RESEARCH PAPER

Topology optimization of structures in unilateral contact

Niclas Strömberg · Anders Klarbring

Abstract In this paper a general framework for topology optimization of structures in unilateral contact is developed. A linear elastic structure that is unilaterally constrained by rigid supports is considered. The supports are modeled by Signorini's contact conditions which in turn are treated by the augmented Lagrangian approach as well as by a smooth approximation. The latter approximation must not be confused with the well-known penalty approach. The state of the system, which is defined by the equilibrium equation and the two different contact formulations, is solved by a Newton method. The design parametrization is obtained by using the SIMP-model. The minimization of compliance for a limited value of volume is considered. The optimization problems are solved by SLP. This is done by using a nested approach where the state equations are linearized and sensitivities are calculated by the adjoint method. In order to avoid mesh-dependency the sensitivities are filtered by Sigmund's filter. The final LP-problem is solved by an interior point method that is available in Matlab. The implementation is done for a general design domain in 2D as well as in 3D by using fully integrated isoparametric elements. The implementation seems to be very efficient and robust.

Keywords SIMP · Contact · SLP

N. Strömberg (✉)
Department of Mechanical Engineering,
Jönköping University, 551 11 Jönköping, Sweden
e-mail: niclas.stromberg@jth.hj.se, stni@jth.hj.se

A. Klarbring
Division of Mechanics, Linköping University,
581 83 Linköping, Sweden
e-mail: anders.klarbring@liu.se

1 Introduction

A proper modeling of boundary conditions is crucial when performing topology optimization of machine components. Improper modeling will result in poor concepts. Often it is not sufficient to use the same boundary conditions in an optimization as in a direct finite element analysis of the state problem. For instance, a boundary which can appropriately be considered fixed in a direct finite element analysis might be too stiff in a topology optimization analysis, resulting in a too weak design concept. In many optimization situations one must also include contact conditions in order to set up a proper model.

In this work a method for topology optimization that includes frictionless contact is proposed. This is modeled by utilizing Signorini's contact conditions: letting ξ stand for a contact force and η for a contact gap, these can be written as

$$\xi \geq 0, \quad \eta \geq 0, \quad \xi\eta = 0. \tag{1}$$

A main difficulty with these conditions is that ξ cannot be seen as a function of η or vice versa. On the other hand it is possible to find functions Ψ of both variables such that $\Psi(\xi, \eta) = 0$ is equivalent to (1). However, see Christensen and Klarbring (1999) and references therein, such reformulations will be either non-smooth or have a singular Jacobian at the origin (intermediate contact states). Thus, the standard theory of Newton's method will not be applicable, but, nevertheless, since the critical points are isolated, this method may still work in practice. We have implemented the smooth function $\Psi(\xi, \eta) = (\xi - \eta)^2 - \xi|\xi| - \eta|\eta|$, suggested by Mangasarian (1976), and the non-smooth function $\Psi(\xi, \eta) = -\xi + (\xi - \eta)_+$, related to the augmented
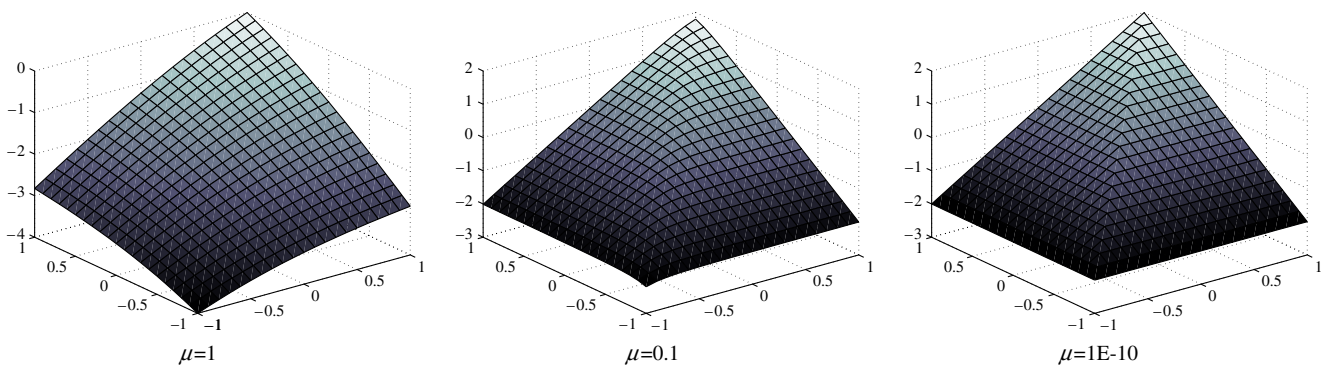
Fig. 1 Λ_μ plotted for different values of μ. A plot similar to the right one is obtained by plotting $2\min(\xi, \eta)$

Lagrangian approach. The index $+$ means that the bracket is zero if the argument is negative. This latter function is equivalent to $\Psi(\xi, \eta) = -\min(\xi, \eta)$. It turns out that an approach based on the non-smooth reformulation of Signorini's contact conditions is significantly more stable and efficient than that based on the smooth reformulation and the presentation in this paper, therefore, concentrates on the non-smooth function.

