Struct Multidisc Optim (2013) 48:73–94
DOI 10.1007/s00158-012-0878-1

RESEARCH PAPER

Local and global Pareto dominance applied to optimal design and material selection of composite structures

Carlos A. Conceição António

Received: 1 August 2012 / Revised: 25 November 2012 / Accepted: 13 December 2012 / Published online: 6 February 2013

Abstract The optimal design of hybrid composite structures considering sizing, topology and material selection is addressed in a multi-objective optimization framework. The proposed algorithm, denoted by Multi-objective Hierarchical Genetic Algorithm (MOHGA), searches for the Pareto-optimal front enforcing population diversity by using a hierarchical genetic structure based on co-evolution of multi-populations. An age structured population is used to store the ranked solutions aiming to obtain the Pareto front. A self-adaptive genetic search incorporating Pareto dominance and elitism is presented. Two concepts of dominance are used: the first one denoted by local non-dominance is implemented at the isolation stage of populations and the second one called global non-dominance is considered at age structured population. The age control emulates the human life cycle and enables to apply the species conservation paradigm. A new mating and offspring selection mechanisms considering age control and dominance are adopted in crossover operator applied to age-structured population. Application to hybrid composite structures requiring the compromise between minimum strain energy and minimum weight is presented. The structural integrity is checked for stress, buckling and displacement constraints considered in the multi-objective optimization. The design variables are ply angles and ply thicknesses of shell laminates, the cross section dimensions of beam stiffeners and the variables associated with the material distribution at laminate level and structure level. The properties of the proposed approach are discussed in detail.

Keywords Multi-objective · Hybrid composites · Non-domination · Co-evolution · Self-adaptive · Age control

1 Introduction

Structural applications of composite materials have been growing due to their excellent specific stiffness, low weight, and reduced energy consumption. One approach for decreasing costs in composite structures is to adopt a hybrid construction where expensive and high-stiffness materials are used together with inexpensive and low-stiffness material. The optimization problem of topology associated with material/stacking sequence design of hybrid composites is very complex when sizing variables, as ply angle and layer thickness are simultaneously considered. Furthermore, since the balance between weight/cost and stiffness is important in the construction of hybrid laminates the use of multi-objective design procedures is necessary.

Only a few researchers have presented multi-objective design approaches for hybrid composites. In particular, Wang et al. (2002) developed a model for the trade-off between manufacturing costs and minimum weight applied to aerospace structures. Rahul et al. (2006) minimize the cost/weight of hybrid laminates while maximizing the strength under impact loads. Also, Pelletier and Vel (2006) proposed two models. In the first model the objectives are to maximize the loading capacity and to minimize the mass, and in the second one the objectives are to maximize the axial and hoop rigidities and to minimize the

C. A. Conceição António (✉)
IDMEC, Faculty of Engineering, University of Porto,
Rua Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal
e-mail: cantonio@fe.up.pt

mass of a graphite/epoxy pressure vessel. Baier et al. (2008) combine strength and manufacturing requirements to optimize structures using metal parts and carbon composites. Ghiasi et al. (2010) proposed a model for simultaneous structural and manufacturing design of composites. Tang et al. (2010) presented a multi-objective optimization model based on dominance considering material selection together with shape and sizing under static and thermal loads. These authors analyse several examples showing the capabilities of the approach based on the use of mixed variables to build the Pareto front overcoming the difficulties associated with convexity and continuity of the Pareto domain. Recently Irisarri et al. (2011) presented a multi-objective optimization strategy of composite stiffened panels. The global optimization is addressed as follows: approximation of objective functions and decomposition of design domain using iteratively the variables associated with the skin and the stiffeners. Other examples of multi-objective optimization of composite structures based on decomposition approaches can be found in António and Hoffbauer (2009), Murugan et al. (2007) and Kassapoglou and Dobyns (2001). A review of optimization techniques used in the design of composite structures presented by Awad et al. (2012) refer some new real scenarios proposals for civil engineering applications.

In general, a multi-objective optimization algorithm leads to a set of optimal solutions known as Pareto-optimal solutions. The reason for this is that no solution can be considered better than other relatively to the objective functions. The principal goal of a multi-objective optimization algorithm is the search of the global Pareto-optimal front keeping population diversity in the Pareto-optimal solutions. In this paper such a challenge is performed using a multi-objective hierarchical genetic algorithm (MOHGA) with co-evolution of multi-populations. This new approach is based on author's previous developments (Conceição António 2006, 2009a, b) now adding dominance concepts but preserving the populations diversity an important issue in multi-objective optimization. Section 2 presents the formulation of the optimization of hybrid composites. The main features of the proposed approach of MOHGA are described in Section 3. Optimization results are provided in Section 4 and the discussion and conclusions on the performance of the proposed approach are presented in Section 5.

2 Multi-objective optimization of hybrid composites

2.1 Multi-objective based design formulation

According to the scope of this paper the multi-objective optimization problem can be mathematically expressed as:

$$\text{Minimize } \mathbf{f}(\mathbf{u}) = \left\{ f_i(\mathbf{u}) : \Re^n \mapsto \Re; \quad i = 1,\ldots,m; \quad m > 1 \right\}$$

subject to

$$g_j(\mathbf{u}) \le 0; \quad \left\{ g_j(\mathbf{u}) : \Re^n \mapsto \Re; \quad j = 1,\ldots, p \right\}$$

and (1)

$$h_k(\mathbf{u}) = 0; \quad \{ h_k(\mathbf{u}) : \Re^n \mapsto \Re; \quad k = 1,\ldots, r \}$$

with contradictory objectives.

There is no unique solution to a problem with more than one conflicting objectives and the existing solutions are denoted by Pareto-optimal solutions. The classification as "Pareto-optimal" depends on the concept of dominance according the following definitions:

Definition 1 (Dominance) Let be $\mathbf{Q} \subseteq \Re^n$ the subset in the minimization problem formulated in (1). A solution $\mathbf{u}_1 \in \mathbf{Q}$ dominates a solution $\mathbf{u}_2 \in \mathbf{Q}$, if the objective value for $\mathbf{u}_1$ is smaller than the objective value for $\mathbf{u}_2$ in at least one objective and is not bigger with respect to the other objectives:

$$\mathbf{u}_1 \prec \mathbf{u}_2 \Leftrightarrow \begin{cases} \forall i : 1 \le i \le m \Rightarrow f_i(\mathbf{u}_1) \le f_i(\mathbf{u}_2) \\ \wedge \\ \exists j : 1 \le j \le m,\ f_j(\mathbf{u}_1) < f_j(\mathbf{u}_2) \end{cases} \tag{2}$$

where $\mathbf{u}_1 \prec \mathbf{u}_2$ denotes $\mathbf{u}_1$ dominates $\mathbf{u}_2$.

Definition 2 (Pareto optimal design) Let be $\mathbf{Q} \subseteq \Re^n$ the subset in the minimization problem formulated in (1). A solution $\mathbf{u}^* \in \mathbf{Q}$ is classified as Pareto optimal design if and only if it is not dominated by any other solution in $\mathbf{Q}$. The set of all Pareto solutions is called the Pareto front, represented by $\mathbf{U}^*$,

$$\mathbf{u}^* \in \mathbf{U}^* \Leftrightarrow \left\{ \mathbf{u} \in \mathbf{Q} : \mathbf{u} \prec \mathbf{u}^* \right\} = \{\varnothing\} \tag{3}$$

The above definitions are essential for further Pareto evolutionary search developments for multi-objective optimization of composite structures.

2.2 Hybrid composite laminates design definition

A composite laminate is a material built with bonded plies aiming to obtain homogenised macro-mechanic behaviour when subjected to external applied loads. Each ply is a *composite system* made of a weakest material denoted by *matrix* reinforced by long or short *fibres* of a strongest material disposed in a random way (short fibres) or with a predefined orientation (long fibres). Here it is considered only composite systems with long fibres with ply orientation determined by the angle of the longitudinal fibre axis relatively to the laminate referential axis.

It is common to identify two kinds of hybrid composite laminates where the structure and the mathematical modelling for mechanical definition are different: interply and intraply hybrid laminates as shown in Fig. 1. In this work only interply hybrid composites are considered. This hybrid

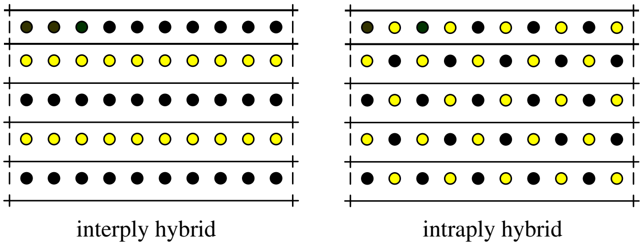

Fig. 1 Hybrid composite laminate constructions

laminate is composed by a stacking sequence of plies using different composite systems for fibre reinforcement. Each ply uses a single composite system.

The composite laminate structures considered in this study are plates or shells reinforced with beam stiffeners. Only one composite system is considered for each beam laminate used in structural reinforcement. However, different materials can be used for each plate or shell laminate. This kind of composites denoted as *interply hybrid laminates* is built using at least two different composite systems for each laminate. Nowadays hybrid composite laminates are commonly used in aeronautical, space and advanced industrial applications. The use of these materials has become competitive since laminate construction based on interply hybrids allows a cost reduction and an increasing performance of the mechanical properties.

In this work optimal design of structures made of hybrid composite materials is performed at the following levels: (1) optimal sizing of plate, shell or beam laminates; (2) optimal stacking sequence of each laminate; (3) optimal laminate distribution for the structure. This natural decomposition of the optimization problem leads to a multimodal optimal design of hybrid composites allowing the possibility of finding several optimal solutions what can be very attractive for the designer.

Hybrid construction used in composite structures deals with the compromise between minimum strain energy and minimum weight, exploring alternative optimal design solutions. On the other hand, weight/cost reduction and maximum structural performance based on the use of alternative optimal design solutions are important challenges for designers concerned with sustainable energy consumption. The multi-objective optimization of composite structures is defined here as a bi-criterion problem formulated as attempting to minimize the weight/cost and the strain energy of the structure subject to constraints related to structural integrity (Conceição António 2002, 2006, 2009a, b).

The structural analysis of hybrid composites was implemented using the same formulation defined previously with the Marguerre shell element and a Timoshenko beam element (Conceição António et al. 2000; Conceição António 2002). The structural system with non linear behaviour is in equilibrium if the internal forces are equal to the external applied loads. Since in the numerical process of solution it is not possible to reach an exact equilibrium situation the goal is to obtain a close state near equilibrium and within a small error. A measure of the error at this equilibrium state based on the Total Lagrangian formulation is the vector of *non-balanced forces* $\boldsymbol{\Psi}(\mathbf{d}, \lambda)$ defined as

$$\boldsymbol{\Psi}({}^t\mathbf{d}, {}^t\lambda) = \mathbf{R}\left({}^t\mathbf{d}\right) - {}^t\lambda\overline{\mathbf{F}} \tag{4}$$

where $\mathbf{R}$ is the vector of equivalent nodal forces associated with the actual stress field on the structure, $\overline{\mathbf{F}}$ is the equivalent nodal forces due to the external applied forces, ${}^t\lambda$ is a scale factor related to the load level t and ${}^t\mathbf{d}$ is the corresponding displacement vector. The equilibrium path is traced using the arc-length method (Crisfield 1997). An approach similar to the one proposed by Budiansky (1974) enables the identification of the load factors ${}^{\lambda}b$ associated with buckling and λ_{FPF} related to first ply failure. The first ply failure is checked using the Huber-Mises law (Conceição António 2002). The same procedure enables to obtain the corresponding critical displacements.

An unified approach (Conceição António 2002) for buckling and first ply failure is used to check the integrity of hybrid composite structures through the concept of *critical load factor* λ_{crit} defined as

$$\lambda_{crit}(\mathbf{x}, \pi) = MIN\left[\lambda_b(\mathbf{x}, \pi), \lambda_{FPF}(\mathbf{x}, \pi)\right] \tag{5}$$

Considering the equilibrium path, the critical displacement d_{crit} can be associated with buckling d_b, or related to first ply failure d_{FPF},

$$d_{crit}(\mathbf{x}, \boldsymbol{\pi}) = MAX\left[d_b\left(\mathbf{x}, \boldsymbol{\pi}\right), d_{FPF}(\mathbf{x}, \boldsymbol{\pi})\right] \tag{6}$$

The design variables $\mathbf{x}$ and $\boldsymbol{\pi}$ are associated with the sizing and material distribution, respectively. The constraints are imposed on the critical load factor λ_{crit}, and on the critical displacement d_{crit}, both of which are associated with buckling and first ply failure (Conceição António 2002).

