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Domain-specific initial population strategy for compliant mechanisms using customized genetic algorithm

Deepak Sharma · Kalyanmoy Deb · N. N. Kishore

Abstract Genetic algorithms (GAs) can precisely handle the discrete structural topology optimization of single-piece elastic structures called compliant mechanisms. The initial population of these elastic structures is mostly generated by assigning the material at random. This causes disconnected or unfeasible designs and further rule-based repairing can result in representation degeneracy. However, the problem-specific initial population can affect the performance of GAs like other operators. In this paper, a domain-specific initial population strategy is developed that generates geometrically feasible structures for path generating compliant mechanisms (PGCMs). It is coupled with the elitist non-dominated sorting genetic algorithm (NSGA-II) which has been customized for structural topology optimization. The performance of initial population strategy over random initialization using customized NSGA-II is checked on single and bi-objective optimization problems. Based on the results, it is observed that the custom initialization outperforms the random initialization by dominating all the solutions and exploring larger area of posed objectives. The elastic structures obtained by solving two examples of PGCMs using domain specific initial population strategy are also presented.

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1 Introduction

Compliant mechanisms (CMs) are flexible elastic structures which can deform to perform assigned tasks. If the task of elastic structures is to trace or generate the user-defined path, then these mechanisms are called as *path generating compliant mechanisms* (PGCMs). These elastic structures can be designed using the pseudo-rigid-body-model (Howell and Midha 1996; Hetrick and Kota 1999) or using topology synthesis method for single-piece CMs (Ananthasuresh et al. 1994; Sigmund 1997). However, CMs are preferred over pseudo-rigid-body mechanisms because CMs are joint-less and monolithic structures, involve less friction, wear and noise (Howell and Midha 1994). They are lightweight devices and also, easy to manufacture without assembly (Ananthasuresh and Kota 1995).

Methods like homogenization method (Bendsøe and Kikuchi 1988; Nishiwaki et al. 1998), material density approach (Yang and Chuang 1994), solid isotropic micro-structure with penalization (SIMP) (Bendsøe 1989; Zhou and Rozvany 1991), regularization method (Belytschko et al. 2003), level-set method (Sethian and Wiegmann 2000; Wang et al. 2003), and evolutionary structural optimization (ESO) (Xie and Steven 1993), are commonly used for topology optimization of structures. Although the homogenization and ESO methods are computationally effective, the convergence to global optimal solution for the given structural optimization problem is not guaranteed (Rozvany 2001; Zhou and Rozvany 2001). Moreover, the topology

optimization of structures based on homogenization, material density and SIMP methods transforms the discrete problem into continuous design variables problem. In this case, threshold value is required to suppress intermediate design variable values that can lead to non-optimal solution. Another issue arises when implementing the large deformation analysis based synthesis with conventional methods to circumvent non-convergence or snap-through during the analysis procedure (Saxena 2005). In that case, the search may not proceed further and may require user intervention to physically remove the sub-region close to their non-existing states that cause bifurcation.

When genetic algorithms (GAs) are used (Chapman et al. 1994; Chapman and Jakiela 1996; Duda and Jakiela 1997; Jakiela et al. 2000), the field of topology optimization of structures has benefited with following properties; (i) precise handling of discrete optimization of structures, (ii) evolution of improved designs, (iii) independence from the design sensitivity information, (iv) suitability in solving complex non-linear structural optimization problems for global optimization, (v) independent handling of multiple objectives etc. Hence, GAs are preferred over the above mentioned methods. However, the structural topology optimization becomes computationally expensive as GAs work on population of structures. A large portion of the computation time is consumed in the finite element analysis (FEA) of structures as compared to GA operators that too increases with the mesh refinement (Schoenauer 1995).

All above methods have their own merits and demerits but they commonly suffer from checkerboard and point singularity problems. These difficulties can be suppressed by using controlling filters used in image processing (Sigmund 1997), using non-conforming finite elements (Jang et al. 2003), using honeycomb representation (Saxena and Saxena 2007) with material-mask overlay strategy (Saxena 2008) etc. However, the disconnected topologies or geometrically unfeasible structures can also evolve using GAs. This problem can arise due to the improper representation of structures at the beginning and during GA iterations. Thus, the material connectivity is required at various parts of the structures to ensure geometrically feasible designs.

Earlier, the material connectivity was ensured by switching the disconnected element to void (Chapman et al. 1994; Chapman and Jakiela 1996; Duda and Jakiela 1997; Jakiela et al. 2000; Deb and Chaudhuri 2005) or by eliminating the bits of disconnected elements using chromosome mask method (Fanjoy and Crossley 2002). But, these approaches can result in representation degeneracy. This issue is handled by penalizing the total area of unusable material (Schoenauer 1995). But the limitation arises when the different combinations of disconnected material have the same area. Using Bézier curve and morphological representation, the connectivity is guaranteed (Tai and Chee 2000), but

the topologies of structures greatly rely on the number and nature of connecting curves. Piece-wise cubic Bézier curves are further used (Wang and Tai 2004), however the morphological representation may hamper the heuristic behavior of GAs for global optimization. The image processing based connectivity analysis with dynamic penalty has also been proposed to handle degeneracy (Wang and Tai 2005). However, it does not eliminate the one-node connectivity. The connectivity among the applied boundary and support conditions of PGCMs can also be ensured by joining the islands of material at the parts of mentioned conditions using straight line (Sharma et al. 2006, 2008c). If any element of material is still not connected to these islands, then it is changed to void. This connectivity scheme can minimize the representation degeneracy up to some extent and does not hamper the working of GA. However, the significantly change in the shape of elastic structures cannot be overruled.

The problem of representation degeneracy can be reduced if GAs start with geometrically feasible or connected structures. In this way, the dependency of any rule based connectivity or repair technique can be avoided and GAs are allowed to work heuristically with some ad-hoc technique. Moreover, the well-suited initial population can speed up the convergence of GAs on specific problems and methods (Haubelt et al. 2005; Hill and Hiremath 2005). In spite of that, the random initialization of population is not always worse at-least for continuous optimization problems (Maaranen et al. 2007). It means that the problem-specific initial population will always affect the convergence, performance and ability of GAs cannot be ensured beforehand. Nevertheless, GA using problem specific knowledge can be a potential tool for global structural topology optimization (Wang et al. 2006). Few studies have concentrated on the initial population for GAs such as Voronoi-based representation (Hamda et al. 2002), morphological representation (Tai and Chee 2000), by introducing the optimal solutions of single-objective optimization (Madeira et al. 2005) and by reducing the structure size with initial population strategy (Toğan and Daloğlu 2008). Otherwise in all studies, the initial structures are generated by assigning the material at random to the grids or elements.

