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Structural optimization of the cross-beam of a gantry machine tool based on grey relational analysis

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Abstract Crossbeam structural design of gantry machine tool is a multi-level, multi-index and multi-scheme decision-making problem. In order to solve the above problem, the optimum seeking model of crossbeam structure was built through using the grey relational analysis and Analytic Hierarchy Process. The finite element analysis of the static and dynamic performance parameters for four kinds of crossbeam structural schemes designed had been done, and the optimal design scheme was selected by using the optimum seeking model. After conducting sensitivity analysis for the optimal crossbeam selected, the reasonable design variables were obtained, and the dynamic optimization design model of crossbeam was established. Six groups of non-inferior solutions were obtained after solving the optimization design model. The optimal solution was selected from the non-inferior solution set through using the crossbeam structural optimization method based on grey relational analysis again, which makes the crossbeam's dynamic performance improving greatly. The dynamic experiments on the crossbeams before and after optimization design were conducted, then the experimental results show that the first four order natural frequencies of the crossbeam increase 17.56 %, 19.36 %, 17.04 % and 19.58 % respectively, which proves that the structural optimization design method based on grey relational analysis proposed in this paper is reasonable and practicable.

Keywords Gantry machine tool · Crossbeam · Grey relational analysis · Optimization design · Sensitivity analysis · Dynamic experiment

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1 Introduction

With the vigorous development of China's aerospace, marine engineering equipment, shipbuilding, automotive and other manufacturing industries, the processing demand of large and complex parts increases greatly. The gantry machine tool with the advantage of extensive machining span, high machining precision and good rigidity has been applied in the above manufacturing industries broadly, whose development is very rapid subsequently. In the Chinese state science and technology major projects of "high-grade CNC machine tools and basic manufacturing equipment", R&D of gantry machine tool has been taken as an important study issue that includes the structural dynamic optimization design techniques for its main components and parts. As an important part of gantry machine tool, crossbeam achieves the Y-axis feed motion, and supports the spindle system to achieve the Z-axis feed motion. The structural and dynamic properties of crossbeam affect the gantry machine tool's overall stiffness and machining precision directly. Therefore, the crossbeam's structural optimization design of the gantry machine tool has become a research focus by more and more scholars. Luo and Li (2006) analyzed the stiffness of a crossbeam with different rib plate shapes by finite element method. Shi (2009) conducted the static analysis, modal analysis and harmonic analysis for the gantry machining centre's crossbeam and found out the weakness for further optimization design. Wang et al. (2009) conducted topology optimization design for the crossbeam through studying the layout of the rib plates, so the static stiffness and anti-vibration performance are greatly improved. Guan et al. (2010) discovered the weak link of a gantry machine tool's crossbeam by using finite element method and put forward a modification scheme, which made the static and dynamic performance of crossbeam improving. Zhao et al. (2008) carried out structural bionic optimization design for stiffener plate of a gantry machine tool's crossbeam based on giant waterlily vein distribution, so the crossbeam achieved lightweight performance design and its dynamic performance was improved further.

Although the above research results have promoted the development of crossbeam's optimization design method, there still exist several problems that need to be solved and are as follows:

(1) The crossbeam's structures were commonly designed by experience, and the optimization design was the comparative analysis and selection among the different shapes or optimizing the key sizes of one structure. The combination of both above methods was seldom considered.
(2) The sensitivity of the structure and design parameters to the crossbeam's static and dynamic performance was rarely considered (Wang et al. 2010), which may result in that the crossbeam's structural optimization design has certain blindness.
(3) How to select the optimal one from many possible crossbeam structural design schemes or/and non-inferior solution set of crossbeam's optimization design, the machine design experts mainly decided by the subjective experience for lacking of a scientific optimum seeking method.

In order to address the above issues, a static and dynamic optimization method for crossbeam's structural design based on grey relational analysis was established in this paper. The comparative analysis of the static and dynamic performance parameters for four kinds of crossbeam structural design schemes was conducted through using CAD/CAE integrated design approach, and the optimal design scheme of crossbeam was selected by using the static and dynamic optimization method. After conducting sensitivity analysis for the optimal crossbeam, the mathematical model of dynamic optimization design was built. Solving the above mathematical model with the optimization method proposed in this paper, the crossbeam of gantry machine tool achieved the dynamic multi-objective optimization design.

2 Grey relational analysis of crossbeam design scheme set

Crossbeam is a major part of gantry machine tool, so its structural property affects the machine tool's machining accuracy greatly (Li et al. 2010). In the process of formulating gantry machine tool's crossbeam structural design scheme, the main consideration is whether the mechanical properties and anti-vibration performance of the crossbeam meet the design requirements, which shows the fact that the crossbeam design scheme optimum seeking is a multi-level and multi-objective decision-making problem with many factors. In the multi-index and multi-scheme decision-making issues, expert assessment method, Analytic Hierarchy Process, fuzzy comprehensive evaluation and grey relational analysis are used usually, whose comparative analysis are as follows.

The expert evaluation method is a kind of method that makes overall judgment to objects based on subjective judgment of experts. This method is relatively simple and gives full play to experts' wisdom and experience. Timely decisions are made with this method, but evaluations are influenced by random factors and evaluation results are more likely to be effected by subjective thoughts of judger and are limited to personnel experience and knowledge, which may lead to personal prejudice and one-sidedness.

Analytic Hierarchy Process is a multi-criteria decision-making method combining of qualitative and quantitative analysis and is proposed by American operations researcher T.L Saaty. Consistency test is necessary for issues with three or more schemes to ensure the rationality. Issues failing to pass the consistency test need to be compared again or evaluating results will be influenced. So the efficiency of Analytic Hierarchy Process decision-making is low for multi-scheme issues. The advantages of Analytic Hierarchy Process are obvious in determining the evaluation index weight. On the one hand, evaluation index weight is the qualitative evaluation of each index made by decision-makers. The intention of the decision-makers can be reflected by using Analytic Hierarchy Process. On the other hand, the evaluation weight is relatively stable once assigned.

The fuzzy comprehensive evaluation method is also a multi-index decision-making method, but the traditional fuzzy evaluation matrix based on factor membership only consider the various factors' contribution to optimal membership degree individually (Cao et al. 2005). The multi-objective optimization decision-making system is an organic whole whose factors have interconnections and jointly affects the system characteristics, so it is a grey information system. Therefore, it should be fully considered that gantry machine tool crossbeam structure design schemes often contain both fuzzy and grey information during the process of design scheme optimum seeking.

The grey relational analysis is also a multi-index and multi-scheme decision-making method, which is proposed by Professor Deng Julong of Huazhong University of Science and Technology of China (Lin and Lin 2002). Grey relational analysis method uses grey relational degree to describe the strength and order of the relationship between the factors. If sample data reflects two' change in trend is basically the same, the relational degree between them is large, conversely, related degree is small. The advantage of grey relational analysis method is that it can greatly reduce the loss due to information asymmetry, data requirements are lower, and computational workload is small. Grey relational analysis is based on the development trends, therefore, the sample number requirement is not too much, and its analysis result is the same with the qualitative analysis.