Thus, the multi-objective optimization of plates and shells of hybrid composite structures reinforced with beams under static loading can be formulated as follows:

$$\text{Minimise} \quad OBJ(\mathbf{x}, \boldsymbol{\pi}) = \left(W(\mathbf{x}, \boldsymbol{\pi}), E\left(\mathbf{x}, \boldsymbol{\pi}\right)\right) \tag{7}$$

subject to

$$\varphi_1(\mathbf{x}, \boldsymbol{\pi}) = 1 - \frac{\lambda_{crit}(\mathbf{x}, \boldsymbol{\pi})}{\overline{\lambda}_a} \leq 0 \tag{8}$$

$$\varphi_2(\mathbf{x}, \boldsymbol{\pi}) = \frac{d_{crit}\left(\mathbf{x}, \boldsymbol{\pi}\right)}{\overline{d}_a} - 1 \leq 0 \tag{9}$$

with $\overline{\lambda}_a$ and $\overline{d}_a$ the allowable values for critical load factor and critical displacement, and the size constraints:

$$x_j^l \leq x_j \leq x_j^u, \quad j = 1, \ldots \bar{N}_x \tag{10}$$

satisfying the equilibrium equation set:

$$\Psi\left({}^t\mathbf{d}, {}^t\lambda, \mathbf{x}, \boldsymbol{\pi}\right) = \mathbf{R}\left({}^t\mathbf{d}, \mathbf{x}, \boldsymbol{\pi}\right) - {}^t\lambda\bar{\mathbf{F}} = 0 \tag{11}$$

and the additional arc-length method equation:

$$\Phi\left({}^t\mathbf{d}, {}^t\lambda, \mathbf{x}, \boldsymbol{\pi}\right) = 0 \tag{12}$$

In equilibrium (11), R(${}^t\mathbf{d}$, $\mathbf{x}$, $\boldsymbol{\pi}$) denotes the internal forces in the structural system reflecting the dependence relatively to design variables. Since the nonlinear geometric behaviour is considered, the equilibrium is reached through an iterative and incremental loading process based on the arc-length method in (11) for a load level t (Conceição António 2002, 2009a, b). In the objective function given by (7) the term W ($\mathbf{x}$, $\boldsymbol{\pi}$) is the weight of the structure used in the laminates, and it is defined as

$$W(\mathbf{x}, \boldsymbol{\pi}) = \sum_{j=1}^{NLam} \sum_{i=1}^{np(j)} \rho_{ij}(\boldsymbol{\pi}) \quad V_{ij}(\mathbf{x}) \tag{13}$$

where ρ_{ij} ($\boldsymbol{\pi}$) is the specific weight of the composite system, V_{ij} ($\mathbf{x}$) is the volume of the *i-th* ply of the *j-th* laminate, *NLam* is the number of laminates of the hybrid composite structure and np (j) is the number of plies of the *j-th* laminate. The term $E(\mathbf{x}$, $\boldsymbol{\pi})$ in (7) represents the energy of the structural system corresponding to the deformed configuration of the structure evaluated in close form as

$$E(\mathbf{x}, \boldsymbol{\pi}) = \frac{1}{2}{}^t\lambda\bar{\mathbf{F}} \cdot {}^t\mathbf{d} \tag{14}$$

corresponding to equilibrium defined in (11) and (12).

The composite structures considered in this approach are composed of plate, shell, or beam laminates. The design variables represented by vector $\mathbf{x}$ are defined as follows: the angle $\theta_{i,j}$ and the thickness $\bar{t}_{i,j}$ of the *i-th* ply of the *j-th* plate or shell laminate, grouped in the vectors $\boldsymbol{\theta}$ and $\bar{\mathbf{t}}$, respectively; the height h_j and the width w_j of the rectangular cross section of the *j-th* beam laminate, grouped in the vectors $\mathbf{h}$ and $\mathbf{w}$, respectively. Figure 2 shows the design variables for the optimization problem.

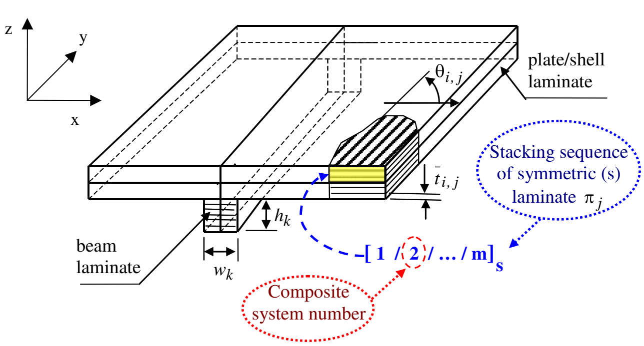

Fig. 2 Definition of the design variables associated with the j-th plate or shell hybrid laminate and *k-th* beam laminate (one composite system)

The macro-mechanical properties of each laminate depend on the design variables previously defined and also on the ply material and on the laminates distribution on the structure. The material distribution at laminate level and structure level is defined through the variable π_j, associated with the *j-th* plate or shell laminate as shown in Fig. 2. These variables related to the hybridisation are important in the optimal design of laminated structures with multi-materials (Conceição António 2006, 2009a, b). In the approach considered in this work, they are grouped in vector $\boldsymbol{\pi}$.

3 Multi-objective genetic algorithm based on dominance

3.1 Co-evolution of populations

The design variables considered in the optimization of composite structures are both continuous and discrete by nature. The representation of these continuous and discrete variables by a single string with an appropriate code format in the GA increases the cost of the space searching. Therefore, decomposition approaches have been proposed in some composite optimization models aiming to increase the search efficiency. This decomposition strategy is independent of the optimization method, for example in Conceição Antonio et al. (1995, 2000) the search performs at a bilevel strategy using gradient based methods at both levels. In a further development, a multilevel GA was proposed (Conceição António 2002) based on a bilevel decomposition performing sequentially with different populations and fitness functions aiming to obtain good convergence properties and better efficiency. Murugan et al. (2007) proposed a decomposition of the optimization problem of aircraft composite structures into global and local levels. In this approach the box-beam cross-sectional dimensions of the composite structure are optimized at global level whereas the optimal ply angles of those box-beams are determined at local level. Conceição Antonio (2006, 2009a, b) proposed a multimodal optimization approach denoted by hierarchical genetic algorithm (HGA) for a single objective considering simultaneously the optimal sizing and material distribution of composite structures. The approach is based on co-evolution of multiple populations using the same fitness function exploring different zones of the search space. In general the previous proposals try to overcome the complexity and specificity associated with the optimization of composite structures aiming to keep the population's diversity in GA search. In particular, in the proposed HGA approach (Conceição Antonio 2006, 2009a, b) the diversity aspect is reinforced throughout the age structure of the population and by using the *Species Conservation* paradigm. The concept of species evolution in multi-island GA has been proposed by Siarry et al. (2002). However the definition of species is different from the author's proposals. Furthermore the control of number of individuals belonging to the same species was introduced into HGA (Conceição Antonio 2006, 2009a, b).

The new proposed algorithm takes advantage of the decomposition used in previous approaches introducing important alterations to preserve the population diversity during evolution. Diversity is an important issue aiming to search the multi-objective optimization of composite structures. So, an approach for structural robust design that simultaneously considers minimum weight/cost and maximum performance/minimum strain energy is proposed in this paper. The trade-off between performance target, depending on given stress, displacement and buckling constraints imposed on composite structures, against minimum weight/cost, is searched. The Pareto front is built and such a challenge is performed here based on a modified version of a previously proposed hierarchical genetic algorithm (HGA) with co-evolution of multi-populations (Conceição António 2006, 2009a, b). The new approach is denoted by MOHGA, a self-adaptive genetic search (Conceição António 2009a, b) incorporating Pareto dominance and an elitist strategy based on survival of non-dominated solutions and individual age control. MOHGA with age structure is a mixed model applying multiple crossover and mutation operators aiming to explore the synergy and adaptive properties in multi-populations topology. The algorithm considers a sequential hierarchical relationship between sub-populations evolving in separated isolation stages followed by migration. Improvements based on the *Species Conservation* paradigm are performed to avoid genetic tendencies due to elitist strategies used in hierarchical sub-populations (Conceição António 2006).

In the proposed MOHGA model described in Fig. 3, three sub-populations are arranged in a ring and have a hierarchical relationship going from the upper level sub-

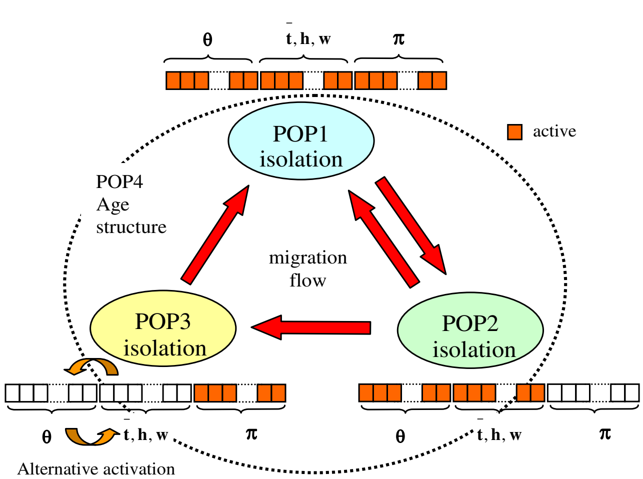

Fig. 3 Hierarchical relationships of sub-populations of the MOHGA and chromosome segmentation

population POP1 to the lower level sub-population POP3. Each MOHGA sub-population has an independent evolution during a time period denoted by *Isolation Stage*, where the crossover and mutation operators are applied in a sequence of operations as described in reference (Conceição António 2006). After isolation, a *Migration Stage* occurs with individuals moving towards the subsequent sub-populations in the ring net as shown in Fig. 3. The initialization of the subsequent populations is performed with migrated individuals together with new randomly generated individuals. A life cycle from POP1 to POP3 is called *epoch*.

The use of elitist strategies in the sub-populations of the MOHGA seems contradictory to the *Species Conservation* paradigm, and to overcome this difficulty an enlarged population POP4 is considered (Conceição António 2006, 2009a, b). Then, all individuals generated as "new" during the evolutionary process can be stored into POP4 with an age structure. In practice to avoid saturation of unfeasible solutions into POP4 a threshold can be imposed (Conceição António 2006). This threshold is associated with the fitness value of the worst individual of the elite group of each sub-population at each generation.

Depending on the evolving sub-population, different design variables are considered in the optimization model corresponding to active and non-active segments of each chromosome as shown in Fig. 3. The use of different active segments of the chromosome corresponds to a decomposition of the design space. Chromosome activation deals with segmentation of the population in niches and is driven together with the use of *species concept*.

A species is a class of individuals with common characteristics. Most of the developments using species conservation techniques are radii-based. In order to differentiate species, a radius parameter is introduced defining a species distance. The previous definitions of species found in literature are based on geometric or landscape concepts (Siarry et al. 2002). These species conservation techniques show difficulties associated with the necessary distance measure and depend on the given radius threshold, especially in high dimensional search spaces. To overcome the referred disadvantage the radii-based concept is eliminated and it is introduced an alternative method based on sub-population differentiation (Conceição António 2006). In this work, the term "species" does not express the idea of a fixed group of forms, but rather it is a concept to integrate features of a group of individuals (i.e. possible solutions).

Definition 3 (Species) Individuals with the same material distribution on shell laminates lay-up and same laminate distribution on composite structure belong to same species.

The above definition means that individuals with identical material distribution topology belong to the same species having equal code value for the 3rd segment of the chromosome associated with that material distribution on laminate and on structure. The variables associated with material distribution are grouped in vector $\boldsymbol{\pi}$ as shown in Fig. 3. To induce niche behaviour, rules for species conservation and species dominance are adopted, and the number of individuals belonging to a species is controlled (Conceição António 2006). More specifically, the implementation of the *Species Conservation* paradigm is considered at the *Isolation* and *Migration* stages based on the following rules at each evolution stage: in *Isolation stage* the number of individuals belonging to the same species is limited; and in *Migration stage* all candidates to migration must belong to different species (Conceição António 2006, 2009a, b).

3.2 Self-adaptive procedures

It is recognized that the efficiency of genetic algorithms improves if some adaptive rules are included. Adaptive rules use additional information related to the behaviour of state and design variables of the structural problem. At each generation, the self-adaptation of the genetic parameters to evolutionary conditions attempts to improve the efficiency of the genetic search. The introduction of adaptive rules occurs at selection, mutation, crossover and migration operators (Conceição António 2009a, b). Self-adaptation has proven to be highly beneficial in automatically and dynamically adjusting evolutionary parameters. The introduction of adaptive rules occurs at three levels: (i) when defining the search domain in each generation; (ii) by considering a crossover operator based on commonality and local improvements; and (iii) by controlling mutation, including behavioural data.

