

# Design optimization of supersonic jet pumps using high fidelity flow analysis

James Eves · Vassili V. Toropov · Harvey M. Thompson ·  
Nik Kapur · Jian Fan · Daniel Copley · Adrian Mincher

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**Abstract** Supersonic jet pumps are simple devices with no moving parts, where a high velocity (primary) flow is used to pump a second fluid. In this paper, Computational Fluid Dynamics (CFD) is combined with an optimization framework in order to develop a tool for the rapid generation of jet pump designs. A key feature of the problem formulation is the transformation of the jet pump design parameters in terms of geometric ratios. This approach dramatically reduces the number of unrealistic designs covered by the Design of Experiments. Optimal Latin Hypercubes for surrogate model building and model validation points are constructed using a permutation genetic algorithm and

design points are evaluated using CFD. Surrogate models of primary and entrained flow rates are built using a Moving Least Squares approach. A series of optimizations for various pump sizes are performed using a genetic algorithm and Sequential Quadratic Programming, with responses calculated from the surrogates. This approach results in a set of optimized designs, from which pumps for a wide range of flow rates can be interpolated.

**Keywords** Jet pump · CFD optimization · Shape optimization · Surrogate model

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J. Eves · V. V. Toropov · H. M. Thompson · N. Kapur  
School of Mechanical Engineering, University of Leeds, Leeds,  
West Yorkshire, LS2 9JT, UK

J. Eves  
e-mail: j.s.v.eves@leeds.ac.uk

V. V. Toropov (✉)  
School of Civil Engineering, University of Leeds, Leeds,  
West Yorkshire, LS2 9JT, UK  
e-mail: v.v.toropov@leeds.ac.uk

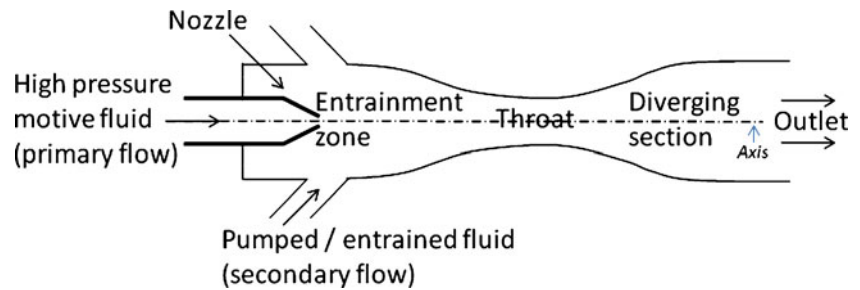
J. Fan  
Northwestern Polytechnical University, Xi'an, Shaanxi, 710072,  
People's Republic of China

D. Copley · A. Mincher  
Parker-Hannifin, Dewsbury, West Yorkshire, WF12 7RD, UK

## 1 Introduction

Supersonic jet pumps are devices that can pump and compress flow without the need for moving parts. Traditional jet pump designs (illustrated in Fig. 1) consist of a high velocity jet (primary or motive flow) that entrains a secondary (pumped) flow. Designs of jet pumps suggest 3 main regions—(i) a converging section known as the entrainment region or zone, (ii) a parallel throat section and finally (iii) a diverging section known as the diffuser. Within the converging section the high velocity jet issuing from a nozzle entrains the secondary flow—the energy for the pumping action of the secondary flow is scavenged from the high velocity primary flow. Mixing of these flows occurs in the throat and in the divergent section the static pressure is raised to equal that downstream. Although originally developed in the 19th century for use in steam engines the use of these devices is again attracting attention as, due to their lack of moving parts, they can be “fit for life”. This makes them an environmentally friendly component in a variety of applications such as refrigeration (Yu et al. 2006) and desalination (Kumar et al. 2007). They are also seen as a

**Fig. 1** Schematic showing the principles of operation of a typical jet pump



highly cost effective method for maintaining pressures in oil reservoirs (Sarshar and Beg 1998).