In this paper, a domain-specific initial population strategy is developed that generates geometrically feasible structures for path generating compliant mechanisms (PGCMs). The performance of proposed initial population over random initialization of structures is checked on single and bi-objective sets. The optimization problem is subjected to Euclidean distance based constraints on precision points (Sharma et al. 2006, 2008a, b, c). The above PGCM problem is solved using the elitist non-dominated sorting genetic algorithm (NSGA-II, Deb et al. 2002), which has been modified with two-dimensional (2-D) structure representation scheme, 2-D crossover, parallel computing and with

additional binary bits to identify applied boundary and support conditions. The remaining paper is organized in five sections. In Section 2, the bi-objective optimization formulation for PGCM is discussed. The multiobjective evolutionary algorithm with proposed initial population strategy and other customized schemes are discussed in Section 3. Section 4 shows the performance of developed initial population over random initialization when coupled with customized NSGA-II. The elastic structures of two PGCMs are also shown and the results are discussed. Section 5 outlines the conclusions drawn from the study with a note on scope of future work.

2 Description of problem formulation

In this section, we describe the problem formulation of path generating compliant mechanisms (PGCMs). Earlier studies show two methods of designing PGCMs. In the first method, Euclidean distance formulation is used as the objective to minimize the gap between the precision points of prescribed path and corresponding points on the actual path traced by the elastic structures (Tai et al. 2002; Saxena 2005). However, this formulation does not limit the maximum gap between the prescribed path and actual path which may result in undesirable performance. Hence, these paths can be apart from each other. Fourier shape descriptor-based objective function is the second method to design PGCMs (Rai et al. 2007). In this formulation, parameters have to be specified a-priori that may affect the optimum solution. We consider that the functional aspect of PGCM is to generate the prescribed path. Hence, this task has to be represented as constraint instead of objective function. The constraints based on Euclidean distance formulation can limit the maximum gap between the prescribed and actual paths for all feasible solutions (Sharma et al. 2006, 2008a, b, c).

A hypothetical case is shown in Fig. 1 to describe the constraint formulation. The user-defined path is represented by N precision points. The corresponding points on the actual path traced by the elastic structure are evaluated from the non-linear finite element analysis based on equal load steps. The constraints are designed by evaluating Euclidean distance (d_1) between the current (i) and previous ($i-1$) precision points and is multiplied by a factor η defined as *percent of allowable deviation*. Then, another Euclidean distance (d_2) between the current precision point (i) and the corresponding point (i_a) on the actual path is calculated. Thereafter, the constraint "$d_2 \leq d_1$" is imposed at each precision point. A pictorial significance is also shown in Fig. 1 where, if a circle of radius d_1 at the current precision point (i) is drawn, then the corresponding point (i_a) of actual path must lie within or on the circle to satisfy this constraint. The mathematical representation of constraints at N precision
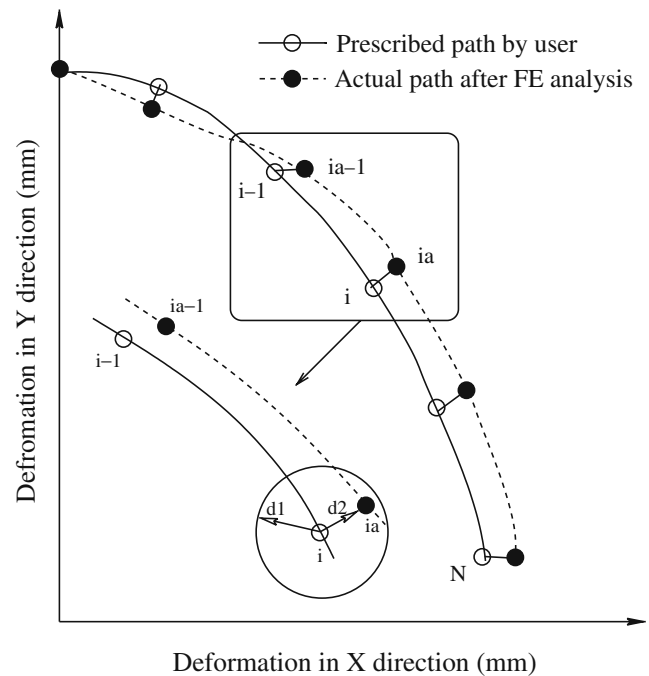

Fig. 1 The prescribed path and an actual path traced by the elastic structure after FE analysis

points is given in (1). Any elastic structure that satisfies the above constraints will trace the user-defined path depending on user-defined allowable deviation (η).

Using the constraint formulation of (1), PGCMs can be designed using various features. In the present study, the minimization of weight is used as primary objective for single-objective optimization. For bi-objective study, the secondary objective of minimum supplied input energy to elastic structure is coupled with the primary objective. The elastic structures are evolved using both the objective sets and the performance of domain specific initial population is checked.

Single-objective optimization:

   Minimize: Weight of structure

Bi-objective optimization:

   Minimize: Weight of structure (primary obj.),

   Minimize: Supplied input energy to structure

        (secondary obj.),   (1)

Both problems are subjected to:

$$1 - \frac{\sqrt{(x_{ia}-x_i)^2+(y_{ia}-y_i)^2}}{\eta \times \sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}} \geq 0, \quad i = 1, 2, ..., N$$

$$\sigma_{\text{flexural}} - \sigma \geq 0,$$

where $\eta = 15\%$, is kept fixed in this paper (Sharma et al. 2009), and σ_{flexural} and σ are the flexural yield strength of
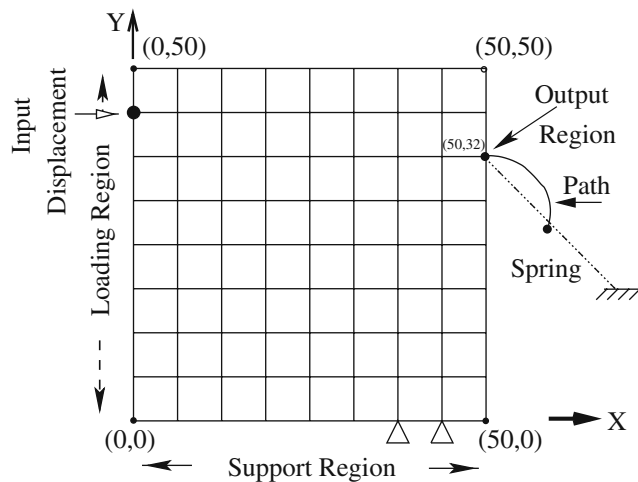
Fig. 2 Design-domain with loading, output and support regions

material and the maximum stress developed in the structure respectively.

In this paper, the design domain (50 mm by 50 mm) of Fig. 2 is used for compliant mechanisms that is mainly categorized into three regions. The first region is called the support region where the nodes of an element of the elastic structure are restrained with zero displacement. In the second region, defined as loading region, an input displacement boundary condition is applied at a node of the element. The output region is the third region, in which a fixed point on the elastic structure traces out the user defined path. In this work, the origin of the design domain is fixed on its left lower corner and the output region is positioned at the coordinate (50, 32) of the structure. As in Fig. 2, a spring of constant stiffness ($\kappa = 0.4$ KN/m) is attached at the output point to provide some resistance during the deformation of elastic structures.

3 Customized NSGA-II algorithm

GA using problem specific knowledge can be a potential tool for global structural topology optimization (Wang et al. 2006). The benefits of these specific information can be used at various places like, to formulate the problem (Sharma et al. 2006, 2008c), to develop problem specific GA operators (Hamda et al. 2002; Madeira et al. 2005; Tai and Chee 2000) etc. The existing NSGA-II algorithm has also been modified by the authors to customize it for solving topology optimization of compliant mechanisms (Sharma et al. 2008a, b, c).