The following crossover operators are used in the MOHGA: *Elitist Hybrid Crossover with genetic improvement* (EHCgi); *Elitist parameterized Uniform Crossover* (EpUC) (Conceição António 2006, 2009a, b); and the new operator *Age-Dominance parameterized Uniform Crossover* (ADpUC). Table 1 shows the mechanisms of recombination of the proposed crossover operators, grouping them according to taxonomy analysis (Conceição António 2009a, b). The crossover operators based on its application to both parents are grouped as: Discrete Crossover Operators (DCOs), Aggregation Based Crossover Operators (ABCOs), and Neighbourhood-Based Crossover Operators (NBCOs). The mechanisms of recombination are the Mating Selection Mechanism (MSM), the Offspring Generation Mechanism (OGM), and the Offspring Selection Mechanism (OSM) (Conceição António 2009a, b).

Some changes in MSM and OSM of previously developed *Age parameterized Uniform Crossover* (ApUC) (Conceição António 2006) are introduced as will be describe in further sections. The changes in MSM are based on concepts of individual age and dominance. The proposed OSM is based on survival of non-dominated individuals (rank 1) in POP4 behind lethal age. According to the referred changes the new operator is denoted by *Age-Dominance parameterized Uniform Crossover* (ADpUC).

The selection of crossover operators depends on the quality of the offspring generated by recombination. During the evolutionary process, the best offspring solution obtained from the crossover is compared with the worst solution of the elite group. This last solution is the best candidate to be eliminated in the next generation. A success event happens when the best-fitted offspring is better than the worst-fitted individual of the elite group of the population. The quality of the solutions obtained from a crossover is then associated with the number of success events divided by the number of generations where the crossover operator is called and is denoted by the success rate. A self-adaptive probability to select a crossover operator scheme can be established depending on the cumulative number of success events (Conceição António 2009a, b). In this work, two

Table 1 Crossover operators for self-adaptive procedure

Crossover	Mechanism of recombination			Taxonomy analysis
	MSM	OGM	OSM	
EHCgi	Elitist based on fitness	Hybrid crossover with genetic improvement	Elite group transferring	NBCO
EpUC	Elitist based on fitness	Uniform parameterised crossover	Elite group transferring	DCO
ADpUC (new)	Individual age and dominance	Uniform parameterised crossover	Lethal age control + non-dominating sorting + survival of non-dominated individuals	DCO

crossover schemes can be selected in each sub-population of MOHGA as follows:

POP1: EpUC or ADpUC
POP2: EpUC or EHCgi
POP3: EpUC or ADpUC

The incorporation of data related to the behaviour of state variables of the structural system is the main objective of the *Controlled Mutation* (Conceição António 2006, 2009a, b). The establishment of a relationship between the stress field in the composite structure and the chosen genes to mutate can control the mutation process. This is also a self-adaptive procedure adopted for MOHGA. The Controlled Mutation is performed in an alternative way with *Implicit Mutation* (Conceição António 2002). This last procedure is based on introduction of new randomly generated individuals into population which changing the chromosomes in further generations are equivalent to mutation process. Both mutation operators are described in references (Conceição António 2002, 2006, 2009a, b).

3.3 Non-dominated sorting at isolation stage

At the isolation stage of each MOHGA sub-population defined here as set $\mathbf{Q} \subseteq \Re^n$, individuals are sorted and ranked according to local non-constrain-dominance. Following the definition by Deb (2001), an individual $\mathbf{u}_i \in \mathbf{Q}$ is said to constrain-dominate an individual $\mathbf{u}_j \in \mathbf{Q}$, if any of the following conditions are verified:

(1) $\mathbf{u}_i$ and $\mathbf{u}_j$ are feasible, with
   (i) $\mathbf{u}_i$ is no worse than $\mathbf{u}_j$ for all objectives, and
   (ii) $\mathbf{u}_i$ is strictly better than $\mathbf{u}_j$ in at least one objective,

(2) $\mathbf{u}_i$ is feasible while individual $\mathbf{u}_j$ is not,
(3) $\mathbf{u}_i$ and $\mathbf{u}_j$ are both infeasible, but $\mathbf{u}_i$ has smaller constraint violation.

The constraint violation of an individual $\mathbf{u} = [\mathbf{x}\ \boldsymbol{\pi}]$ is defined to be equal to the sum of the violated constraint function values in the multi-objective optimization problem formulated from (7) to (12):

$$\xi(\mathbf{u}) = \sum_{i=1}^{2} \Gamma_i(\mathbf{u}) \tag{15}$$

where $\Gamma_i(\mathbf{u}) = \Gamma_i\,[\varphi_i(\mathbf{x}, \boldsymbol{\pi})]$, with

$$\Gamma_i\,[\varphi_i(\mathbf{x}, \boldsymbol{\pi})] = \begin{cases} 0 & \text{if} \quad \varphi_i(\mathbf{x}, \boldsymbol{\pi}) \le 0 \\ \varphi_i(\mathbf{x}, \boldsymbol{\pi}) & \text{if} \quad \varphi_i(\mathbf{x}, \boldsymbol{\pi}) > 0 \end{cases} \tag{16}$$

The concept of constrain-domination enables to compare two individuals in problems having multiple objectives and constraints, since if $\mathbf{u}_i$ constrain-dominates $\mathbf{u}_j$, then $\mathbf{u}_i$ is better than $\mathbf{u}_j$. If none of the three conditions referred above are verified, then $\mathbf{u}_i$ does not constrain-dominate $\mathbf{u}_j$.

The problem of stacking sequence design of composite structures is well known for having many local optima, and so, dominated solutions are expected. The approach proposed in this work uses a mixture of developed techniques and new techniques in order to find multiple Pareto-optimal solutions in parallel. The principal aspects are: the storage of the obtained Pareto-optimal solutions, the use of the concept of Pareto dominance in order to assign scalar fitness values to individuals, and the clustering through the co-evolution of sub-populations to reduce the number of non-dominated solutions stored without destroying the characteristics of the Pareto-optimal front.

3.4 Fitness assignment procedure at isolation stage

To build the Pareto front at isolation stage of each sub-population it is necessary to rank the individuals according to non-dominance definitions. In isolation stage this is named *local dominance*. The proposal of Fonseca and Fleming (1993) is adopted in this paper, a scheme in which the rank of a certain individual corresponds to the number of chromosomes in the current population by which it is dominated. So, lets consider an individual/solution $\mathbf{u}_i$, which is dominated by p_i individuals in the current generation. Its current position in the individual's ranking can be given by:

$$r_i = 1 + p_i \tag{17}$$

All non-dominated individuals are assigned rank one (rank 1), denoted by $\mathbf{r}^1$. Rank one is temporarily disregarded from the population and the non-constrain-dominated solutions of the remaining population are founded and designated as non-constrain-dominated set of rank 2. The procedure continues until all the individuals are ranked. A short analysis reveals that any population, must have at least one solution with rank 1 and that the maximum rank of any population individual cannot be larger than the population size, N.

After the ranking is performed in ascending order of magnitude, a raw fitness is assigned to each solution based on its rank. The raw fitness is obtained using a linear mapping function taking values between N for the best ranked solution and 1 for the worst ranked solution. The raw fitness is averaged considering at a time the solutions in each rank. So, for any solution $\mathbf{u}_i$, the following average fitness assignment is obtained:

$$F_i^{\text{aver}} = \begin{cases} N - 0.5(\eta(1) - 1) & \text{if} \quad r_i = 1 \\ N - \sum\limits_{k=1}^{r_i - 1} \eta(k) - 0.5\,(\eta(r_i) - 1) & \text{if} \quad r_i \ne 1 \end{cases} \tag{18}$$

being r_i the rank of $\mathbf{u}_i$ as established in (17) and $\eta(r_i)$ the number of solutions in rank r_i. As follows F_i^{aver} is the average fitness of all solutions having the same rank r_i. The mapping and averaging procedures ensures that better ranked solutions have a higher fitness and non-dominated solutions play the most important role in a population.

In order to guarantee the diversity of non-dominated solutions the concept of niching among solutions belonging to each rank is adopted as proposed by Fonseca and Fleming (1993). In the proposed approach a solution located in a less-crowded region will have a better shared fitness. The shared fitness F_i^{shar} of a solution $\mathbf{u}_i$ is obtained dividing the corresponding average fitness calculated using (18) by the niche count $nc(\mathbf{u}_i)$,

$$F_i^{shar} = \frac{F_i^{aver}}{nc(\mathbf{u}_i)} \tag{19}$$

The niche count of a solution $\mathbf{u}_i$ is calculated as

$$nc(\mathbf{u}_i) = \sum_{j=1}^{\eta(r_i)} Shar(\delta_{ij}) \tag{20}$$

being $Shar\,(\delta_{ij})$ the sharing function defined as

$$Shar(\delta_{ij}) = \begin{cases} 1 - \left(\dfrac{\delta_{ij}}{\sigma_{share}}\right)^{\alpha} & \text{if} \quad \delta_{ij} \le \sigma_{share} \\ 0 & otherwise \end{cases} \tag{21}$$

where δ_{ij} is the normalized distance between any two solutions $\mathbf{u}_i$ and $\mathbf{u}_j$ (including itself) in rank r_i, α is the shape parameter of the sharing function and σ_{share} is a reference distance associated with the sharing effect. It is possible that several solutions have the same rank position and to distinguish their quality the metric distance is calculated using the objective values:

$$\delta_{ij} = \sqrt{\sum_{k=1}^{M} \left(\frac{f_k^{(i)} - f_k^{(j)}}{f_k^{\max} - f_k^{\min}} \right)^2} \tag{22}$$

where $f_k^{\max}$ and $f_k^{\min}$ are the maximum and the minimum objective function values of the k-th objective. In the proposed MOHGA approach those objective function values are evaluated in age structured population POP4.

In a rank, the average shared fitness value of solutions should remain the same average assigned fitness value before sharing (Deb 2001). In order to preserve the referred property a scaling is performed as follows

$$\overline{F}_i^{shar} = \frac{F_i^{aver}\eta\,(r_i)}{\sum\limits_{k=1}^{\eta(r_i)} F_k^{shar}} F_i^{shar} \tag{23}$$

The described procedure will continue for all ranks and the fitness assignment process can be described as follows:

1st Step: Initialize $\eta(j) = 0$ for all possible ranks j = 1, . . . , N. Do solution counter i = 1.

2nd Step: Calculate the number of solutions p_i that dominate solution $\mathbf{u}_i$. The rank of $\mathbf{u}_i$ is computed as $r_i = 1 + p_i$. Increment the number of solutions in rank as $\eta(r_i) \leftarrow \eta(r_i) + 1$.

3rd Step: If $i < N$ do $i \leftarrow i + 1$ and go to Step 1, else go to Step 4.

4th Step: Identify the maximum rank $\overline{r}$ verifying $\eta(r_i) > 0$. Based on rank sort and calculate the average fitness F_i^{aver} for any solution $\mathbf{u}_i$ and for i = 1, . . ., N, according to (18). Set a rank counter $r = 1$.

5th Step: For each solution $\mathbf{u}_i$ in rank r, calculate the niche count $nc(\mathbf{u}_i)$ with the other solutions in the same rank by using (20) and the shared fitness F_i^{shar} by using (19). Scale the shared fitness using (23).

6th Step: If $r < \overline{r}$, do $r \leftarrow r + 1$ and go to Step 5, else stop.

In the proposed approach a dynamically updated procedure is adopted for σ_{share} the reference distance in sharing function defined by (21). For two objectives (Deb 2001) the updating expression is:

$$\sigma_{share} = \frac{f_1^{\max} - f_1^{\min} + f_2^{\max} - f_2^{\min}}{N - 1} \tag{24}$$

where $f_k^{\max}$ and $f_k^{\min}$ are the maximum and minimum values of each objective function evaluated in age structured population POP4 where the *global dominance* is evaluated. However the above described fitness assignment process is performed at isolation stage and is based on *local dominance*.

3.5 Building Pareto front based on age-structured population

A continuous model of generation of individuals was adopted for age-structured population. An enlarged population with age structure POP4 and performing in parallel with the hierarchical topology of MOHGA is considered in this model (Conceição António 2006). Each individual belonging to population POP4 is characterized by two parameters: *individual age* and *lethal age*. The individual age increases one unit after each generation. Any individual removed from MOHGA sub-populations either by elitist strategy or by finishing of *Isolation stage* of evolution and not selected for migration, will survive in the population with age structure POP4. Furthermore, its *individual age* will continue increasing until removed definitively due to *lethal age*.

The population POP4 with age structure is ranked according to non-dominance concepts defined in Section 3.3. This is denoted by *global dominance* and the corresponding global Pareto front is built. All non-dominated individuals (rank 1) will survive after reaching the lethal age. These individuals stay in age structured population POP4 while they belong to Pareto front. Once they change their status to dominated (rank > 1) they are eliminated from age-structured population. The last defined condition means a continuously update of global Pareto front along evolutionary process with an increasing number of solutions of rank 1.

