RESEARCH PAPER

A hyper-heuristic approach to aircraft structural design optimization

Jonathan G. Allen · Graham Coates · Jon Trevelyan

Abstract The conceptual design of an aircraft is a challenging problem in which optimization can be of great importance to the quality of design generated. Mass optimization of the structural design of an aircraft aims to produce an airframe of minimal mass whilst maintaining satisfactory strength under various loading conditions due to flight and ground manoeuvres. Hyper-heuristic optimization is an evolving field of research wherein the optimization process is continuously adapted in order to provide greater improvements in the quality of the solution generated. The relative infancy of hyper-heuristic optimization has resulted in limited application within the field of aerospace design. This paper describes a framework for the mass optimization of the structural layout of an aircraft at the conceptual level of design employing a novel hyper-heuristic approach. This hyper-heuristic approach encourages solution space exploration, thus reducing the likelihood of premature convergence, and improves the feasibility of and convergence upon the best solution found. A case study is presented to illustrate the effects of hyper-heuristics on the problem for a large commercial aircraft. Resulting solutions were generated of considerably lighter mass than the baseline aircraft. A further improvement in solution quality was found with the use of the hyper-heuristics compared to that obtained without, albeit with a penalty on computation time.

Keywords Aircraft conceptual design · Structural optimization · Hyper-heuristic optimization

J. G. Allen (✉) · G. Coates · J. Trevelyan
School of Engineering and Computing Sciences,
Durham University,
Durham DH1 3LE, UK
e-mail: j.g.allen@durham.ac.uk

1 Introduction

At the conceptual stage of aircraft design, structural optimization can be employed with great effect to establish good quality solutions. Structural optimization aims to minimize airframe mass whilst maintaining structural integrity under loading conditions laid out in airworthiness requirements, e.g. European Aviation Safety Agency (2012), which is important in reducing manufacturing and operation costs. This requires an efficient process to find a good quality solution from a typically large number of possible variants in a reasonable amount of time (Raymer 2006).

Multidisciplinary optimization (MDO) within the field of aerospace design was reviewed by Sobieszczanski-Sobieski and Haftka (1997), and Allen et al. (2010); describing a common interest on the aerodynamic and structural optimization of the aircraft for minimum drag and mass respectively. Increased computational demand is a typical challenge of MDO, commonly resulting in the decomposition or approximation of the problem, or the consideration of only a single discipline of optimization (Sobieszczanski-Sobieski and Haftka 1997). Decomposition has often led to the sole consideration of single aircraft section, e.g. wing, resulting in a failure to obtain a complete aircraft configuration (Allen et al. 2010). An alternative approach for the reduction of computational demand uses a multi-tier optimization framework. Such an approach typically employs a population-based optimization technique initially to obtain an approximation to a solution prior to the application of a gradient-based technique for local improvement of the solution (Raymer 2002; Hansen and Horst 2008). Surrogate modelling, commonly using a Latin hypercube, provides another method of reducing computational demand, leading to an approximate solution to the problem (Hu and Yu 2009; Neufeld et al. 2010).

A common optimization process is followed for most studies within the field of aircraft design. A period of initialization is followed by mission definition to determine requirements of the aircraft. Mass estimation obtains an initial estimate of aircraft mass for use during the design process through empirical methods. Optimization of the aircraft design is then conducted within the selected engineering disciplines through the generation and subsequent analysis of many variations of aircraft designs (Kesseler and Vankan 2006; Raymer 2006). Optimization is often performed over a single discipline, such as for the aerodynamic or structural design of the aircraft. When MDO is employed, optimization is performed within the disciplines either in series, such as through initial optimization of the aerodynamic profile prior to structural optimization, or simultaneously. A pareto frontier is typically used for evaluation of a multi-objective function (Bartholomew 1998; Amadori et al. 2007). An additional design objective of minimum cost is considered in some cases (Gantois and Morris 2004; Kaufmann et al. 2010). The often unpredictable, multi-modal solution space leads to common use of genetic algorithms (GAs) to solve the problem (Raymer 2002).

The optimization of a solution to a problem is highly dependent on the process followed. In unpredictable domains with no known solution, the development and tuning of high quality, problem-specific optimization techniques can be difficult, requiring extensive investigation and validation. Hyper-heuristic optimization evaluates the performances of such techniques such that they may be applied more effectively to the problem (Burke et al. 2010). Hyper-heuristic optimization is performed across two independent domains: the *problem* and *hyper-heuristic domains*, as shown in Fig. 1. Within the problem domain, heuristics (wherein the term also considers meta-heuristics) search for a near-optimal solution to a given problem, and are labelled *low-level heuristics*. Conversely, *hyper-heuristics* are applied in the higher-level domain to improve the performance of the optimization process within the problem domain and promote further solution improvement. As such, a hyper-heuristic was introduced as *"an approach that operates at a higher level of abstraction than current meta-heuristic approaches"* (Cowling et al. 2000). A barrier restricts data flow between domains to allow the passage of solely problem-independent information to inform the actions of hyper-heuristic optimization (Chakhlevitch and Cowling 2008). Such hyper-heuristic actions are dependent on the hyper-heuristic approach employed, itself defined by specific aspects in which the process within the problem domain is controlled. Four common aspects are: *heuristic selection*, *population distribution*, *parameter control*, and *perturbation analysis*.

Heuristic selection chooses the most appropriate low-level heuristic for application in the problem domain from a set of candidate heuristics, leading to an alternative description of hyper-heuristics as *"heuristics to choose heuristics"* (Burke et al. 2010). Such hyper-heuristics may be *constructive* or *perturbative*, where the former creates a new solution incrementally whilst the latter evolves an existing solution over a period of generations (Burke et al. 2010). As the optimization of an aircraft design is typically performed through the evolution of a baseline design (Raymer 2002; Maute and Allen 2004), constructive heuristics are not described herein. Perturbative heuristics employ *move acceptance* to define rules for the approval of selection, where common methods include all moves (AM), improving or equal (IE), only improving (OI), and Monte Carlo (MC) methods. AM permits selection regardless of performance, OI only permits selection with an improvement in solution quality, whilst IE permits selection with solutions of better or equal quality. MC methods allow beneficial moves and randomly permit negative moves with linearly (LMC) or exponentially (EMC) decreasing probability, providing promising results when combined with a counter of iterations since improvement (EMCQ) (Ayob and Kendall 2003; Özcan et al. 2008).

Population distribution divides a set of solutions between multiple low-level heuristics for a generation. Distribution is performance-based, random, or equal, such that each heuristic optimizes solely individuals within its *sub-population*. When a single-solution heuristic is selected, each individual within the sub-population is optimized independently. Such an approach aims to overcome limitations of individual heuristics through the availability of alternatives (Rafique et al. 2011). Sub-populations must be adequately sized to allow sufficient opportunity for improvement. This can be addressed by a dynamic population size, such as in Arabas et al. (1994) where the fitness-driven lifetime of individuals enabled variation in population size.

Parameter control adapts low-level heuristics during application based on the history of the problem (Eiben et al.