A key issue facing the designers of jet pumps is the onset of thermodynamic shocks in the throat section, causing severe losses in efficiency and pressure lift. To overcome this, some modern jet pump designs depart from the conventional configuration described earlier. Analytical approaches using one-dimensional compressible flow theory exist for determining a smooth diffuser profile to prevent these shocks (Eames 2002). However these approaches have been shown to lack the fidelity needed to model the complexity of flows within these pumps, which involve thermodynamic shock interactions and turbulent mixing of the primary and entrained flows (Fan et al. 2011). Because of this, Computational Fluid Dynamics (CFD) has become the most common tool for analyzing jet pumps (He et al. 2009).

Although CFD generally offers higher accuracy than analytical methods there is continued debate about how turbulence should be modeled to improve accuracy further. Several models, such as the RNG  $k-\varepsilon$  and SST  $k-\omega$ , have been analyzed in the context of jet pumps (Bartosiewicz et al. 2006) and the choice of turbulence model has been shown to effect local flow structure (Hemidi et al. 2009). However recent validation of numerical models using experimental data has shown that the standard  $k-\varepsilon$  model is capable of predicting global performance responses such as pressure lift and entrainment ratios with accuracies comparable to the more complex Reynolds-Stress-Model.

Several approaches for improving jet pump efficiencies using high fidelity flow analysis have been proposed. These include studies of operating conditions (Wang and Dong 2010) or, more commonly, the jet pump geometry. Investigations into improved jet pump designs range from simple parameter studies (Prabkeao and Aoki 2005) to formal optimization frameworks (Helali and Kueny 2008), with many of these studies reporting significant improvements in terms of entrainment ratios and pressure lift over conventional designs. Studies into the effect of the nozzles used in jet pumps have also been published. These include the use of elliptical nozzles (Guillaume and Judge 1999) and multiple nozzles (Narabayashi et al. 2006).

This paper is concerned with the development of surrogate models of jet pump performance based on key geometry parameters. Surrogate models can be built using CFD analysis of a series of pump designs, and once built give simple approximations of entrained flow rates and pressure lift. Using surrogate models allows computationally inexpensive optimization of jet pump designs, without the need for additional CFD analyses. A very recent study has considered the problem of jet pump optimization for a given set of operating conditions (Fan et al. 2011), however the present study focuses on industrial applications where the pumps are part of a larger system, and design requirements are subject to change. The intention here is to build surrogate models for a wider range of operating conditions, which allows the rapid optimization of pump designs to meet these ever changing requirements.

## 2 Optimization strategy

Initial jet pump designs are produced using the analytical and semi-empirical methods proposed by Eames (2002). A constant rate of change of momentum is prescribed within the diffuser to prevent thermodynamic shocks occurring. Flow variables are then calculated based on this assumption using 1D compressible flow theory and the required diffuser profile to achieve this flow can be determined. High fidelity analysis of the jet pump is then performed using the steady-state Navier-Stokes equations for 2D axisymmetric flow. Second order upwind schemes are used for all flow variables and solutions are computed using the SIMPLE (Patankar and Spalding 1972) algorithm. Following Hemidi et al. (2009), turbulence is modeled using the standard  $k-\varepsilon$  transport model.

A surrogate modeling approach was adopted for the optimization study. Design of experiments (DoE) is carried out using a nested Optimal Latin Hypercube containing build and validation points. This is achieved via a permutation genetic algorithm (Narayanan et al. 2007), applied to the multi-objective problem of optimizing the uniformity of the model building and validation points, and the combined DoEs. For each DoE the Audze-Eglais optimality criterion

(Audze and Eglais 1977) shown in Eq. 1 is used resulting in the objective function defined by Eq. 2.

$$U = \sum_{p=1}^p \sum_{q=p+1}^q \frac{1}{l_{pq}} \tag{1}$$

$$F = W_b U_b + W_v U_v + W_c U_c \tag{2}$$

where  $U$  is a pseudo-potential energy of DoE points,  $l_{pq}$  is the distance between points  $p$  and  $q$  where  $p \neq q$ ,  $F$  is the objective function to be minimized,  $W$  are weighting factors, and  $b$ ,  $v$  and  $c$  denote model building, model validation and combined DoEs respectively.

