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Optimum thickness of curtain grouting on dam foundation with minimum seepage pressure resultant

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Abstract The distribution of hydraulic head on the dam foundation plane with curtain grouting is analyzed by the simplified one-dimensional seepage model, also is studied the effect of various parameters of curtain grouting on seepage pressure on the foundation plane. The theory of the optimum thickness of curtain grouting is proposed from the viewpoint of the minimum seepage pressure resultant and proved by the two-dimensional seepage model and the finite element method, which includes two cases of homogenous foundation and layered foundation.

Keywords One-dimensional seepage model · Curtain grouting · Seepage pressure · Optimum thickness

1 Introduction

At present, the reduction factor of uplift pressure at the drainage curtain is adopted in the design to reflect the effect of the curtain grouting on seepage pressure on the foundation plane (Qi 1997). By means of this empirical factor, only the whole effect of curtain grouting and drainage curtain is reflected comprehensively (Mao 2003). The quantitative effect of permeability coefficient, thickness and depth of the curtain grouting on seepage pressure on foundation plane is not taken into account (Chai et al. 2005a). Based on the one-dimensional seepage theory, the effect of curtain grouting on seepage pressure on foundation will be analyzed in this paper and the theory of the optimum thickness of curtain grouting is proposed from the viewpoint of the minimum seepage pressure resultant and proved by the two-dimensional seepage model and the finite element method, which includes two cases of homogenous foundation and layered foundation.

Most optimization problems can be formulated as a formal mathematical problem of the form min f(x) subject to g(x) < 0 (Piermatei Filho and Leontiev 2009; Akbari et al. 2010; Fuchs and Shemesh 2004; Zhang et al. 2008). But in this paper, we propose the optimum thickness of curtain grouting on dam foundation from the viewpoint of the minimum seepage pressure resultant (min f(x)) by equating the first derivative to zero, the constraint conditions g(x) < 0 (various parameters of curtain grouting) will be meet by engineering experiences.

2 Effect of the curtain grouting on seepage pressure on the foundation plane

The sketch of a dam foundation with a complete curtain grouting is shown as Fig. 1a, and the symbols and notations are given in Table 1.

The flow in differential depth dz, which is close to the foundation plane, is assumed to be one-dimensional in order to obtain the seepage velocity and unit discharge directly by Darcy's law.

The unit seepage discharge q_1 through AB is

$$q_1 = v_1 \cdot dz = kJ_1 dz = k\frac{H - H_1}{a_1}dz \tag{1}$$

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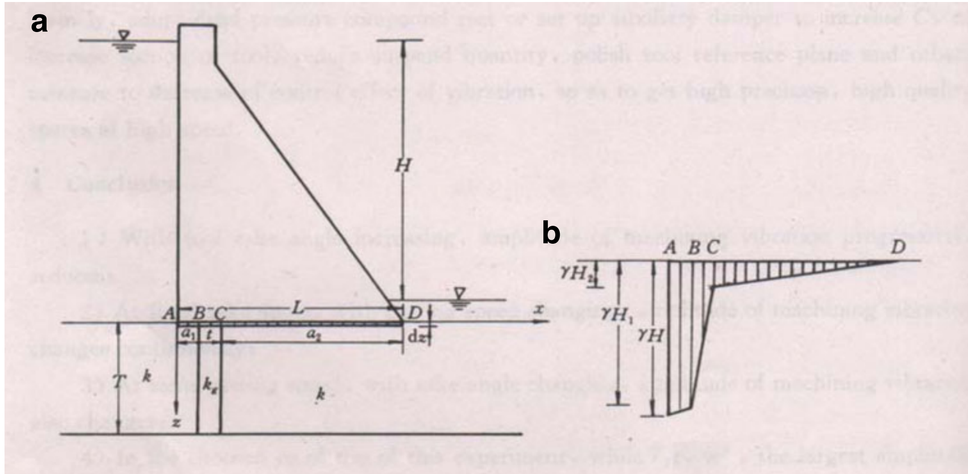

Fig. 1 Complete curtain grouting. **a** Complete curtain grouting. **b** Hydrostatic seepage pressure distribution on foundation plane

where v_1 and J_1 are the seepage velocity and hydraulic gradient of *AB*, respectively.