Based on the above analysis, the gantry machine tool crossbeam's structure design scheme optimum seeking is a grey information system and is a multi-index decision-making issue, therefore, the grey relational analysis (Cheng et al. 2011) and Analytic Hierarchy Process (Wang and Zhou 2009) were introduced to build the crossbeam's design scheme optimum seeking model.

Assuming that n crossbeam design schemes compose a scheme set of grey system, and each design scheme has M indexes. The grey system is decomposed into m subsystems according to different attributes, each grey subsystem has $m_1, m_2, \ldots, m_m$ indexes respectively and they satisfy the following equations.

$$M = \bigcup_{i=1}^{m} m_i, m_i \bigcap m_k = \Phi \qquad i \neq k \tag{1}$$

In the above equations, ∪ is union set, ∩ is intersection set, and Φ is the empty set.

Assuming that n index values of the j-th index in the i-th grey subsystem can be expressed as vector x_j^i

$$x_j^i = \left(x_{j1}^i, x_{j2}^i, \cdots, x_{jn}^i\right) \tag{2}$$

m_i indexes of n schemes in the i-th grey sub-system can be expressed as the following matrix

$$X_{m_i \times n}^i = \begin{bmatrix} x_{11}^1 & x_{12}^1 & \cdots & x_{1n}^1 \\ x_{21}^2 & x_{22}^2 & \cdots & x_{2n}^2 \\ \vdots & \vdots & \ddots & \vdots \\ x_{m_i 1}^{m_i} & x_{m_i 2}^{m_i} & \cdots & x_{m_i n}^{m_i} \end{bmatrix} \tag{3}$$

Among them, $i = 1, 2, \ldots, m$.

In order to conduct grey relational analysis conveniently, non-dimensional normalized treatment is made on all evaluation index values of crossbeam design schemes. The treatment methods are as follows:

(1) For the bigger the better ("Benefit-type") evaluation index

$$r_{j,k}^j = \frac{x_{j,k}^j - \min\left(x_{j,k}^j\right)}{\max\left(x_{j,k}^j\right) - \min\left(x_{j,k}^j\right)} \tag{4}$$

(2) For the smaller the better ("Costs-type") evaluation index

$$r_{j,k}^j = \frac{-x_{j,k}^j + \max\left(x_{j,k}^j\right)}{\max\left(x_{j,k}^j\right) - \min\left(x_{j,k}^j\right)} \tag{5}$$

Among them, $j = 1, 2, \cdots, m_i, k = 1, 2, \cdots, n$.

After conducting non-dimensional normalized treatment, matrix (3) is as follows:

$$R_{m_i \times n}^i = \begin{bmatrix} r_{11}^1 & r_{12}^1 & \cdots & r_{1n}^1 \\ r_{21}^2 & r_{22}^2 & \cdots & r_{2n}^2 \\ \vdots & \vdots & \ddots & \vdots \\ r_{m_i 1}^{m_i} & r_{m_i 2}^{m_i} & \cdots & r_{m_i n}^{m_i} \end{bmatrix} \tag{6}$$

As the crossbeam design scheme optimum seeking has relativity of comparison, and scheme optimum seeking in the *i-th* grey sub-system is relative to the m_i evaluation indexes of this sub-system, an ideal reference sequence is selected firstly, which is denoted as follows:

$$F_i^0 = \left[f_1^0, f_2^0, \cdots, f_{m_i}^0\right]^T \tag{7}$$

In the formula, $f_h^0 = \max\left(r_{h1}^h, r_{h2}^h, \cdots, r_{hn}^h\right)$, $h = 1,2, \ldots, m_i$, namely each evaluation index of F_i^0 is the maximum value of corresponding evaluation indexes which participate optimum seeking of n schemes. F_i^0 is the ideal design scheme or reference sequence, while n schemes are comparison sequence. Close degree between reference sequence and comparison sequence is usually measured through the grey relational coefficient (Rao et al. 2009). $\xi_{h,j}^h$ is the grey relational coefficient of the *j-th* comparison sequence relatives to the *h-th* index of reference sequence ($j = 1,2, \ldots, n$). $\xi_{h,j}^h$ can be obtained from (8).

$$\xi_{h,j}^h = \frac{\min\limits_h \min\limits_j \left|f_h^0 - r_{h,j}^h\right| + \rho \max\limits_h \max\limits_j \left|f_h^0 - r_{h,j}^h\right|}{\left|f_h^0 - r_{h,j}^h\right| + \rho \max\limits_h \max\limits_j \left|f_h^0 - r_{h,j}^h\right|},$$
$$h = 1, 2, \cdots, m_i, \quad j = 1., 2, \cdots, n \tag{8}$$

In the formula, $\rho \in [0,1]$, this paper takes $\rho = 0.5$, thus the grey relational coefficient matrix of sub-system i is as following.

$$\Xi_{m_i \times n}^i = \begin{bmatrix} \xi_{11}^1 & \xi_{12}^1 & \cdots & \xi_{1n}^1 \\ \xi_{21}^2 & \xi_{22}^2 & \cdots & \xi_{2n}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \xi_{m_i 1}^{m_i} & \xi_{m_i 2}^{m_i} & \cdots & \xi_{m_i n}^{m_i} \end{bmatrix} \tag{9}$$

Using the above method, grey relational coefficient matrix of each subsystem can be obtained. The grey relational coefficient matrix of gantry machine tool's crossbeam design scheme obtained through joining the grey relational coefficient matrix of each subsystem is as follows.

$$\Xi = \begin{bmatrix} \Xi^{1}_{m_1 \times n} \\ \Xi^{2}_{m_2 \times n} \\ \vdots \\ \Xi^{i}_{m \times n} \\ \vdots \\ \Xi^{m}_{m_m \times n} \end{bmatrix} \quad (10)$$

3 Establishing optimum seeking model of crossbeam's design schemes

There are many kinds of crossbeam structure schemes that meet the design requirements usually, and the machine tool designers need to select the best one whose comprehensive performance is optimal. In order to meet the development requirements of modern CNC machine tool, the static properties, the anti-vibration performance and the lightweight performance should be considered when we determine the optimal crossbeam structure scheme, so it is a multi-objective optimum seeking decision-making problem, and a reasonable optimum seeking model need be established.

3.1 The objective system of crossbeam design scheme

Based on the grey system theory, and according to design requirements, the objective functions of optimum seeking model are the static properties $S(Y)$, anti-vibration performance $V(Y)$ and lightweight performance $W(Y)$. The decision-making requirements of these three objective functions are as follows, the static properties and lightweight performance indexes are the lower the better and the anti-vibration performance indexes are the higher the better. The above three decision-making objectives are both independent and interrelated, each of them contains of different indexes that constitute the crossbeam design scheme optimum seeking decision-making system. All the index vectors decomposition of this decision-making system is shown in Fig. 1.