Surrogate models were built using a Moving-Least-Squares (MLS) method where the weighting of points in the regression coefficients calculation are determined using a Gaussian decay function:

$$w_i = \exp\left(-\theta r_i^2\right) \tag{3}$$

where  $w$  is the weighting of the DoE build point  $i$ ,  $r_i$  is the normalized distance from the current point to model building point  $i$ , and  $\theta$  is a closeness-of-fit parameter. This parameter is optimized to minimize the  $R^2$  value for the obtained surrogate model, as calculated on the validation DoE. The surrogate is then rebuilt using the combined building and validation DoEs. Global optimization is performed using a genetic algorithm (GA) with responses calculated using the surrogate models. The optimized design variables from the GA are then fine-tuned using a Sequential Quadratic Programming method to ensure an optimum has been reached.

### 3 Optimization problem formulation

In earlier work (Fan et al. 2011) the shape and size of a jet pump were optimized to minimize the primary flow rate

while achieving an entrained flow rate of 200 L/min and a pressure lift of 132.5 mbar. The radius of the diffuser as a function of the axial co-ordinate was re-defined as:

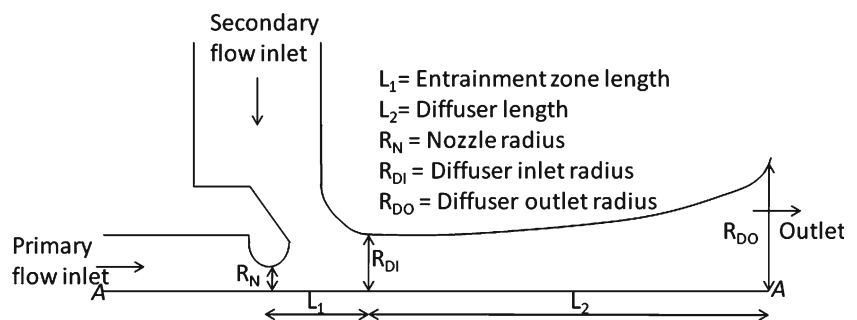
$$R(x) = a_1 \tanh\left(a_2 \frac{x}{L_2} - a_3\right) + a_4 \tag{4}$$

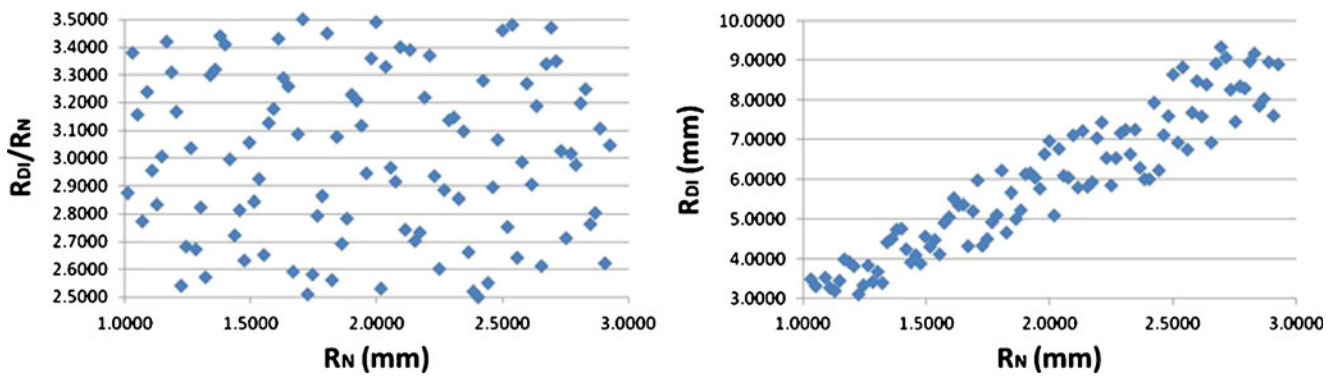
where  $R$  is the diffuser radius,  $L_2$  is the diffuser length.  $x = 0$  at the diffuser inlet shown with radius  $R_{DI}$  in Fig. 2, and  $a = [7.5, 3.5, 3.5, 10.6]$ .