We can obtain the unit discharge q_2 and q_3 through *BC* and *CD*, respectively, by the same theory as follows,

$$q_2 = v_2 \cdot dz = k_g J_2 dz = k_g \frac{H_1 - H_2}{t} dz \tag{2}$$

$$q_3 = v_3 \cdot dz = k J_3 dz = k \frac{H_2}{a_2} dz \tag{3}$$

where v_2 and J_2 are the seepage velocity and hydraulic gradient of *BC*, respectively; v_3 and J_3 are the seepage velocity and hydraulic gradient of *CD*, respectively.

From the continuity of fluid flow we have

$$q_1 = q_2 = q_3 \tag{4}$$

From (1–3) and (4), we have the following equations

$$\begin{cases} k\dfrac{H - H_1}{a_1} dz = k_g \dfrac{H_1 - H_2}{t} dz \\ k_g \dfrac{H_1 - H_2}{t} dz = k\dfrac{H_2}{a_2} dz \end{cases} \tag{5}$$

Since H_1 and H_2 are unknown, the equations may be solved as follows.

$$\begin{cases} H_1 = \dfrac{\frac{k}{k_g} t + a_2}{\frac{k}{k_g} t + a_1 + a_2} H \\ H_2 = \dfrac{a_2}{\frac{k}{k_g} t + a_1 + a_2} H \end{cases} \tag{6}$$

which quantitatively reflects the effect of the thickness of curtain grouting t and permeability coefficient k_g of curtain

Table 1 The symbols and notations

Symbol	Notation	Remark	Symbol	Notation	Remark
A	Point of the dam heel		a_2	Distance from the toe to curtain downstream	$a_2 = \overline{CD}$
B	Junction of curtain upstream and foundation		L	Total width of the foundation	$L = a_1 + t + a_2$
C	Junction of curtain downstream and foundation		T	Foundation depth	
D	Point of the dam toe		k	Average permeability coefficient of foundation	
H	Total head between up and downstream		k_g	Average permeability coefficient of curtain	
H_1	Head at point B		x	Horizontal ordinate	Downstream as positive
H_2	Head at point C		z	Vertical ordinate	Down as positive
a_1	Distance from heel to curtainupstream	$a_1 = \overline{AB}$	dz	Differential depth which is close to foundation plane	
T	Thickness of the curtain	$t = \overline{BC}$	γ	Gravity density of water	

on head distribution of the foundation plane. From (6), the equivalent seepage travel length of curtain is approximately equal to $\frac{k}{k_g}$ times the actual thickness of curtain t via similar triangles.

The total head h at any point in the seepage field is composed of the pressure head ($\frac{p}{\gamma}$), elevation head z and velocity head ($\frac{av^2}{2g}$), that is (Chai et al. 1996),

$$h = \frac{p}{\gamma} + z + \frac{av^2}{2g} \tag{7}$$

where p is the seepage pressure, γ is the gravity density of water, z is the vertical ordinate, v is the seepage velocity, a is the velocity head modification coefficient, and g is the gravity acceleration.

In general, the seepage velocity v is very small and $\frac{av^2}{2g}$ is so much smaller that it can be neglected. On the foundation plane the elevation head $z = 0$. So, the seepage pressure on foundation plane p can be expressed as follows.

$$p = \gamma h \tag{8}$$

Therefore the seepage pressure distribution in the case of the complete curtain grouting can be obtained as shown in Fig. 1b. The seepage pressure p_1 and p_2 at the upstream and downstream curtain face can be expressed as follows.

