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INDUSTRIAL APPLICATION

Application of SEUMRE global optimization algorithm in automotive magnetorheological brake design

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Abstract In this paper, the superior performance of a novel space exploration and unimodal region elimination global optimization algorithm, SEUMRE, is demonstrated through comparisons with other well known global optimization techniques, including genetic algorithm (GA), simulated annealing (SA), and a highly nonlinear design problem—the optimal design of automotive magnetorheological brake (MRB). Unlike the conventional brakes, an MRB employs the interaction between a magnetorheological fluid and an applied magnetic field to generate the retarding braking torque. The SEUMRE design optimization algorithm was used to maximize the braking torque and minimize the weight of the brake structure. The computation time and optimized design parameters illustrated SEUMRE's capability to converge to an accurate result faster than the conventional global optimization methods. However, SA provided significantly better optimization results than GA and SEUMRE in terms of the cost function.

Keywords Global optimization · Magnetrohelogical brake system · Space exploration · Metamodeling · Engineering design

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1 Introduction

Global optimization algorithms are effective methods in finding optimal solutions for given problems. They have found success in many applications and have been extensively used in many fields such as engineering, economics, and mathematics. Many optimization algorithms have been introduced, and new and improved algorithms appear in the literature every year. Generally, optimization algorithms can be divided into the following two main categories: gradient based (deterministic) and non gradient based (Zabinsky and Smith 1992; Hendrix 1998). Many engineering design problems are highly complex in nature. Therefore, deriving the gradient is very expensive, and requires intensive computational efforts and resources. Accordingly non-gradient optimization algorithms that are stochastic and heuristic based (Gill et al. 1981; Baritompa and Hendrix 2005) have become more attractive. These algorithms have become popular and have found lots of interest because of their superior performance and their capability in extracting information on the search space without experiencing computational complexities. Today, more efficient, effective and robust non-gradient optimization algorithms are continuously being introduced to handle complex engineering problems from various applications.

In the authors recent work (Younis and Dong 2009a), a new stochastic and heuristic based global optimization search method, Space Exploration Unimodal Region Elimination (SEUMRE), has been introduced. The optimization algorithm starts the search by spreading sampling points to explore the design space and based on the preliminary information obtained to predict where global solution may exist and where the search must be focused. Such optimization algorithm is particularly suitable for highly nonlinear and complex design optimization problems involving expensive analysis and simulation processes such as finite element analysis (FEA) and computational fluid dynamics (CFD).

In this work, the SEUMRE algorithm is applied to solve a highly nonlinear and complex real-life engineering design optimization problem—the optimal design of automotive magnetorheological brake (MRB). The method is also compared with other well known stochastic optimization algorithms, including genetic algorithm (GA) and simulated annealing (SA). The main objective of the study is to further explore the potential of the new global optimization algorithm through a practical industrial design problem.

2 Global optimization algorithms

The well known global optimization algorithms, genetic algorithm (GA) and simulated annealing (SA), as well as the new space exploration and region elimination global optimization algorithm, SEUMRE, are briefly reviewed and discussed in this section. Nevertheless, there are other heuristic optimization algorithms that have proven to perform well and converge to accurate solutions with reasonable computational cost, Particle Swarm Optimization (PSO) (Kennedy and Eberhart 1995) and Ant Colony Optimization (ACO) (Dorigo et al. 1996) algorithms are examples.

2.1 Genetic algorithm

Genetic algorithm (GA) is one of the most efficient metaheuristics that has been employed in a wide variety of problems (Goldberg 1989; Michalewicz 1996). However, GA, like other metaheuristics, suffers from the slow convergence that brings about the high computational cost. GA is a class of search procedures based on the mechanics of natural genetics and natural selection (Goldberg 1989). Generally, the GA mechanism consists of three fundamental operations: reproduction, crossover and mutation. Reproduction is the random selection of copies of solutions from the population according to their fitness value to create one or more offspring. Crossover defines how the selected chromosomes (parents) are recombined to create new structures (offspring) for possible inclusion in the population. Mutation is a random modification of a randomly selected chromosome. Its function is to guarantee the possibility to explore the space of solutions for any initial population and to permit the freeing from a zone of local minimum. Generally, the decision of the possible inclusion of crossover/mutation offspring is governed by an appropriate filtering system. Both crossover and mutation occur at every cycle, according to an assigned probability. The aim of the three operations is to produce a sequence of population which, in average, tends to improve.

2.2 Simulated annealing

Simulated annealing (SA) was developed by Kirkpatrick et al. (1983) to deal with highly nonlinear problems. SA is a generalization of the Monte Carlo method for examining the equations of state and frozen states of n-body systems. The algorithm emulates the annealing process on how liquid freezes or metal recrystalizes in cooling. SA is easy to implement, although the method converges slowly and it is difficult to find an appropriate stopping rule. As a random-search technique which exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure, SA searches for a minimum in a more general system, forming the basis of an optimization technique for combinatorial and other problems.