A potential application of jet pump technology has been identified where a range of pumps are required, each capable of entraining a different volume of air. The required flow rates range between 200 L/min and 1,200 L/min. All of these pumps must generate 150 mbar of pressure lift with 2 bar of pressure at the primary flow inlet. The aim of the current study is to develop a design tool that will allow the rapid optimization of designs for this wide range of entrained flow rates with low computational effort. To achieve this the jet pump design was parameterized into as few design variables as possible. A previous optimization study for a 600 L/min pump (Fan et al. 2011) resulted in the shape of the diffuser remaining relatively constant and just the diffuser radius increasing. For this reason the shape of the pump was maintained and only the size was optimized, i.e.  $a_2$  and  $a_3$  were fixed and  $a_1$  and  $a_4$  were allowed to vary. Furthermore, Fan et al. (2011) showed pressure lift to be quite insensitive to the length of the diffuser and entrainment zone, shown as  $L_2$  and  $L_1$  in Fig. 2 respectively, so these were not included as design variables. The jet pump is therefore described by only three variables, the nozzle radius  $R_N$ , the diffuser inlet radius  $R_{DI}$ , and the diffuser outlet radius  $R_{DO}$ . The total length of the diffuser is related to the nozzle radius so that pumps of different sizes maintain similar proportions to the original design.

Certain combinations of these three variables are undesirable and are known to give poor performance. For example a large diffuser with a small nozzle is inefficient and a large nozzle with a small diffuser causes the pump to stall. Because of this, the design variables used in this

**Fig. 2** Axi-symmetric model of a jet pump geometry, axis is along line A-A





**Fig. 3** (left) Distribution of design points in design variable space, and (right) corresponding jet pump dimensions, for design variables one and two

optimization study are defined relative to each other, rather than as explicit dimensions, and are given by:

$$DV_1 = R_N \quad (5)$$

$$DV_2 = R_{D1}/R_N \quad (6)$$

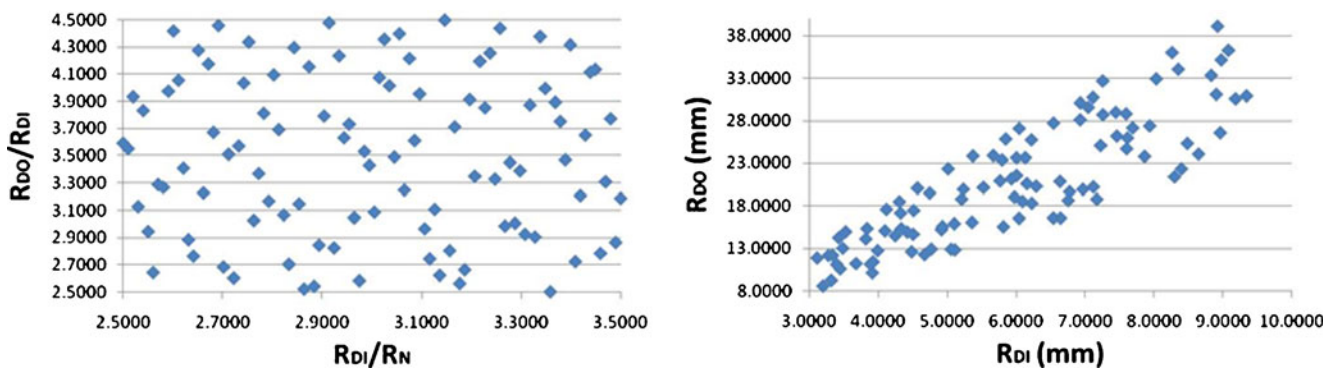
$$DV_3 = R_{D0}/R_{D1} \quad (7)$$

where  $DV$  are the three design variables.