$$\begin{cases} p_1 = \dfrac{\frac{k}{k_g}t + a_2}{\frac{k}{k_g}t + a_1 + a_2}H\gamma = \dfrac{L + \left[\frac{k}{k_g} - 1\right]t - a_1}{L + \left[\frac{k}{k_g} - 1\right]t}H\gamma \\ p_2 = \dfrac{a_2}{\frac{k}{k_g}t + a_1 + a_2}H\gamma = \dfrac{L - a_1 - t}{L + \left[\frac{k}{k_g} - 1\right]t}H\gamma \end{cases} \tag{9}$$

We can see that p_1 increases and approaches to $H\gamma$ and p_2 reduces and approaches to zero when $\frac{k}{k_g}$ increases gradually or the permeability coefficient k_g of curtain grouting decreases gradually.

According to (9) and Fig. 1b, we can get the resultant force P (the area of the map 1(b)) of seepage pressure within unit width of dam foundation as follows.

$$\begin{aligned} P &= \frac{a_1}{2}(\gamma H + p_1) + \frac{t}{2}(p_1 + p_2) + \frac{a_2}{2}p_2 \\ &= \frac{\gamma H}{2} \cdot \frac{t^2 + 2a_1 t + \beta L^2}{t + \beta L} \end{aligned} \tag{10}$$

in which

$$\beta = \left[\frac{k}{k_g} - 1\right]^{-1} \tag{11}$$

Now we analyze the effect of a_1, k_g and t on P, respectively.

(1) From (10), P will increase correspondingly while a_1 increases. When $a_1 = 0$, the value of P is minimum. But in engineering practice, the curtain grouting cannot begin from heel due to the limits of the construction conditions and so on. So it is better to make a_1 as small as possible in order to reduce the seepage pressure resultant P.

(2) Taking P differentiating with respect to β, we have

$$\frac{\partial P}{\partial \beta} = \frac{\gamma H}{2} \cdot \frac{Lt\,(L - t - 2a_1)}{(t + \beta L)^2} = \frac{\gamma H}{2} \cdot \frac{Lt\,(a_2 - a_1)}{(t + \beta L)^2} \tag{12}$$

Generally, $a_2 > a_1$, so $\frac{\partial P}{\partial \beta} > 0$ and P reduces if β reduces. That is to say, β and total seepage pressure resultant P reduces while k_g reduces and the value of $\frac{k}{k_g}$ increases.

(3) Taking P differentiating with respect to t, we have

$$\frac{\partial P}{\partial t} = \frac{\gamma H}{2} \cdot \frac{t^2 + 2\beta L t + \left(2\beta L a_1 - \beta L^2\right)}{(t + \beta L)^2} \tag{13}$$

We can see from the above equation that when $t < \left[\sqrt{\beta^2 + \beta - 2\beta\frac{a_1}{L}} - \beta\right]L$, $\frac{\partial P}{\partial t} < 0$, so the seepage pressure resultant P decreases with the thickness of curtain grouting t increasing. But $\frac{\partial P}{\partial t} > 0$ in the case of $t > \left[\sqrt{\beta^2 + \beta - 2\beta\frac{a_1}{L}} - \beta\right]L$, so the seepage pressure resultant P increases with the thickness of curtain grouting t increasing. Therefore, the thickness of grouting can not be increased infinitely. In theory, the optimum thickness of grouting t_{opt} can be given as follows.

$$t_{opt} = \left[\sqrt{\beta^2 + \beta - 2\beta\frac{a_1}{L}} - \beta\right]L \tag{14}$$

Table 2 The optimum thickness of curtain grouting

Case	Introduction to case	β	t_{opt}	$P_{\min}$
1	$\frac{k}{k_g} = 10, a_1 = 0$	1/9	$0.240L$	$0.240\gamma HL$
2	$\frac{k}{k_g} = 100, a_1 = 0$	1/99	$0.091L$	$0.091\gamma HL$
3	$\frac{k}{k_g} = 10, a_1 = 0.1L$	1/9	$0.207L$	$0.307\gamma HL$
4	$\frac{k}{k_g} = 100, a_1 = 0.1L$	1/99	$0.080L$	$0.180\gamma HL$
5	$\frac{k}{k_g} = 10, a_1 = 0.05L$	1/9	$0.224L$	$0.274\gamma HL$