3.2 The mathematical model of crossbeam design scheme optimum seeking

Assuming that the design scheme set of crossbeam is P (P_1, $P_2, \ldots, P_n$), and each design scheme is likely to be selected or not selected, so it can be used 1 or 0 to indicate. And 0- 1 variables y_i is used to describe the choice state of P_i.

$$y_i = \begin{cases} 1, \text{ scheme } P_i \quad \text{selected} \\ 0, \text{ scheme } P_i \quad \text{not selected} \end{cases} \quad (11)$$

So 0–1 vector $Y = (y_1, y_2, \ldots, y_n)$ is used to describe the design scheme set vector P of crossbeam.

$$\begin{cases} y_i(y_i - 1) = 0 \\ \sum_{i=1}^{n} y_i = 1 \end{cases} \quad (12)$$

Equation group (12) ensures that only one in y_1, y_2,..,y_n is 1. The different values of $y_1, y_2, \ldots, y_n$ represent different crossbeam design schemes, so the original problem turns into getting the solution $Y^* = (y_1^*, y_2^*, \cdots, y_n^*)$. For example, there are three crossbeam design schemes (y_1, y_2, y_3) that can be respectively expressed as (l, 0,0), (0, l, 0), (0,0, l), if the scheme y_3 is selected, then the solution is $Y*$ = (0, 0, l).

Based on the preceding analysis, the mathematical model of multi-objective optimum seeking decision-making is as follows:

For a certain type of crossbeam's design scheme optimum seeking:

Design scheme set exists: $Y = [y_1, y_2, \ldots, y_n]$

$$\begin{cases} y_i(y_i - 1) = 0 \\ \sum_{i=1}^{n} y_i = 1 \\ g_u(Y) \le 0(u = 1, 2, \cdots, k) \\ h_v(Y) = 0(v = 1, 2, \cdots, l < n) \end{cases}$$

$$\begin{aligned} &\text{Brought optimum } [S(Y), V(Y), W(Y), T(Y), E(Y)] \\ &\quad = [S(Y*), V(Y*), W(Y*), T(Y*), E(Y*)] \end{aligned}$$

$$\begin{cases} Y \in R^n, y_1, y_2, \cdots, y_n, \text{design schemes} \\ \text{Y} * \text{is the best design scheme} \end{cases}$$

In the model, the length, the width, the height, the strength, the stiffness and the first-order natural frequency of the crossbeam are main constraints.

(1) Length constraint: $h_1(Y) = -L(Y) + L = 0$
(2) Width constraint: $h_2(Y) = -D(Y) + D = 0$
(3) Height constraint: $h_3(Y) = -H(Y) + H = 0$
(4) Strength constraint: $g_1(Y) = \sigma_{\max}(Y) - [\sigma] \le 0$
(5) Stiffness constraint: $g_2(Y) = K_{\max}(Y) - [K] \le 0$
(6) First-order natural frequency constraint $g_3(Y) = -f_1(Y) + [f] \le 0$

In the formula, L, D, H and $[f]$ stand for design requirements of length, height, width and minimum first-order natural frequency of the crossbeam. $[\sigma]$, $[K]$ stand for allowable stress and allowable stiffness of crossbeam material.

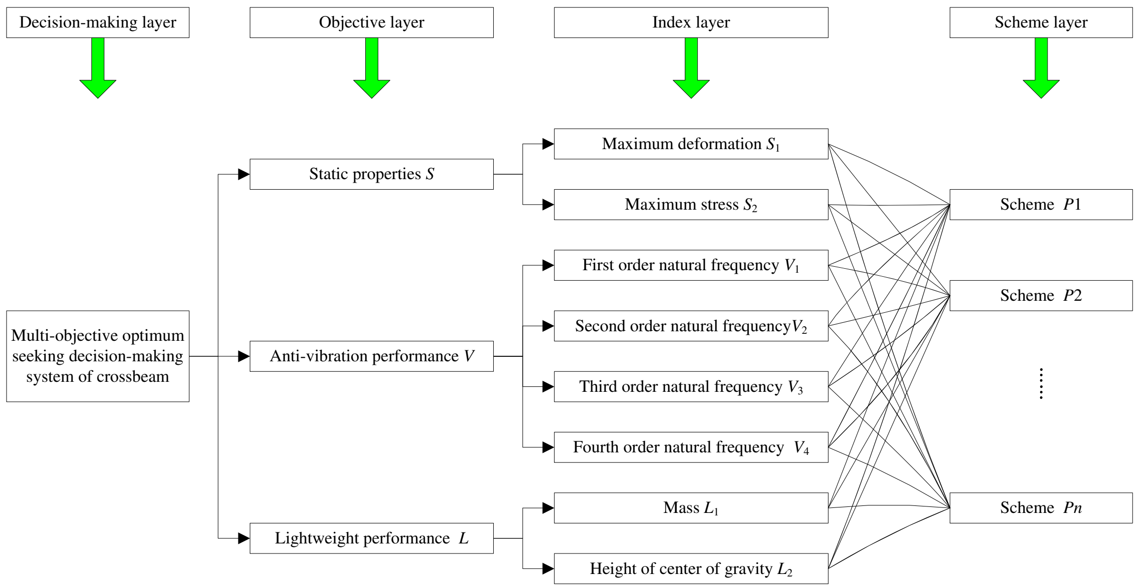


Fig. 1 Decision-making system of crossbeam design scheme

3.3 Solving method for optimum seeking model

The solving process of crossbeam design optimum seeking model is as follows: obtaining crossbeam design schemes that meet the constraints firstly, then evaluating the scheme set by using grey relational analysis method according to the optimum seeking model, determining the optimal design scheme of the crossbeam finally. Grey relational analysis method is used to process data, and the data processing is carried out in accordance with (3)–(10) of this paper. Analytic Hierarchy Process (Li and Zhang 2009) is used to determine the weight coefficient.