Fig. 1 Domains of hyper-heuristic optimization
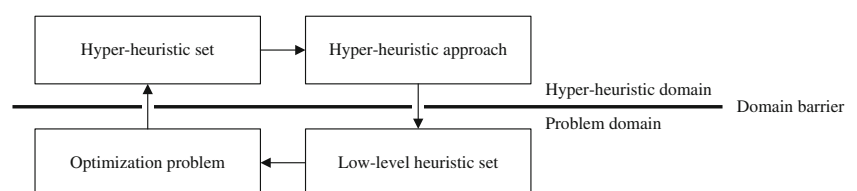
2007). Such changes may be made either through perturbation of existing values or selection of the better performing settings; the latter is referred to as *operator selection* (Burke et al. 2010; Maturana et al. 2010).

Local solution space learning is encouraged through perturbation analysis of individuals using a memetic algorithm (Özcan et al. 2008). Analysis frequency and duration, and the choice of solutions to perturb are key to the success of this aspect. A set proportion of the population or only improved solutions are typically perturbed, continuing until no further improvement is made or for a specified duration (Ong et al. 2006).

A learning mechanism drives a hyper-heuristic approach, commonly using reinforcement learning to reward improvements in a hyper-heuristic objective function formed by measures of process performance within the problem domain (Burke et al. 2010). One such function, a *choice function*, measures performance against a set of criteria, including improvements in solution quality and computational time taken (Cowling et al. 2000).

The focus of this paper is the design of a hyper-heuristic approach including the above aspects for the hyper-heuristic structural optimization of an aircraft conceptual design. The remainder of the paper is organized as follows. Section 2 describes the hyper-heuristic approach developed to assist aircraft structural optimization, leading to the presentation of an optimization framework to address the problem in Section 3. The framework is demonstrated through a case study in Section 4 prior to concluding remarks in Section 5.

2 Hyper-heuristic approach

A hyper-heuristic approach for aircraft design optimization has been developed to improve solution quality and feasibility compared to those achievable through traditional methods. Whilst most hyper-heuristic approaches only incorporate a single aspect of process control, this approach includes the integration of the following four aspects of hyper-heuristics:

1. Selection of appropriate low-level heuristics for use in the problem domain based on earlier performance;
2. Biased distribution of the population towards the better performing low-level heuristics selected;
3. Control of process parameters for promotion of more efficient optimization and increased solution quality;
4. Perturbation analysis of the running best solution upon discovery for local solution space learning.

Reinforcement learning promotes beneficial application of the hyper-heuristic approach. Thus, exploration of the solution space is encouraged during early generations prior to later promotion of convergence upon the best solution found. This reduces the likelihood of premature convergence on local optima and permits analysis of the solution space neighbouring good solutions.

2.1 Heuristic selection

Heuristic selection ensures the application of appropriate low-level heuristics to the problem at a given process generation. This enables the encouragement of diversity or convergence, achieved through the ranking of heuristics by performance against the hyper-heuristic objective function (to be described in Section 2.5). The heuristic set is presented in Table 1, comprised of those commonly applied within the domain of aircraft design.

Single-solution and population-based heuristics may be employed as low-level heuristics, the latter including random, evolutionary algorithm (EA), swarm intelligence (SI), and GA heuristics. Individuals assigned to single-solution heuristics are optimized independently. A similar list of heuristics applied within the hyper-heuristic domain is presented in Table 2, with ticks indicating which heuristics may be applied for each aspect of the hyper-heuristic approach. Note that some heuristics are available as both low-level and hyper-heuristics.

Hyper-heuristics perform heuristic selection as follows: (i) SR randomly selects a low-level heuristic from the heuristic set; (ii) PE performs random selection from a predefined number of well-performing heuristics; (iii) GR selects the low-level heuristic that generated the best solution or a random heuristic if the current heuristic is the best; (iv) HC, SA, and TS select heuristics by ranking the heuristic set by solution quality and randomly selecting the closest best or worst candidate; (v) RW and TS similarly rank the heuristic set and probabilistically select and mutate a heuristic by rank.

Table 1 Candidates within low-level heuristic set

Category		Low-level heuristic
Single-solution		Hill climbing (HC)
		Simulated annealing (SA)
		Tabu search (TS)
Population-based	Random	Monte Carlo (MC)
		Random immigration (RI)
	GA	Roulette wheel (RW)
		Tournament selection (TO)
		Breeder pool (BP)
	EA	Killer queen (KQ)
		Differential evolution (DE)
	SI	Particle swarm (PSO)

Table 2 Candidates within hyper-heuristic set

Hyper-heuristic	Heuristic selection	Parameter control	Perturbation analysis
Simple random (SR)	✓	✓	
Peckish (PE)	✓	✓	
Greedy (GR)	✓	✓	
Hill climbing	✓	✓	✓
Simulated annealing	✓	✓	✓
Tabu search	✓	✓	✓
Roulette wheel	✓		
Tournament selection	✓		

Move acceptance controls heuristic selection, with the AM, IE, and EMCQ methods available, as well as a SA approach. The latter two are preferred as they probabilistically permit negative moves, hence limiting the likelihood of dominance by well-performing low-level heuristics. This is necessary as convergence-encouraging heuristics may be expected to converge prematurely within a multi-modal solution space, but are desired towards the end of the process for efficient convergence.

2.2 Population distribution

For generations with multiple low-level heuristics selected, the population is distributed between the low-level heuristics with greater probability of being assigned to those with a better performance history. The initial population is either randomly or equally distributed. The total population size is increased by a factor of the number of selected low-level heuristics to ensure an adequate sub-population size per low-level heuristic to allow suitable opportunity for solution improvement. The maximum number of heuristics per generation is therefore limited to reduce the required increase in computation time for population analysis.

A dynamic population size limits sub-population size to prevent excessively large sub-populations. Sub-populations exceeding the imposed limit are reduced through the random rejection of individuals, whilst additional solutions are generated randomly within a sub-population to increase the population in generations following such reductions, thus preventing elitism.

Population-based low-level heuristics are applied to a sub-population in the same manner as when traditionally applied to a full population of individuals, albeit with no knowledge of individuals outside the sub-population. When a single-solution low-level heuristic is employed, each individual within the sub-population is optimized independently for one iteration per process generation. Thus the sub-population effectively contains a set of independent solutions optimized in parallel.

2.3 Parameter control

A set of control parameters listed in Table 3 drives the operation of the optimization process. These parameters are used by low-level heuristics when chosen during heuristic selection. Parameter control is performed to assist the optimization process in (i) preventing premature convergence on local optima through encouraged solution space exploration; (ii) improving convergence on the best solution obtained; (iii) focusing on critical variables without requiring excessive computation; (iv) avoiding convergence on an infeasible solution.

The ranges given in Table 3 have been determined through an earlier study of typical values found in the literature (Grefenstette 1986; Bean and Hadj-Alouane 1992; Clerc and Kennedy 2002; Pedersen 2010) to ensure appropriate limits for the variation of parameters. The penalty coefficient controls the severity of penalty applied to infeasible solutions to promote convergence on a feasible solution. The lengths of binary chromosomes of converging design variables are extended to allow optimization at increased resolution. The remaining parameters promote exploration, diversity, or negative moves during early generations before discouraging such actions in later generations.