Though many multiobjective evolutionary algorithms (MOEAs) have been developed, there are only a few dominance ranking based algorithms that are really effective to solve multiobjective optimization problems. Mostly, these

MOEAs differ in their ranking methods which helps to select and propagate good individuals to the next iteration. Among them, NSGA-II is the fastest. It has also shown to have a good convergence property to the global 'Pareto-optimal' front as well as to maintain the diversity of population on the 'Pareto-optimal' front for various two objective test and engineering problems (Deb 2001). Thus, NSGA-II is used as a global search and optimizer in this paper. In this section, we draw our attention to discuss different customization schemes that modify the existing NSGA-II for topology optimization of structures.

A local search method is also coupled with the customized NSGA-II algorithm as a post-processing method to further refine the non-dominated compliant mechanisms. The basic detail of local search based customized NSGA-II algorithm is shown in Fig. 3 which is also discussed in the subsequent sections.

3.1 GA parameters

The NSGA-II parameters are given in Table 1. In this work, the binary string representing a structural member of the population is made of two sets as shown in Fig. 4. First set of 625 bits represents the material distribution as described in Section 3.2. The decoded value of second set identifies the support and loading positions in their respective regions (refer to Fig. 2), and the magnitude of input displacement. For the same, 12 bits of second set are further
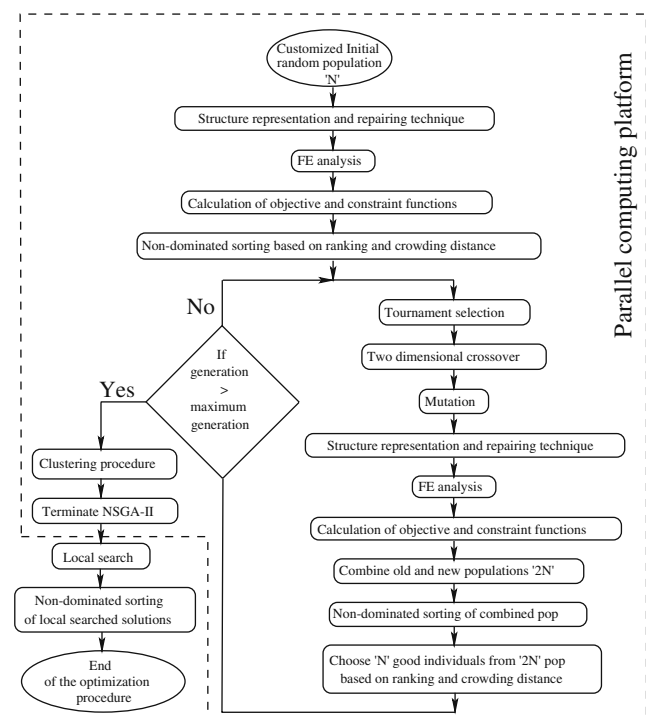

Fig. 3 A flow chart of customized NSGA-II algorithm

Table 1 GA parameters

Population	240	Generation	100
Crossover probability	0.95	Mutation probability	1/string length
String length for a structure	625	String length for boundary and support conditions	12

divided into three groups of five, three and four bits respectively as shown in Fig. 4. The decoded value of first five bits indicates the location of an element from the origin where the elastic structure is to be supported. The decoded value of subsequent three bits helps to determine the loading position, that is, a node where the input load is applied. The decoded value of last four bits are used to evaluate the magnitude of input displacement which can vary from 1 to 16 mm at step of 1 mm. Using this flexibility, the optimization algorithm will find the optimum set of these applied boundary and support conditions to promote non-dominated solutions during NSGA-II run. The additional significance of this flexibility will be discussed later along with the results presented in the study.

3.2 Structure representation scheme

A binary string of 625 bits is used to represent the material distribution for the elastic structure. First, a binary string is copied to two dimensional representation as shown in Fig. 5. Thereafter, the material-void representation of each grid is chosen based on the binary bit value. For example, the bit value '1' signifies that material is present whereas, '0' represents the void. This scheme divides a design domain of structure into $25 \times 25 (= 625)$ grids in x and y directions, respectively.

3.3 Domain specific initial population strategy

As discussed in Section 1, the evolved structures using GA can have disconnected or unfeasible geometrical topologies. This issue arises when GA operators are performed on the population. However, it can also appear at the beginning of GA when material is assigned at random. Earlier studies
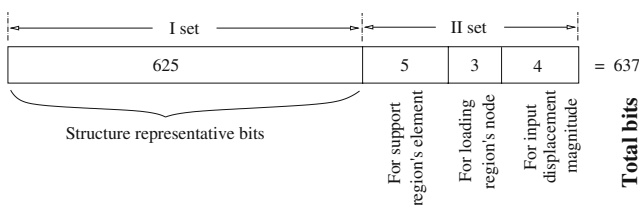
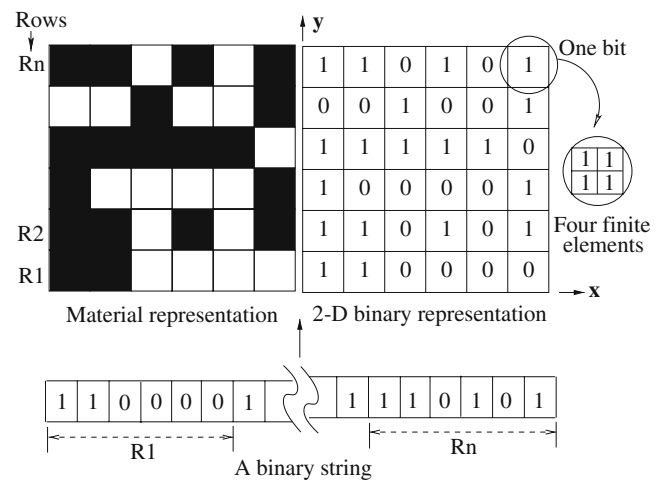**Fig. 4** A binary string comprises of two sets

Fig. 5 A representation of structure using binary string

described in Section 1 concentrated on the connectivity techniques. However, it can be suppressed at the beginning by evolving initial population of geometrically feasible structures. In this case, the representation degeneracy issue can be reduced which may come with various rule-based connectivity techniques existing in the literature (discussed in Section 1).

Nevertheless, the problem-specific initial population can also affect the performance of GA as other operators do. However, this feature is not much explored in the topology optimization of structures even though it may look intuitive. Only a few studies (discussed in Section 1) incorporate the initial representation. On the contrary, Maaranen et al. (2007) showed that the initial random population of GA is not always worse, at least for continuous optimization. In this conflicting scenario, the performance of developed initial population is checked on single and bi-objective frames over random initialization of material. The statistical quantitative analysis is done to support results for global optimization. In the following paragraphs, the developed initial population strategy is discussed for PGCMs.

For describing the initial population strategy, a hypothetical case is shown in Fig. 6 which shows the connectivity between the support and loading regions. Here, the element positions of support and loading regions are calculated after decoding second set of binary string (refer to Section 3.1 and Fig. 4). The location of output region is fixed in this study because this point will trace the user-defined path.