The above referred properties of enlarged population POP4 based on age control and updating global dominance imply changes in the original approach of *Age parameterized Uniform Crossover* (ApUC) (Conceição António 2006). Thus, the new operator is denoted by *Age-Dominance parameterized Uniform Crossover* (ADpUC) according to the performed changes. The changes in mating selection mechanism (MSM) are based on concepts of individual age and dominance. Thus, the MSM is conditioned by two rules: (i) the candidate age, and (ii) the rank position in the non-dominating ranking. One parent comes from the enlarged population according its age and the second parent is selected based on the global dominance.

Assuming that population maturity and potentiality follow a Normal distribution the parent selection is probability dependent and this is applied to first parent choice. Figure 4 presents the Normal probability density function, $f_z(z)$, applied to the first parent selection in this crossover process. Individuals with ages located at the tails of the Normal density function are the youngest and the oldest of the scale, and they have a very low probability to be selected as parents. Then the reproduction rate by crossover depends on the maturity and degrades as the life cycle goes on till the end.

The selection of the second parent is based on its dominance. An individual of rank $r(\mathbf{p}_i)$ lower than $\bar{r}$, is a mating candidate. The group of individuals obeying to this conditions is denoted by Ω. The second parent is select from the set group Ω with a Uniform probability distribution function. This guarantees the improvement of global Pareto front during the evolutionary process.

The hybrid nature of enlarged population POP4 also implies changes in the offspring selection mechanism (OSM) of ADpUC. Indeed, the OSM is now controlled by dominance: (i) individuals with rank > 1 are eliminated after reaching lethal age; (ii) non-dominated individuals in POP4 will survive after reaching lethal age while keeping their status.

The offspring generation mechanism (OGM) performs a multipoint combination of genes from both parents' chromosomes $\mathbf{p_1}$ and $\mathbf{p_2}$. The genetic material exchange is based on the technique "*Parameterised Uniform Crossover*" proposed by Spears and DeJong (1991). However, due to co-evolution of multiple populations and the corresponding chromosome segmentation with gene activation as referred in Section 3.1, adaptive rules are considered. So, the genetic material exchange is implemented as follows:

(1) For active segments of the chromosome, each offspring gene is selected in a biased way (Spears and DeJong 1991) given a probability for choosing the corresponding gene from the progenitor with best rank position obtained according to dominance concepts. If both progenitors belong to the same rank the selection is done according to assigned fitness.
(2) The genes of non-active segments of the offspring chromosome are equal to the genes of the corresponding segments in the chromosome of one of the parents selected randomly.

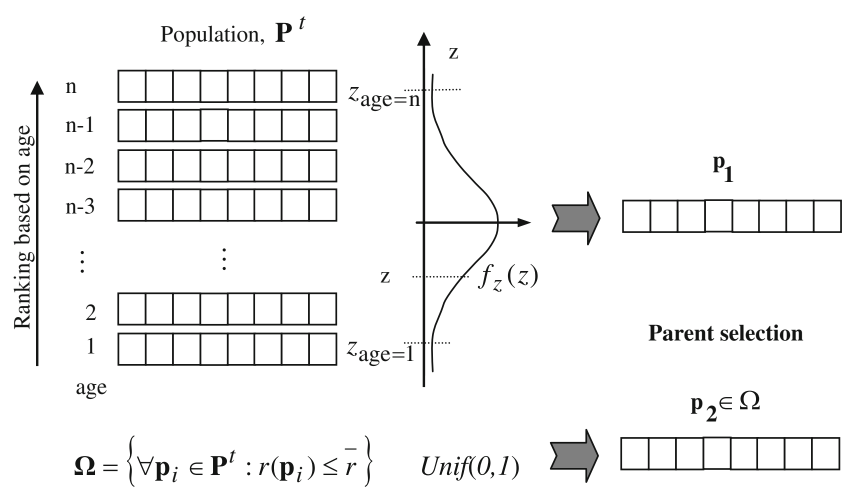

Fig. 4 Parent selection based on age and dominance degree (MSM)

3.6 MOHGA approach development

The proposed model is based on the introduction of major changes in the hierarchical genetic algorithm (HGA) previously presented by the author (Conceição António 2006, 2009a, b). To be able to apply the concepts of multi-objective optimization to the optimal design of composite structures three fundamental aspects were considered in proposed MOHGA:

1. *Local dominance*: The introduction of the concept of non-dominance in the evolution of each sub-population at its stage of isolation is one major change. The dominance is applied only to the restricted set of individuals of such sub-population being so denoted by local dominance. As was stated in Section 3.4 the definition of the fitness of each individual no longer depends on an absolute value related to the individual's performance but on the concept of dominance. The individual fitness is calculated according to the niche occupied by the individual in the sub-population and also depending on the number of individuals with the same level of dominance in its neighbourhood. So, the concept of shared fitness is adopted. This aims to obtain a balanced distribution of solutions along the constructed local Pareto front and updated during each isolation stage. The elitist strategy adopted at that stage is based on fitness as also on the concept of dominance albeit implicitly.
2. *Building of global Pareto front*: The age structured population POP4 now plays a key role in building the global Pareto front. Indeed the enlarged population POP4 is used to store the ranked solutions. According to the principles of the *Species Conservation* paradigm, any new individual generated by genetic operators belongs inherently to this population. The life cycle of each individual is emulated being characterized by the variation of reproductive capacity with age and ends when it reaches the lethal age. Beyond the age control, the enlarged population POP4 is organized based on the concept of dominance applied in each generation of the evolutionary process. Given the size and history of this population, the dominance is applied in the global sense, which together with the age control allows the progressive construction of global Pareto front. Indeed, the dimension of POP4 is controlled by the age of its members and only those non-dominated individuals (rank 1) will survive beyond the lethal age. As the process is continuously applied at every generation, it is possible that an individual with non-dominated status when it reaches the lethal age will be subsequently dominated and consequently eliminated. This leads to an increased historical record of individuals/non-dominated solutions (rank 1) in the course of the evolutionary process obtaining finally the global Pareto front.
3. A third important aspect is the inclusion of the concept of dominance in parallel with the individual age through the new genetic operator *Age-Dominance parameterized Uniform Crossover* (ADpUC) applied to the enlarged age-structured population POP4. The non-elitist access and further emulation of individual life cycle into the enlarged population POP4 come from the *Species Conservation* paradigm application. The OSM and MSM mechanisms of ADpUC are not fitness-based contrary to other crossover operators. Indeed, the age and dominance nature of ADpUC operator is an innovative procedure of MOHGA and it is very important to reinforce the population diversity.

The proposed Multi-objective Hierarchical Genetic Algorithm (MOHGA) performs according to the flow diagram presented in Fig. 5. The organization and interaction of multiple populations in the co-evolution process and the most important aspects are highlighted.

This new approach is based on author's previous developments of HGA (Conceição António 2006, 2009a, b) now adding dominance concepts in order to allow the multi-objective optimization approach. The new MOHGA is composed by a set of operations: some were introduced by the referred papers, some others by this manuscript, which are new. The review Table 2 exhibits the difference between HGA and MOHGA referring the most important implemented changes.

MOHGA shows advantages when compared with other Island model GAs (Siarry et al. 2002) that come from the application of different strategies aiming to balance exploitation and exploration effects. The most important differences are pointed as follows:

Speciation Species definition based on material distribution on shell laminates lay-up and laminate distribution on structure is very interesting for multi-objective optimization of hybrid composite structures considering sizing, topology and material selection. On other hand material-based definition is more realistic than radii-based definition proposed in island model GAs (Siarry et al. 2002).

Species Conservation paradigm Actually in island model GAs, there are many factors that make them fail to identify and to maintain niches as expected. The most salient fact is that the migration policy in the island model is good for exploration, but may also result in dominance and take over of a subpopulation by intruders (Li and Kou 2008). To overcome such behaviour the following features are considered in MOHGA: (i) Limitation of the number of individuals belonging to the same species during migration stage and at the isolation stage of sub-populations avoiding genetic tendencies due to elitist strategies traduced by

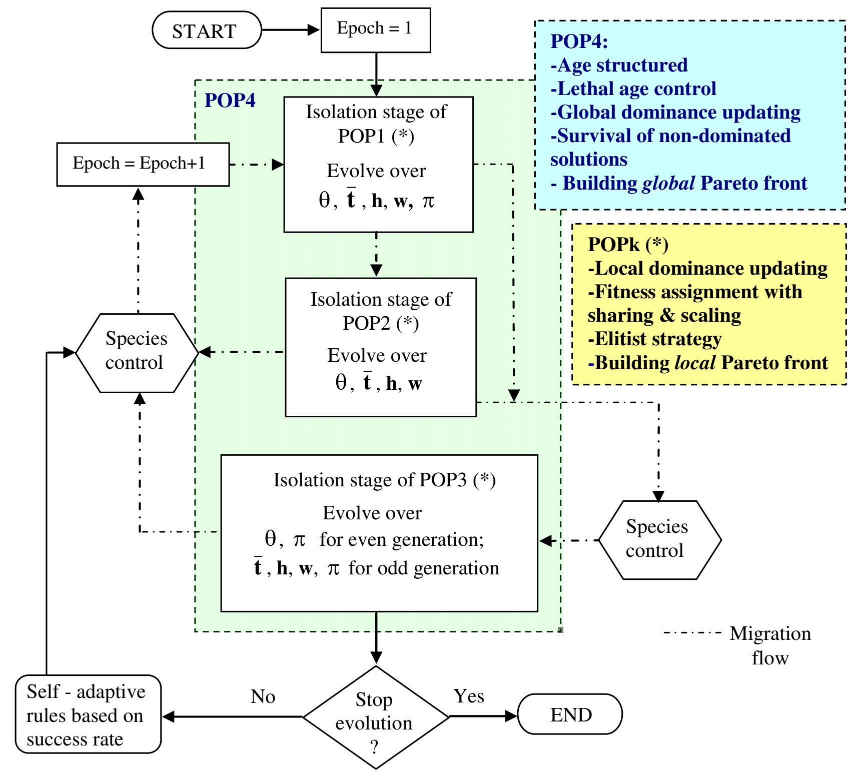

Fig. 5 Flow diagram of MOHGA

species supremacy over all. (ii) Access to enlarged population is free to enable the survival of weakest species. (iii) Species diversity is the first principle to guarantee the population diversity and alternative designs in multi-objective optimization.

Enlarged population with multiple species (POP4) This concept is not presented in Island model GAs. It enables the species conservation concept application, it is used to emulate the life cycle together with dominance properties for multi-objective optimization and it enables the building of global Pareto front in a dynamic way. The competition in the enlarged and age structured population (POP4) is the basis for co-evolution of multiple species as opposite to sub-populations evolving at isolation stage where the number of different species is reduced.

Diversity control MOHGA searches for the Pareto-optimal front enforcing population diversity by using a hierarchical genetic structure based on individual's chromosome segmentation and gene activation. This hierarchical structure is completely different of common island mode GAs where the decomposition in multi-populations is based on geometric arguments. The non-dominating sorting used in the multi-objective strategy can deal with the loss of diversity. Nevertheless the use of local dominance at isolation

Table 2 Comparison between HGA and the new proposed MOHGA

Algorithm	Optimization problems		Crossover operators				Mutation operators	
	Single objective	Multi-objective	EHCgi	EpUC	ApUC	ADpUC	Implicit	Controlled
HGA	X		X	X	X		X	X
MOHGA		X	X	X		X	X	X

Algorithm	Fitness assignment	Elitism	Self-adaptive rules	Age Control
HGA	Direct	Fitness-based	X	Lethal age
MOHGA	1. Rank-based dominance	Based on non-dominanted sorting	X	Survival of rank 1 individuals/ global Pareto front building
	2. Average fitness			
	3. Niche count	Local dominance/global dominance		Lethal age (rank > 1)
	4. Shared fitness			
	5. Scaled fitness			

stage of sub-populations and of global dominance at age structured population together with species conservation paradigm enables the maintenance of good diversity. As it will be proven in the numerical discussion the entropy stays above a high threshold at the final epochs of the evolutionary process based on MOHGA.

4 Numerical results

4.1 Problem definition

Aiming to discuss the capabilities of MOHGA to deal with multi-objective optimization of composite structures a numerical example is presented. A cylindrical shell with beam stiffeners both made of laminated composite materials is considered as shown in Fig. 6. The structural analysis of hybrid composites was implemented using the same formulation defined previously with the Marguerre shell element and a Timoshenko beam element (Conceição António et al. 2000; Conceição António 2002) as referred in Section 2.2. The non linear geometric behaviour is considered for the structural system.

The shell is hinged on straight sides and free at its curved boundaries. A central point load $F_{\max} = 4\ kN$ is applied. Since geometric and material symmetry conditions are considered only a quarter of the structure is considered for the structural optimization problem as shown in Fig. 6. The construction of the composite structure is based on shell interply hybrid laminates with n_p plies and beam laminates with n_b plies for the stiffners. In this example ten laminates were taken into account for the structure, four interply hybrid laminates (from 1 to 4) for the shell elements and the other beam laminates (from 5 to 10) for the elements of stiffeners. All laminates are symmetric and composed by six plies for the implemented example. The shell and beam laminates are identified by bracketed numbers in Fig. 6. The beam stiffeners are connected below the shell elements as shown in Fig. 6.