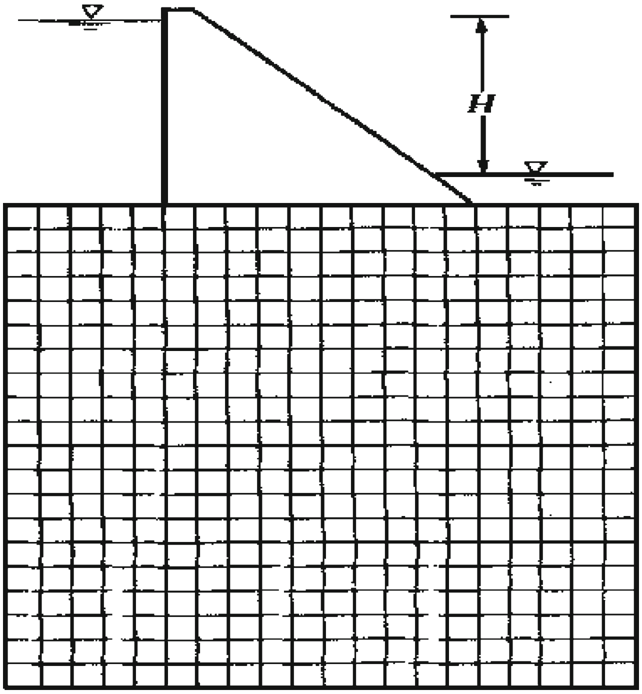

Fig. 2 The computation model and FE mesh of the example

The corresponding minimum seepage pressure resultant $P_{\min}$ is

$$P_{\min} = \gamma HL\left[\sqrt{\beta^2+\beta-2\beta\frac{a_1}{L}}+\frac{a_1}{L}-\beta\right] = \gamma Ht_{opt}+\gamma Ha_1 \quad (15)$$

The optimum thickness of curtain grouting t_{opt} and the corresponding minimum seepage pressure resultant $P_{\min}$ are calculated and listed in Table 2 for several cases.

3 Example and validation

The two-dimensional computational model for calculation example is shown as Fig. 2. In the case of complete curtain grouting, the above model is calculated by two-dimensional finite element method. The dam foundation (seepage area) is divided into $20 \times 20 = 400$ elements and $21 \times 21 = 441$ nodes. The one-dimensional seepage model theory is verified by two-dimensional finite element method (Chai and Deng 2004; Chai and Li 2004; Chai et al. 2004, 2005b; Piermatei Filho and Leontiev 2009; Akbari et al. 2010; Fuchs and Shemesh 2004; Zhang et al. 2008). The solutions of seepage pressure distribution and the resultant force by two methods are listed in Table 3 under four different cases with different k/k_{g} and t.

It can be seen from Table 3 that the result has a small error and the same law of seepage pressure distribution and resultant force, comparing one-dimensional seepage model with two-dimensional finite element method. Comparing case 1 with case 2, we can see that the seepage pressure resultant P reduces when the thickness of curtain t increases from $0.1L$ to $0.2L$, which is because the optimum thickness t_{opt} is $0.207L$ under these two cases, P reduces with t arising when $t < t_{\text{opt}}$. Comparing case 3 with case 4, P increases when t increases from $0.1L$ to $0.2L$, which is because t_{opt} is $0.080L$ under these two cases, P increases with t arising when $t > t_{\text{opt}}$. This indicates the rationality of the optimum thickness theory of curtain grouting from the viewpoint of the minimum seepage pressure resultant.

In addition, comparing case 1 with case 3, and comparing case 2 with case 4, we can know that the total seepage pressure P reduces with k/k_{g} increasing, which also supports the above conclusion.