For connecting the support and loading regions, a random integer number between $1-5$ is generated to decide the number of intermediate points through which these two regions get connected. Depending upon the number of intermediate points, the coordinates of each intermediate point is randomly generated within the design domain. For the hypothetical case of Fig. 6, four random points (P1, P2,
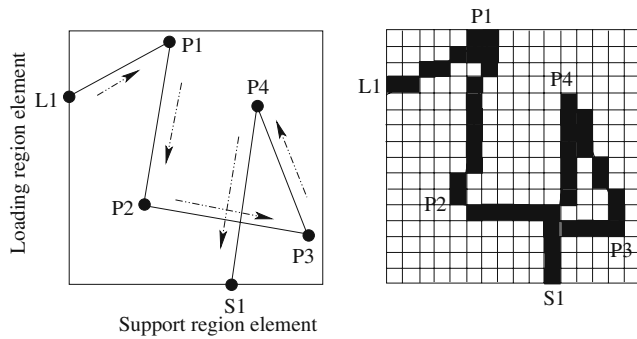
Fig. 6 Connectivity between support and loading regions

P3 and P4) are generated within the design domain and the support (S1) and the loading (L1) positions are connected through these points by straight lines. Thereafter, a material is assigned to those elements where these straight lines pass. A material connectivity of the above mentioned regions is also shown in Fig. 6. Similarly, a set of piece-wise linear line segments between the support and output regions and another set between the loading and output regions are explained. Depending on the randomly generated intermediate points, an initial population for the customized NSGA-II is developed. The above-defined strategy not only incorporates the diversity in the initial population, but also ensures the well-connected continuum structures at support, loading and output positions.

3.4 GA operators

In this study, a two-dimensional crossover operator is used that swaps either a patch of row or column as shown in Fig. 7 by flipping a coin (Deb and Goel 2001; Deb and Chaudhuri 2005). The width and location of patch are found randomly and it is swapped between the two parents. For the crossover of remaining 12 bits of binary string, a standard single point crossover operator is used.

In this paper, the mutation of each bit of binary string representing the structure is done with a probability of 1/string length. For mutating the remaining 12 bits of same binary
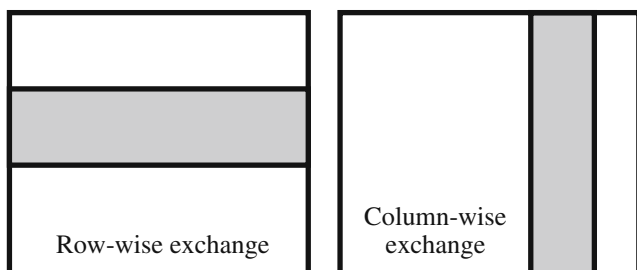
string, the values of support and loading regions, and magnitude of input displacement are decoded and then perturbed in the range of $\{-2, 2\}$ at their decoded values. It is also ensured that the perturbed values of above three conditions do not fall outside their respective bounds. After perturbation, these mutated values are again coded into the binary string of 12 bits to get the nearest integer number at the decoded value.

3.5 Connectivity and repairing techniques

Crossover and mutation operators of GA create new solutions that differ from the parent solutions. These GA operators are responsible for the search aspect of GA. However, the new solutions may suffer from disconnected topology problem. In this paper, the connectivity technique is discussed using a hypothetical case of Fig. 8 where the support region (**S**) is disconnected from the loading (**L**) and the output (**O**) regions. In this disconnected scenario, the individual distances are calculated from the centroid of each grid of material of **S** to the centroid of each grid of material of **L** and **O**. Then, the straight lines (**L1, L2**) are drawn from the centroid of those two grids which show minimum distances between **S-L** and **S-O**. In this way, the connectivity among **S**, **L** and **O** regions of a structure is checked and ensured.

The point singularity between the two material element's may arise due to the developed initial population strategy, GA operators and after the connectivity technique. An ad-hoc repairing technique is employed in this paper which is motivated from the image processing concept (Sigmund 1997). For any element of material, there are eight possible neighborhoods as shown in Fig. 9. Among them, material at positions 2, 4, 6 and 8 can create point singularity. To eliminate this problem, an extra material has to be filled. Suppose position 2 creates point connectivity, then an extra material
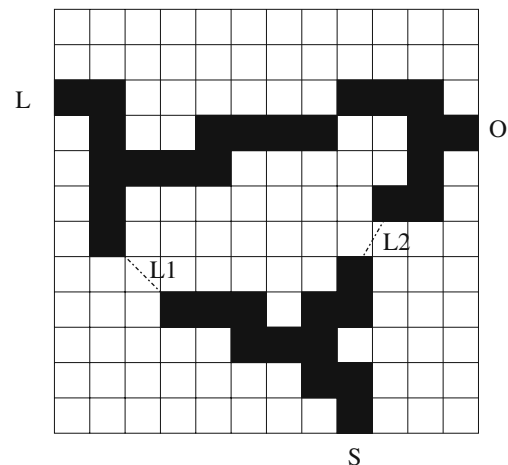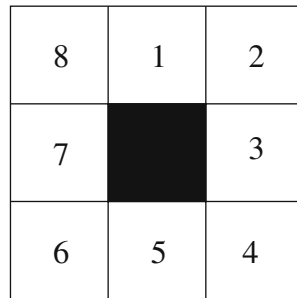

Fig. 7 A two dimensional crossover which swaps a patch of row/column between the two parent solutions

Fig. 8 Disconnected topology: a hypothetical case

can be filled at 1 or 3 with equal probability. In this way, the point singularity for each element of material is checked and eliminated.

Due to the mutation operator, any floating material element can appear which is not connected to the topology. In this case, this isolated element is changed to void by assigning value '0'.

3.6 Finite element analysis (FEA)

After the custom initialization, structure representation, connectivity and repairing techniques, the elastic structures are geometrically feasible without checkerboard, point singularity and floating element problems. These elastic structures are now undergo for FEA. In this study, one grid of a structure (as described in Section 3.2) is further discretized into four finite elements with same Boolean variable value as shown in Fig. 5. Therefore in the present process, the structure is discretized with 4×625 ($= 2,500$) 4-node rectangular finite elements and analyzed through a non-linear large deformation FE analysis using *ANSYS* package. However, the GA operations are performed on 625 bits representing the same structure.

3.7 Parallel computing

A distributed computing platform is used in the present study to reduce the computational time involved in the topology optimization of compliant mechanisms. In this parallelization process, the root processor first initializes a random population. Then, it divides the entire population into different sub-populations in proportion to the number of processors available. After this, each sub-population is sent to different slave processors. These slave processors further perform finite element computation and evaluate the objective functions and constraints. These values are then sent to the root processor. Thereafter, the root processor performs the GA operators, like selection, crossover and mutation operators, non-dominated front ranking etc. on the population and replaces it with good individuals.

The above process is repeated till the termination criterion of NSGA-II is met. A MPI based Linux cluster with 24 processors is used in the present study.