2.3 Space exploration and region elimination algorithms

Metamodeling based search, space exploration, and region reduction/elimination methods are effective optimization schemes for computationally intensive global design optimization problems. The space exploration and region reduction methods start their search by sending agents or sampling points to explore the entire design space. These agents will yield some information (values of objective and constraint functions) about the design space. The obtained information provide hints for ranking different multimodal regions based on their potentials to contain the global optimum, allowing more attention being put on these promising regions during subsequent searches using more samples to further explore the regions and to refine the unimodal model over each region. Exploration of the design space is also very valuable for a better understanding of the design problem. A surrogate model, or metamodel, is easy to construct and "cheaper" to calculate, comparing to the calculation of the original "expensive" objective function. For optimization problems in which the calculations of the objective and constraint functions require extensive numerical analysis and simulation, introduction of the metamodel and use of the cheap points to replace the expensive points can considerably reduce computation time and make global optimization feasible.

Region elimination schemes intend to keep the size of the design space under control. Many region elimination methods have been recently proposed. Among those, the most representative group is the *multivariable elimination* procedures (Gill et al. 1981), which reduce the design space by discarding the unpromising regions that do not contain the optimum. Another approach, *metaheuristics* (Weise 2009) which uses a complex probability model to identify the promising regions with a higher chance to contain the design optimum. However, the strategy is computationally less efficient due to the complexity of the probabilistic models used.

In general, metamodeling and region elimination based search methods have been found promising for applications that require intensive computations (Jones et al. 1998; Simpson et al. 2001; Wang et al. 2001, 2004; Shan and Wang 2004; Wang and Shan 2004; Kaymaz and McMathon 2005; Huang et al. 2006; Younis and Dong 2009b). Due to the nature of the MRB design optimization problem, the top performing metamodeling and region elimination based search method, SEUMRE (Younis and Dong 2009a), is used in this work to identify the optimal design. Extended from its predecessor, the Approximated Unimodal Region Elimination (AUMRE) algorithm (Younis et al. 2009), SEUMRE consists of the following key elements: (a) dividing the design space into several unimodal regions using design experiment data; (b) identifying and ranking the regions that most likely contain the global optimum; (c) fitting a Kriging model (Cressie 1988) with additional design experiments using Latin Hypercube designs (McKay et al. 1997) over the most promising region and identifying its minimum; and (d) moving to the next most promising region. The process is carried on until all promising unimodal regions are processed, and most likely the global optimum is obtained from identified local optima. The pseudo codes and flowchart (Fig. 1) of the algorithm are given in the following.

Pseudo code for SEUMRE:

1. Generate experimental designs in the design space.
2. Divide the design space into sub-spaces.
   (a) Use min function values obtained as center points for unimodal regions.
   (b) Roughly identify the boundaries for the unimodal regions.
   (c) Refine the space by generating more experimental designs.
3. Construct Kriging meta-model.
4. Find its optimum.
5. Move to the next design sub-space and repeat steps 2.a–4.
6. If all design space is processed and termination criteria are met, Compare obtained optimum values and find the global optimum.
7. Terminate.

SEUMRE was tested on many representative benchmark problems (Younis and Dong 2009a). Results of SEUMRE on the representative benchmark problems and the required computations represented by the number of objective function evaluations, and CPU time are shown and summarized in Appendix 1. The formulas for the benchmark test problems can be found in Younis and Dong (2009a).

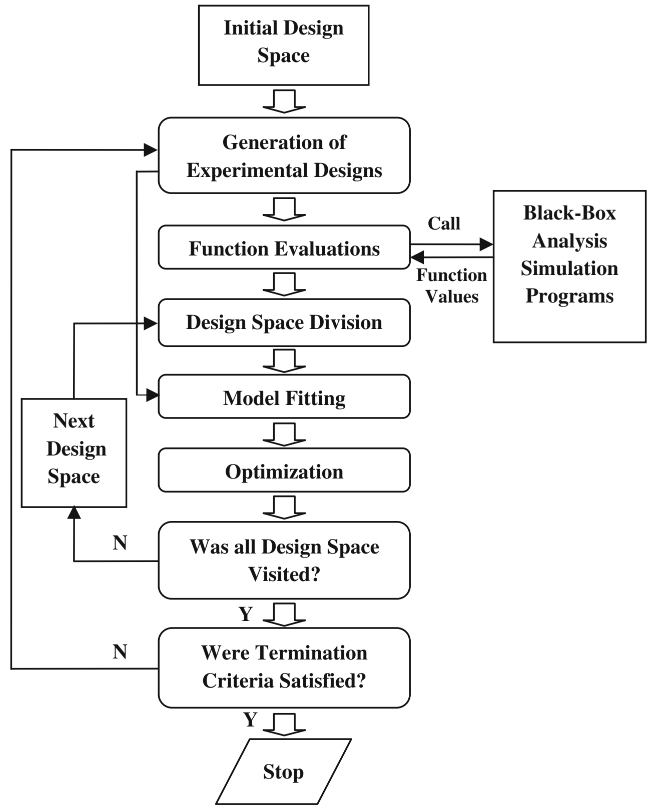

Fig. 1 SEUMRE algorithm flowchart (Younis and Dong 2009a)

3 Magnetorheological Brake (MRB) design optimization problem

Conventional hydraulic brakes (CHB) are used to provide required braking torque in stopping a vehicle. The CHB system consists of brake pedal, hydraulic fluid, transfer pipes and brake actuators (disk and drum brakes). When the driver presses on the brake pedal, the hydraulic brake fluid provides the pressure in the brake actuators that squeezes the brake pads onto the rotor. Unlike their conventional counterparts, electromechanical brake (EMB) systems offer potential improvements such as faster response, easy controller implementation and parameter control, reduced weight and wiring, reduced maintenance etc, due to their purely electrical nature. Magnetic particle/fluid brake (or MRB) is one of the commonly known electromechanical brakes introduced as possible substitutes of the CHB systems.