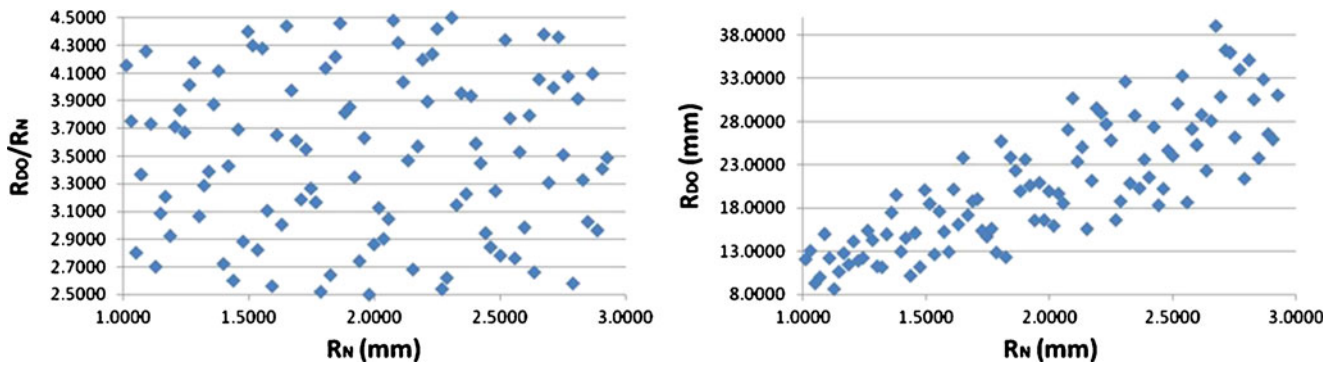
A 100 point optimal Latin hypercube DoE is constructed with three dimensions using the approach described earlier. Of the 100 points, 70 are building points and 30 are validation points. Equal weights are used in Eq. 2. The levels of the Latin hypercube are then scaled to correspond to the ranges:  $0.0 \text{ mm} \leq DV_1 \leq 2.9 \text{ mm}$ ;  $2.5 \leq DV_2 \leq 3.5$ ; and  $2.5 \leq DV_3 \leq 4.5$ . The distribution of points in the design variable space is shown in Figs. 3, 4 and 5 along with the distribution corresponding to the explicit jet pump

dimensions. This illustrates the benefits of this parameterization method, as all design points describe designs that are expected to be feasible, and designs that are known to be ineffective are avoided.

An initial CFD mesh is morphed to match each set of design variables. This is achieved by altering the location of boundary nodes, followed by linear interpolation on all interior nodes. Mesh checks and smoothing ensure mesh quality is maintained. The method of parameterization used also helps reduce large deformations of the cells' shapes, as the proportions of the pump remain broadly similar. CFD analysis is performed at each design point, using the approach described in section II. Pressures of 2 bar at the primary flow inlet boundary,  $-150 \text{ mbar}$  at the entrained flow inlet, and 0 bar at the outlet are specified as boundary conditions, relative to atmospheric pressure. Mesh sizes of 40,000 cells are used, based on a study of the dependence of the global performance responses on mesh density. Calculation time for each design point is approximately 6 CPU hours on two 2.6 GHz processors.



**Fig. 4** (left) Distribution of design points in design variable space, and (right) corresponding jet pump dimensions, for design variables two and three



**Fig. 5** (left) Distribution of design points in design variable space, and (right) corresponding jet pump dimensions, for design variables one and three

Volumetric flow rates at both primary and entrained flow inlets for each design point are extracted from the CFD data. Moving Least Squares approximations of these responses are then constructed, using a second order base polynomial and the 70 model building points. The closeness of fit parameter in Eq. 3 is optimized using the 30 model validation points. The primary flow rate is almost entirely dependent on the nozzle radius and is easily modeled with negligible error. The entrained flow rate response is also modeled well using this approximation, resulting in a validation matrix  $R^2$  value of 0.9998 with a maximum absolute error of 8 L/min. The surrogates are then rebuilt using all design points.

The optimization problem is formulated as follows:

$$\begin{aligned} &\text{maximize: } \dot{V}_{ent} \\ &\text{subject to: } \dot{V}_{pri} \leq c_0 \\ &\quad c_i^L \leq DV_i \leq c_i^U, \quad i = 1, 2, 3 \end{aligned} \tag{8}$$