3.8 Clustering procedure

For an adequate convergence near to the global 'Pareto-optimal' front, the evolutionary algorithms (EAs) need a fairly large population and generations depending upon the problem complexity. At the end, EA evolves large number of non-dominated solutions. The clustering procedure employed in the study selects a few solutions for end users. In this procedure, the neighboring solutions are grouped together and solutions from each group representing that region of the non-dominated front are chosen as representative solutions (Zitzler 1999). Figure 10 shows the procedure pictorially.

3.9 Local search method: post-processing

The local search method used in the present work is a combination of mutation and hill climbing process. As a single objective function is needed for hill climbing, the bi-objective problem is reduced to single-objective using weighted sum method. The scaled single-objective function is minimized in the present study as in (2).

$$Minimize\, F(x) = Minimize \sum_{j=1}^{n} \frac{\overline{w}_j^x \left(f_{j_{\max}}^x - f_j^x \right)}{f_{j_{\max}}^x - f_{j_{\min}}^x}, \quad (2)$$

where f_j^x is jth objective function, $f_{j_{\min}}^x$ and $f_{j_{\max}}^x$ are minimum and maximum values of jth objective function in the population respectively, n is number of objectives and $\overline{w}_j^x$ is the corresponding weight to the jth objective function which is computed as:

$$\overline{w}_j^x = \frac{\left(f_{j_{\max}}^x - f_j^x \right) \setminus \left(f_{j_{\max}}^x - f_{j_{\min}}^x \right)}{\sum_{k=0}^{M} \left(f_{k_{\max}}^x - f_k^x \right) \setminus \left(f_{k_{\max}}^x - f_{k_{\min}}^x \right)}, \quad (3)$$

where M is the number of representative solutions after clustering procedure. In (2), the values of the objective functions
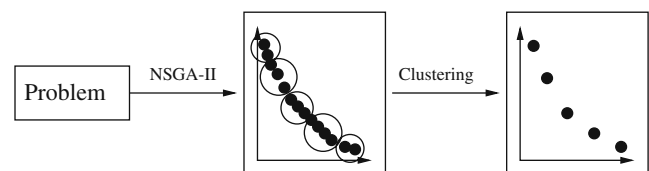

Fig. 10 A clustering procedure

are normalized to avoid bias towards any objective function. In this approach, the weight vector decides the importance of different objectives, in other words it gives the direction to local search in the objective space (Deb 2001). As (3) suggests, these weights are calculated based on their positions in two-objective space after the termination of NSGA-II.

In the local search method, the weighted sum of scaled fitness of a selected representative solution is evaluated as given in (2). Thereafter, a binary string of the solution is converted into a two-dimensional array and checked for the grids having a material. For each material's grid, there are maximum of eight possible neighborhoods as shown in Fig. 9. One by one, all neighboring bits including its own bit, value are mutated. The new elastic structure is now extracted on which the finite element computations are performed for objective function and constraints values. If this new structure does not satisfy any of the constraint, then the change in the new string is discarded and the old values are restored. Otherwise, the weighted sum of scaled fitness of the new string is calculated and compared with that of the old string's value. If mutating a bit brings an improvement in the scaled fitness, then the change is accepted. Else, the change is discarded and the previous values are restored. When all the bits having material are mutated along with their neighborhoods, the grids of the new elastic structure are again checked for material and mutated as discussed above. When the scaled fitness of elastic structure before checking the material's grid is same as after mutating all bits having material with their neighborhood, the local search method is terminated. In the same way, all representative solutions are mutated. This post-processing technique is an exhaustive method that may require considerable computation time to refine the solutions. It is expected that the use of proposed initial population strategy can assist the customized NSGA-II to evolve the improved set of representative non-dominated solutions. Thereafter, the local search gets boosted to refine these improved solutions in short span of time. Note that the aim of executing this exhaustive local search is to evolve the optimum solutions which cannot be further improved in their objective values.

4 Results and discussion

In this section, the performance of domain-specific initial population is checked on single and bi-objective optimization problems over random initialization. Statistical performance assessment tools are used for which 10 independent runs of customized NSGA-II are taken. Two examples of PGCMs are solved and their elastic structures are presented.

In this paper, a few parameters are kept constant such as, a material with Young's modulus of 3.3 GPa, flexural yield stress of 6.9 MPa, density of 1.114 gm/cm^3 and

Table 2 Solutions of single-objective optimization

Solutions (weight in gm)→	GA	Local search
Custom initialization	0.682	0.525
Random initialization	1.043	0.645

Poisson ratio of 0.40, is assumed. The direction of input displacement is fixed along x-axis of design domain. The prescribed path is represented by five precision points and the trajectory traced by output point of elastic structures is evaluated through a geometric nonlinear FE analysis using $ANSYS$ package. The stress concentration near the support is ignored in this work. Maximum six representative solutions are chosen from the non-dominated solutions using clustering procedure.

4.1 Comparison and performance assessment

In this section, the example of curvilinear path generating compliant mechanism is solved using single and bi-objective optimization formulations as given in (1). The customized NSGA-II algorithm is run with domain specific initial population strategy ('custom initialization'). Then, the same algorithm with random initialization scheme is run in which material is assigned at random to initialize the population. The results of single-objective optimization of the minimum weight is shown in Table 2. The custom initialization evolves the lighter weight elastic structure than the random initialization after the termination of GA. When the local search is performed independently on both the solutions, the solution generated using custom initialization is again lighter weight than the random initialization. Thereafter, the bi-objective optimization problem is solved. In this case, the non-dominated solutions of custom initialization dominate all solutions of random initialization after the completion of NSGA-II. The
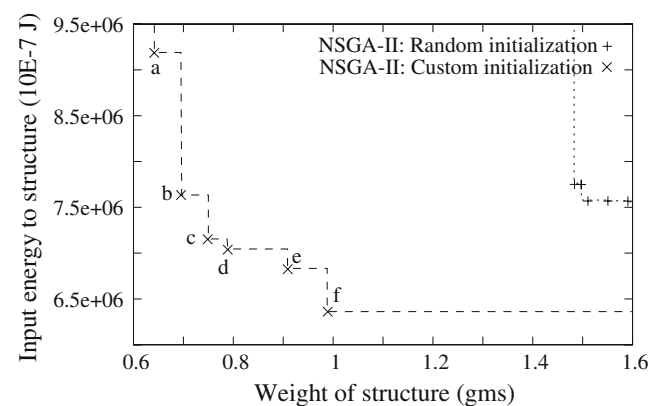

Fig. 11 Non-dominated solutions from both population initializations after the completion of NSGA-II
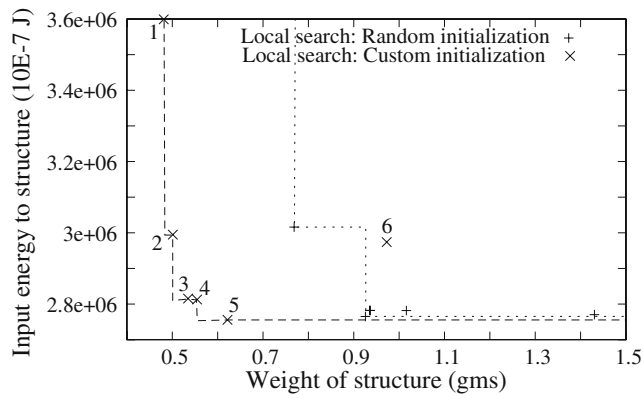
Fig. 12 Local search solutions from both population initializations

solutions are shown in Fig. 11 which also reveal largely explored area by the non-dominated solutions of custom initialization in the two-objective space. As a good platform of solutions is provided by the customized NSGA-II using the domain specific initial population strategy, the non-dominated local search solutions (1–5) outperform the results of random initialization as shown in Fig. 12. Hence, the solutions 1–5 become the part of 'Pareto-optimal' front. Note that the local search is executed independently on the representative NSGA-II solutions which does not follow the domination principle of multi-objective optimization. However, only the non-dominated PGCMs are presented in this paper.