MRB employs magnetorheological fluids (MRFs) that have controllable rheological characteristics through variation in magnetic field. MRFs are created by adding micron-sized iron particles to an appropriate carrier fluid such as oil, water or silicon. Their rheological behavior is almost the same as that of the carrier when no external magnetic field is present. However, when exposed to a magnetic field, the iron particles acquire a dipole moment aligned with the applied magnetic field to form linear chains parallel to the field (Phillips 1969; An and Kwon 2003). This reversibly changes the liquid to solid-like that has a controllable yield strength, whose magnitude depends directly on the magnitude of the applied magnetic field. A basic configuration of MRB was introduced by (Park et al. 2006). As shown in Fig. 2, in this configuration, a rotating disk (3) is enclosed by a static casing (5), and the gap (7) between the disk and casing is filled with the MR fluid (a typical MR fluid is a type of functional fluid which has a suspension of magnetic particles in inert carrier liquids). A coil winding (6) is embedded on the perimeter of the casing and when electrical current is applied to it, magnetic fields are generated, and the MR fluid in the gap becomes solid-like instantaneously. The shear friction between the rotating disk and the solidified MR fluid provides the controllable retarding torque and the magnitude of the shear friction is directly related to the applied magnetic flux density on the fluid.

The literature presents a number of MR fluid-based brake designs, e.g., (Carlson et al. 1998; Webb 1998; Carlson 2001; Lord Corporation Material Division 2003; Liu et al. 2006; Park et al. 2006). In (Carlson et al. 1998; Carlson 2001), Carlson of Lord Corporation proposed and patented general purpose MRB actuators, which subsequently became commercially available (Lord Corporation Material Division 2003). In Webb (1998), an MRB design was proposed for exercise equipment (e.g., as a way to provide variable resistance). Recently, an MRB was designed and prototyped for a haptic application as well (Liu et al. 2006).

Detailed analysis and design considerations of an MRB can be found in our previous work (Karakoc et al. 2008). In this work, practical design criteria such as material selection, sealing, working surface area, viscous torque generation, applied current density, and MR fluid selection were considered to select a basic automotive MRB configuration. Then, a detailed finite element analysis (FEA) is performed to analyze the resulting magnetic circuit and heat distribution within the MRB configuration. This is followed by design optimization to obtain optimal design parameters that can generate the maximum braking torque in the chosen configuration. A prototype MRB was subsequently built and tested and the experimental results showed a good correlation with the finite element simulation predictions.

4 MRB model

The MRB design optimization is based on the model for calculating the braking torque resulted from different MRB configurations. The idealized characteristics of the MR fluid can be described effectively by using the Bingham plastic model (Phillips 1969; An and Kwon 2003; Kordonsky 1993; Weiss et al. 1994). According to this model, the total shear stress τ is:

$$\tau = \tau_H \operatorname{sgn}(\dot{\gamma}) + \mu_p \dot{\gamma} \tag{1}$$

where τ_H is the yield stress due to applied magnetic field, μ_p is the no-field plastic viscosity of the fluid and $\dot{\gamma}$ is the shear rate. The braking torque for the geometry shown in Fig. 2 can be defined as follows:

$$T_b = \int_A \tau r dA = 2\pi N \int_{r_1}^{r_2} (\tau_H \operatorname{sgn}(\dot{\gamma}) + \mu_p \dot{\gamma}) r^2 dr \tag{2}$$

where A is the working surface area (the domain where the fluid is activated by applied magnetic field intensity), r_2 and r_1 are the outer and inner radii of the disk, N is the number of disks used in the enclosure and r is the radial distance from the centre of the disk.

Assuming the MR fluid gap in Fig. 2 to be very small (e.g., ∼1 mm), the shear rate can be obtained by:

$$\dot{\gamma} = \frac{rw}{h} \tag{3}$$

with the additional assumption of linear fluid velocity distribution across the gap and no slip conditions. In (3), w is the angular velocity of the disk and h is the thickness of the MR fluid gap. In addition, the yield stress, τ_H, can be approximated in terms of the magnetic field intensity applied specifically onto the MR fluid, H_{MRF} , and the MR fluid dependent constant parameters, k and β, i.e.

$$\tau_H = k H_{MRF}^{\beta} \tag{4}$$

By substituting (3) and (4), the braking torque equation in (2) can be rewritten as:

$$T_b = 2\pi N \int_{r_1}^{r_2} \left(k H_{MRF}^{\beta} \operatorname{sgn}(\dot{\gamma}) + \mu_p \frac{rw}{h} \right) r^2 dr \tag{5}$$