A laminate is built with plies each one made of a *composite system* of a polymeric matrix reinforced with longitudinal fibres. Several *composite systems* are used in shell interply hybrid laminate construction as is detailed in Figs. 1 and 2 presented in Section 2.2. However, the same *composite system* is used for all beam laminates. The mechanical properties of the *composite systems* used for ply laminates are taken from Tsai (1987) and presented in Table 3 with longitudinal strength, X, transversal strength, Y, and shear strength, S, longitudinal Young modulus, E_1, transversal elastic modulus, E_2, shear modulus, G_{12}, Poisson's ratio,s ν, and specific weight of the material, ρ.

One material from Table 3 is selected for each ply of each shell laminate. Four composite systems of fibre/matrix are considered as possible for each ply of the shell interply hybrid laminates in this study: one carbon/epoxy composite (CFRP), two glass/epoxy composites (GFRP) and one Kevlar/epoxy composite (KFRP). The Kevlar/epoxy is considered as a possible material choice only for the inner ply of the symmetric construction of the shell interply hybrid laminates. The remaining materials are free selection and at

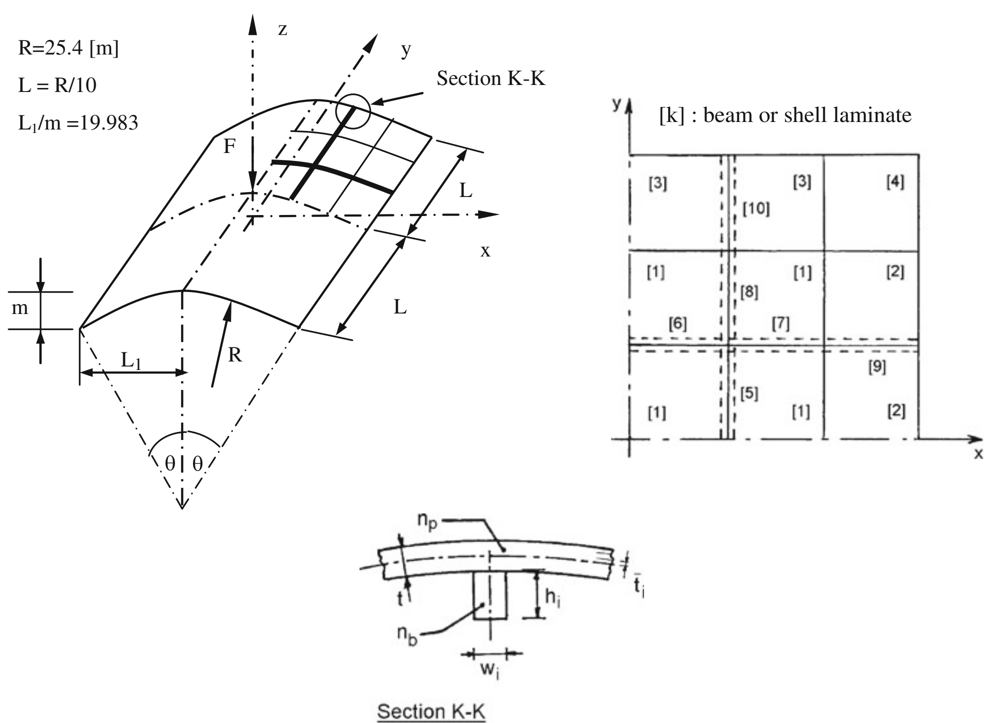

Fig. 6 Geometry and laminates of the cylindrical shell reinforced with beams

Table 3 Mechanical properties of the materials used in the composite laminates

Composite system		E_1	E_2	G_{12}	ν	X	Y	S	ρ
Type	Code	[GPa]	[GPa]	[GPa]		[MPa]	[MPa]	[MPa]	[kg/m^3]
CFRP: T300/N5208	1	181.00	10.3	7.17	0.28	1500	40	68	1600
GFRP: Scotchply 1002	2	38.60	8.27	4.14	0.26	1062	31	72	1800
GFRP: E-glass/epoxy	3	43.00	8.90	4.50	0.27	1280	49	69	2000
KFRP: Kev 49/epoxy	4	76.00	5.50	2.30	0.34	1400	12	34	1460

least two composite systems must be considered for hybrid composite shell laminate construction. Then there are 33 possible combinations of these four composite systems for the stacking sequence π_j considering the defined rules and six plies in the symmetric *j-th* composite shell laminate construction.

The beam laminates have six plies made of composite system number 2 from Table 3 and this kind of material does not change during the optimization process. The maximum allowed value for critical displacement in buckling failure or first ply failure (FPF) as defined in Section 2.2 is $\overline{d}_a = 1.3 \times 10^{-1}$m. The lower bound for the critical load factor defined in (8) is $\overline{\lambda}_a = 0.45$. The size constraints on the design variables are established as:

$$\begin{aligned} -90^\circ &\le \theta_{i,j} \le 90^\circ \\ 1.2 \times 10^{-3}\text{ m} &\le \overline{t}_{i,j} \le 2.4 \times 10^{-3}\text{ m} \\ 2.0 \times 10^{-2}\text{ m} &\le h_j \le 4.0 \times 10^{-2}\text{ m} \\ 5.0 \times 10^{-3}\text{ m} &\le w_j \le 1.5 \times 10^{-2}\text{ m} \end{aligned} \tag{25}$$

The genetic parameters of MOHGA are presented in Table 4. Nine individuals belonging to different species participate in each migration flow between the three MOHGA sub-populations as shown in Figs. 3 and 5. The number of digits in code format refers to the binary coding of the first two segments and the last number refers to the integer code used in third segment of the chromosome associated with laminate identification and laminates distribution on the composite structure as shown in Figs. 2 and 3.

In the age-structured population POP4, the *lethal age* is equal to 35 generations. Aiming to avoid storing a large number of unfeasible solutions, the updated access to POP4 is constrained to solutions with fitness higher than 50 % of the fitness of the worst individual from the elite group in each sub-population as referred in Section 3.1. This reference threshold is updated at each generation.

Table 4 Genetic algorithm parameter definitions

Sub-population	POP1	POP2	POP3
Population size	27	27	27
Elite group size	9	9	9
Mutation group size	9	9	9
Nr. digits/binary or integer code	4/3/4	4/3/4	4/3/4
Generations/isolation time	6	6	6

In the self-adaptive crossover procedure the parameters are according to reference (Conceição António 2009a, b). The mutation operators *Implicit Mutation* and *Controlled Mutation* used in this work have the same probability to be selected as defined previously in author's previous research (Conceição António 2009a, b). The shape parameter in (21) is $\alpha = 1$.

4.2 Analysis of Pareto-optimal front construction

The MOHGA performs during thirty epochs. Figure 7 shows the Pareto fronts (rank 1) for different epochs of

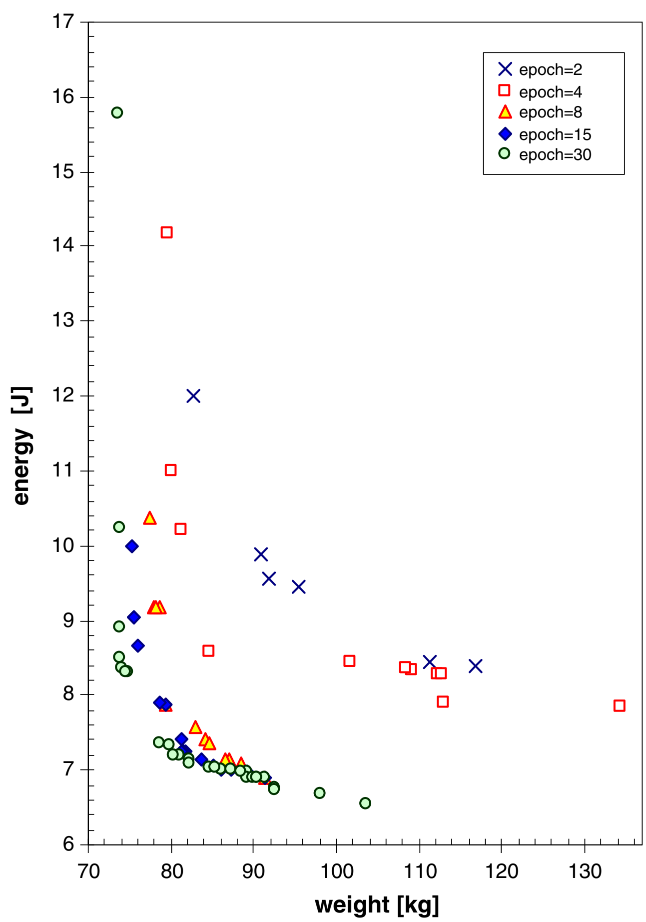

Fig. 7 Non-dominated solutions in Pareto fronts during MOHGA evolution (measured in POP4)

Table 5 Decoding results of the Pareto-optimal front solutions for the first two segments of chromosome ($\bar{t}_{i,j}$, h_i, w_i [mm] and $\theta_{i,j}$ [degrees])

Laminate		Design variables	Pareto optimal solutions				
Shell	Beam		1st	2nd	3rd	4th	5th
1		$\bar{t}_{1,1}/\theta_{1,1}$	1.20/6	1.20/30	1.89/30	1.89/30	1.89/42
1		$\bar{t}_{2,1}/\theta_{2,1}$	1.20/18	1.20/18	1.20/66	1.20/66	2.06/78
1		$\bar{t}_{3,1}/\theta_{3,1}$	1.20/−42	1.20/−42	1.20/−42	1.89/−42	1.20/−42
2		$\bar{t}_{1,2}/\theta_{1,2}$	1.20/−6	1.20/−6	1.20/−6	1.20/−6	1.54/−6
2		$\bar{t}_{2,2}/\theta_{2,2}$	1.20/−30	1.20/−30	1.20/−30	1.20/−78	2.06/−30
2		$\bar{t}_{3,2}/\theta_{3,2}$	1.20/78	1.20/42	1.37/78	1.20/42	1.20/−54
3		$\bar{t}_{1,3}/\theta_{1,3}$	1.20/90	1.20/90	1.20/90	1.20/90	1.37/42
3		$\bar{t}_{2,3}/\theta_{2,3}$	1.20/−18	1.20/78	1.20/78	1.20/78	1.54/90
3		$\bar{t}_{3,3}/\theta_{3,3}$	1.20/−66	1.20/−66	1.20/−66	1.20/−66	1.20/−66
4		$\bar{t}_{1,4}/\theta_{1,4}$	1.20/90	1.20/−78	1.20/−78	1.20/90	1.20/6
4		$\bar{t}_{2,4}/\theta_{2,4}$	1.20/−66	1.20/−66	1.20/−66	1.20/−66	1.54/−54
4		$\bar{t}_{3,4}/\theta_{3,4}$	1.20/18	1.20/18	1.54/18	1.54/18	1.54/18
	5	h_1/w_1	25.7/10.7	25.7/13.6	37.1/10.7	37.1/12.1	37.1/12.1
	6	h_2/w_2	20.0/7.9	25.7/7.9	25.7/7.9	25.7/7.9	20.0/13.6
	7	h_3/w_3	28.6/7.9	28.6/7.9	40.0/7.9	28.6/9.3	40.0/13.6
	8	h_4/w_4	20.0/6.4	20.0/6.4	20.0/6.4	25.7/6.4	25.7/12.1
	9	h_5/w_5	34.3/5.0	40.0/5.0	40.0/10.7	40.0/13.6	31.4/5.0
	10	h_6/w_6	20.0/5.0	20.0/5.0	31.4/10.7	31.4/5.0	31.4/6.4
Objectives:		Weight [kg]	73.646	74.893	82.146	91.352	103.529
		Energy [J]	15.791	8.298	7.132	6.888	6.547
Individual age [generations]			3	25	105	366	69

the evolutionary process. The results are obtained at the end of each epoch after isolation stages of sub-populations POP1, POP2 and POP3. The global dominance measured in age structured population POP4 is used to trace the associated Pareto front. The performance of the proposed approach to search for Pareto front's solutions considering the multi-objective optimization problem can be observed.

Some Pareto-optimal solutions belonging to rank 1 for the proposed structural problem are presented in Tables 5 and 6. The solutions are captured at the 30th epoch of age structured population POP4. There are similarities among ply angle solutions for different composite laminates. Also it can be noticed that for solutions of the Pareto-optimal front corresponding to minimum weight (first and second solutions) most of the thickness design variables associated with shell laminates take minimum values from size constraints in (25). This means that variations in optimal values for the weight objective function depend on variables associated with stiffeners as considered in the composite structure.