Figure 1 shows that the evaluation system of crossbeam design scheme optimum seeking has four levels that are decision layer, objective layer, index layer and scheme layer. The index composition of each objective can be expressed as follows: $S(S_1,S_2)^T$, $V(V_1,V_2,V_3,V_4)^T$ and $W\ (W_1,W_2)^T$. There are n crossbeam design schemes. After quantifying the all performance indexes, a reference index set can be chosen, which is composed by choosing the best index value of all the crossbeam design schemes. The reference index set describes an ideal design scheme of crossbeam. Then grey relational coefficient matrix Ξ of n design schemes relative to reference design scheme can be obtained further. In

Fig. 2 3-D parameter model of the gantry machine tool

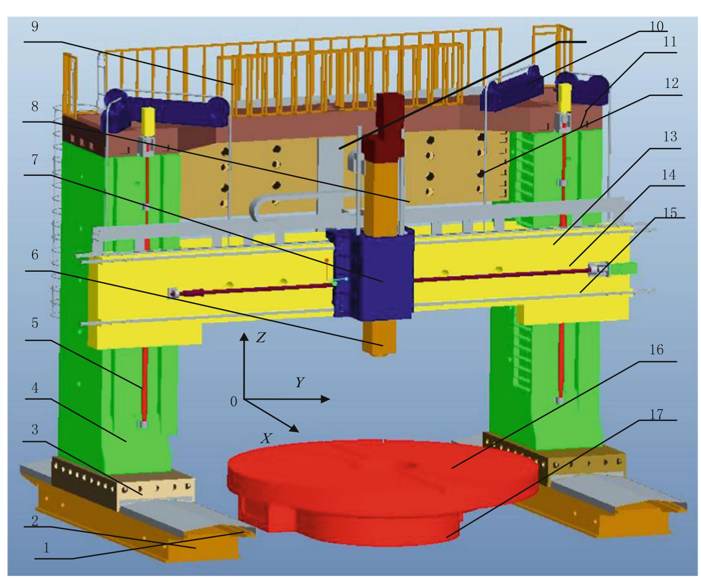
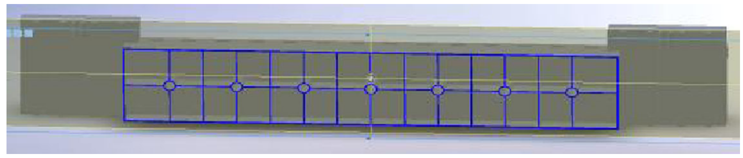
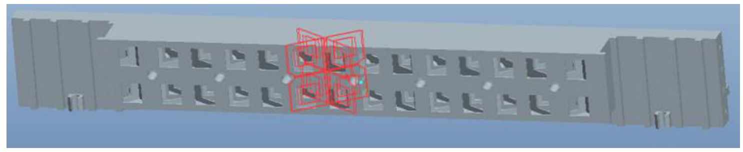
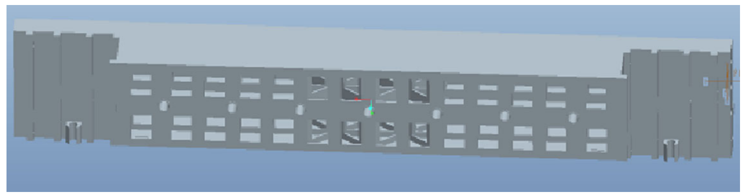
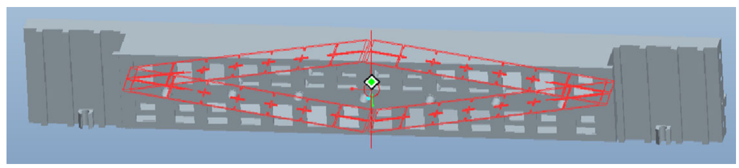


Table 1 The four kinds of beam structure schemes

Scheme number	Structural characteristics	Schematic diagram
Scheme A	#-type rib plate structure	
Scheme B	X-type rib plate structure	
Scheme C	Integrated structure of #-type and X-type rib plate structure	
Scheme D	X-type rib plate structure and reinforcing rib plates	

the (13), ξ is the grey relational coefficient of each crossbeam's evaluation index relative to the reference index set.

$$\Xi = \begin{bmatrix} \xi^{s1}_{s_1,1} & \xi^{s1}_{s_1,2} & \cdots & \xi_{s_1,n^{s1}} \\ \xi^{s2}_{s_2,1} & \xi^{s2}_{s_2,2} & \cdots & \xi^{s2}_{s_2,n} \\ \xi^{v1}_{v_1,1} & \xi^{v1}_{v_1,2} & \cdots & \xi^{v1}_{v_1,n} \\ \xi^{v2}_{v_2,1} & \xi^{v2}_{v_2,2} & \cdots & \xi^{v2}_{v_2,n} \\ \xi^{v3}_{v_3,1} & \xi^{v3}_{v_3,2} & \cdots & \xi^{v3}_{v_3,n} \\ \xi^{v4}_{v_4,1} & \xi^{v4}_{v_4,2} & \cdots & \xi^{v4}_{v_4,n} \\ \xi^{w1}_{w_1,1} & \xi^{w1}_{w_1,2} & \cdots & \xi^{w1}_{w_1,n} \\ \xi^{w2}_{w_2,1} & \xi^{w2}_{w_2,2} & \cdots & \xi^{w2}_{w_2,n} \end{bmatrix} \quad (13)$$

By using the Analytic Hierarchy Process, the objective layer's weight coefficient W (w_s, w_v, w_w, w_t, w_e) and each index layer's weight coefficient W_s (w_{s1}, w_{s2}), W_v (w_{v1}, w_{v2}, w_{v3}, w_{v4}), W_w (w_{w1}, w_{w2}) can be obtained. Then objective layer's grey relational vector R_i (r_{i1}, r_{i2}, r_{i3}, ..., r_{in}) ($i = 1, 2, \ldots, n$) and decision-making layer's grey relational vector R (r_1, r_2, ..., r_n) also can be figured out.

Take calculating an-vibration performance's grey relational vector R_v of all schemes' objective layer as example:

$$R_v = (w_{v1}, w_{v2}, w_{v3}, w_{v4}) \bullet \begin{bmatrix} \xi^{v1}_{v1,1} & \xi^{v1}_{v1,2} & \cdots & \xi^{v1}_{v1,n} \\ \xi^{v2}_{v2,1} & \xi^{v2}_{v2,2} & \cdots & \xi^{v2}_{v2,n} \\ \xi^{v3}_{v3,1} & \xi^{v3}_{v3,2} & \cdots & \xi^{v3}_{v3,n} \\ \xi^{v4}_{v4,1} & \xi^{v4}_{v4,2} & \cdots & \xi^{v4}_{v4,n} \end{bmatrix} = (r_{v1}, r_{v2}, \cdots, r_{vn}) \quad (14)$$

Similarly, R_s, R_w, R_t and R_e can be obtained, then the grey relational vector R of decision-making layer can be calculated by using (15).

$$R = (w_s, w_v, w_w) \bullet \begin{bmatrix} r_{s1} & r_{s2} & \cdots & r_{sn} \\ r_{v1} & r_{v2} & \cdots & r_{vn} \\ r_{w1} & r_{w2} & \cdots & r_{wn} \end{bmatrix} = (r_1, r_2, \cdots, r_n) \quad (15)$$

Table 2 The material parameters of crossbeam

Material	Density kg/m^3	Elastic modulus /GPa	Poisson ratio
HT300	7800	180	0.3

The best crossbeam design scheme Y^* can be obtained by comparing $r_1, r_2, \ldots, r_n$.