2.4 Perturbation analysis

Perturbation analysis is performed by a single-solution heuristic chosen by heuristic selection and Lamarckian evolution to further improve solution quality. This is performed solely for newly-discovered optima to limit the required computational effort through repeated perturba-

Table 3 Hyper-heuristically controlled parameters

Parameter	Affected low-level heuristic	Range	
		Min.	Max.
Penalty coefficient	–	0.25	2.00
Strand length	–	4-bits	16-bits
Crossover probability	RW, TO, BP	0.50	1.00
Crossover points	RW, TO, BP	1	Random
Mutation probability	RW, TO, BP	0.00	0.01
Breeder pool intake	BP	0.10	0.30
Indigenous population	RI	0.10	0.40
Differential weight	DE	0.00	2.00
Crossover probability	DE	0.00	1.00
Cognitive parameter	PSO	1.40	2.10
Social parameter	PSO	0.90	1.80
Inertia weight	PSO	0.55	0.75
Cooling rate	SA	0.00	0.95
Length of tabu list	TS	0	100

tion of randomly-selected variables and re-analysis of the solution until no further improvement is made.

2.5 Learning mechanism

Reinforcement learning drives the hyper-heuristic approach through continuous evaluation of process performance measured using the following criteria:

1. Objective value of best solution;
2. Mean objective value;
3. Population variance;
4. Convergence rate.

Population variance is measured as the mean variance of design variables (Morrison and De Jong 2002) and convergence as the magnitude of change in objective value. These criteria form a choice function based on that of Cowling et al. (2000), differing in its design for the encouragement of solution quality and variance or convergence rather than low-level heuristic effectiveness and computation speed whilst also being used for multiple aspects of the hyper-heuristic approach rather than solely heuristic selection. As such, this function defines a maximization hyper-heuristic objective function, ϕ^n

$$\phi^n = a_1^n \phi_1^n + a_2^n \phi_2^n + a_3^n \phi_3^n + a_4^n \phi_4^n \tag{1}$$

where $\phi_1^n = \dfrac{1}{\min \Phi(X^{\Delta n})}$

$$\phi_2^n = \dfrac{1}{\overline{\Phi(X^{\Delta n})}}$$

$$\phi_3^n = \left(1 - \dfrac{n}{N}\right) \overline{\sigma^2(X^{\Delta n})}$$

$$\phi_4^n = n \overline{\delta(X^{\Delta n})}$$

$$a_k^n = \dfrac{0.25}{\max\left(\phi_k^n, \phi_k^{n-\Delta n}\right)} \text{ for } k = 1, 2, 3, 4$$

at generation n of N. Herein the superscripts n and N indicate the generation and Δn a period of generations over which a value is recorded. Therefore $\Phi(X^n)$, $\sigma^2(X^n)$, and $\delta(X^n)$ respectively denote the problem objective value, population variance, and magnitude of convergence rate during the period for a population set X of μ solutions at generation n. The function consists of four components, ϕ_1^n, ϕ_2^n, ϕ_3^n, and ϕ_4^n, to measure the aforementioned criteria over a period of Δn generations immediately preceding generation n. The

component values are compared against those for the previous period of Δn generations in order to establish whether an improvement has been made in these aspects of process performance, i.e. if $\phi^n > \phi^{n-\Delta n}$. Coefficient a_k^n normalizes the kth component of (1) to restrict its value to the range $0 \le a_k^n \phi_k^n \le 0.25$ for both periods, thus bounding the function such that $0 \le \phi^n, \phi^{n-\Delta n} \le 1$ and ensuring homogeneity. Components ϕ_1^n and ϕ_2^n promote improvement in the quality of the best solution and population set throughout the process. Conversely, the n-based scaling factors in the expressions for ϕ_3^n and ϕ_4^n weight these components for encouraged variance, and thus solution space exploration, during early generations before promoting a high convergence rate for later generations.

3 Framework for aircraft structural design optimization with a hyper-heuristic approach

The hyper-heuristic approach described in Section 2 has been embedded within a framework for aircraft conceptual design optimization (Allen et al. 2010). This framework provides the ability for adaption of the optimization process based on its performance such that solutions of higher quality may be obtained than through traditional methods. The key stages of the framework are presented in Fig. 2. The framework initializes the process to define the requirements of the aircraft (stages 0.1 and 0.2 in Fig. 2), optimization process (0.3), and finite element analysis (FEA) (0.4). A design is subsequently obtained through the following modules.

3.1 Mission definition

Given the requirements input during initialization, a mission profile is generated (1.1) to permit definition of the selected loading conditions using the airworthiness requirements (1.2). The aircraft payload is also defined based on the requirements of the design (1.3).

3.2 Mass estimation

Empirical methods (Roskam 1986; Raymer 2006) are employed (2.1) to determine payload mass (2.2), and an estimate of aircraft mass at stages during the mission (2.3). The necessary mission fuel is also estimated (2.4).

3.3 Design optimization

Empirical formulae (Torenbeek 1982; Raymer 2006) are used to generate the external profile of the aircraft to meet the requirements for flight (3.1). Structural optimization is then performed within the profile for an objective
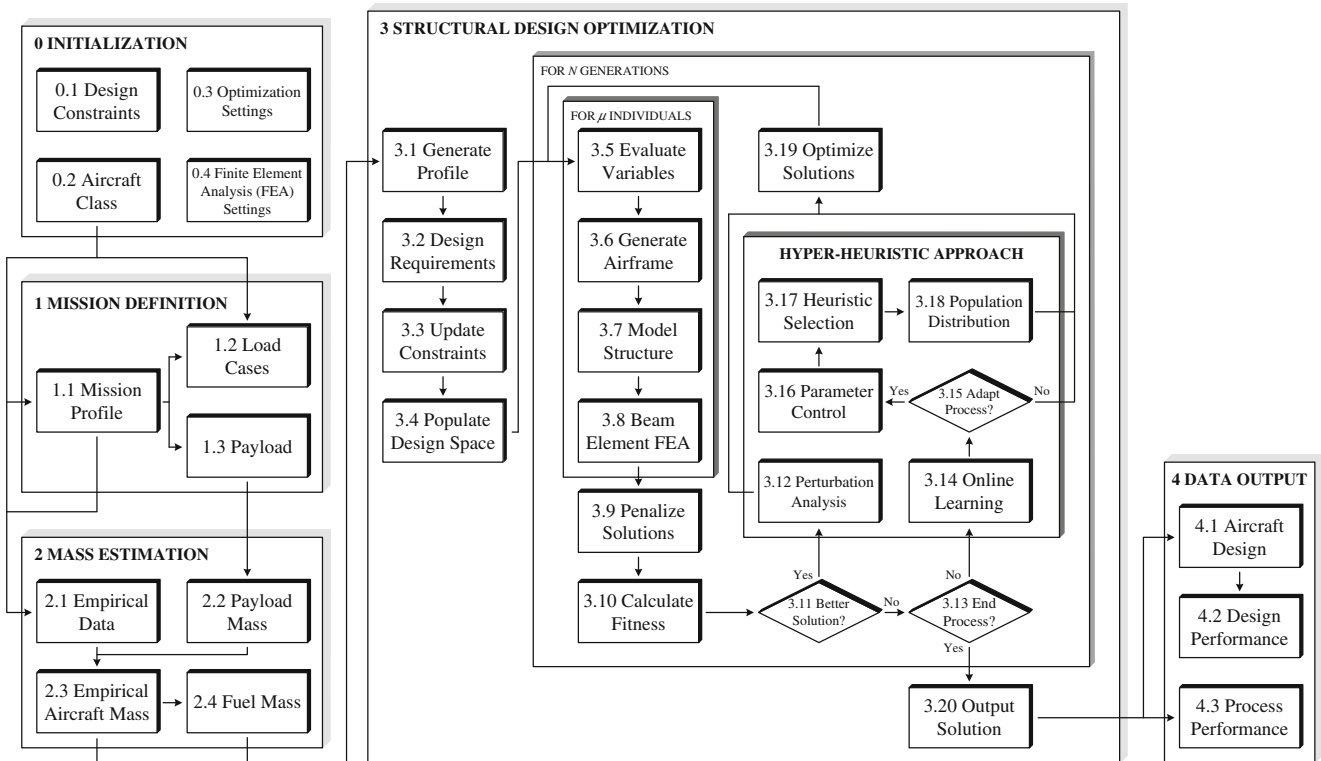
Fig. 2 Framework for optimization of aircraft structural design with embedded hyper-heuristic approach (HHA)