Table 6 presents the description of the third segment of the chromosome for the five Pareto-optimal front solutions (rank 1) of Table 5. The optimal stacking sequence at laminate level and optimal laminate distribution at structure level are searched. The optimal laminate distribution in the structure is based on different laminates. In this example four laminates are considered for the composite structure. The stacking sequence of each symmetric shell laminate is given in closed brackets using the composite system numbering defined according to Table 3.

It is observed that all composite systems are used in the Pareto-optimal stacking sequence. However the composite system based on Carbon/Epoxy (CFRP: T300/N5208) and the composite system based on Kevlar/Epoxy (KFRP: Kev 49/epoxy) are the most frequently used in Pareto's optimal designs. Most of the solutions use the KFRP composite sys-

Table 6 Decoding results of the Pareto-optimal front solutions corresponding to the 3rd segment of each chromosome (ply material properties defined in Table 2)

Pareto-optimal solutions	Optimal stacking sequence at laminate level & optimal laminates distribution on structure						
	π_1	;	π_2	;	π_3	;	π_4
1st	$[1/1/4]_S$	;	$[1/1/4]_S$	;	$[1/1/4]_S$	;	$[1/1/4]_S$
2nd	$[1/1/4]_S$	;	$[1/1/4]_S$	;	$[2/1/4]_S$	;	$[1/2/4]_S$
3rd	$[1/1/4]_S$	;	$[1/1/4]_S$	;	$[1/1/4]_S$	;	$[1/2/4]_S$
4th	$[1/1/4]_S$	;	$[1/3/4]_S$	;	$[2/1/4]_S$	;	$[3/2/2]_S$
5th	$[1/1/4]_S$	;	$[1/3/4]_S$	;	$[2/1/4]_S$	;	$[3/2/2]_S$

tem at the inner ply of the laminate due to its light weight. Also the CFRP composite system is preferred for outer plies due to its higher strength and lower weight. This example shows the efficiency of MOHGA supported by adaptive rules and non-dominated sorting for the construction of the Pareto-optimal front.

The global dominance concepts applied to age-structured population POP4 at 4th epoch and 8th epoch produce a set of results shown in Fig. 8. The rank-based dominance definitions presented in Section 2.1 are shown for non-dominated individuals/solutions of rank 1 and dominated solutions from rank 2 until rank 4. An improvement is observed from epoch 4 to epoch 8 in ranked solutions. The minimization of both objectives drives the ranked solutions toward the left and lower corner of the graph. The differences of objective values associated with different rank positions of solutions based on dominance concepts are minimized at the 8th epoch as shown in Fig. 8. This improvement in quality of partial objectives as weight and strain energy aiming the multi-objective minimization is observed within the first epochs of the evolutionary search.

A measure of quality for the evolutionary process is the number of solutions belonging to Pareto's front inside age-structured population POP4 denoted by rank 1 solutions. As previously referred, all non-dominated individuals (rank 1) will survive after reaching the lethal age while keeping their status of global dominance. This means that the percentage of those individuals in POP4 is associated with the success of building the global Pareto front. Also the percentage of individuals with lower rank number (rank<4) indicates the quality of solutions in POP4. The behaviours of both referred indicators are represented in Fig. 9. The percentage of rank 1 solutions is over 6 % after the 16th epoch. The percentage of individuals or solutions with rank less than 4 is kept above 8 % after the 7th epoch of the evolutionary process. There is an oscillation in percentage of rank 1 solutions that is compensated by an increase of dominated solutions of rank<4. This last group determines good candidates for the Pareto front when the crossover and mutation operators are applied.

After the 2nd epoch the number of individuals at age-structured population POP4 is between 293 and 340. This number is controlled by lethal age and dominance as previously referred in Section 3.5. Figure 10 shows the age of rank 1 individuals belonging to POP4 for different times of evolutionary search. Most of rank 1 individuals or solutions at 4th epoch and 8th epoch are below the dotted line corresponding to lethal age. The number of rank 1 individuals above lethal age is increasing with evolutionary search. Also the longevity of those individuals is increasing. The number and the longevity of old rank 1 individuals in age-structured population are important to build the global Pareto front. Both aspects reflect the quality of the solutions produced by the proposed approach of MOHGA based on local and global dominance concepts.

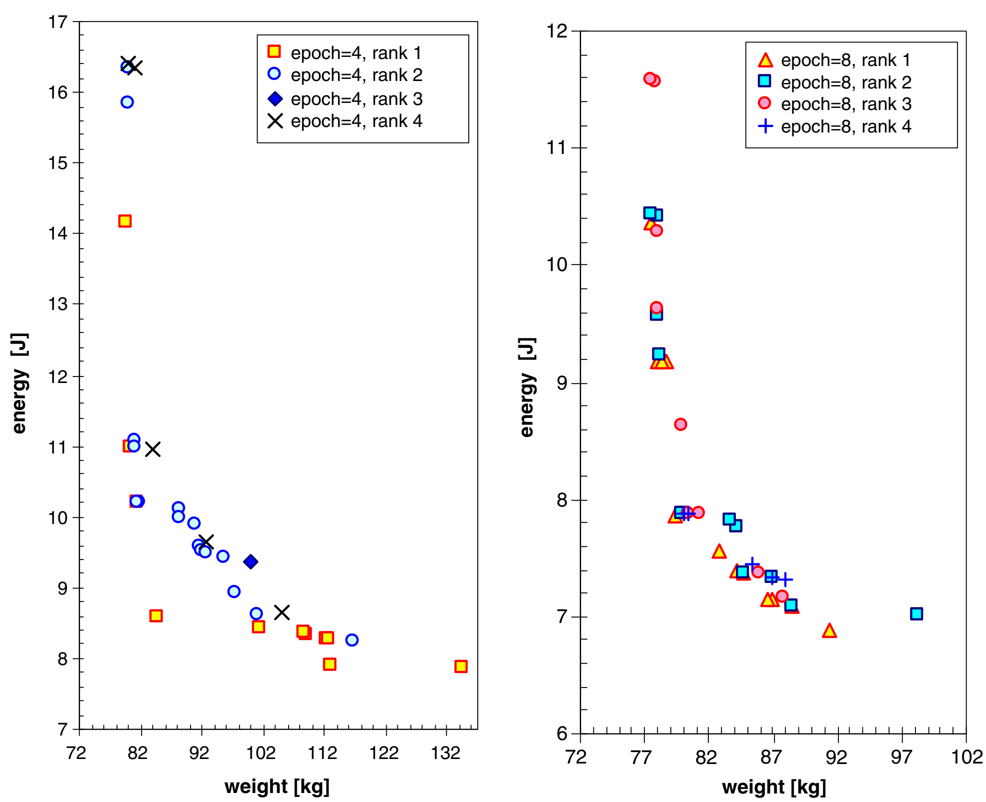

Fig. 8 Ranking of solutions of POP4 at two times of MOHGA evolution

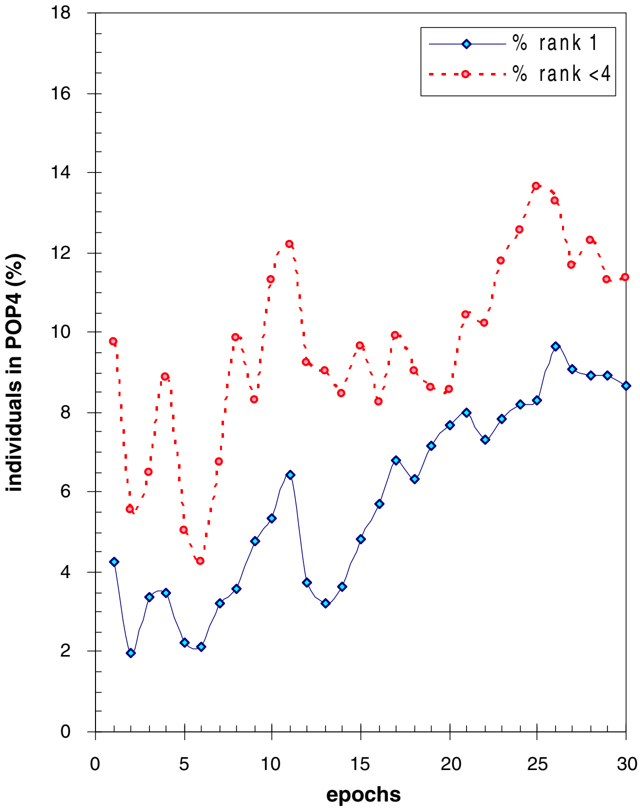

Fig. 9 Percentage of best solutions (rank <4) in POP4 during evolution process

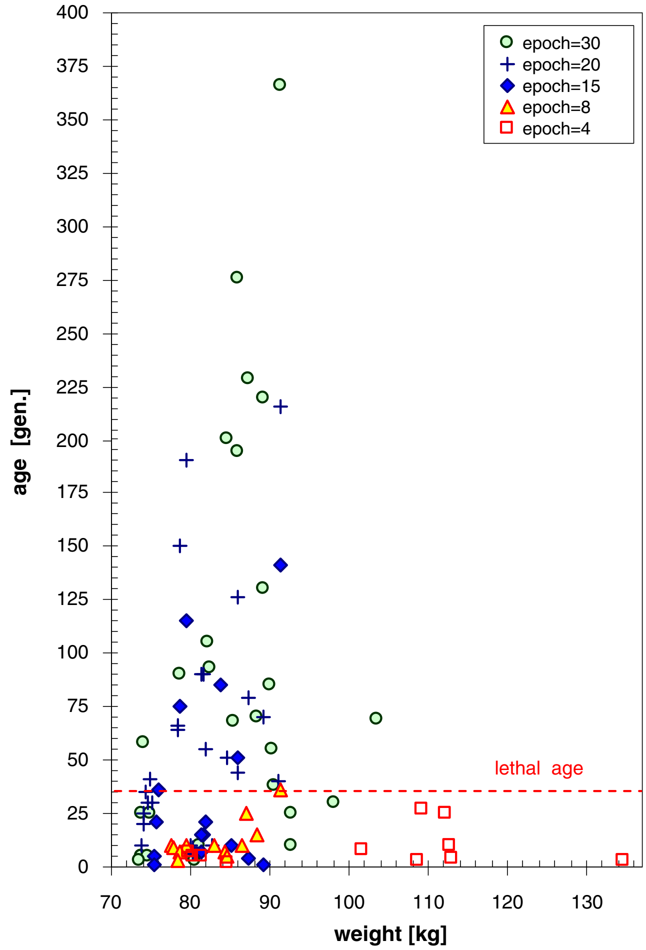

Fig. 10 Age of individuals of Pareto front (rank 1) in POP4 for different times of MOHGA evolution

Figure 11 shows the age of individuals/solutions belonging to Pareto front (rank 1) of age-structured population POP4 at the end of evolutionary process. Some of those solutions are presented in Tables 5 and 6 (age in rectangular frame). The inner picture shows different individual age classes in generations. The percentage of individuals having age upper than lethal age is equal to 60 %. This high percentage of rank 1 individuals into POP4 identifies the maturation of the population. However, with further evolution, there is some probability to appear new rank 1 individuals since six individuals are generated during the last isolation stage in POP3.

4.3 Influence of hierarchical organization and local dominance

To illustrate the local dominance evaluation at isolation stage for different sub-populations considered in the proposed hierarchical approach some results of the fitness assignment procedure are presented. Table 7 shows the results of MOHGA fitness assignment procedure described in Section 3.4 and based on local dominance. The results correspond to the last generation of 30th epoch in isolation stage at sub-population POP3. The non-dominated sorting definition of Section 3.3 has been previously applied to all solutions into each population and ranked according to its local dominance. In rank column it can be seen that some ranks have no associated solutions. The last nine solutions do not satisfy the constraints in problem formulation presented in (8) and (9).

The use of an elitist strategy at isolation stage of POP1, POP2 and POP3 sub-populations guarantees that rank 1 solution group drives the evolutionary process. In general it is observed that for POP1 the size of the elite group is larger than rank 1 group. Differently for POP2 the size of rank 1 group is larger than the elite group and for POP3 the size of both groups is equal. The enlarged age-structured population POP4 has 2–4 times more rank 1 individuals/solutions than other sub-populations of the hierarchical proposed approach. This can be observed in Fig. 12 at two different epochs of the evolutionary process. In particular, the identification of individuals in Pareto fronts for POP3 based on local dominance and for POP4 based on global dominance is presented.