4 Crossbeam's overall structural design scheme optimum seeking

The 3-D parametric model of the gantry machine tool studied in this paper is shown in Fig. 2. In order to ensure the machining precision, the gantry machine tool must have good rigidity. As a major part of the gantry machine tool, crossbeam's structure plays the role of supporting the square ram and spindle system. Therefore, it is very important to select a reasonable crossbeam to improve the rigidity of the whole machine tool. In the process of research and development of gantry machine tool, four kinds of crossbeam structures were designed, all of which meet the design requirements. Traditionally, the designers select the crossbeam design scheme by using experience, so it is difficult to obtain the best structure. In order to solve the above problem, this paper will use the optimum seeking model to select a crossbeam structure with optimal comprehensive performance.

4.1 The description of crossbeam design schemes

The four kinds of crossbeams designed in this paper are shown in Table 1, whose material properties are shown in Table 2. In this paper, the structural and modal finite element analysis of crossbeams were carried out by simulating the actual load condition and constraints in CAE software, then the index values of static properties, anti-vibration performance and lightweight performance were obtained, which are shown in Table 3. Table 3 shows that the selection of the crossbeam structural design is a comprehensive decision-making problem with multi-level, multi-indexes and multi-scheme. If the designers adopt the traditional selection method, it will be difficult to guarantee the selected scheme is the best due to the subjectivity and randomness.

Table 4 Judgment rules

Scale a_{ij}	Meaning
1	Both elements are equally important.
3	The former element is slightly important than the latter.
5	The former element is obviously important than the latter.
7	The former element is strongly important than the latter.
9	The former element is extremely important than the latter.
2,4, 6,8	Represent the median value of the adjacent judge.

4.2 Getting the optimal solution of the crossbeam overall design

4.2.1 Determining the weight coefficient of each index

As mentioned above, $P = (A, B, C, D)$ is the four kinds of crossbeam structural design schemes. In order to select the best crossbeam from the four schemes, the weight coefficients of indexes at all levels should be determined firstly. There are many ways to determine the weight coefficient, and this paper will use the Analytic Hierarchy Process to determine the index weight coefficient.

Firstly, the evaluation matrix $I = (a_{ij})_{n\times n}$ should be built according to the comprehensive performances of crossbeams by using the rules as shown in Table 4, where a_{ij} is the important scale coefficient that element i relative to element j. All the evaluation matrixes obtained are (16)–(19) respectively. I_1 is the evaluation matrix for the crossbeam's first level index (S,V,L). I_{21} is the evaluation matrix for the secondary index of $S(S_1, S_2)$. I_{22} is the evaluation matrix for secondary index of $V(V_1, V_2, V_3, V_4)$. I_{23} is the evaluation matrix for the secondary index of $L\,(L_1, L_2)$.

$$I_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1/2 & 1 \end{bmatrix}, \tag{16}$$

Table 3 Performance data of four crossbeams

Objective	Index	Scheme A	Scheme B	Scheme C	Scheme D
Static properties S	Maximum deformation $S_1/10^{-5}$ m	4.72	4.12	4.59	4.59
	Maximum stress S_2/Mpa	3.31	3.15	3.33	3.16
Anti-vibration performance V	First-order natural frequency v_1/Hz	53.65	53	52.18	53.144
	Second-order natural frequency v_2/Hz	73.72	73.446	72.04	70.03
	Third-order natural frequency v_3/Hz	116.33	120.08	114.74	111.93
	Fourth- order natural frequency v_4/Hz	143.36	146.49	146.16	136.59
Lightweight performance L	Mass L_1/kg	30274	32098	31079	32080
	Height of center of gravity L_2/mm	758	816	782	809

Table 5 Weight coefficient of each index

Objective layer	Weight coefficient	Index layer	Weight coefficient
Static properties	0.4126	Maximum deformation	0.675
		Maximum stress	0.325
Anti-vibration performance	0.3275	first-order natural frequency	0.381
		second-order natural frequency	0.298
		third-order natural frequency	0.211
		fourth-order natural frequency	0.110
Lightweight performance	0.2599	Mass	0.5
		height of center of gravity	0.5

$$I_{21} = \begin{bmatrix} 1 & 3 \\ 1/3 & 1 \end{bmatrix} \tag{17}$$

$$I_{22} = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 1/2 & 1 & 2 & 3 \\ 1/2 & 1/2 & 1 & 3 \\ 1/2 & 1/3 & 1/3 & 1 \end{bmatrix}, \tag{18}$$

$$I_{23} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{19}$$

Secondly, according to $IW^{\mathrm{T}} = \lambda_{\max} W^T$, the maximum eigenvalue $\lambda_{\max}$ and the corresponding eigenvectors W^{T} can be calculated for the comparison matrix I. The weight vectors obtained by the consistency test are $W_1 = (0.3275, 0.4126, 0.2599)$, $W_{21} = (0.675, 0.325)$, $W_{22} = (0.381, 0.298, 0.211, 0.110)$, $W_{23} = (0.5, 0.5)$. Therefore, the weight coefficient of each index is shown in Table 5.

4.2.2 Obtaining optimal crossbeam design scheme

After processing all the data in Table 3 according to (3)–(10), all the grey relational coefficients of each index were obtained, which are shown in Table 6.

After using the model solving method of Section 3.3 to calculate the data of Table 5 in MATLAB software, the result obtained is that all the schemes' comprehensive performance grey relational degree which relatives to the reference index set (ideal scheme) is $R = (0.6797, 0.6831, 0.5733, 0.5608)$. So the comprehensive evaluation order of crossbeams is B > A > C > D, which means that the design scheme B is the best option and the optimal solution is $Y^* = (0, 1, 0, 0)$.

5 The sensitivity analysis and size optimization design of crossbeam

Considering that the crossbeam has a complex structure and many design variables, the design parameters which have great influences on the static and dynamic performances of the crossbeam structure should be found out to carry on the structure optimization design effectively, which can avoid the blindness for the structural optimization. For the selected crossbeam structure (design scheme B), the reasonable design variables can be obtained for the optimization design through the sensitivity analysis, which is targeted to carry out the structural optimization design to obtain the optimal size of the crossbeam structure.

Table 6 Grey relational coefficient and weight coefficient of each index

Sub-system and ts weight coefficient	Index	Weight coefficient	Grey relational coefficient			
			Scheme A	Scheme B	Scheme C	Scheme D
Static properties (0.4126)	Maximum deformation	0.675	0.333	1	0.750	0.697
	Maximum stress	0.325	0.347	0.362	0.333	1
Anti-vibration performance (0.3275)	The first-order natural frequency	0.381	1	0.870	0.524	0.333
	The second-order natural frequency	0.298	1	0.531	0.333	0.592
	The third-order natural frequency	0.211	0.521	1	0.578	0.333
	The fourth-order natural frequency	0.110	0.613	1	0.938	0.333
Lightweight performance (0.2599)	Mass	0.5	1	0.333	0.556	0.358
	Height of center of gravity	0.5	1	0.333	0.586	0.4