In this paper, the quantitative performance assessment of the customized initial population strategy over the random initialization is also done. The customized NSGA-II is executed for ten different runs with both population initializations. $R-$ and $hypervolume$ indicators are used (Hansen and Jaszkiewicz 1998; Zitzler and Thiele 1999) that can signify the proximity and spread of non-dominated front with respect to the reference set. The reference set of non-dominated solutions is designed by choosing the non-dominated solutions over the different runs of both population implementations. The statistical values of these indicators are given in Table 3 in which the value -1 is

Table 3 $R-$ and $hypervolume$ indicators values of customized NSGA-II algorithm for different initializations

Initialization→	Random		Customized	
Indicators→	$R-$	Hyp	$R-$	Hyp
Mean	3.61E−01	7.43E−01	1.25E−01	2.78E−01
Median	3.85E−01	7.94E−01	1.27E−01	2.72E−01
Std .Dev.	0.10795	0.20667	0.03897	0.08750
Best	1.08E−01	2.48E−01	7.64E−02	1.80E−01
Worst	4.87E−01	9.45E−01	1.45E−01	4.09E−01

(a) Sol. 1: Undeformed (b) Sol. 1: Final deformed

(c) Sol. 2: Undeformed (d) Sol. 2: Final deformed

(e) Sol. 3: Undeformed (f) Sol. 3: Final deformed

(g) Sol. 4: Undeformed (h) Sol. 4: Final deformed

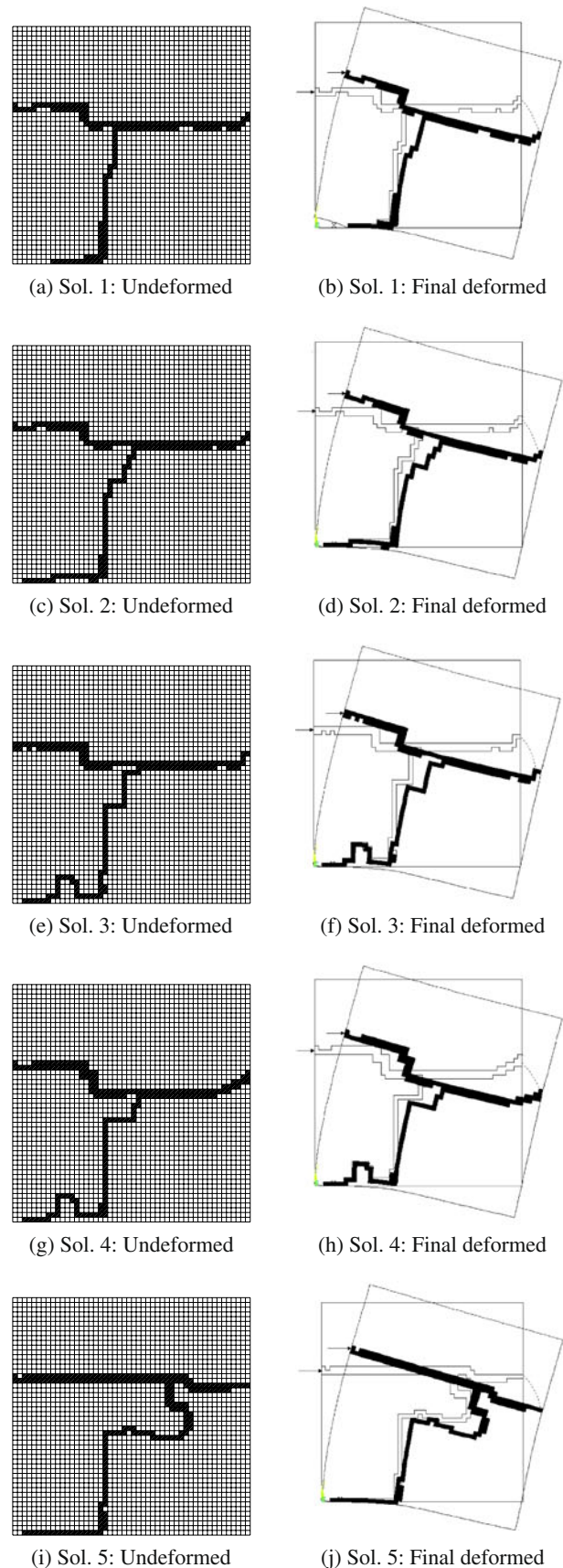(i) Sol. 5: Undeformed (j) Sol. 5: Final deformed

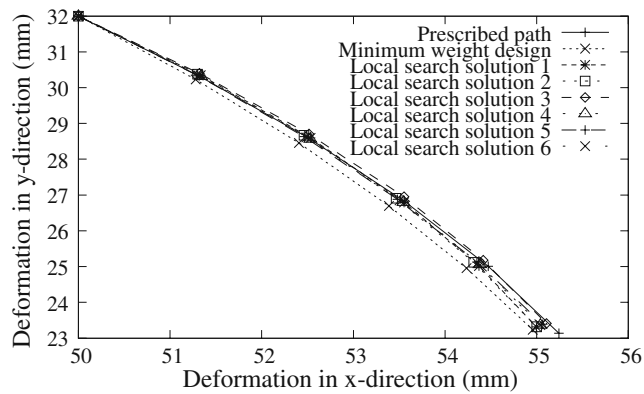**Fig. 13** Non-dominated PGCMs generating curvilinear path

Fig. 14 Prescribed path and path traced by all local search solutions of single- and bi-objective studies

the best and $+1$ is considered as worst (Zitzler et al. 2003; Knowles et al. 2006). The statistical values of the custom initialization are not only better than the random initialization but also close to zero. This observation supports the role of domain specific initial population strategy for global optimization.

4.1.1 Non-dominated PGCMs and their properties

In the last section, the solutions 1–5 are evolved as 'Pareto-optimal' solutions which are shown in Fig. 13. The solution 1 evolves as minimum weight PGCM, but at the same time it requires larger input energy to deform the elastic structure and to follow the prescribed path. While the solution 5 requires minimum supplied input energy but it is heavier in all five. Similarly, other solutions show 'trade-off' between the posed objectives. Although these solutions are topologically same, but they have different shapes. For example, the solution 5 has a 'winding shape' (refer to Fig. 13(i)) that may reduce the requirement of supplied input energy to elastic structure. Similarly, the solutions 3 and 4 also have some

'winding shapes' at two places. The solutions 1 and 2 seem rigid.