function of minimum structural mass of solutions within the population set

$$\min \Phi(X^n) \text{ for } n = 1, 2, \ldots, N \qquad (2)$$

Structural design requirements to satisfy geometric constraints imposed by the external profile, such as limits on member positions, are calculated (3.2). The ranges of design variables are then evaluated to ensure they comply with these constraints (3.3). An initial population of designs is then either seeded or randomly generated (3.4). Optimization is performed over a series of generations, where for each generation the population is firstly analyzed to determine structural performance. For each individual the aircraft design represented by the design variables is determined (3.5), from which an airframe is generated (3.6), and finite element (FE) model constructed (3.7). In order to reduce the computational effort required to solve the FEA problem for each individual, the FE model is constructed of one-dimensional beam elements, with multiple similar structural members grouped within elements to reduce the sizes of the FEA matrices. Such member combinations include multiple ribs, frames, floor beams, and fuselage stringers grouped into a lower number of respective beam elements. Similarly, lifting surface stringers are merged within spar elements, with skin lumped within stringers. The number of members grouped within elements is determined by the fidelity of the model as defined during initialization, with element

properties subsequently determined through the smearing of the properties of the members within. The feasibility of the individual against design constraints for minimum factor of safety against yield using the von Mises criterion, $c_{i,1}$, and maximum wingtip deflection, $c_{i,2}$, is then established by solving the FEA problem (3.8). These constraints are determined by the airworthiness requirements (European Aviation Safety Agency 2012) and the maximum allowable deflection before ground strike (Grasmeyer et al. 1998) respectively for individual $i \in X$ at generation n

$$\begin{aligned} c_{i,1}(X^n) &\geq 1.5 \\ |c_{i,2}(X^n)| &\leq 7.5 \text{ m} \end{aligned} \qquad (3)$$

The objective value of each solution is initially calculated as the structural mass of the design, where ρ, A, and l denote the density, area, and length of member k of K_i^n for the ith individual of the population

$$f_i(X^n) = \sum_{k=1}^{K_i^n} (\rho A l)_{i,k}^n \qquad (4)$$

for the unpenalized objective value, $f_i(X^n)$. An exterior penalty function penalizes infeasible solutions (3.9)

$$\Phi_i(X^n) = f_i(X^n) \left\{ 1 + \lambda^n \sum_{j=1}^{m} g_{i,j}^2(X^n) \right\} \qquad (5)$$

where λ^n is the adaptive penalty coefficient, and $g_{i,j}(X^n)$ the measure of violation of constraint j of m defined as

$$g_{i,j}(X^n) = \max(0, c_{i,j}(X^n)) \qquad (6)$$

The severity of penalty applied to infeasible solutions is determined by the penalty coefficient, defined through parameter control. For applications without the use of parameter control, the coefficient is adapted through static rules to increase or decrease the coefficient depending on the feasible proportion of the population (Bean and Hadj-Alouane 1992) as

$$\lambda^{n+1} = \begin{cases} (1/\beta_1)\lambda^n & \text{if } \beta(X^n) > 80\,\% \\ \beta_2\lambda^n & \text{if } \beta(X^n) < 20\,\% \\ \lambda^n & \text{otherwise} \end{cases} \qquad (7)$$

where $\beta(X^n)$ represents the feasible proportion of the population during generation n, and $\beta_{1,2}$ are constants, where $\beta_{1,2} > 1$ and $\beta_1 \neq \beta_2$ to avoid cycling. Individual fitness, $F_i(X^n)$, is then calculated by ranking the population by objective value, $\tau(\Phi_i(X^n))$ (3.10)

$$F_i(X^n) = \frac{1 + \mu - \tau(\Phi_i(X^n))}{\displaystyle\sum_{j=1}^{\mu} \tau(\Phi_j(X^n))} \qquad (8)$$

Improved solutions within the population are identified through comparison of fitness (3.11). If a better solution is discovered, perturbation analysis is performed to the individual until no improvement is made (3.12). Termination criteria are then checked, including a generation limit, number of generations since improvement in objective function, and population variance (3.13). These criteria aim to end the optimization process if no improvement is being made to the current best solution or if the population has converged.

The learning mechanism evaluates the performance of the optimization process during the generation using the hyper-heuristic objective function of (1) (3.14). The measurement period over which the function is measured, Δn, is set during initialization and the value calculated by (1) compared against that of the previous period. If the value at the end of the current period is lower (3.15), hyper-heuristics are applied to modify the process in an attempt to improve its performance. In such cases modifications are applied through parameter control of the process settings (3.16), heuristic selection (3.17), and population distribution (3.18). Move acceptance is applied midway through this period to permit rejection of newly-selected heuristics and population distribution failing to satisfy the rule of move acceptance chosen during initialization. The population is then optimized using the selected low-level heuristics within the defined sub-populations (3.19). This optimization process is repeated until a termination criterion is satisfied, at which point the best solution obtained is output (3.20).

3.4 Data output

Upon the completion of the optimization process, a model of the best design solution is output (4.1) along with the performance of the aircraft during FEA (4.2). Finally, the performance of the process if output (4.3), detailing the selection of low-level heuristics, control of parameters, distribution of the population, population feasibility, and process convergence during execution.

4 Case study

The application of the framework is demonstrated using a computational implementation called AStrO (Aircraft Structural Optimizer). The baseline aircraft design for structural optimization is the Airbus A340-300, with Table 4 listing a selection of properties of the aircraft (Jackson 2009; Airbus SAS 2012). Table 4 also includes characteristics of the mission profile, a cruise between two aerodromes, and the loading conditions to be simulated. Cabin pressurization, engine thrust, and gravity are encompassed within these load cases.