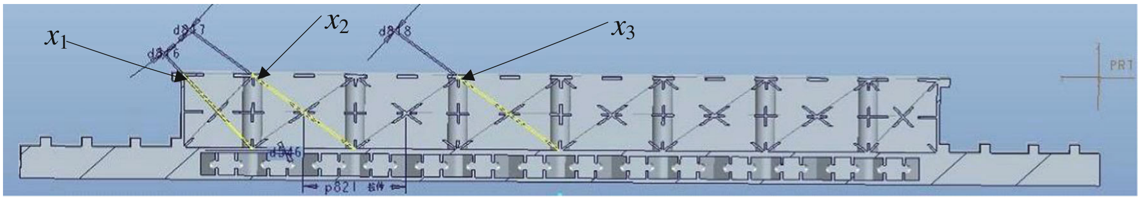


Fig. 3 Design parameters of the crossbeam

5.1 The sensitivity analysis of the crossbeam

Using the optimal gradient method, the sensitivities of various parameters for the crossbeam structure to the maximum deformation, the maximum stress, the first four order natural frequencies, the mass and the height of center of gravity were obtained separately. By the calculation and the comparison analysis, the thickness of the crossbeam's rib plates P_2-d816, P_2-d817 and P_2-d818 (as shown in Fig. 3) have great influence on the static and dynamic performances of the crossbeam, as shown in Fig. 4.

From the sensitivity analysis results, we can know that the sensitivity of the rib plate thickness of different crossbeam sections Sec6to the static and dynamic performances is different. The rib plate thickness P_2-d816 has more great influence on the maximum stress, the second order natural frequency and the third order natural frequency of the crossbeam. The rib plate thickness P_2-d817 has more great influence on the maximum stress, the third order natural frequency and the forth order natural frequency of the crossbeam. The rib plate thickness P_2-d818 has more great influence on the maximum deformation, the first order natural frequency and the second order natural frequency of the crossbeam. Of course, the thickness of all the rib plates affects the crossbeam's lightweight performance indexes greatly. Therefore, in order to improve optimization design efficiency, P_2-d816, P_2-d817 and P_2-d818 are defined as design variables, whose initial values and range are shown in Table 7.

5.2 Optimization design modeling of the crossbeam structure

As can be seen from the design results of the scheme B, the maximum stress of the crossbeam is far smaller than the allowable stress of the material, the maximum deformation is also very small, which means that the static performance has already met the design requirement of the crossbeam. Therefore, the ant-vibration performance of the crossbeam is defined as the optimization objective function, the mass and static performances are defined as the constraint functions. The traditional optimization design method of gantry machine tool crossbeam is increasing the crossbeam's low orders natural frequencies respectively without considering the weight coefficient of each natural frequency (Wu et al. 2009), which usually results in crossbeam's higher material consumption and cost. In order to solve the above problem, we give the corresponding weight coefficient to the first four natural frequencies f_1, f_2, f_3 and f_4. Then the dynamic optimization model of the crossbeam is defined as follows.

$$\begin{cases} \min[f(x_1, x_2, x_3)] = \dfrac{10000}{\sqrt{\alpha_1 f_1^2 + \alpha_2 f_2^2 + \alpha_3 f_3^2 + \alpha_4 f_4^2}} \\ s.t \begin{cases} M(x_1, x_2, x_3) \le 33000kg \\ \sigma_{\max}(x_1, x_2, x_3) \le [\sigma] = 60Mpa \\ \delta_{\max}(x_1, x_2, x_3) \le 0.05mm \\ 16mm \le x_1 \le 24mm \\ 16mm \le x_2 \le 24mm \\ 18mm \le x_3 \le 30mm \end{cases} \end{cases} \tag{20}$$

Fig. 4 The sensitivity of the thickness of the rib plates

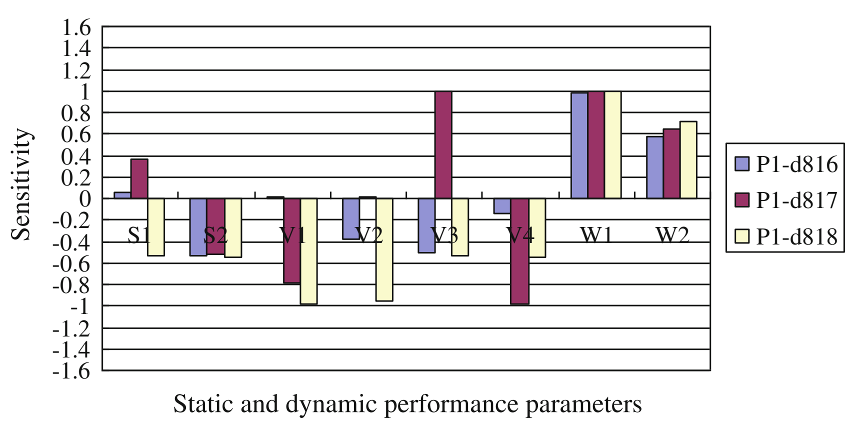


Table 7 The range of design variables

Design variable	Initial value/mm	Minimum value/mm	Maximum value/mm
x_1(P1-d816)	20	16	24
x_2(P1-d817)	20	16	24
x_3(P1-d818)	20	18	30

Where f_1, f_2, f_3 and f_4 are the first four order natural frequencies of the crossbeam, whose corresponding indexes are v_1, v_2, v_3 and v_4 respectively. α_1, α_2, α_3 and α_4 are the corresponding weight coefficients of f_1, f_2, f_3 and f_4, therefore, $(a_1, a_2, a_3, a_4) = A_{21} = (0.381, 0.298, 0.211, 0.110)$.

5.3 Crossbeam's structural optimization results

The dynamic optimization model for the crossbeam structure was solved by the optimization design module of the ANSYS software. After iterating many times, a total of 6 kinds of optimization design schemes are obtained, which are shown in Table 8. This article will use the proposed structural optimization method based on grey relational analysis to select the optimal solution from the six kinds of crossbeam's optimization design results again.

Because the purpose of size optimization design is to improve the dynamic performance of the crossbeam, the optimal solution of optimization design will be selected according to the vibration analysis results of crossbeams. The anti-vibration performance's grey relational degree of the six kinds of crossbeams is $R'' = (0.4979, 0.6566, 0.6731, 0.4340, 0.3841, 0.5836)$, which is calculated from the data in Table 8 by using the optimum seeking model. So the optimization scheme III is the best one. The comparative results of crossbeam's static and dynamic parameters before and after optimization design show that the crossbeam's mass is controlled, the maximum stress and deformation are reduced and the anti-vibration performance is improved further, which means that dynamic multi-objective optimization design purpose is achieved.

The overall structure and rib plate arrangement of the crossbeams are alike before and after optimization design, as is shown in Fig. 5. However, the crossbeams' inner rib plates are different in thickness x_1, x_2, x_3 before and after optimization design, and their contrast is shown in Table 9. Figure 6 shows the crossbeam's physical model that was manufactured according to the optimization design scheme III. Figure 7 shows the gantry machine tool's physical model after the crossbeam's optimization design. The crossbeam after optimization design assembles so well on whole machine that ensures the machining precision of the gantry machine tool, which proves that the crossbeam's optimization design method based on grey relational analysis proposed in this paper has high engineering practicability.