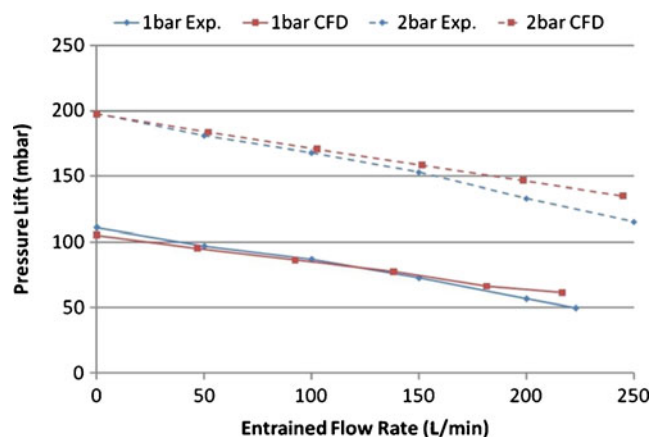
where  $\dot{V}_{pri}$  and  $\dot{V}_{ent}$  are the volumetric flow rates at the primary and entrained flow inlets respectively,  $c_0$  is the constraint imposed on the primary flow rate, and  $c_i^L$  and  $c_i^U$  are the lower and upper bounds of the design variables. Solutions to the problem, with various values of  $c_0$  ranging from 50 L/min to 320 L/min, are computed using the combination of a genetic algorithm and Sequential Quadratic Programming. In all cases, the constraints on design variables correspond to the maximum and minimum values used in the scaled design of experiments.

### 4 Optimization results

An initial pump design was developed using the analytical approach described in section II. An experimental model of

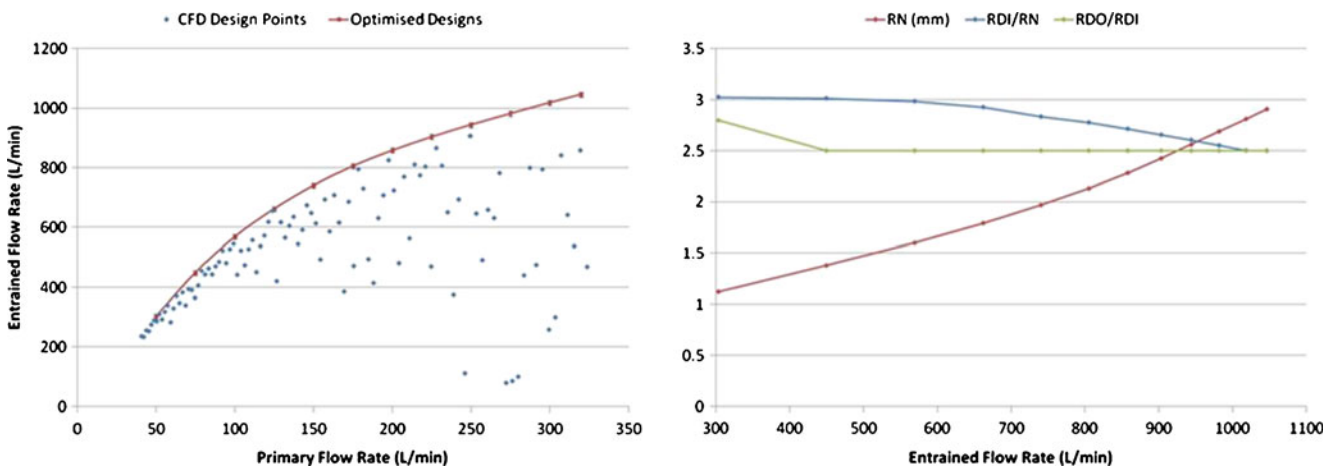
this pump was built using rapid prototyping and experimental data for various primary inlet pressures, entrained flow rates and pressure lifts was produced in order to validate the CFD model. This study is presented in more detail in a previous paper (Fan et al. 2011), but a comparison between the pressure lift (defined as the pressure difference between the inlet of the secondary flow and the outlet of the jet pump) and the entrained flow-rate for two different primary flow rates (controlled by the pressure) is included here for completeness (Fig. 6).