Table 4 Selected properties of aircraft and mission

Property	Value
Wing span	60.30 m
Sweep	30.0°
Tail span	19.40 m
Height	16.99 m
Fuselage length	63.69 m
Width	5.64 m
Undercarriage track	10.69 m
Wheelbase	25.37 m
Powerplant	4× CFM International 56-5C4
Mass operating empty	130,200 kg
Maximum takeoff	276,500 kg
Cruise altitude	35,000.00 ft
Range	5,000.00 nm
Speed	0.82 M
Number of flight crew	2
Passengers	335
Aircraft class	Civil transport
Load case in flight	+2.5 g pull-up manoeuvre
On ground	2-point landing

The FE model is constructed at a level of fidelity of 10 %, i.e. 1 in 10 ribs, frames, floor beams, and fuselage stringers are modelled as elements with remaining members grouped within the closest respective element, leading to smeared properties within the element. Critical members, e.g. spars and members with attachments, are exceptions that are modelled in isolation, whilst floor beams are modelled at the same position as frames within the cabin. Further, each lifting surface is constrained to having two spars. This level of fidelity is chosen having established through previous experiments that a model of such fidelity provides substantial gains in computational speed without generating unacceptable errors in the results of the FEA compared to a model of 100 % fidelity, i.e. when all structural members are modelled individually. More specifically, during these experiments the low fidelity model required less than 2 % of the computation time to model and solve the FEA problem for one load case, whilst the difference in values of the design constraints were never greater than 4 % and on no occasion resulted in a different number of constraint violations within the aircraft design. Due to a hardware constraint imposed by the computational resources on the problem runtime, such increased computation speed with reasonable accuracy of results permits the consideration of a much greater number of design solutions. Table 5 lists additional constraints on the design, typical of those of a large commercial aircraft (Niu 1999).

The design variables of the study for the structural layout of the aircraft are listed in Table 6. Variables V1-8 define the number of structural members within the airframe, whilst V9-11 determine the proportion of fuselage frames within the nose, tail, and wingbox. The height and width of the spars at the root relative to the tip are defined by V12-13, with linear spanwise variation, and V14 sets the front wing spar position as a fraction of chord, c. The spanwise distributions of ribs within the lifting surfaces are defined by V15-17 for a greater number of ribs at the root to react expected stress concentrations under bending loads. Thus,

Table 5 Constraints on structural members

Structural member	Material	Profile	Thickness
Lifting surface rib	Al 7075-T6	I	10.0 mm
Spar	Al 7178-T6	I	4.0 mm
Stringer	Al 2014-T6	Z	2.0 mm
Fuselage frame	Al 7075-T6	T	10.0 mm
Stringer	Al 2014-T6	Z	5.0 mm
Floor beam	Al 7075-T6	I	20.0 mm
Skin	Al 2014-T6	–	3.0 mm
Floor	Al 7075-T6	–	20.0 mm

Table 6 Constrained ranges of design variables

ID	Design variable	Range	
		Min.	Max.
V1	Number of wing ribs	10	100
V2	Wing stringers	20	120
V3	Horizontal tail ribs	10	40
V4	Horizontal tail stringers	10	80
V5	Vertical tail ribs	10	40
V6	Vertical tail stringers	10	80
V7	Fuselage frames	20	160
V8	Fuselage stringers	30	180
V9	Frames in nose	5.0 %	15.0 %
V10	Frames in tail	5.0 %	15.0 %
V11	Frames in wingbox	5.0 %	20.0 %
V12	Height of wing spars at root	1.0	4.0
V13	Width of wing spars at root	1.0	4.0
V14	Position of wing front spar	0.2 c	0.35 c
V15	Wing ribs	1.0	3.0
V16	Horizontal tail ribs	1.0	3.0
V17	Vertical tail ribs	1.0	3.0

for a surface of span b with the root rib of R ribs positioned at y_0, the spanwise position of the rth rib is given by

$$y_r = \frac{r^{\alpha-1}(Cb - y_0)}{R^\alpha} + y_0$$

where $C = \begin{cases} 0.5 \text{ for wing, horizontal tail} \\ 1.0 \text{ for vertical tail} \end{cases}$

$\alpha = $ V15, V16, V17 as required   (9)

The case study is performed through a series of optimization runs, differing as described by Table 7 with ticks denoting active aspects of the hyper-heuristic approach. Parameter control is setup as in Table 3 with initial values generated using the SR hyper-heuristic. Hyper-heuristics for runs with multiple hyper-heuristic aspects are applied as: i) heuristic selection, ii) parameter control, and iii) perturbation analysis. No more than three low-level heuristics per generation are selected to prevent the need for an excessively large population to provide opportunities for improvement within sub-populations. Run 8 is an exception where a dynamic population size limits sub-populations to 100 individuals. All runs are seeded within an identical initial population, and uniform crossover and EMCQ move acceptance are used. Termination criteria include a limit of 1,000 process generations, minimum population variance of 2.0 %, and 250 successive generations without solution improvement. This final criterion is raised to 350 generations when using parameter control to permit opportunity for changes in

Table 7 Setup of hyper-heuristic approach for runs performed for case study and required population size

	Settings of variables for runs of case study							
	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8
Population size	100	100	100	100	100	300	300	Dynamic
Heuristic selection					✓	✓	✓	✓
Population distribution						✓	✓	✓
Parameter control				✓			✓	✓
Perturbation analysis			✓				✓	✓
Low-level heuristics	MC	RW	RW	RW	All	All	All	All
Hyper-heuristics			SA	SA	GR	RW	i) RW	i) RW
							ii) SA	ii) SA
							iii) TS	iii) TS

parameters to take effect. These values are selected having performed well during earlier preliminary investigations.

4.1 Results

Ten experiments were conducted for each run listed in Table 7 to account for the variability in the results due to the stochastic nature of the heuristics employed. Table 8 presents the results from the experiment generating the best solution for each run. For the best solution during each run, Table 8 includes the objective value, Φ_{min}, percentage difference from the best solution overall, $\Delta\Phi_{min}$, worst values for constraints, $c_{1,2}$, and feasible proportion of structural members, η. Final population feasibility, $\beta(X^N)$, and variance, $\sigma^2(X^N)$, are also given, as are the generation number at termination, N, and computation time taken as a proportion of that required for the run that generated the overall best solution, ΔT. Also included are the maximum, mean, and standard deviation of the best objective values across all experiments, Φ_{max}, $\overline{\Phi}$, and $s(\Phi)$ respectively. Key findings of the runs include:

Run 1 Random search, worst solution, poor convergence and population feasibility;

Run 2 Traditional application of GA, convergence on local optimum, largely feasible final population;

Run 3 Large improvements in solution quality, premature termination limited quality of final solution;

Run 4 Promoted beneficial actions to correct initial poor performance, promoted solution feasibility;

Run 5 Better solution than Runs 1–4, large variations in population feasibility and variance;

Run 6 Poorer final solution than Run 5, but with improved convergence and population feasibility;

Run 7 Solution quality further improved, general lack of convergence due to sub-population independence;

Run 8 Best solution, lack of convergence or population feasibility, large increase in computation time.