6 Experimental verifying of the crossbeam's optimization design

6.1 Experimental purpose

In order to verify the correctness of the crossbeam's optimization design method based on grey relational analysis proposed in this paper, the dynamic experiments on the crossbeams before and after the optimization design had been carried out in a closed constant temperature laboratory. The purpose of the dynamic experiment is to obtain the first four order natural frequencies of crossbeam.

6.2 Experimental principle

The principle of crossbeam's dynamic experiment is shown in Fig. 8. After selecting proper test points on crossbeam,

Table 8 The non-inferior solution of crossbeam's optimization design

Optimization schemes		I	II	III	IV	V	VI
Design variables/mm	x_1	16.75	23.25	16.75	23.25	16.74	23.25
	x_2	23.25	23.25	16.75	16.745	23.25	23.25
	x_3	19.12	19.12	28.88	28.88	28.88	28.87
Optimization objectives	f_1	63.46	63.47	64.37	63.44	63.28	63.29
(natural frequency /Hz)	f_2	87.68	87.97	87.73	87.82	87.69	87.98
	f_3	144.34	144.31	143.81	143.86	144.11	144.13
	f_4	170.82	170.77	171.87	171.53	171.00	170.90
Constraint functions	M/Kg	32377	32425	32347	32395	32464	32511
	σ_{max}/Mpa	3.13	3.08	3.09	3.11	3.12	3.13
	δ_{max}/mm	0.0411	0.0409	0.0411	0.0410	0.0408	0.0409

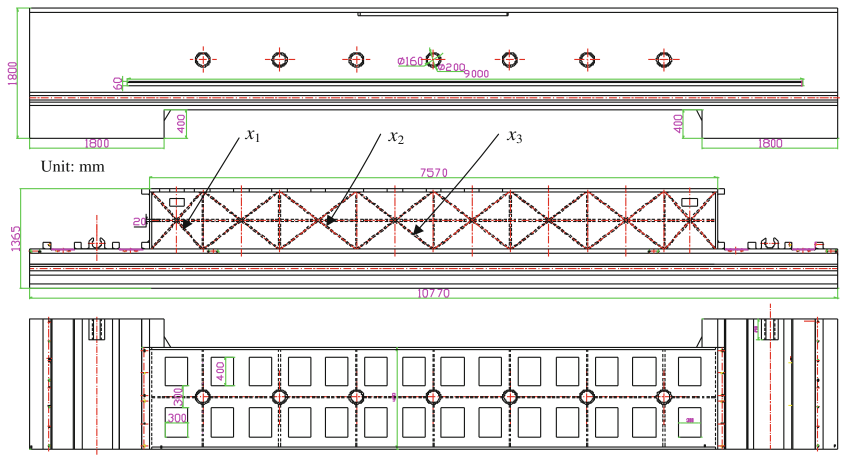


Fig. 5 Crossbeam's overall structure and rib plate arrangement

the dynamic test and analysis system was installed properly according to the principle model shown in Fig. 8, then the excitation force signal and corresponding response signal of each point can be obtained. With the modal analysis software, the frequency response function of test points can be analyzed, and the crossbeam's natural frequencies can be indentified through analyzing the frequency response function curve (Lei et al. 2009).

Fig. 6 Physical model of crossbeam

6.3 Experimental set up and process

The crossbeam's dynamic experimental equipment is shown in Fig. 9. During the experiment process, the ambient temperature was 25 °C. The experimental apparatus are as follows: the hammer (The top of hammer is force sensor, sensitivity: 0.2216 mv/N), the acceleration sensors (Sensitivity: 99.1 mv/g in the X-axis direction, 99 mvg in Y-axis direction, and 106.1 mv/g in Z-axis direction), the dynamic test system, and computer. The hammer was used to excite the crossbeam. The force sensor was used to pick up the excitation signal and convert it into charge signal. The acceleration sensors were used to pick up the response signal and convert it into charge signal. The dynamic test system produced by the Belgium LMS company was used to acquire and process experimental data. The modal analysis tool is the LMS Test Lab analysis system that matches the LMS

Fig. 7 Physical model of gantry machine tool

Table 9 Rib plates' thickness contrast

Design variables	Initial design scheme	Optimization design scheme
x_1/mm	20	16.75
x_2/mm	20	16.75
x_3/mm	20	28.88

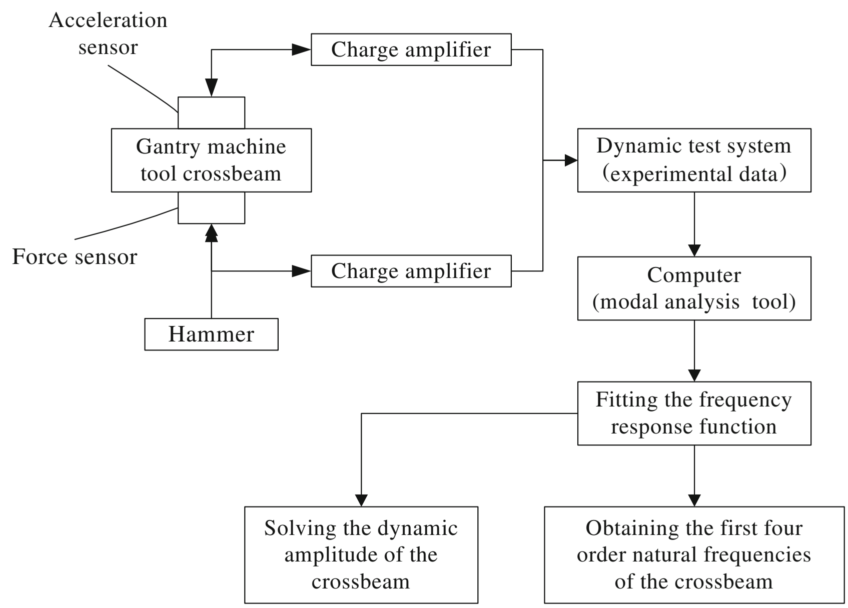


Fig. 8 Principle model of crossbeam's dynamic experiment

dynamic test system. The experimental typical process is as follows:

(1) The appropriate test points on the crossbeam were selected, which are shown in Fig. 10. The dynamic experiment system was installed in according to the dynamic experimental principle Fig. 8.
(2) The dynamic experiment on crossbeam was conducted by using a method of single-point excitation and multi-point response. The impact force was applied to the excitation point by hammer, and the acceleration sensors were fixed on test points to pick up the acceleration response signal.
(3) After setting the related parameters in modal analysis software, the excitation force signal and acceleration response signal of the test points on crossbeam were collected.
(4) The experimental data obtained was input to the computer by A/D conversion of LSM dynamic test system, then the frequency response function curve of crossbeam was fitted through modal analysis software.
(5) According to the crossbeam's frequency response function curve, the first four order natural frequencies of crossbeam were identified.