Then, (5) can be divided into the following two parts after the integration:

$$T_H = \frac{2\pi}{3} N k H_{MRF}^{\beta} \left(r_2^3 - r_1^3 \right) \tag{6}$$

$$T_\mu = \frac{\pi}{2h} N \mu_p \left(r_2^4 - r_1^4 \right) w \tag{7}$$

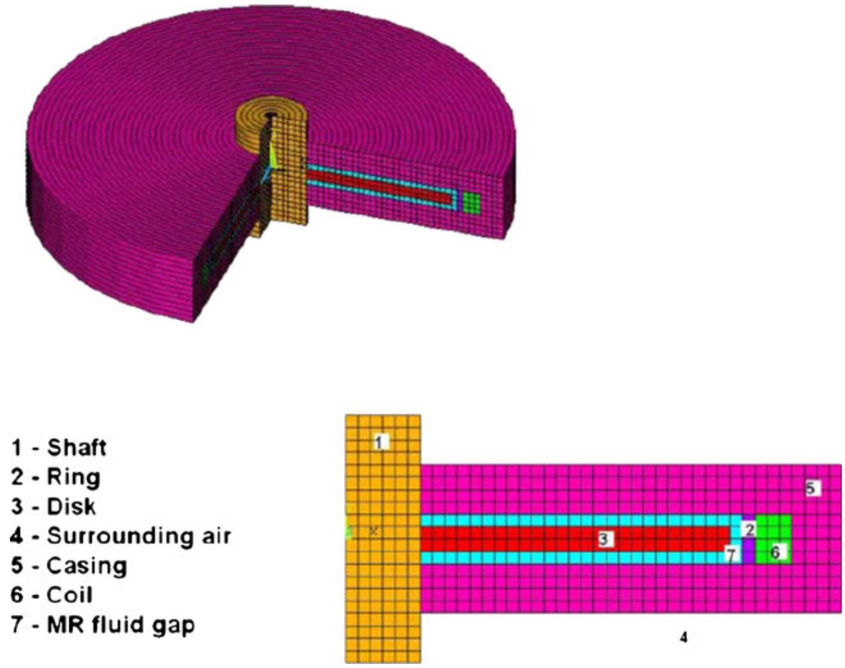

Fig. 2 Basic MRB design (Park et al. 2006)

where T_H is the torque generated due to the applied magnetic field and T_μ is the torque generated due to the viscosity of the fluid. Finally, the total braking torque is $T_b = T_\mu + T_H$. From the design point of view, the parameters that can be varied to increase the braking torque generation capacity are: the number of disks (i.e. N), the dimensions and configuration of the magnetic circuit (i.e. r_2, r_1, and other structural design parameters shown in Fig. 2), and H_{MRF} that is directly related to the applied current density in the electromagnet and materials used in the magnetic circuit.

The flux density is radially changing with the distance from the symmetry axis. Thus, the shear values are integrated over the working surface area to obtain the braking torque. H_{MRF} is not constant and the magnetic field in MRF is not homogenous. The flux varies in the radial and axial direction. In the simulation, in order to calculate the shear stress, the flux density in the vicinity of the shear disk is used. So the values are taken from the fluid-shear disk boundary and used in order to solve for the braking torque calculations.

In order to solve for the braking torque, the magnetic field distribution over the MRB domain has to be calculated. Therefore, a nonlinear finite element model (FEM) for the MRB is created and, using this model, the braking torque generation for different configurations can be calculated. For the current comparison study of the global optimization

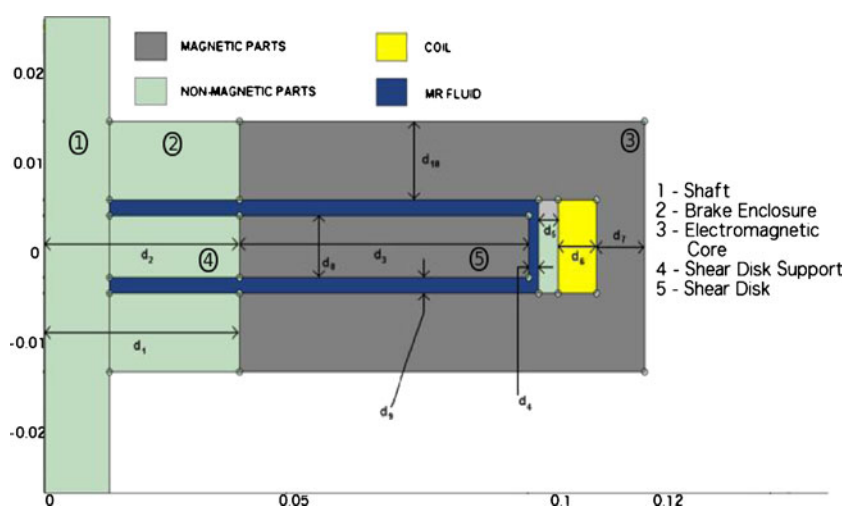

Fig. 3 Chosen MRB based on the design criteria and dimensional parameters related to magnetic circuit design

algorithms, a one disk MRB configuration, shown in Fig. 3, is adapted as the benchmark design.

5 Formulation of the optimization problem

The cross-section of the selected one disk configuration of the MRB is shown in Fig. 3. As a next step, the chosen design configuration was optimized for higher braking torque and lower weight. In setting up such an optimization problem for the MRB, a cost function was defined by including the braking torque and weight as functions of the dimensional parameters of the magnetic circuit shown in Fig. 3.