Table 8 Results obtained for experiments generating best design solutions and variability of solution quality

	Values for best solution obtained during run and variability across all experiments							
	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8
Φ_{min}, kg	57,922	50,234	50,609	50,465	47,069	47,659	46,228	45,323
$\Delta\Phi_{min}$, %	24.41	10.28	11.02	10.74	3.78	5.02	1.98	0.00
c_1	1.63	1.52	1.58	1.51	1.58	1.46	1.51	1.59
c_2, m	−0.46	−0.23	−0.54	−0.27	−0.28	−0.23	−0.23	−0.24
η, %	100.00	100.00	100.00	100.00	100.00	96.06	100.00	100.00
$\beta(X^N)$, %	9.00	85.00	72.00	97.00	11.00	98.78	29.27	12.50
$\sigma^2(X^N)$, %	43.73	3.44	9.84	8.90	42.97	3.99	26.27	38.08
N	489	691	386	1,000	994	938	1,000	1,000
ΔT	0.08	0.04	0.04	0.04	0.06	0.19	0.22	1.00
Φ_{max}, kg	63,021	64,150	60,418	60,344	50,940	50,829	48,266	46,644
$\overline{\Phi}$, kg	60,744	56,583	53,880	53,604	48,466	48,763	47,028	45,931
$s(\Phi)$, kg	1,661	5,673	3,053	4,068	1,151	1,042	669	480

Run 1 provided an indication of a random search for a solution using the MC low-level heuristic, whilst Run 2 exemplified a typical existing approach to the problem through the application of a GA using a RW. These results provided useful benchmarks for comparison against the runs involving the application of hyper-heuristics in Runs 3 to 8. A significant improvement in the quality of the best solution, measured by objective value, was obtained against these benchmarks with heuristic selection enabled. Parameter control in Run 4 led to an increase in final population feasibility, with a feasible solution obtained for all runs except Run 6. The ability to vary chromosome strand lengths for greater resolution of optimization permitted a solution closer to the constraint boundaries to be found for Runs 4 and 7, as did perturbation analysis during the latter.

Perturbation analysis in Run 3 provided instances of significant improvement in the solution. This is illustrated in Fig. 3a, which shows the objective value of the running best solution, alongside the variance and feasibility of the population for Runs 3 and 4. Improvements in the objective function of at least 10 % are labelled PA for perturbation analysis. A local minimum was discovered after 136 generations of Run 3 leading to termination due to an inability to further improve the solution. Thus perturbation analysis provided the opportunity for rapid evaluation of the local design space, although was susceptible to premature convergence.

The improvement in the objective value of the best solution during Run 4 initially followed a similar trend to that of Run 3. However, periods of deteriorating population feasibility led to the application of parameter control to promote feasibility by increasing the penalty coefficient. Four instances of such control are labeled PC in Fig. 3a, showing decreasing feasibility over generations preceding increases in the penalty coefficient and subsequently population feasibility. This adaption improved solution quality, population feasibility, and reduced population variance. Ultimately, the solution obtained during Run 4 was poorer than that for Run 2. However, final population feasibility was improved and a solution closer to the constraint boundaries found.

Heuristic selection during Runs 5 to 8 provided a noticeable improvement in the quality of the solution generated. The elitist nature of the GR hyper-heuristic during Run 5 produced a better solution than the RW hyper-heuristic in Run 6 due to the possibility of selecting poorer performing low-level heuristics using RW. However, this elitist behaviour led to the dominance of exploration-encouraging low-level heuristics, a factor that EMCQ move acceptance did not overcome. This resulted in poor convergence and population feasibility for Run 5 as indicated in Table 8 and Fig. 3b. Such dominance was further reduced during Run 6 by population distribution leading

to improved population feasibility and variance. Figure 3b is annotated to indicate the selection of low-level heuristic during Run 5. Influences of different heuristics on population diversity can be seen by the patterns in variance and feasibility. A selection of short periods indicate where move acceptance rejected heuristic selection, e.g. between generations 675 to 725 where DE was rejected in favour of PSO. Similar trends were seen during Run 6 with the corresponding objective value also plotted in Fig. 3b.

Perturbation analysis within Run 7 enabled improvement in the final objective value compared to Run 6. However, parameter control did not provide the expected benefits in population feasibility and convergence due to sub-populations converging independently. This poor convergence led to large variations in population feasibility, as shown in Fig. 3c alongside the objective values of Runs 7 and 8. Figure 3c is annotated by HS to indicate generations at which heuristic selection was accepted. Additional annotations indicate perturbation analysis, PA, as in Fig. 3a, and generations at which parameter control was applied with notable effects, PC. Poor convergence was also apparent during Run 8, with an increase in computation time due to a population size of up to 1,100 individuals. The dynamic population size reduced the population to 848 individuals, however this was still much larger during all other runs. As such, increased time for FEA was required and there existed a greater possibility of finding a good solution. Run 8 did, therefore, provide the best solution to the problem. Runs using parameter control terminated upon reaching the limiting number of optimization generations whilst all other runs terminated due to successive generations without improvement. Nevertheless, the number of solution evaluations, defined by the population size and number of generations, was comparable to those of similar studies (Raymer 2002; Özcan et al. 2008; Rafique et al. 2011), ranging from 38,600 evaluations for Run 3 to 857,272 evaluations for Run 8.

The behavior discussed above was consistent across all experiments; however variability in the quality of the solutions generated by each run was evident. The mean objective values of the best solutions of Runs 5 to 8 were lower than for Run 2, the best performing traditional method, with analysis indicating the differences in objective values were statistically significant under t-tests for means with assumed unequal variance ($P < 0.001$). However, no statistically significant difference existed between Run 2 and Runs 3 and 4 ($P \approx 0.1$) due to the prevailing influence of the same GA employed during these runs. Runs 7 and 8 generated the lowest means and standard deviations of solution quality, indicating the greatest repeatability of high quality solutions when using the complete hyper-heuristic approach.