As a third approach to the treatment of (1), we have also investigated a smoothing technique suggested by Facchinei et al. (1999) and used by Hilding (2000) for solving contact problems. It should not be confused with the more familiar penalty approach, but is instead based on the fact that the smooth relationship

$$\Lambda_\mu = \Lambda_\mu(\xi, \eta) = \xi + \eta - \sqrt{(\xi - \eta)^2 + 4\mu^2} = 0 \qquad (2)$$

is equivalent to $\xi \geq 0$, $\eta \geq 0$, $\xi\eta = \mu^2$, which in turn approximates (1) for small values of μ, see Facchinei et al. (1999). Furthermore, if $\mu \to 0$, then $\Lambda_\mu = 2\min(\xi, \eta)$. This is also illustrated in Fig. 1, where Λ_μ is plotted for different values of μ.

A review of papers concerning topology optimization of structures in unilateral contact gives the conclusion that the number of such papers is surprisingly few. The list is longer, however, if one considers structural optimization for contact problems in general. A review on that subject can be found in Hilding et al. (1999). A pioneering work on topology optimization of structures in unilateral contact is Petersson and Patriksson (1997). A more recent work is Fancello (2006). Another recent work is Mankame and Ananthasuresh (2004), where compliant mechanisms were generated by including contact conditions.

In this work the minimization of compliance for a linear elastic structure in unilateral contact with a rigid support that is modeled by Signorini's contact conditions is considered. As discussed above, Signorini's con-

tact conditions are treated by the smoothing approach of Facchinei as well as by the augmented Lagrangian approach. The state equations are then solved by the Newton method that was used in Strömberg (1997) for solving an augmented Lagrangian formulation of frictional contact. This approach has also been used to solve 3D contact problems (Strömberg 1999), thermo-mechanical friction problems (Ireman et al. 2002), dynamic stick-slip problems (Strömberg 2003), gear noise problems (Klarbring et al. 2004), rigid body impact problems (Johansson 2001) and frictional contact in non-linear elasticity (Strömberg 2005, 2006).

A density approach for the design parametrization, where the SIMP-model is used for penalization of intermediate values, is utilized. The resulting optimization problem is treated by SLP, using an interior point method for the corresponding LP-problem. The sensitivities of the objective function are obtained by the adjoint method. This implies linearizing the state problem as seen in Klarbring and Rönnqvist (1995). The stiffness is calculated by using fully integrated isoparametric elements and Sigmund's filter (Sigmund 2001) is used in order to avoid mesh dependency and checkerboards. The method is implemented in the in-house toolbox Topo2D/3D by using Matlab and Intel Fortran.

The outline of the paper is as follows: in Section 2 the governing equations of the state problem are given, in Section 3 the optimization problem is defined, in Section 4 the numerical treatment is discussed, in Section 5 numerical examples in both 2D and 3D are considered and, finally, some concluding remarks are presented.

2 The state equations

Let us consider a linear elastic body that is represented by a stiffness matrix $\boldsymbol{K}$ obtained by using finite elements. The body is constrained by at least one rigid
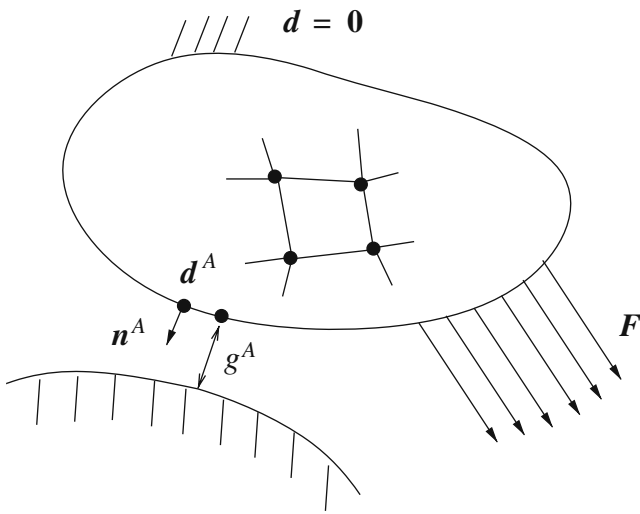
Fig. 2 A linear elastic body unilaterally constrained by a rigid support

support as shown in Fig. 2. The contact between the support and the elastic body is considered to be frictionless. The potential contact surface is defined by a set of potential contact nodes. The normal of the contact surface at contact node A is denoted n^A. The undeformed distance between the contact node and the support in the normal direction is given by g^A. The kinematic constraint representing impenetrability of the rigid support can now be formulated as

$$d^A \cdot n^A - g^A \leq 0, \tag{3}$$

where d^A is the displacement vector of node A. These conditions for all contact nodes can be summarized as the following inequality:

$$C_N d - g \leq 0, \tag{4}$$

where d is the displacement vector, C_N is a transformation matrix containing the normal directions n^A and the column vector g contains all gaps g^A. We will also use the notation C_N^A to represents a row of C_N such that $C_N^A d = d^A \cdot n^A$.