Considering the size of rank 1 group in different environments it can be concluded that the search based on

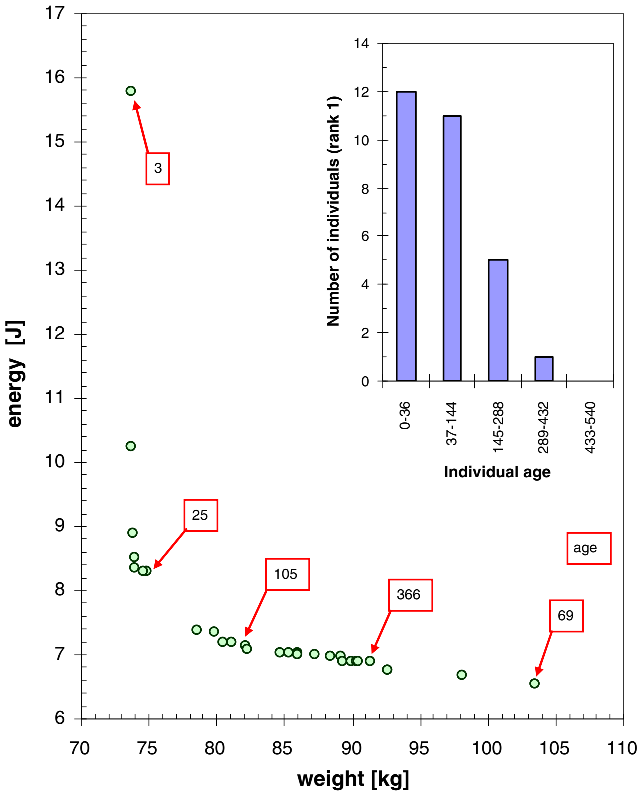

Fig. 11 Age of individuals belonging to Pareto front in POP4 at 30th epoch

local dominance used in sub-populations exhibit exploitative properties while the behaviour of age-structured population based on global dominance is more explorative. The hierarchical structure of MOHGA improves the synergies of exploitation and exploration properties that are beneficial for the evolutionary process and effective in optimal Pareto front search.

4.4 Diversity analysis of age-structured population

In order to preserve the diversity of Pareto optimality a population entropy control is proposed. Entropy definition is based on information theory as suggested by Chun et al. (1997). Being M the size of the population, the entropy of the *j-th* gene is defined as:

$$S_j^M = \sum_{i=1}^{M} \sum_{k=i+1}^{M} -P_{ik} \log (P_{ik}) \tag{26}$$

where P_{ik} is the probability of phenotype value z_{ij} of the *j-th* gene of the *i-th* chromosome to be different from value z_{kj} of the *j-th* gene of the *k-th* chromosome and is calculated as follows,

$$P_{ik} = 1 - \frac{|z_{ij} - z_{kj}|}{B_j - A_j} \tag{27}$$

being $[A_j, B_j]$ the range of phenotype values for the *j-th* gene. The average entropy S^M of the population is equal to the average of entropies of different genes and is defined as:

$$S^M = \frac{1}{n} \sum_{j=1}^{n} S_j^M \tag{28}$$

The diversity of the population can be associated with S^M denoted here by *population entropy*. Therefore the higher the variability of the chromosomes, the higher the entropy of the population and the better is its quality. Further, the quality of a population can be established in terms of convergence of the evolutionary process towards the global optimal solution.

Since the age-structured population POP4 is formed by individuals generated as “new”, a good measure of diversity is the entropy of that population (Conceição António 2009a, b). Figure 13 shows the entropy changes of the age-structured population POP4 along the evolutionary process. The entropy of the population increases until the 3rd epoch and is kept relatively high during the next three epochs. From the 7th epoch until the 12th epoch the entropy values decrease. A slightly recovery in entropy occurs during the next ten epochs. The entropy stays above a threshold at the final epochs of the evolutionary process.

The size variation of POP4 is controlled by lethal age and rank 1 group size. Furthermore, both the imposed threshold and the updated access constraint as referred in Section 3.1 avoid storing a large number of unfeasible solutions in POP4. To consider the size of age-structured POP4 on the entropy evaluation the *specific entropy* as follows is considered

$$\overline{S}^M = \frac{S^M}{M} \tag{29}$$

being M the size of POP4. Figure 13 shows the behaviour of this last diversity measure having a similar behaviour as S^M. Also the specific entropy kept high threshold at the final epochs of the evolutionary process. This means that the diversity of populations is guaranteed in the multi-objective optimization performed by MOHGA.

4.5 Validation of results and decision-making design rules

In order to validate the results obtained from MOHGA a comparison with Non-dominated Sorting Genetic Algorithm (NSGA) proposed by Deb (2001) is established in the same multi-objective optimization framework. The implementation of NSGA algorithm is based on dominance concepts established in Section 3.3 and on the fitness assignment procedure described in Section 3.4. According to Deb (2001) developments the NSGA does not use the hierarchical topology, the self-adaptive procedures and the species conservation principles. Also the individual age is

Table 7 Fitness assignment procedure at the last generation of isolation stage of POP3 corresponding to the end of 30th epoch (* with constraints violation)

Solution	Weight [kg]	Energy [J]	Rank	Average fitness	Niche count	Shared fitness	Scaled fitness
1	103.5286	6.5473	1	23.0000	5.3519	4.2975	29.7974
2	73.6463	15.7910	1	23.0000	6.2241	3.6953	25.6220
3	92.6707	6.7439	1	23.0000	6.6079	3.4807	24.1339
4	73.7239	10.2460	1	23.0000	7.3946	3.1104	21.5662
5	81.0937	7.1299	1	23.0000	7.4930	3.0695	21.2832
6	80.4273	7.1986	1	23.0000	7.5207	3.0582	21.2047
7	73.8705	8.8959	1	23.0000	7.5371	3.0516	21.1584
8	73.9113	8.5033	1	23.0000	7.5404	3.0503	21.1494
9	74.5521	8.3005	1	23.0000	7.5634	3.0409	21.0849
10	81.1089	7.1944	2	18.0000	1.0000	18.0000	18.0000
11	107.1201	6.9644	3	17.0000	1.0000	17.0000	17.0000
12	75.4562	14.3460	5	15.0000	2.0980	7.1496	16.5980
13	100.6693	8.0166	5	15.0000	2.4116	6.2199	14.4396
14	95.0681	8.0351	5	15.0000	2.4940	6.0143	13.9624
15	90.6556	9.3422	7	13.0000	1.0000	13.0000	13.0000
16	90.6298	19.6210	10	12.0000	1.0000	12.0000	12.0000
17	97.4469	16.5030	13	11.0000	1.0000	11.0000	11.0000
18	112.2460	21.0280	18	10.0000	1.0000	10.0000	10.0000
19*	112.5514	9.5026	19	9.0000	1.0000	9.0000	9.0000
20*	99.6246	7.2828	20	8.0000	1.0000	8.0000	8.0000
21*	82.0200	5.8952	21	7.0000	1.0000	7.0000	7.0000
22*	89.0116	5.0019	22	6.0000	1.0000	6.0000	6.0000
23*	80.9253	5.6113	23	5.0000	1.0000	5.0000	5.0000
24*	85.6814	4.3935	24	4.0000	1.0000	4.0000	4.0000
25*	84.8893	3.4606	25	3.0000	1.0000	3.0000	3.0000
26*	85.3015	2.9784	26	2.0000	1.0000	2.0000	2.0000
27*	85.5905	3.0617	27	1.0000	1.0000	1.0000	1.0000

merely used for information and comparative analysis with MOHGA.

In NSGA implementation the age structured population does not intervene in the evolution being used only to store the non-dominated solutions. The crossover operator used in NSGA search is the *Elitist parameterized Uniform Crossover* (EpUC) with the properties defined in Table 1. The mutation is based on two operators, the *Controlled* Mutation and the *Implicit Mutation* performed in an alternative way as referred in Section 3.2.

Aiming to use similar conditions for the problem solved by MOHGA, only an evolving population with 27 individuals is considered. The genetic parameters of Table 4 corresponding to sub-population POP1 are used in NSGA. This means that all design variables are addressed simultaneously in the evolutionary process. A time evolution of 540 generations is considered for NSGA. This time evolution is equivalent to 30 epochs times 18 generations per epoch used in MOHGA. This way, the same number of fitness evaluations is used in both algorithms.

The comparison of results between NSGA and MOHGA is made in Fig. 14 where both Pareto fronts are plotted. From this figure it can be concluded that the proposed MOHGA to obtain the Pareto-optimal front is efficient. The number of non-dominated solutions obtained using MOHGA is larger than the number obtained using NSGA as shown by Fig. 14. The number of solutions with lower age than lethal age (35 generations) is larger in MOHGA than in NSGA. This emphasises the highest capacity of MOHGA to improve its results.

The optimization problem formulated in this study has design variables $\mathbf{x}$ and $\boldsymbol{\pi}$ associated with sizing and material distribution, respectively, as defined in Section 2.2. Although the use a discrete binary code format, the design variables $\mathbf{x}$ correspond to a continuous space design originally. On other hand the distribution of material represented by variable vector $\boldsymbol{\pi}$ is associated with a discrete domain encoding the composite systems with possible choice for optimal design. From previous considerations the optimization problem has mixed variables and so the feasible domain

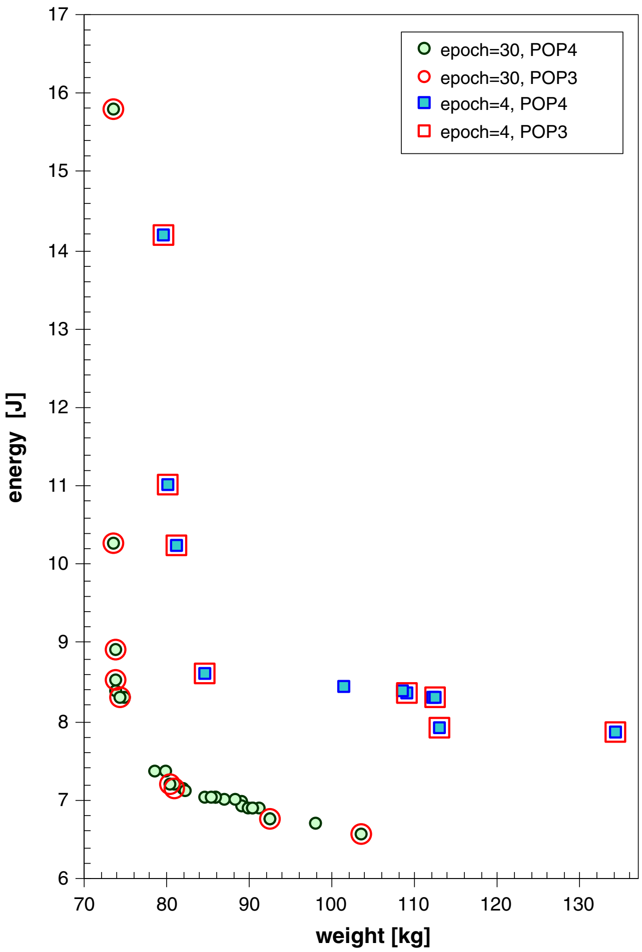

Fig. 12 Identification of individuals in Pareto fronts for POP3 (local dominance) and for POP4 (global dominance) at two different epochs of the evolutionary process