Fig. 3 Running best solution objective value, population variance, and population feasibility during runs. **a** Comparison of effects of pertubation analysis and parameter control during Runs 3 and 4. **b** Effects of GR hyper-heuristic when employed for heuristic selection during Run 5 compared to RW for Run 6. **c** Effects of complete hyper-heuristic approach during Run 7 compared to with dynamic population size for Run 8
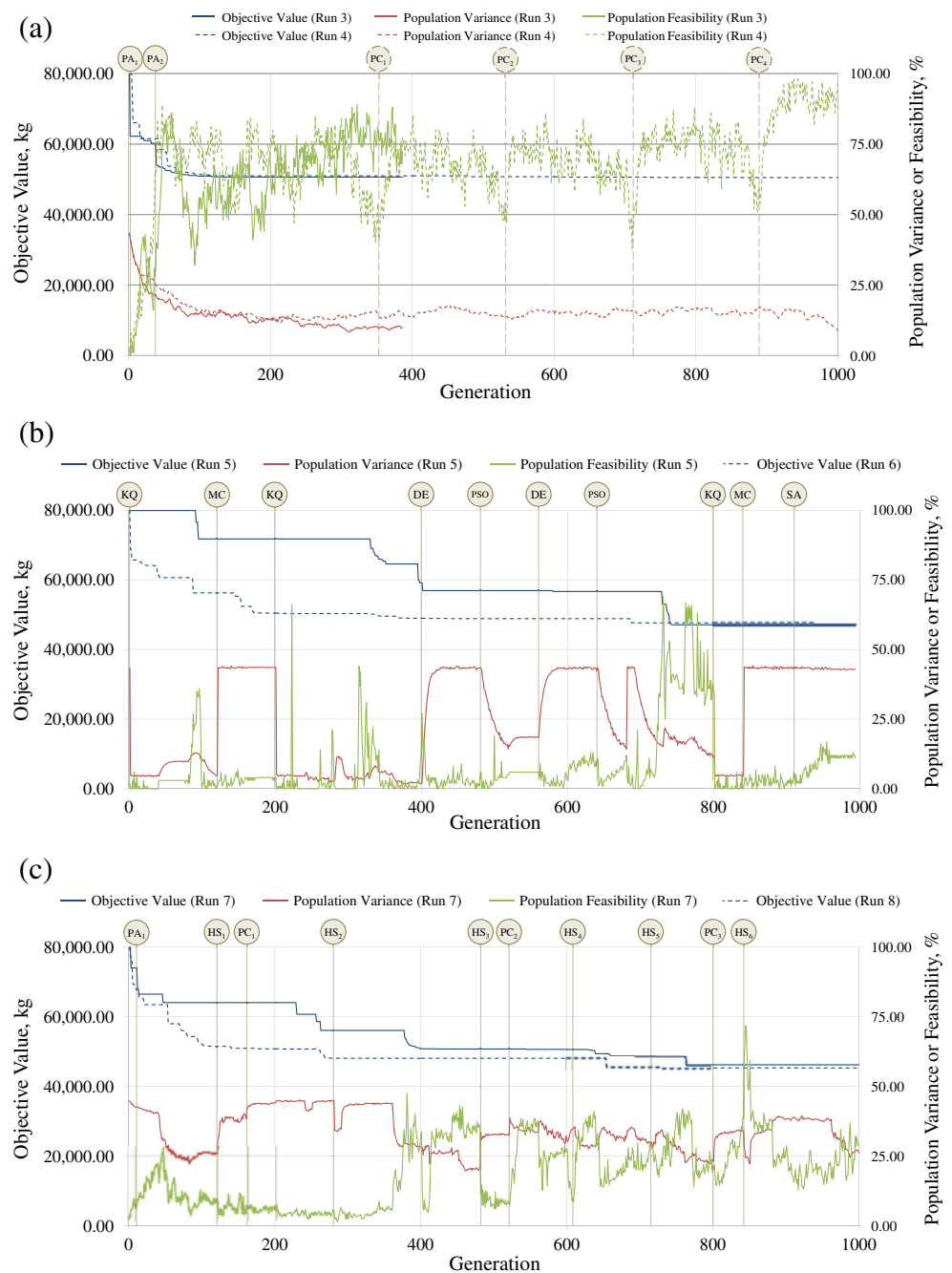
The evolution of the solution during Run 8 is shown in Fig. 4 for the (a) initial, (b) to (e) intermediate, and (f) final aircraft designs. Ribs and frames are shown in blue, spars and floor beams in red, and stringers in green. A decrease in the number of ribs and frames and variation in the distributions of both throughout the aircraft are visible. Although not shown, an increase in spar thickness at the root was constant throughout, leading to a strengthening of the wing at the root to react the bending loads imposed by the load cases. As skin properties were lumped within stringers, the number of fuselage stringers was driven by the pressurization of the cabin and bending due to the landing load.

There were noticeable differences in the design generated during Run 8 compared to the existing design of the Airbus A340-300. The front wing spar
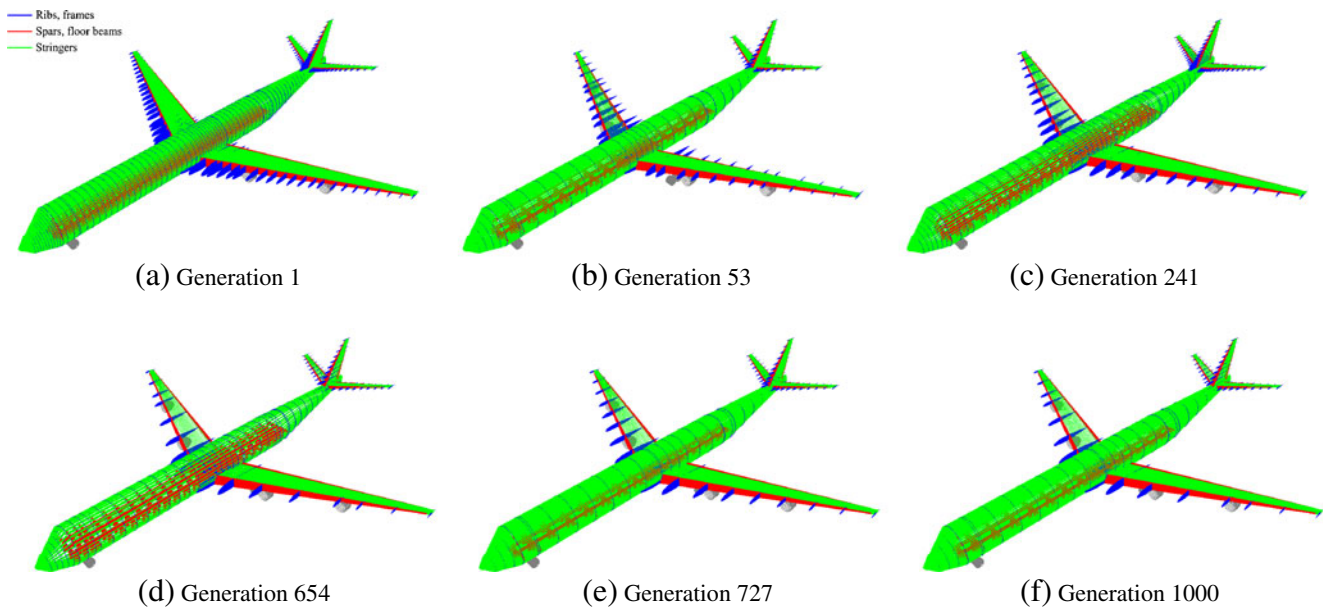
Fig. 4 Evolution of solution for Airbus A340-300 airframe design during Run 8

was positioned at $0.34c$, a value closer to that of the Airbus A380 rather than approximately $0.22\ c$ for the A340-300 (Sensmeier and Samareh 2004). This aftward position reduced the sweep of the spars, leading to lower shear due to the angle between the applied load and member. The wing root was strengthened through a greater concentration of ribs and increased spar thickness, leading to the aircraft design containing fewer than 30 % of the number of wing ribs within the existing aircraft design (Sensmeier and Samareh 2004). The operating empty mass (OEM) of the Airbus A340-300 is stated as 130,200 kg (Jackson 2009). However, this includes the masses of non-structural aspects of the design, e.g. systems and powerplants. As such, an estimate of the structural mass of the airframe, determined using empirical formulae (Torenbeek 1982), of 52,293 kg provides a better value for comparison with the results of this study. Thus, the resulting designs were between 86.67 % and 110.76 % of the estimated structural mass of the existing design. However, it should be noted that the data for the Airbus A340-300 is an estimate, based on the final aircraft designed to a greater level of detail than would be expected during conceptual design. Furthermore, this study only considered two loading conditions, whereas the application of a greater number of load cases may be expected to require a further increase in structural mass for satisfactory strength under load. Therefore, all runs using the hyper-heuristic approach, except the prematurely converged Run 3, indicated an improvement in the structural mass of the aircraft when engineered at a conceptual level of design.