The objective function of the MRB optimization problem is defined as

$$\text{Minimize}: f(\mathrm{d}) = k_w \frac{W}{W_{ref}} - k_t \frac{T_H}{T_{ref}} \tag{8}$$

where;

$$k_w + k_T = 1 \tag{9}$$

subject to:

$$W < 150\,\mathrm{N} \quad \text{and} \quad D_{brake} < 240\,\mathrm{mm} \tag{10}$$

in which W(N) is the weight of the actuator, T_H (Nm) is the braking torque generated due to applied magnetic field, D_{brake} is the overall diameter of the actuator, d = [d$_1$, d, ..., d$_{10}$]^T is the design variable vector that consists of the dimensional parameters shown in Fig. 3 and k_w and k_T are the weighting coefficients. In order to solve the objective function, k_w and k_T were set to be 0.1 and 0.9, respectively, as the maximum torque generation is of our primary concern. In addition, the reference weight value W_{ref} was obtained considering the overall system weight of the CHB (Karakoc et al. 2008). Moreover, since the braking torque

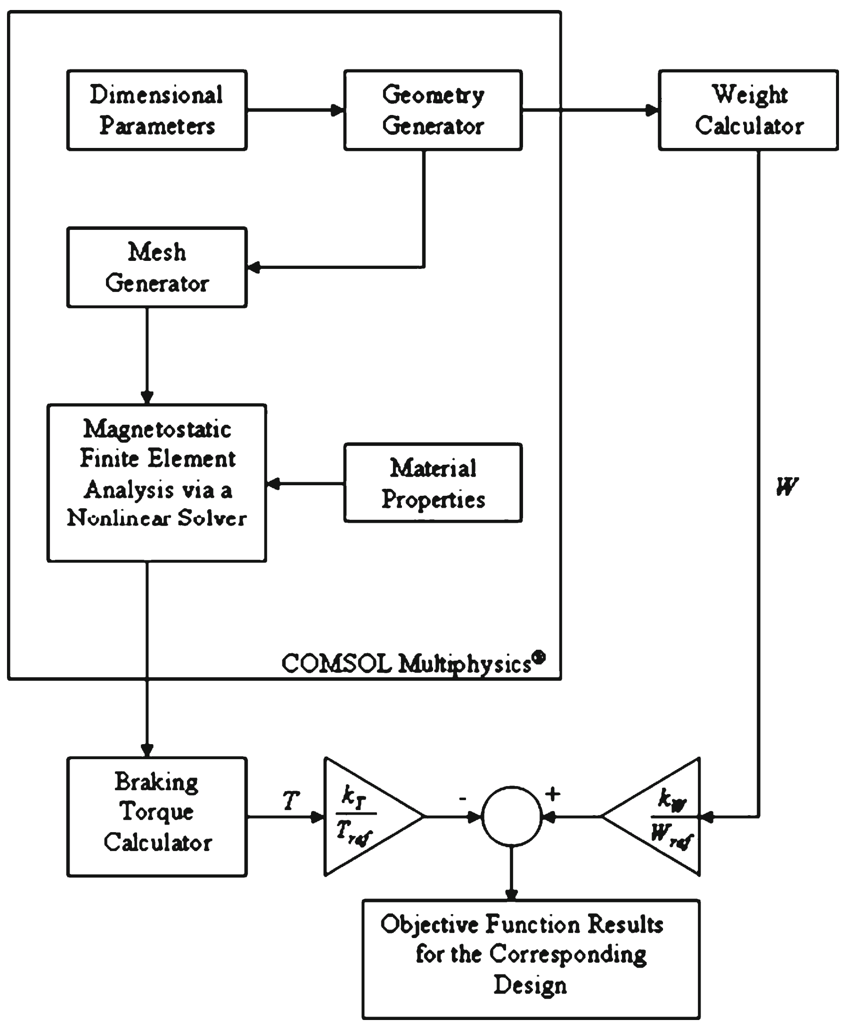

Fig. 4 MDO procedure for computing the cost function a random design

Table 1 Comparison of the resulting optimum design parameters and computational efficiency

Global optimization algorithm	Obtained optimum function value	Obtained optimum design parameters (cm)	Number of iterations	CPU time (h)	Relative CPU time (h)
Simulated annealing (SA)	−1.3623	2.28, 3.99, 3.75, 0.27, 0.50, 2.35, 1.10, 1.97, 0.10, 1.27	2500	4.0348	1.344
Genetic algorithms (GA)	−1.0998	3.84, 2.47, 5.01, 0.30, 0.76, 1.85, 1.15, 2.00, 0.10, 1.23	1100	3.002	1.000
Space exploration (SEUMRE)	−1.1630	1.04, 2.92, 4.71, 0.15, 0.73, 2.15, 1.13, 1.82, 0.11, 1.34	115	1.145	0.381

generated by the proposed MRB configuration is comparably less than that of the CHB, T_{ref} was selected to be 20 Nm. This reference torque value was selected by checking a number of random MRB designs which satisfied the constraints.