The path traced by all elastic structures are shown in Fig. 14 along with a prescribed curvilinear path. The constraints at precision points impose the adherence of actual paths with the prescribed path. The values of maximum allowed gap d_1 and actual distance between the prescribed and actual paths d_2 are given in Table 4. It can be seen here that the d_2 value increases for all solutions as they follow precision points 1–5. This shows that the precision points defining the extreme part of prescribed path are more critical. But the importance of constraints on initial precision points cannot be ignored that assisted the customized NSGA-II to evolve feasible compliant mechanisms (Sharma et al. 2009). The prescribed path for this example is designed such that the output point of each elastic structure has to deform 10.48% in x-direction and 17.72% in y-direction with respect to the size of design domain.

In this paper, the applied boundary and support conditions are identified by customized NSGA-II. Table 5 shows these conditions for single and bi-objective optimization. The identical loading position and same input displacement magnitude are observed for all solutions in both studies. However, the support positions are different. These conditions are evolved without any priori information to assist and propagate non-dominated solutions. Moreover, the designer and decision makers can benefit from such flexibility to explore the non-optimum conditions which might have been considered in the previous practices.

The computational time of customized NSGA-II with local search is reported in Table 6. The FEA of elastic structures is done in parallel because it consumes maximum computational time than the GA operators. During the NSGA-II run 24 processors are used that reduced the computational time approximately in the proportion of processors used. However, the local search is performed independently on different processors that consumes considerable time to improve the solutions.

Table 4 Deviation at precision points

Precision points	1	2	3	4	5
Max. allowed d_1	0.3196	0.3142	0.3074	0.3084	0.3092
Single-objective study 'Minimum weight design'					
d_2	0.1032	0.1739	0.2127	0.2470	0.3091
Two-objective study					
Solution 1: d_2	0.0290	0.0301	0.0324	0.1047	0.3058
Solution 2: d_2	0.0531	0.0829	0.1220	0.1979	0.3091
Solution 3: d_2	0.0761	0.1084	0.1262	0.1812	0.3091
Solution 4: d_2	0.0226	0.0407	0.0256	0.1064	0.3091
Solution 5: d_2	0.0278	0.0516	0.0939	0.1766	0.3091
Solution 6: d_2	0.0161	0.0217	0.0588	0.1502	0.3083

Table 5 Evolved applied boundary and support conditions of curvilinear PGCMs

Conditions → Study ↓	Support position (mm) (from the origin)	Loading position (mm) (from the origin)	Input displacement (mm)
Single-obj.	20	32	7
Bi-obj.	Sol. 1 at: 8	32	7
	Sols. 2−5 at: 2		

4.2 Straight line generating PGCMs

In this section, another example of PGCMs generating straight line path is solved. The compliant mechanisms are generated using bi-objective set and domain specific initial population with customized NSGA-II. Note that the same categorization of design domain in Fig. 2 is used that introduces difficulty for optimization technique. Because such type of categorization of applied boundary and support conditions allow elastic structures to follow some curvilinear path instead of straight line.

For this example of compliant mechanism, five representative NSGA-II solutions are evolved (*a−e*) as shown in Fig. 15. After the local search, only two solutions (1 and 2) are evolved as non-dominated solutions. It is worthwhile to mention again that the local search method improves the solution based on single-objective function which does not follow the definition of domination. Thus, the solutions 1 to 5 are evolved independently.

The elastic structures of non-dominated solutions are shown in Fig. 16. Both PGCMs look very similar. They have two common closed loops and two similar segments. However, it is interesting to observe how they show 'trade-off' between the posed objectives. The smaller closed loop and the segments of solution 1 look thinner than solution 2 that makes the difference in weight. For supplied input energy, two regions of solution 1 are highlighted by circle and rectangle in Fig. 16(b). A relatively larger deformation can be seen at the region with circle of solution 1 than

Table 6 Computation time of curvilinear PGCM problem

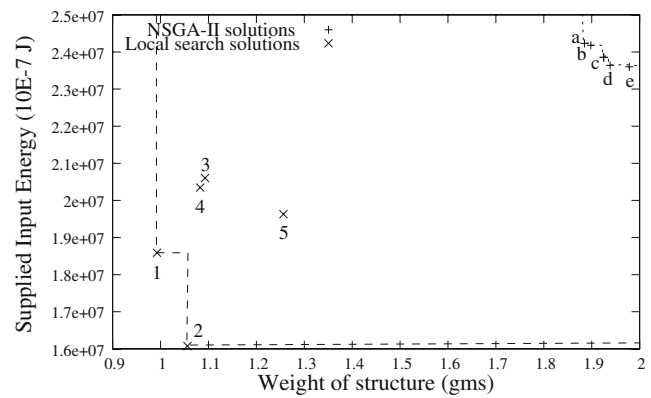| Problem	NSGA-II time (hrs)	Local search time (hrs)
Single-objective	5.59	12.19
Two-objective	5.47	Solution 1: 8.65
		Solution 2: 9.15
		Solution 3: 9.61
		Solution 4: 10.93
		Solution 5: 11.52
		Solution 6: 25.07

Fig. 15 NSGA-II and local search solutions for straight line PGCM

the corresponding region of solution 2. This causes a difference in the styles of segment joining the bigger closed loop and straight line tracing output point for both solutions. The same segment of solution 2 looks more rigid than that of solution 1. As more thinner regions appear in solution 1, these regions are deformed relatively more and thus, require larger supplied input energy so that the output of the elastic structure can generate the prescribed straight line path.

The straight line path traced by all local search solutions along with the prescribed path is shown in Fig. 17. It shows that the elastic continuum structures do not trace the exact straight-line path. However, the imposed constraints at precision points limit the maximum gap between the prescribed and actual paths. The real values of maximum allowed gap
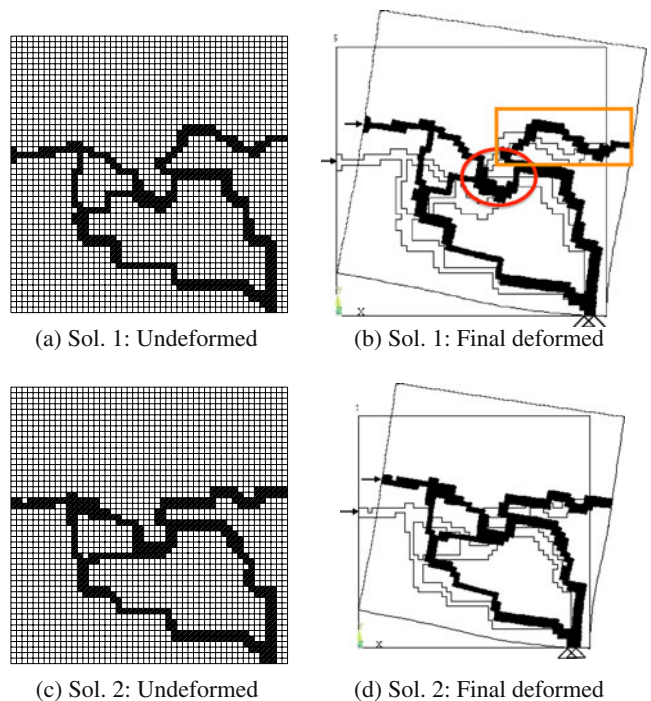
(a) Sol. 1: Undeformed   (b) Sol. 1: Final deformed