Table 3 The seepage pressure distribution and the resultant force on homogenous foundation

Case	Introduction to case	$\frac{H_1}{H}$	$\frac{H_2}{H}$	$\frac{P}{\gamma HL}$	Remark
1	$\frac{k}{k_g}=10, a_1=0.1L, t=0.1L$	0.947	0.421	0.334	One-dimensional seepage theory
		0.887	0.465	0.394	Two-dimensional finite element method
2	$\frac{k}{k_g}=10, a_1=0.1L, t=0.2L$	0.964	0.250	0.307	One-dimensional seepage theory
		0.912	0.336	0.371	Two-dimensional finite element method
3	$\frac{k}{k_g}=100, a_1=0.1L, t=0.1L$	0.991	0.073	0.182	One-dimensional seepage theory
		0.968	0.141	0.228	Two-dimensional finite element method
4	$\frac{k}{k_g}=100, a_1=0.1L, t=0.2L$	0.995	0.034	0.214	One-dimensional seepage theory
		0.981	0.075	0.240	Two-dimensional finite element method

Table 4 Seepage pressure distribution and resultant force on layered foundation

Case	Introduction to case	$\frac{H_1}{H}$	$\frac{H_2}{H}$	$\frac{P}{\gamma HL}$	Remark
1	$\frac{k}{k_g} = 100, a_1 = 0.1L, t = 0.1L$	0.991	0.073	0.182	One-dimensional seepage theory
	k_1: k_2: k_3: k_4 = 10:5:2.5:1	0.973	0.145	0.213	Two-dimensional finite element method
2	$\frac{k}{k_g} = 100, a_1 = 0.1L, t = 0.1L$	0.991	0.073	0.182	One-dimensional seepage theory
	k_1: k_2: k_3: k_4 = 1:2.5:5:10	0.964	0.147	0.213	Two-dimensional finite element method
3	$\frac{k}{k_g} = 100, a_1 = 0.1L, t = 0.2L$	0.995	0.034	0.214	One-dimensional seepage theory
	k_1: k_2: k_3: k_4 = 10:5:2.5:1	0.984	0.083	0.235	Two-dimensional finite element method
4	$\frac{k}{k_g} = 100, a_1 = 0.1L, t = 0.2L$	0.995	0.034	0.214	One-dimensional seepage theory
	k_1: k_2: k_3: k_4 = 1:2.5:5:10	0.977	0.085	0.233	Two-dimensional finite element method

4 Discussions about the layered foundation

The above discussions are all about simple and homogeneous foundations, but in reality most foundations are layered. The theory of OPTIMUM thickness of curtain grouting based on the one-dimensional theory for a homogeneous case could not apply directly to the multi-layer scenario. Here we show the error and effectivity of the theory of OPTIMUM thickness of curtain grouting in case of the multi-layer scenario. Now we assume that the foundation of the computation model (in Fig. 2) is divided into four layers averagely, and that k_1, k_2, k_3, k_4 are the permeability coefficients of four layers, respectively, from up to down. By the one-dimensional theory and two-dimensional finite element method, we can get the calculating results as Table 4.

We can see from Table 4 that the permeability of layered foundation has certain effect on seepage pressure. The seepage pressure mainly depends upon the permeability of the top layer. Comparing case 1 with case 3, and case 2 with case 4, it can be seen that the theory of the optimum thickness of curtain grouting on dam foundation is also effective from the viewpoint of the minimum seepage pressure resultant.

5 Conclusions

In this paper we analyze the seepage pressure on the foundation plane with curtain grouting by the one-dimensional theory and propose the optimum thickness of curtain grouting from the viewpoint of the minimum seepage pressure resultant and proved by the two-dimensional seepage model and the finite element method, which includes two cases of homogenous foundation and layered foundation. In engineering practice, it is better to make the distance from curtain grouting to the heel of dam as near as possible (to make a_1 minimum), to make the highest quality of the curtain grouting (to make k_g minimum), and to make the thickness of curtain grouting near the optimum thickness t_{opt}, so that the seepage pressure resultant can be minimized. The conclusions have an important value for engineering design of the curtain grouting on dam foundation.

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