As the constraints for the optimization problem, the weight of the actuator was set to be smaller than the weight of the CHB, i.e. $W < 150$ N. In addition to this, since the brake should fit into a wheel, the diameter of the MRB is set to be smaller than the inner diameter of the wheel. In this study, the brake is optimized for the standard 13 in. wheel whose inner diameter is 240 mm, i.e. $D_{brake} < 240$ mm.

Due to the multidisciplinary nature of the design problem, the Multidisciplinary Design Optimization (MDO) procedure is used in the calculation of the objective function for each design as illustrated in Fig. 4. Detailed analysis for forming the MDO problem can be found in (Karakoc et al. 2008). The optimization problem is then solved using the SEUMRE algorithm. To assess the performance of the algorithm, two other commonly used global optimization search methods, GA and SA, are also used as comparison tests. The results are presented and discussed in detail in the following section.

6 Results of the optimization and discussions

For a given MRB design configuration, COMSOL Multiphysics™, a commercial FE software package, was used to calculate the magnetic flux density within MRB. MATLAB™ was then used to solve for the weight and the braking torque using the given brake configuration and the FEA result. Each of the three global optimization programs has been run multiple times and the best optimum result is accepted and used as the measure for search efficiency, as given in Table 1. A PC with dual core Intel Xeon (E5140) processor and 8 GB memory is used during the design optimization. All computations were carried out in MATLAB environment. The global optimization programs are either downloaded from the referenced sites or based on the research codes written by the authors.

The obtained optimum function values are −1.3623, −1.0998 and −1.1630 from SA, GA and SEUMRE, respectively. It appears that SA outperformed the rest in terms of the solution accuracy. However it took SA approximately 4.035 h to reach the optimum solution with high accuracy. If we take the computation cost into consideration, SEUMRE performed well in comparison to SA and GA. Because GA

Table 2 Optimized MRP prototype specifications

Main specifications of MRP	Optimization methods used		
	SA	GA	SEUMRE
Diameter (mm)	240.00	240.00	240.00
Coil wire size	AWD 21 (dia. 0.77 mm)		
MR fluid used	MRF-132DG		
Maximum current applied (A)	1.80	1.80	1.80
Maximum current density applied (A/m^2)	2.54E+6	2.54E+6	2.54E+6
Magnetic materials used	Steel 1018	Steel 1018	Steel 1018
Weight (kg)	14.8708	14.4875	13.7014
Amount of MR fluid used (m^3)	6.725E-05	6.77E-05	5.44E-05
Maximum braking torque (Nm)	33.9896	27.92	30.04

is well known and widely used algorithm it was considered as a reference to measure the performance of all. The last column in Table 1 represents optimum solutions. SEUMRE and SA required 1/3 and 1.3 times of the CPU time, comparing with GA. Table 1 illustrates the needed computation time and number of iterations by these three algorithms to converge to the global optimum. SEUMRE used less computation time and required least number of iterations (115 iterations) to converge to the design optimum. SA and GA required 2500 and 1100 iterations, respectively. SA is the worst in terms of both the number of iterations and CPU time although it does lead to a significantly better minimum in the global optimization.

The optimal design parameters from the global optimization are given in Table 2. One of the very important considerations of MRB system with many rotating parts is the weight. The optimal design using SEUMRE leads to a minimum weight of 13.7 kg, compared to 14.67 and 14.49 Kg from SA and GA, respectively.

Tables 4 and 5 in Appendix 2 showed the results for one disk and at different weight factors. These two scenarios were tested and shown even though they are not of concern in this application since maximizing the torque and minimizing the weight is the goal of this paper. In Table 4 the weight was emphasized over the torque so a higher weigh factor is assigned for the weight while a low weight factor is assigned for the torque. It can clearly be observed that the weight and the torque results are not good. In Table 5 same weight factor has been assigned to the weight and the torque. The results have a little improvement compared to the pervious scenario.

In this application, the SA algorithm with a deterministic local optimization core search scheme led to more accurate optimum regarding the obtained optimum value of the objective function. While the SEUMRE algorithm uses the Kriging search scheme and GA uses the stochastic search method, excellent for high order, large dimensional problems, came slightly short on the convergence accuracy. The

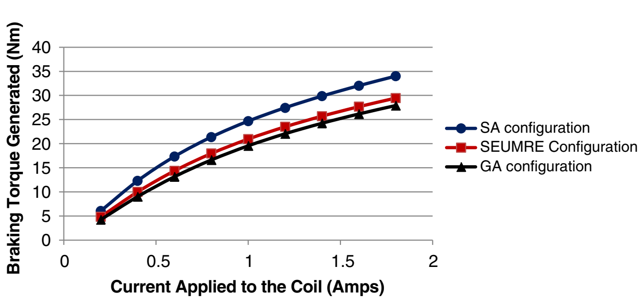

Fig. 5 MRB design optimization results: braking torque comparisons

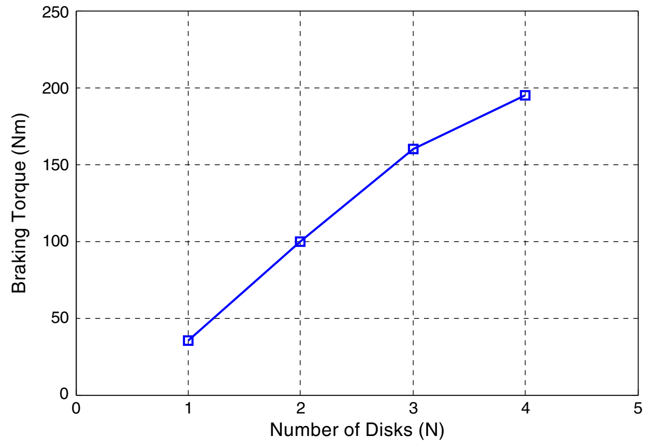

Fig. 6 Braking Torque versus the number of disks

resulting small accuracy difference may be quite different for another problem.