5 Conclusions

A hyper-heuristic approach for use within an optimization framework has been presented to aid aircraft structural design at a conceptual level. The aspects of the approach have been described, including selection and control of heuristics, distribution of a population, and perturbation analysis. This hyper-heuristic approach encourages solution space exploration before focussing on convergence on the best solution obtained, leading to improved solution quality and process efficiency.

The results of a case study have shown that an improvement in solution quality can be obtained using the hyper-heuristic approach with a cost to computation time. Heuristic selection offered possible improvements in the solution generated, with parameter control providing additional gains in solution quality and feasibility. Perturbation analysis allowed exploration of the design space closer to the constraint boundaries, whilst population distribution reduced the limitations inherent to heuristics when applied in isolation.

Future research will improve the efficiency of the framework, most notably the hyper-heuristic approach to reduce the time penalty of applying hyper-heuristics to the problem. Further investigation is required into the choice function to better encourage convergence, as well as the dynamic population size to provide greater reductions in population size, thus focussing on fewer low-level heuristics per generation. As such, the framework will better apply hyper-heuristics to the problem, providing further benefits over traditional approaches.

References

Airbus SAS (2012) A340-200/-300 airplane characteristics for airport planning. Tech. rep. Rev. 19, Airbus SAS

Allen JG, Coates G, Trevelyan J (2010) A theoretical framework for the optimisation of the structural layout of an aircraft using deterministic and stochastic optimisation. In: Proceedings 8th ASMO-UK/ISSMO conference, pp 19–25

Amadori K, Jouannet C, Krus P (2007) A framework for aerodynamic and structural optimization in conceptual design. In: Proceedings 25th AIAA applied aerodynamischen conference, pp 4061–4070

Arabas J, Michalewicz Z, Mulawka J (1994) GaVaPS—a genetic algorithm with varying population size. In: Proceedings 1st IEEE conference evolution computer, pp 73–78

Ayob M, Kendall G (2003) A Monte Carlo hyper-heuristic to optimise component placement sequencing for multi head placement machine. In: Proceedings international conference intelligent technique, pp 19–25

Bartholomew P (1998) The role of MDO within aerospace design and progress towards an MDO capability. In: 7th AIAA/USAF/NASA/ISSMO symposium on multidisciplinary analysis optimization

Bean JC, Hadj-Alouane AB (1992) A dual genetic algorithm for bounded integer programs. Tech. rep. TR 92-53, University of Michigan

Burke EK, Hyde M, Kendall G, Ochoa G, Özcan E, Qu R (2010) Hyper-heuristics: a survey of the state of the art. Tech. rep. NOTTCS-TR-SUB-0906241418-2747, University of Nottingham

Chakhlevitch K, Cowling P (2008) Hyperheuristics: recent developments. Adapt Multilevel Metaheuristics 136:3–29

Clerc M, Kennedy J (2002) The particle swarm-explosion, stability, and convergence in a multidimensional complex space. IEEE Trans Evol Comput 6(1):58–73

Cowling P, Kendall G, Soubeiga E (2000) A hyperheuristic approach to scheduling a sales summit. In: Practice theory autom timetabling III, pp 176–190

Eiben AE, Michalewicz Z, Schoenauer M, Smith JE (2007) Parameter control in evolutionary algorithms. Parameter Setting Evol Algorithm 54:19–46

European Aviation Safety Agency (2012) Certification specifications and acceptable means of compliance for large aeroplanes, CS-25. Ammend, p 12

Gantois K, Morris AJ (2004) The multi-disciplinary design of a large-scale civil aircraft wing taking account of manufacturing costs. Struct Multidisc Optim 28:31–46

Grasmeyer JM, Naghshineh-Pour A, Tetrault PA, Grossman B, Haftka RT, Kapania RK, Mason WH, Schetz JA (1998) Multidisciplinary design optimization of a strut-braced wing aircraft with tip-mounted engines. Tech. rep. MAD 98-01-01, MAD Center

Grefenstette JJ (1986) Optimization of control parameters for genetic algorithms. IEEE Trans Syst Man Cybern 16:122–128

Hansen LU, Horst P (2008) Multilevel optimization in aircraft structural design evaluation. Comput Struct 86:104–118

Hu TY, Yu XQ (2009) Aerodynamic/stealthy/structural multidisciplinary design optimization of unmanned combat air vehicle. Chin J Aeronaut 22:380–386

Jackson P (2009) Jane's all the world's aircraft 2009–2010. Jane's Information Group, Coulsdon

Kaufmann M, Zenkert D, Wennhage P (2010) Integrated cost/weight optimization of aircraft structures. Struct Multidisc Optim 41:325–334

Kesseler E, Vankan WJ (2006) Multidisciplinary design analysis and multi-objective optimisation applied to aircraft wing. WSEAS Trans Syst Control 1(2):221–227

Maturana J, Lardeux F, Saubion F (2010) Autonomous operator management for evolutionary algorithms. J Heuristics 16(6):881–909

Maute K, Allen M (2004) Conceptual design of aeroelastic structures by topology optimization. Struct Multidisc Optim 27:27–42

Morrison RW, De Jong KA (2002) Measurement of population diversity. In: Proceedings 5th international conference LNCS 2310, pp 31–41

Neufeld D, Behdinan K, Chung J (2010) Aircraft wing box optimization considering uncertainty in surrogate models. Struct Multidisc Optim 42:745–753

Niu MYC (1999) Airframe stress analysis and sizing, 2nd edn. Conmilit Press, Hong Kong

Ong YS, Lim MH, Zhu N, Wong KW (2006) Classification of adaptive memetic algorithms: a comparative study. IEEE Trans Syst Man Cybern B 36(1):141–152

Özcan E, Bilgin B, Korkmaz EE (2008) A comprehensive analysis of hyper-heuristics. Intell Data Anal 12(1):3–23

Pedersen MEH (2010) Good parameters for differential evolution. Tech. rep. HL1002, Hvass

Rafique AF, He L, Kamran A, Zeeshan Q (2011) Hyper heuristic approach for design and optimization of satellite launch vehicle. Chin J Aeronaut 24(2):150–163

Raymer DP (2002) Enhancing aircraft conceptual design using multidisciplinary optimization. Dissertation, Kungliga Tekniska Högskolan

Raymer DP (2006) Aircraft design: a conceptual approach, 4th edn. AIAA Education Series, Reston

Roskam J (1986) Rapid sizing method for airplanes. J Aircr 23(7):554–560

Sensmeier MD, Samareh JA (2004) A study of vehicle structural layouts in post-WWII aircraft. In: 45th AIAA/ASME/ASCE/AHS/ASC structure, structure dynamics materials conference

Sobieszczanski-Sobieski J, Haftka RT (1997) Multidisciplinary aerospace design optimization: survey of recent developments. Struct Optim 14:1–23

Torenbeek E (1982) Synthesis of subsonic airplane design. Kluwer Academic Publishers, Dordrecht