6.4 Experimental results analysis

After completing the dynamic experiments for the crossbeams according to scheme B before and after optimization design, we obtained their frequency response function curves that are shown in Figs. 11 and 12 respectively. The comparative results of crossbeam's dynamic parameters before and after optimization design are shown in Table 10, which are obtained according to the dynamic experiments. If the natural frequencies' experimental values of the crossbeam increase after optimization design, it means that the optimization design method is feasible. From

Fig. 9 Crossbeam's dynamic experimental equipment

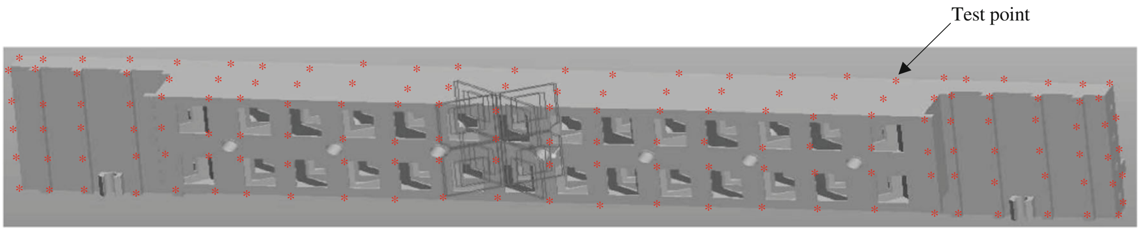


Fig. 10 Test points on the crossbeam

Fig. 11 Frequency response function curve of initial crossbeam (scheme B)

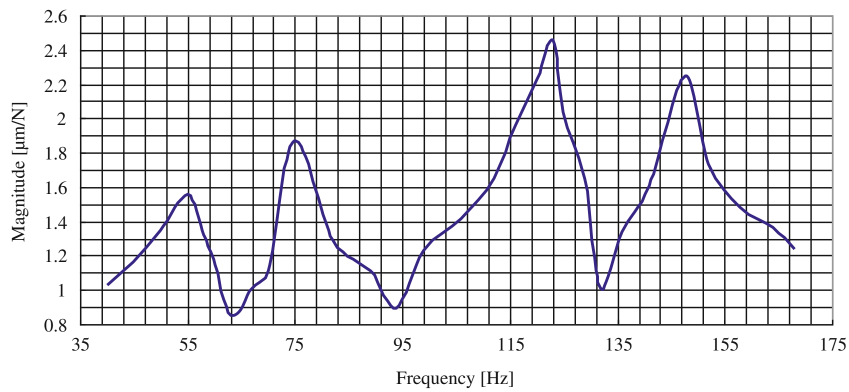


Fig. 12 Frequency response function curve of crossbeam after optimization

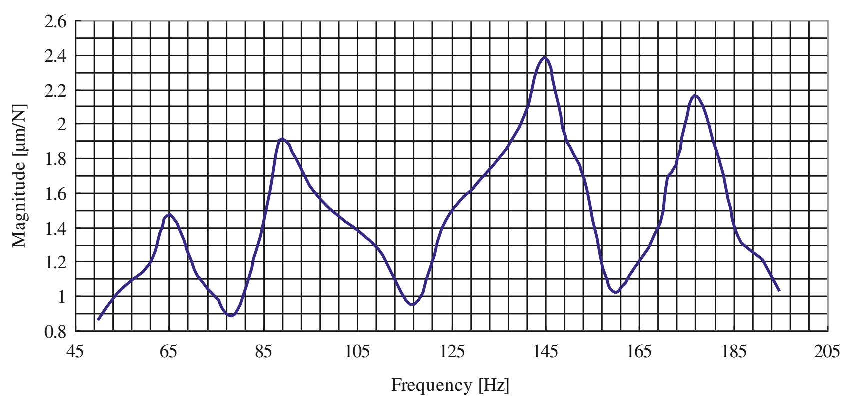


Table 10 Dynamic parameters contrast

Comparison of indexes	Natural frequency		
	Initial scheme	Optimization scheme	Rate of change
The first order	55.43 Hz	65.16 Hz	17.56 %
The second order	74.56 Hz	88.92 Hz	19.36 %
The third order	123.78 Hz	144.87 Hz	17.04 %
The fourth order	147.65 Hz	176.56 Hz	19.58 %

Table 10 we can know that the first four order natural frequencies of the crossbeam increase 17.56 %, 19.36 %, 17.04 % and 19.58 % respectively, which further proves that the optimization design method for crossbeam based on grey relational analysis proposed in this paper is reasonable and feasible.

7 Conclusions and future work

(1) Based on the multi-index and multi-scheme features of crossbeam structural design, a target system for crossbeam structure design schemes optimum seeking was established, which includes three aspects of static properties, anti-vibration performance and lightweight performance and the decision-making target of target system was decomposed. On this basis, a mathematical model for crossbeam structure design schemes optimum seeking was established and an algorithm of combining grey relational analysis and Analytic Hierarchy Process to solve the model was proposed as well. By applying this optimum seeking model on gantry machine tool crossbeam selection to choose an optimal comprehensive performance for overall structure of crossbeam, subjectivity and randomness of traditional designers' empirical method were avoided.

(2) In order to improve the crossbeam's anti-vibration performance, the size sensitivity analysis and dynamic optimization design of selected crossbeam were conducted. A crossbeam structure with optimal rib plate size was obtained by solving the dynamic optimization model, which makes the static properties and mass of the crossbeam meeting the design requirements and the anti-vibration performance improving markedly. Through dynamic experiments, the effectiveness of the optimization results was verified, which indicates that structural optimization design method based on grey relational analysis has strong engineering practicality and certain reference value for structural optimization design of other machine tool.

(3) The novelty of this paper is that the gantry machine tool crossbeam's static and dynamic performance optimum seeking model is established through using Analytic Hierarchy Process and grey relational analysis. The optimum seeking model not only can pick out the design scheme with optimal comprehensive performance from a lot of crossbeams that have different overall structures, but also can be used to select the optimal solution from the non-inferior solution set of a specific structural crossbeam's multi-objective optimization design. In this paper, the sensitivity analysis was used to select the design variables for the crossbeam's dynamic multi-objective optimization, so the efficiency and effectiveness of the optimization design are improved. The above optimization strategy not only expands the engineering applications field for Analytic Hierarchy Process, grey relational analysis and sensitivity analysis, but also provides a new idea for the multi-objective optimization design of the machine tool's structural parts.

(4) The structure selection and multi-objective optimization design for gantry machine tool crossbeam is a method of obtaining optimal solution based on finite element analysis results, thus, the optimization design effect is highly depended on the accuracy of finite element model. Therefore, how to increase the accuracy of finite element model will be a research focus in next step work. In addition, the structural optimization design method based on grey relational analysis proposed in this paper is very theoretical, in order to put this method into a more widely use in machine tool design engineering, the technology how to program this method also is a in-depth needed study issue.

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