The other important consideration is the maximum torque generated during braking. Figure 5 shows the relation between the current applied to the coil in *Amps* and the generated braking torque in *Nm* obtained using the three global optimization algorithms. Optimization result from SA gave better results than SEUMRE and GA. The amount of the generated braking torque obtained by SA is 33.47 Nm compared to 30.04 and 27.92 Nm from SEUMRE and GA, respectively (see Table 2).

Also in this paper, the optimal design parameters of MRB for different number of disks (2, 3, and 4 disks) were determined. The optimization results of different number

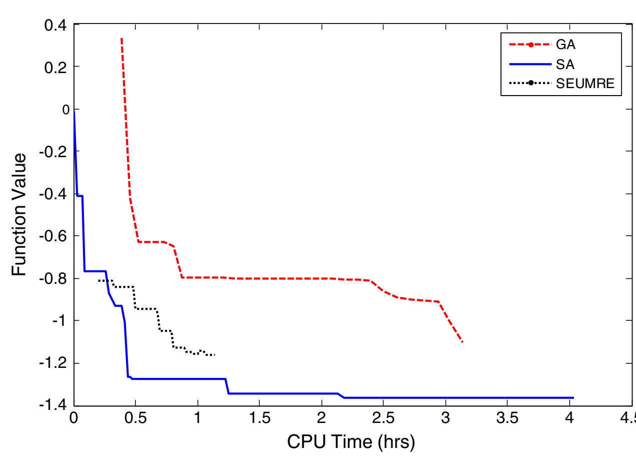

Fig. 7 Accuracy and computation cost relationship for one disk MRB ($K_T = 0.1$ and $K_w = 0.9$)

of disks and different weight coefficients can be found in Appendix 2 in this paper. The results presented in Tables 6, 7 and 8 in Appendix 2 shows that as the number of the disks increases the torque generated increases. Figure 6 supports that claim. It also reports that SA outperformed the other optimization methods in terms of the obtained optimum function value and the torque generated. SA obtained the max torque values. SEUMRE obtained the best weight with comparably good Torque values and at less computational cost as can be seen in Tables 6, 7, and 8 in Appendix 2.

The overall performance of an optimization algorithm for computation intensive design optimization problems, such as the MRB design, is judged using the following two criteria.

- Computational efficiency

The number of iterations and computation time needed by SEUMRE is always less than that for GA and SA. It is well reflected by the results from the design optimizations using SEUMRE, GA and SA.

- Performance or convergence accuracy

SEUMRE can locate the global optimum quickly and reach comparable accuracy for intensive computation optimization applications. In this work, SA converged to the global optimum with higher accuracy due to its capable, deterministic local optimization search scheme. It also performs comparably well regarding the value of the objective function. The optimum objective function of SA was 17% better than that of SEUMRE and 24% better than that of GA. However, much greater computation efforts (high number of iterations and CPU time) are needed to reach this accuracy.

Figure 7 shows the tradeoff between accuracy and computational cost. It can be clearly observed from Fig. 7 that both SA and GA take significant time to converge to a minimum. Among these methods, SEUMRE has the best accuracy to computational time ratio; however SA converges to results better than SEUMRE results faster than SEUMRE. Although the SA results are significantly enhanced, significant improvements in the computational time makes SEUMRE a faster and accurate alternative for existing stochastic optimization algorithms.

7 Conclusions

In this work, the new space exploration and region elimination based SEUMRE, and the conventional GA and SA global optimization algorithms were used for the design optimization of an automotive MRB system. The optimal design parameters of the MRB system were identified with maximum braking torque generated at the rotating disks and minimum brake system weight. The results obtained from the three optimization algorithms were compared against each other in terms of computational efficiency and accuracy. The results indicated that SEUMRE was computationally advantageous than GA and SA. However, it was also found that SA provided significantly better optimization results than GA and SEUMRE regarding the obtained value of the objective function. The advantages of SEUMRE as a new promising global optimization algorithm for complex engineering applications requiring intensive numerical computation are demonstrated.