The optimized responses, along with the design points evaluated using CFD and used to build the surrogate models, are shown in Fig. 7 (left). A front, along which optimal designs exist, can clearly be seen. This is analogous to a Pareto front for the multi-objective problem of minimizing primary flow rate and maximizing the entrained flow rate. The decreasing gradient of this front indicates that there are



**Fig. 6** CFD validation study showing pressure lift from experimental and CFD models for two primary flow pressures and a range of entrained flow rates

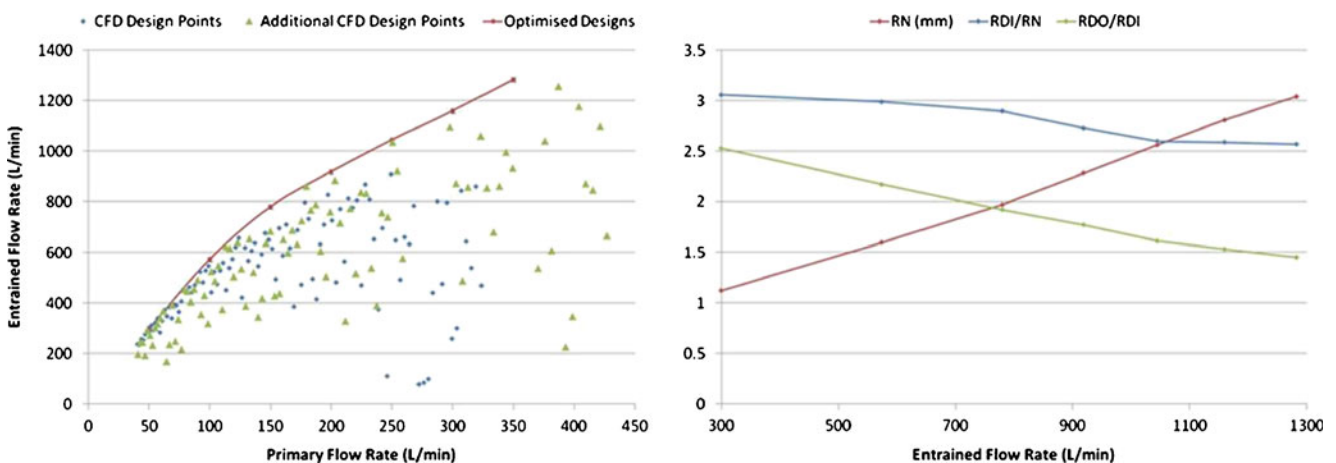




**Fig. 7** (left) Responses from CFD evaluations with a series of optimized designs. (right) Optimized design variables for various entrained flow rates

losses in efficiency at larger jet pump sizes, furthermore the maximum required flow rate of 1,200 L/min was not reached due to the constraints imposed by the selection of the design variable space. The optimized design variables for various entrained flow rates are shown in Fig. 7 (right). The required nozzle radii increase with the larger entrained flow rates, as would be expected. The ratio of diffuser inlet radius to nozzle radius starts at 3.0 for low flow rates and reaches its lower bound of 2.5 at the largest pump size. The ratio of diffuser outlet to inlet reaches its lower bound for all but the smallest pump size. This explains some of the losses in efficiency at the higher flow rates, as the values of the third design variable are not truly optimal.

For these reasons a second optimal Latin hypercube DoE is constructed with 100 design points. The upper bound on  $DV_1$  is increased to 3.3 mm, the lower bound on  $DV_2$  is decreased to 2.0, and the lower bound on  $DV_3$  is decreased to 1.25. Pre-existing points from the first DoE are included in the objective function calculations in Eq. 1. CFD analysis is performed on the new design points and response surfaces are rebuilt using both data sets. The optimization study is then repeated. Figure 8 (left) shows that at higher flow rates the front of optimal designs is now shifted forward and has a steeper gradient. This confirms that some of the loss in efficiency at the high flow rates in the previous study is due to the limits imposed on the design variables.



**Fig. 8** (left) Responses from CFD evaluations of original and additional DoEs, showing new set of optimal designs shifted forward with steeper gradient. (right) Design variables optimized using surrogates built from both DoEs

Figure 8 (right) shows that none of the design variables reach their limits.