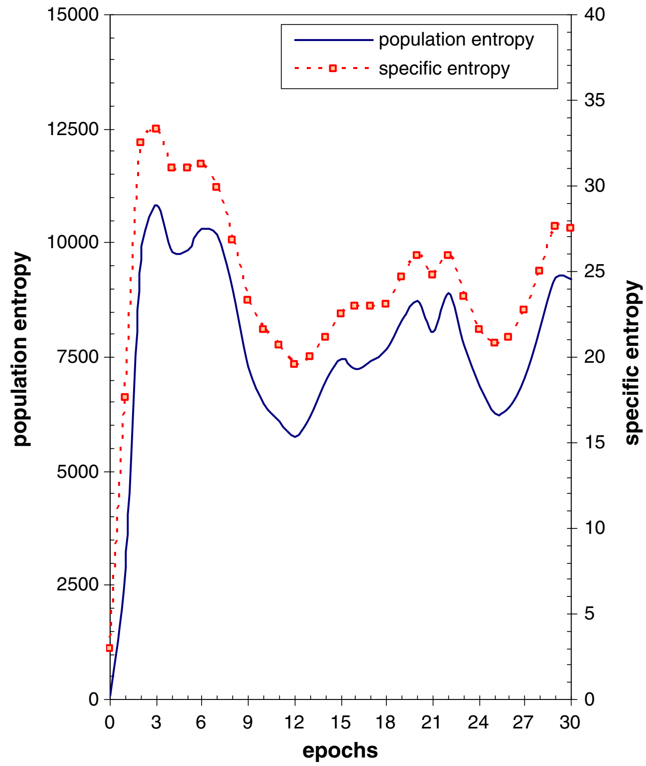

Fig. 13 Diversity of population POP4 measured by entropy

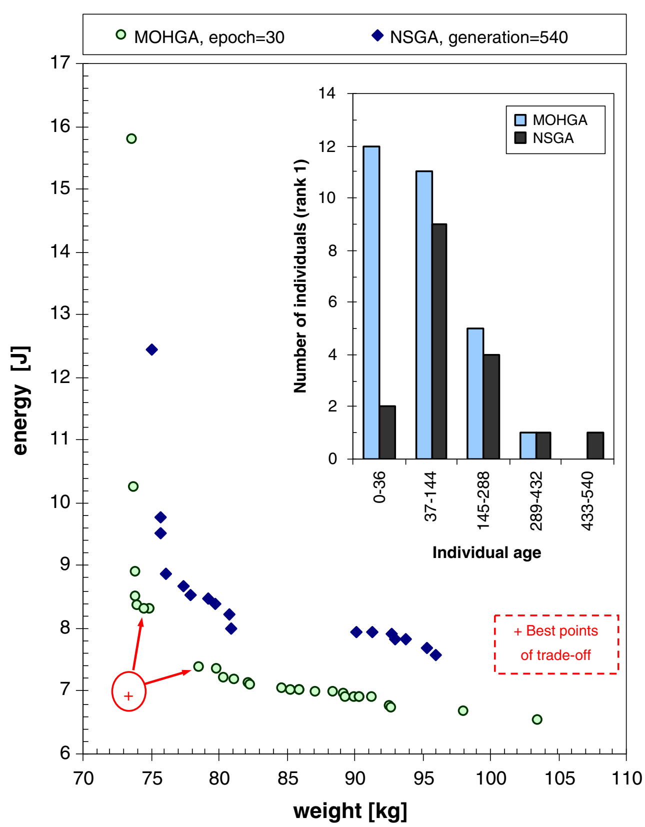

Fig. 14 Comparison of Pareto front and age of solutions using MOHGA and NSGA

includes holes or discontinued regions representing the discrete coding (Tang et al. 2010). As referred by Deb (2001) the sharing function method is used in NSGA with the purpose to make sure that less crowded regions in a front are appropriately explored. Also the same methodology is used by MOHGA fitness assignment procedure described in Section 3.4. Nevertheless, Fig. 14 exhibits portions of Pareto-optimal fronts with absence of solutions what might be associated with the use of mixed variables. However the proposed MOHGA shows better results in construction of Pareto-optimal front than NSGA.

The application of MOHGA to hybrid composite structures requiring the compromise between minimum strain energy and minimum weight is analysed aiming to establish design rules. The trade-off between weight, depending on given stress, displacement and buckling constraints imposed on composite structures, against minimum strain energy, is searched.

The solutions of the Pareto-optimal front obtained by MOHGA are equally valid from the optimal design

Table 8 Best points of mathematical trade-off from Pareto-optimal front

Pareto-optimal solution	Design variables (1st and 2nd segment) $[\theta_{1,1}/\cdots/\theta_{3,1}]_S;\cdots;[\theta_{3,4}/\cdots/\theta_{3,4}]_S$ [°]	$[\bar{t}_{1,1}/\cdots/\bar{t}_{3,1}]_S;\cdots;[\bar{t}_{3,4}/\cdots/\bar{t}_{3,4}]_S[h_1/w_1];\cdots;[h_6/w_6]$ [mm]
1st	$[30/66/-42]_S$; $[-6/-78/42]_S$; $[90/78/-66]_S$; $[90/-66/18]_S$	$[1.5/1.2/1.2]_S$; $[1.2/1.2/1.2]_S$; $[1.2/1.2/1.2]_S$; $[1.2/1.2/1.2]_S$ [37.1/15.0] ; [25.7/7.9] ; [40.0/9.3] ; [25.7/6.4] ; [40.0/13.6] ; [31.4/5.0]
2nd	$[30/18/-42]_S$; $[-6/-30/42]_S$; $[90/78/-66]_S$; $[-78/-66/18]_S$	$[1.2/1.2/1.2]_S$; $[1.2/1.2/1.2]_S$; $[1.2/1.2/1.2]_S$; $[1.2/1.2/1.2]_S$ [25.7/13.6] ; [25.7/7.9] ; [28.6/7.9] ; [20.0/6.4] ; [40.0/5.0] ; [20.0/5.0]

Pareto-optimal solution	3rd segment of design variables							Objectives		Distance to utopia point	Individual age [generations]
	π_1	;	π_2	;	π_3	;	π_4	Weight [kg]	Energy [J]		
1st	$[1/1/4]_S$	;	$[1/1/4]_S$	;	$[1/1/4]_S$	;	$[1/2/4]_S$	78.571	73.663	0.891	90
2nd	$[1/1/4]_S$	;	$[1/1/4]_S$	;	$[2/1/4]_S$	;	$[1/1/4]_S$	74.552	83.005	0.892	5

point of view. Furthermore, a detailed analysis of solutions/individuals in Pareto-optimal front reveals that they belong to fourteen different species. From this it can be concluded that MOHGA successfully preserves the population diversity. Furthermore, MOHGA is capable to indicate alternative optimal designs based on different species as shown in Tables 5 and 6. This last consideration is very important for design applications.

Finally, a mathematical trade-off can be established using the Pareto-optimal front results. Normalizing the objective values by using the maximum of each objective function it is obtained the Pareto normalized front. The point on the Pareto normalized front associated with the minimum distance can be defined as the best mathematical trade-off between weight and strain energy (António and Hoffbauer 2009). In the present case study two solutions are associated with minimum distance. The corresponding optimal design values are described in Table 8. The two solutions belong to different species and so represent alternative designs in terms of material distribution on composite structure.

5 Conclusions

A multi-objective hierarchical genetic algorithm denoted by MOHGA with age structure and based on local and global dominance concepts applied to multi-objective optimization of composite structures is proposed. The approach based on multi-populations evolution uses the species conservation technique to address the optimal stacking sequence and material distribution on composite structures in multi-objective optimization problems. Thus, individuals corresponding to the same material selection and topology of the hybrid composite structure belong to the same species. The material distribution of hybrid composite structures is performed at two levels: laminate level and structural topology level.

A structural robust design problem that simultaneously considers minimum weight/cost and minimum strain energy related to maximum performance is presented. The trade-off between the performance target, depending on given stress, displacement and buckling constraints imposed on composite structures, against robustness, is searched. The global Pareto-optimal front is built at age structured population using the concept of Pareto dominance. The concept of local dominance and a sharing function in order to assign scalar fitness values to individuals is used at isolation stages of sub-populations. Such a challenge calls for a multi-objective optimization and is performed here using the proposed hierarchical genetic algorithm with co-evolution of multi-populations. Self-adaptive rules are incorporated in Pareto front design based on genetic search. The search method adopts an elitist strategy storing non-dominated solutions found during the evolutionary process in an age-structured population.

Results show that MOHGA is promising dealing with multi-objective optimization of hybrid composite structures. From the numerical results some important conclusions can be presented:

– The improvement in quality of partial objectives as weight and strain energy aiming the multi-objective minimization is observed within the first epochs of the evolutionary search.

- The number and the longevity of non-dominated individuals in age-structured population are important to build the global Pareto front.
- The high percentage of non-dominated individuals identifies the maturation of the population. However, with further evolution, there is some probability to appear new non-dominated individuals.
- The use of an elitist strategy at isolation stage of sub-populations guarantees that a non-dominated group of individuals drives the evolutionary process.
- Search based on local dominance used in sub-populations exhibits exploitative properties while the behaviour of age-structured population based on global dominance is more explorative.
- The specific entropy kept high threshold at the final epochs of the evolutionary process. This means that the diversity of populations is guaranteed in the multi-objective optimization performed by MOHGA.
- In order to validate the results obtained from MOHGA a comparison with Non-dominated Sorting Genetic Algorithm (NSGA) proposed by (Deb 2001) is established in the same multi-objective optimization framework. The number of non-dominated solutions obtained using MOHGA is larger than the number obtained using NSGA. The ability to refresh the population with non-dominated individuals is better in MOHGA than in NSGA.
- Both approaches NSGA and NOHGA exhibit portions of Pareto-optimal fronts with absence of solutions being this associated with the use of mixed variables. However the proposed MOHGA shows better results in the construction of the Pareto-optimal front than NSGA.
- A detailed analysis of solutions/individuals at the Pareto-optimal front reveals that they belong to fourteen different species. From this it can be concluded that MOHGA is successful in preserving the population diversity. Furthermore, MOHGA is able to indicate alternative optimal designs based on different species what might be very important for the designers.
- The point on the Pareto normalized front associated with the minimum distance can be defined as the best mathematical trade-off between weight and strain energy.

The hierarchical structure of MOHGA based on local and global dominance concepts improves the synergies of exploitation and exploration properties that are beneficial for the evolutionary process and efficient in Pareto-optimal front determination.

Acknowledgment The authors acknowledge the financial support provided by the Foundation for Science and Technology (FCT), Portugal, through the strategic project PEst-OE/EME/UI0225/2011.

References

António CC, Hoffbauer LN (2009) An approach for reliability-based robust design optimisation of angle-ply composites. Compos Struct 90:53–59

Awad ZK, Aravinthan T, Zhuge Y, Gonzalez F (2012) A review of optimization techniques used in the design of fibre composite structures for civil engineering applications. Mater Des 33: 534–544

Baier H, Huber M, Langer H (2008) Design optimization of hybrid material structures. Struct Multidisc Optim 36:203–213

Budiansky B (1974) Theory of buckling and post-buckling of elastic structures. Adv Appl Mech 14:2–63

Chun J-S, Kim M-K, Jung H-K (1997) Shape optimization of electromagnetic devices using Immune Algorithm. IEEE Trans Magnet 33:1876–1879

Conceição António CA (2002) A multilevel genetic algorithm for optimization of geometrically non-linear stiffened composite structures. Struct Multidisc Optim 24:372–386

Conceição António CA (2006) A hierarchical genetic algorithm with age structure for multimodal optimal design of hybrid composites. Struct Multidisc Optim 31:280–294

Conceição António C (2009a) Self-adaptation procedures in genetic algorithms applied to the optimal design of composite structures. Int J Mech Mater Des 5:289–302

Conceição António CA (2009b) A study on synergy of multiple crossover operators in a hierarchical genetic algorithm applied to structural optimization. Struct Multidisc Optim 38:117–135

Conceição António CA, Torres Marques A, Soeiro AV (1995) Optimization of laminated composite structures using a bilevel strategy. Compos Struct 33:193–200

Conceição António CA, Trigo Barbosa J, Simas Dinis L (2000) Optimal design of beam reinforced composite structures under elasto-plastic loading conditions. Struct Multidiscipl Optim 19:50–63

Crisfield MA (1997) Non-linear finite element analysis of solids and structures, vol 2. Wiley, Chichester

Deb K (2001) Multi-objective optimization using evolutionary algorithms. Wiley, Chichester

Fonseca CM, Fleming PJ (1993) Genetic algorithms for multi-objective optimization: formulation, discussion and generalization. In: Forrest S (ed) Proceedings of the fifth international conference on genetic algorithms, San Mateo, California, University of Illinois at Urbana-Champaign. Morgan Kauffman Publishers, San Francisco, pp 416–423

Ghiasi H, Pasini D, Lessard L (2010) Pareto frontier for simultaneous structural and manufacturing optimization of composite part. Struct Multidisc Optim 40:497–511

Irisarri F-X, Laurin F, Leroy FH, Maire J-F (2011) Computational strategy for multiobjective optimization of composite stiffened panels. Compos Struct 93:1158–1167

Kassapoglou C, Dobyns AL (2001) Simultaneous cost and weight minimization of postbuckled composite panels under combined compression and shear. Struct Multidisc Optim 21:372–382

Li M, Kou J (2008) Crowding with nearest neighbours replacement for multiple species niching and building blocks preservation in binary multimodal functions optimization. J Heurist 14: 243–270

Murugan MS, Suresh S, Ganguli R, Mani V (2007) Target vector optimization of composite box beam using real-coded genetic algorithm: a decomposition approach. Struct Multidisc Optim 33:131–146

Pelletier JL, Vel SS (2006) Multi-objective optimization of fiber reinforced composite laminates for strength, stiffness and minimal mass. Comput Struct 84:2065–2080

Rahul Sandeep, G, Chakraborty D, Dutta A (2006) Multi-objective optimization of hybrid laminates subjected to transverse impact. Compos Struct 73:360–369

Siarry P, Pétrowski A, Bessaou M (2002) A multipopulation genetic algorithm aimed at multimodal optimization. Adv Eng Softw 33:207–213

Spears WM, DeJong SK (1991) On the virtues of parameterized uniform crossover. In: Belew RK, Booker LB (eds) Proceedings of the fourth international conference on genetic algorithms. Morgan Kaufmann, San Mateo, pp 230–236

Tang XG, Bassir DH, Zhang WH (2010) Shape, sizing optimization and material selection based on mixed variables and genetic algorithm. Optim Eng 12:111–128

Tsai SW (1987) Composites design. Think composites. Dayton, USA

Wang K, Kelly D, Dutton S (2002) Multi-objective optimisation of composite aerospace structures. Compos Struct 57:141–148