Appendix 1: Sample of test results of SEUMRE method

Table 3 Results of representative benchmark problems using SEUMRE method

Test problem	Number of dimension	Global optimum solution: Analytical	Global optimum solution: Obtained	# Function evaluation	CPU time (s)	RMSE
Shubert	2	−186.7309	−186.730	56	4.10	1.2e-06
Sphere	10	0.0000	1.3059	282	158.3	2.8e-05
Hartmann, H6	6	−3.3220	−3.3222	79	14.2	0.0151
Hartmann, H16	16	25.875	26.8800	262	433.2	0.0042

RMSE root mean square error

Appendix 2: Comparison of the resulting optimum design parameters and computational efficiency for different number of disks and weight coefficients

Table 4 Optimization results for one disk when $K_T = 0.1$ and $K_w = 0.9$

Global optimization algorithm	Obtained optimum function value	Obtained optimum design parameters (cm)	#Fun. evaluations	CPU time (h)	Weight (Kg)	Torque (Nm)
Simulated annealing (SA)	−0.057	0.50, 0.50, 2.00, 0.10, 0.30, 0.50, 0.50, 0.20, 0.10, 0.20	2602	10.78	0.2327	0.1684
Genetic algorithms (GA)	0.0343	4.85, 2.32, 2.28, 0.25, 0.42, 0.71, 0.59, 0.24, 0.15, 0.22	1420	1.88	0.5901	0.2384
Space exploration (SEUMRE)	0.0842	2.42, 0.97, 2.04, 0.18, 0.51, 1.25, 0.69, 0.41, 0.22, 0.82	280	0.38	1.4329	0.3784

Table 5 Optimization results for one disk when $K_T = 0.5$ and $K_w = 0.5$

Global optimization algorithm	Obtained optimum function value	Obtained optimum design parameters (cm)	#Fun. evaluations	CPU time (h)	Weight (Kg)	Torque (Nm)
Simulated annealing (SA)	−0.3835	4.26, 4.93, 2.83, 0.23, 0.30, 3.00, 0.63, 1.98, 0.1, 0.58	2582	9.80	6.940	25.822
Genetic algorithms (GA)	−0.2983	4.04, 4.68, 2.99, 0.28, 0.78, 2.15, 1.01, 2.00, 0.13, 0.97	1040	2.32	10.119	26.696
Space exploration (SEUMRE)	−0.2694	0.67, 4.83, 2.31, 2.60, 0.66, 2.81, 0.66, 1.90, 0.22, 0.28	245	0.36	4.8468	18.101

Table 6 Optimization results for two disks when $K_T = 0.9$ and $K_w = 0.1$

Global optimization algorithm	Obtained optimum function value	Obtained optimum design parameters (cm)	#Fun. evaluations	CPU time (h)	Weight (Kg)	Torque (Nm)
Simulated annealing (SA)	−4.2300	2.44, 3.71, 3.69, 0.10, 0.51, 1.04, 1.71, 1.76, 0.10, 0.45, 1.36	2514	9.80	15	101.042
Genetic algorithms (GA)	−2.4904	2.40, 2.47, 4.55, 0.10, 0.72, 2.54, 0.70, 0.63, 0.12, 1.08, 0.83	1060	1.53	13.9832	60.2853
Space exploration (SEUMRE)	−3.9608	1.55, 4.38, 4.19, 0.16, 0.60, 1.16, 1.00, 0.88, 0.16, 0.59, 1.25	798	1.88	14.9894	94.75

Upper and Lower Boundaries for Multi-Disk Cases: UB = [0.05, 0.05, 0.12, 0.003, 0.01, 0.03, 0.02, 0.02, 0.003, 0.02, 0.02]; LB = [0.005, 0.005, 0.02, 0.001, 0.003, 0.005, 0.005, 0.002, 0.001, 0.002, 0.002]
UP upper boundaries, *LB* lower boundaries

Table 7 Optimization results for three disks when $K_T = 0.9$ and $K_w = 0.1$

Global optimization algorithm	Obtained optimum function value	Obtained optimum design parameters (cm)	#Fun. evaluations	CPU time (h)	Weight (Kg)	Torque (Nm)
Simulated annealing (SA)	−6.7697	3.50, 4.34, 2.89, 0.28, 0.40, 1.99, 0.96, 0.40, 0.10, 0.70, 1.31	2525	9.84	14.999	160.2925
Genetic algorithms (GA)	−3.8378	2.02, 3.65, 4.24, 0.13, 0.34, 1.35, 0.70, 0.69, 0.10, 0.86, 0.74	1360	2.28	14.0575	91.7346
Space exploration (SEUMRE)	−5.7366	0.90, 3.60, 3.46, 0.16, 0.41, 1.49, 1.34, 0.50, 0.11, 0.71, 1.23	609	1.03	14.9525	136.1700

Table 8 Optimization results for four disks when $K_T = 0.9$ and $K_w = 0.1$

Global optimization algorithm	Obtained optimum function value	Obtained optimum design parameters (cm)	#Fun. evaluations	CPU time (h)	Weight (Kg)	Torque (Nm)
Simulated annealing (SA)	−8.2925	1.82, 0.50, 2.71, 0.10, 0.54, 2.11, 1.17, 0.32, 0.10, 0.29, 1.05	2536	7.27	14.999	195.8254
Genetic algorithms (GA)	−2.7125	4.81, 0.94, 4.76, 0.20, 0.57, 2.14, 0.70, 0.34, 0.10, 1.08, 0.74	1040	1.52	14.9702	65.61
Space exploration (SEUMRE)	−5.5800	1.15, 2.84, 2.83, 0.12, 0.56, 1.93, 0.85, 0.50, 0.11, 0.80, 1.17	637	1.14	14.9981	132.6464

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