## 5 Conclusion

High fidelity flow analysis has been combined with a formal optimization framework in order to optimize a set of jet pumps capable of entraining various flow rates. The jet pump design is parameterized by three key design variables and a number of designs are produced using an Optimal Latin Hypercube DoE. Considering the flow physics enabled the number of unrealistic designs examined by the DoE to be dramatically reduced by the use of a simple transformation of the design variable space. Each design is evaluated using CFD analysis and Moving Least Squares surrogate models of primary and entrained flow rates are built. Finally a set of optimization runs using a genetic algorithm and sequential quadratic programming is performed to define a front along which optimal designs exist. This approach generates data that enables the rapid optimization of jet pump designs for a wide range of entrained flow rates. Designs can now be simply interpolated between the existing optimized designs, with almost no computational effort, or for more accuracy they can be extracted from MLS surrogate models using a genetic algorithm, which takes a few minutes on a standard desktop computer.

There are currently manufacturing tolerances and surface roughness associated with the manufacturing of the optimized jet pump designs, which can result in a reduction in efficiency (Yamazaki et al. 2006). For this reason, it would be advantageous to incorporate reliability-based optimization into the approach. This could be achieved by accounting for uncertainties in the design variables. There is also scope for combining high fidelity flow analysis with the analytical methods of jet pump design in a multi-fidelity framework.

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## References

- Audze P, Eglais V (1977) New approach for planning out of experiments. *Prob Dyn Strengths* 35:104–107
- Bartosiewicz Y, Aidoun Z, Mercadier Y (2006) Numerical assessment of ejector operation for refrigeration applications based on CFD. *Appl Therm Eng* 26(5–6):604–612
- Eames IW (2002) A new prescription for the design of supersonic jet-pumps: the constant rate of momentum change method. *Appl Therm Eng* 22(2):121–131
- Fan J, Eves J, Thompson HM, Toropov VV, Kapur N, Copley D, Mincher A (2011) Computational fluid dynamic analysis and design optimization of jet pumps. *Comput Fluid* 46(1):212–217
- Guillaume DW, Judge TA (1999) Improving the efficiency of a jet pump using an elliptical nozzle. *Rev Sci Instrum* 70(12):4727–4729
- He S, Li Y, Wang RZ (2009) Progress of mathematical modeling on ejectors. *Renew Sustain Energy Rev* 13(8):1760–1780
- Helali A, Kueny JL (2008) Geometric optimisation of a jet pump. Theoretical and experimental aspects of fluid mechanics. In: *Proceedings of the 5th WSEAS international conference on fluid mechanics*, pp 233–238
- Hemidi A, Henry F, Leclaire S, Seynhaeve JM, Bartosiewicz Y (2009) CFD analysis of a supersonic air ejector. Part I: experimental validation of single-phase and two-phase operation. *Appl Therm Eng* 29(8–9):1523–1531
- Kumar RS, Kumaraswamy S, Mani A (2007) Experimental investigations on a two-phase jet pump used in desalination systems. *Desalination* 204(1–3):437–447
- Narabayashi T, Yamazaki Y, Kobayashi H, Shakouchi T (2006) Flow analysis for single and multi-nozzle jet pump. *JSME Int J B Fluid Therm Eng* 49(4):933–940
- Narayanan A, Toropov VV, Wood AS, Campean IF (2007) Simultaneous model building and validation with uniform designs of experiments. *Eng Optim* 39(5):497–512
- Patankar SV, Spalding DB (1972) Calculation procedure for heat, mass and momentum-transfer in 3-dimensional parabolic flows. *Int J Heat Mass Trans* 15(10):1787–&
- Prabkeao C, Aoki K (2005) Study on the optimum mixing throat length for drive nozzle position of the central jet pump. *J Vis* 8(4):347–355
- Sarshar MM, Beg NA (1998) A review of the application of jet pump technology in oil and gas industry to increase production. In: *1st North American conference on multiphase technology*, pp 143–160
- Wang XD, Dong JL (2010) Numerical study on the performances of steam-jet vacuum pump at different operating conditions. *Vacuum* 84(11):1341–1346
- Yamazaki Y, Nakayama T, Narabayashi T, Kobayashi H, Shakouchi T (2006) Effect of surface roughness on jet pump performance. *JSME Int J B Fluid Therm Eng* 49(4):928–932
- Yu JL, Chen H, Ren YF, Li YZ (2006) A new ejector refrigeration system with an additional jet pump. *Appl Therm Eng* 26(2–3):312–319