(c) Sol. 2: Undeformed   (d) Sol. 2: Final deformed
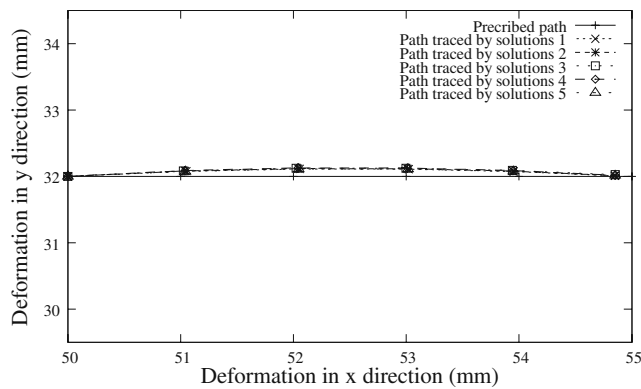
Fig. 16 Non-dominated straight line PGCMs

Fig. 17 Prescribed path and path traced by all local search solutions

d_1 and the actual gap between the paths at precision points d_2 are given in Table 7. For this example, the d_2 value for all local search solutions increases till the precision point 2 and then, it decreases. Finally, the maximum d_2 value can be seen at precision point 5 which makes it critical. The output point of elastic structures are deformed by 10.0% in x-direction and 0.0% in y-direction with respect to the size of design domain to trace the given straight line path.

The identical boundary and support conditions of solutions 1 and 2 are evolved for this example. Both the PGCMs are supported at 46 mm and require 5 mm of input displacement which is applied at 28 mm. An interesting thing can be seen here that these solutions are supported on their right-hand side when tracing the straight-line path. However in the previous example of curvilinear path tracing, PGCMs were supported on their left-hand side. This is quite expected that the elastic structures supported on their right-hand side show minimum tendency to generate higher curvilinear paths for the given categorization of applied boundary and support conditions. On the other hand, left-hand side supported elastic structures tend to trace the curvilinear trajectories. The customized NSGA-II also acknowledges the same support conditions.

4.3 Discussion on performance of customized GA and local search method

In both the examples, the local search method significantly improved the non-dominated solutions of NSGA-II. This justifies its role to further improve the elastic structures. However, it raises a question that the customized GA is not effective and local search method can alone generate 'Pareto-optimal' solutions. In the literature, several limitations of reducing the multi-objective optimization problem into single-objective for generating the non-dominated or 'Pareto-optimal' solutions are discussed. However, a few of them are discussed here that are mainly associated with structural topology optimization. First, the chances of premature convergence is more for single-objective optimization while solving non-linear and multi-modal problems (Deb 2001). Similar fact can be observed particularly for example 2, when the single-objective local search method was executed on five non-dominated solutions ($a−e$ as in Fig. 16). The three solutions (3–5) prematurely converged and only two solutions (1 and 2) were evolved as non-dominated. Second, the local search starting with different initial points can converge to the same final solution (Grosan et al. 2007). It was also observed in earlier studies (Sharma et al. 2006, 2008a, c) that the few representative solutions were converged to the same optimal elastic structure. In that case the performed computations were wasted. Last but not the least, the computations in local search method can be expensive than the present customized GA. It is because (1) has some constraints and the local search method has to first find the feasible solution that must satisfy all the constraints. Only after that, it can improve the solution in the objective value.

On the other hand, the solutions of mulitobjective GAs can also trap in local optimal region. Similar observation is also noticed in this work when the history of non-dominated solutions is plotted in Fig. 18 after every 20 generations. It

Table 7 Deviation at precision points

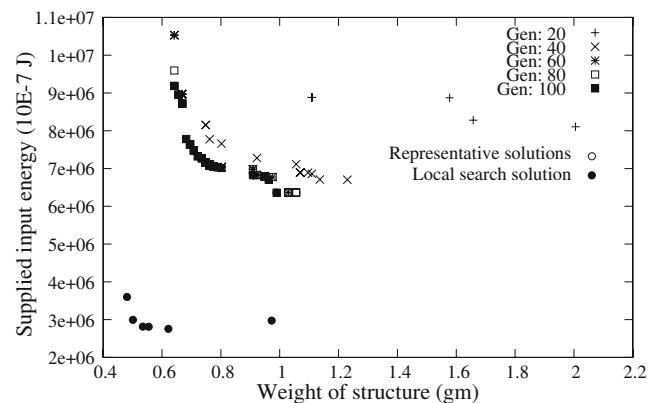| Precision points	1	2	3	4	5
Max. allowed d_1	0.15	0.15	0.15	0.15	0.15
Two-objective study					
Solution 1: d_2	0.0930	0.1260	0.1119	0.0876	0.1498
Solution 2: d_2	0.0901	0.1257	0.1169	0.0957	0.1497
Solution 3: d_2	0.0874	0.1262	0.1243	0.1071	0.1499
Solution 4: d_2	0.0962	0.1350	0.1277	0.1045	0.1498
Solution 5: d_2	0.0869	0.1209	0.1113	0.0911	0.1499

Fig. 18 The history of non-dominated solutions after 20 generations and local search solutions

shows that the progress gets reduced after generation #60. A possible remedy is to use large population and/or iterations depending on the problem complexity (Deb 2001). But it will result in higher computational cost. Thus, the concept of hybrid algorithm is used with NSGA-II that not only improves the non-dominated solutions but also helps them to come-out from the sub-optimal region. Such hybrid techniques are well discussed in the literature and interested readers can refer book by Grosan et al. (2007).

5 Conclusions

In this paper, the initial population strategy has been developed that created geometrically feasible elastic structures by joining the regions of applied boundary and support conditions of path generating compliant mechanisms. The same initial population strategy can be used for various structure topology optimization problems where these applied boundary and support conditions are known or unknown. The results and statistical test demonstrated the successful application of proposed initial population strategy over random initialization. Using proposed initial population strategy, the customized NSGA-II algorithm successfully solved the two examples of PGCMs. The local search further refined the solutions of NSGA-II and assisted them to come out from the sub-optimal region. The elastic structures of both the examples were also presented in this paper. The parallel computing platform reduced the computation time of NSGA-II. However, the local search consumed the considerable computation time. The constraints on the precision points showed the different behavior of PGCMs as curvilinear PGCMs showed active or critical constraints on the extreme part of the prescribed path. In case of the straight line PGCMs, the middle and extreme parts of the path were critical. The criterion of finding the applied boundary and support conditions by customized NSGA-II abided the expected rules of support positions and generated these conditions without a-priori information. These conditions can further be beneficial to check the optimality of predefined conditions in the previous practices for evolving the non-dominated solutions.

In the future work, the effort can be made to generate topologically diverse and improved elastic structures. This can be done by using another objective function and also by developing knowledge-based GA operators.

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