le w_2 \le 1 \\
&\text{Subject to} \quad P\left(g_{open_a} \ge 1\right) \ge R_{target} = \Phi(-\beta) \\
&\qquad P\left(0 \le g_{\max_ef} \le 20\right) \ge R_{target} = \Phi(-\beta) \\
&\qquad P\left(-20 \le g_{\min_ef} \le 0\right) \ge R_{target} = \Phi(-\beta) \\
&\qquad P(g_{roll_a} \le 0.3) \ge R_{target} = \Phi(-\beta) \\
&\qquad P(g_{settl_t} \le 0.4) \ge R_{target} = \Phi(-\beta) \\
&\qquad x_i^L \le x_i \le x_i^U where, \mathrm{i} = 1, 3, 4, 5, 6, 10
\end{aligned} \tag{9}$$

5.4 Result of the RBRDO solutions

The DDO result is used as a reference, and thus the value of f_{DDO} in Table 8 is used for μ_f^* and σ_f^* of the objective function, and μ_f^* corresponds to 2.924 and σ_f^* corresponds to 0.680 and 1.089. The results for RBRDO based on the above formulation with same weights are shown in Table 10. The value of μ_f increases with a conservative solution and σ_f decreases with robustness as an increase in the reliability index (β) and the standard deviation of the input parameter (σ). We examined five steps according to the weights of the mean (w_1) and the weights of standard deviation (w_2) from zero to one, and the results in the case of σ=0.5 are illustrated in Fig. 7. The w_1 of sample points are increased by 0.25 from the left side to the right side of each curve. When w_1 increases, μ_f increases and σ_f decreases. The variation of σ_f is higher, and thus f_{RBRDO} finally decreases. In case 2 (β=3), the slope of variance and mean exceeds those in case 1 (β=2). Therefore, when

Table 10 Result of RBRDO

w_1=0.5, w_2=0.5		Case 1 (β = 2, σ = 0.5)	Case 2 (β =3, σ = 0.5)	Case 3 (β =2, σ = 0.8)	Case 4 (β =3, σ = 0.8)
		μ_f^* =2.924, σ_f^* = 0.680		μ_f^* =2.924, σ_f^* = 1.089	
Design variables	x_1	−0.918	−0.957	−0.919	−0.617
	x_3	−0.083	−0.120	−0.077	−0.224
	x_4	0.860	0.858	0.857	1
	x_5	−0.171	−0.085	−0.128	0.113
	x_6	0.541	0.415	−2.984	0.366
	x_{10}	−0.692	−0.651	−0.695	−0.654
Objective function	f_{RBRDO}	1.421	1.410	1.401	1.335
	μ_f	3.079	3.130	3.109	3.234
	σ_f	1.216	1.189	1.894	1.703

β increases, the mean value is shifted to a conservative value, and the variance value decreases. Cases 3 and 4 depict a sharp slope of the variations. Specifically, in case 4, the curve returns to the direction in which σ_f increases when the w_1 exceeds 0.5.

From the results, it is confirmed that the optimal design variables consider two objectives of structural design methodologies with respect to various uncertainties as follows: the reliability-based design optimization deals with the probability of failure while a robust design optimization minimizes the product quality loss. Additionally, the trend of response variation shows that the overall objective f decreases when the response f_3 with a negative term experiences a significant increase. However, in the problem, f is considered based on a series of calculation for the three responses in the objective. These results generally correspond to a conservative solution, and the values decreased instead of increasing with respect to the objective value of RBDO. The use of a multi-objective evolutionary genetic algorithm, such as non-dominated sorting genetic algorithm (NSGA-II), as an optimizer makes it possible to perform multi-objective optimization that considers all the responses.

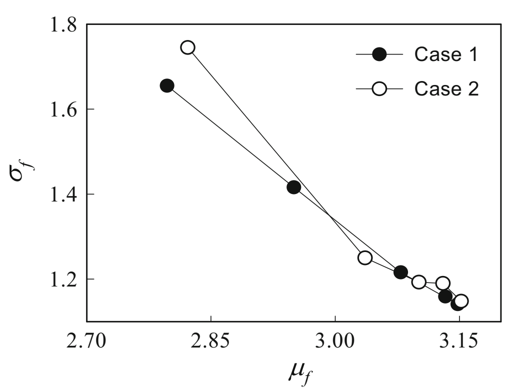


Fig. 7 Pareto optimal solutions for multi-objective function (σ = 0.5)

6 Concluding remarks

In the study, the methods of reliability based design optimization and reliability-based robust design optimization of commercial vehicle cab suspension are implemented with influence factor analysis. The design of experiments of a L27 array is used as a meta-model with 12 control factors and random sampling noise factors. Specifically, we apply principal component analysis to solve the multi-response problem and overcome the limitations of the Taguchi method that are applicable only to a single-response problem. The results of PCA indicate that the multi-response performance index (MPI) value was calculated from the variance percentage of the control factors by considering the uncertainties of noise on each response through an uncorrelated linear combination of PC. Subsequently, an analysis of variance is performed to determine the importance of various design variables and to reduce the costs involved in the design optimization process.

In the case of the deterministic design optimization, uncertainty and variance of design variables are not considered by using only a single fixed representative value, and thus reliability relative to the optimum value is low. Therefore, a stochastic approach, such as robust design and reliability based design, is a more conservative approach for the stability of the structure that reflects the uncertainty of the design variables. In the case of the DDO, the optimal combination value based on DOE meta-model was used as the default criterion, and the optimal objective was derived through STDQAO method. Additionally, RBDO is a representative optimization technique that considers the uncertainty of the objective is performed with the DDO result as the initial value. We examined the objective function with increases in the reliability index and standard deviation of input variables and the trend of variation, and the objective is gradually increased. Both DDO and RBDO were analyzed in the cases of reduced design variables based on the results of the PCA and ANOVA and all design variables. The results confirmed that the cost and time for design optimization are reduced by analyzing the influence factor by using PCA. Furthermore, RBRDO is another reliability based approach that makes it possible to obtain a conservative solution by providing design robustness to the objective function and providing the reliability conditions required by the designer to the constraints. We examined five cases along with the weight of mean and variance as well as the changes in β and σ. We used the values of μ_f^* and σ_f^* in the DDO as the reference for the objective function.

In the present study, both the noise factors in the experimental design method were considered as a normal distribution with a 15% variance in the mean value. A future study will involve more reliable results by estimating the appropriate distribution and considering the uncertainty of the noise factor in the experimental data and reflecting the corresponding parameters such as mean and variance. Moreover, we performed the optimization for a single function f based on a series of responses in this study, and thus a future study will involve using a multi-objective optimization for each response by using an optimizer such as NSGA-II.

Acknowledgements This research is supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF), funded by the Ministry of Science, ICT & Future Planning (2017R1A2B4009606). This work is supported by the Korea Institute of Energy Technology Evaluation and Planning (KETEP) and the Ministry of Trade, Industry & Energy (MOTIE) of Republic of Korea (20163030024420). This research is supported by NGV & Hyundai Motor Group (2016-11-0906).

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