The equilibrium equation[1] of the system reads

$$K d + C_N^T P_N = F, \tag{5}$$

where F is the external forces and P_N is a column vector containing normal contact forces P_N^A, which are

---

[1]Sliding friction might be added by modifying the equilibrium equation as $K d + C_N^T P_N + \mu C_T^T P_N = F$, where μ is a friction coefficient and C_T is a transformation matrix containing tangential contact directions.

subject to Signorini's contact conditions. As discussed in the introduction these conditions read

$$P_N^A \geq 0, \quad C_N^A d - g^A \leq 0, \quad P_N^A \left(C_N^A d - g^A \right) = 0. \tag{6}$$

The displacement state of the contact system is defined by the solution of (5) and (6), which turn out to be the Karush-Kuhn-Tucker conditions for the following minimization problem:

$$\begin{cases} \min_{d} \Pi(d) \\ \text{s.t. } C_N d \leq g, \end{cases} \tag{7}$$

where

$$\Pi(d) = \frac{1}{2} d^T K d - F^T d \tag{8}$$

is the potential energy of our system.

3 The optimization problem

We are interested in minimizing the compliance

$$c = F^T d \tag{9}$$

for the system defined in the previous section. The design parametrization is made by using the SIMP-model, which we briefly describe below; for a more complete presentation we refer to Bendsøe and Sigmund (2003) or Christensen and Klarbring (2009). Thus, the stiffness matrix $K = K(\rho)$ is generated by the following assembly procedure:

$$K(\rho) = \prod_e \rho_e^n k_e, \tag{10}$$

where $\prod$ is an assembly operator, the vector ρ contains density (design) variables ρ_e, such that $\epsilon \leq \rho_e \leq 1$ for each element e. Here ϵ is a small positive number that is set to 0.001 in the calculations. The value $\rho = 1$ represents presence of material and $\rho = \epsilon$ represents no material. The reason for not using $\epsilon = 0$ is that such a choice gives singular stiffness matrices for some designs. k_e is the stiffness matrix of element e for a unit value of ρ_e. In order to obtain close to "0-1"-designs experience has shown $n = 3$ to be a good choice.

The total volume of the design $V = V(\rho)$ is obtained as

$$V = \sum_e \rho_e V_e, \tag{11}$$

where V_e represent the volume of element e. This total volume is constrained by

$$V(\rho) - V_0 \leq 0, \tag{12}$$

where V_0 is the amount of material that can be distributed over the design domain.

Summarizing, the following optimization problem is considered:

$$\begin{cases} \min\limits_{\rho, d} \boldsymbol{F}^T \boldsymbol{d} \\ \text{s.t.} \begin{cases} \begin{cases} \min\limits_{\boldsymbol{d}} \frac{1}{2} \boldsymbol{d}^T \boldsymbol{K}(\rho) \boldsymbol{d} - \boldsymbol{F}^T \boldsymbol{d} \\ \text{s.t. } \boldsymbol{C}_N \boldsymbol{d} \leq \boldsymbol{g} \end{cases} \\ V(\rho) - V_0 \leq 0 \\ \boldsymbol{\epsilon} \leq \rho \leq \mathbf{1}, \end{cases} \end{cases} \quad (13)$$

where $\boldsymbol{\epsilon}^T = \{\epsilon, \ldots, \epsilon\}$ and $\mathbf{1}^T = \{1, \ldots, 1\}$.

4 The numerical treatment

The optimization problem in (13) is solved by SLP. This is done by using a nested approach, such that the problem is solved in the density variables only. A linearization is performed at the current iteration point by using the adjoint method in order to obtain an approximating LP-problem. However, since (13) has the form of a bilevel optimization problem, the linearization is better performed on an alternative system that is equivalent to (13) in the sense that it has the same solution. This is achieved by first using the Karush-Kuhn-Tucker equations (5) and (6) instead of the optimization problem (7), and then reformulating the second of these equations (Signorini's contact conditions) by means of non-linear equations as discussed in the introduction. We will consider two such reformulations: the first one, suggested by Facchinei et al. (1999), is approximate but smooth and has consequently no critical points; the second one, related to the augmented Lagrangian method, is non-smooth, exact and has already been used in a large number of papers on contact, friction and wear as discussed in the introduction. The former may be seen as a regularization of the latter and has theoretical advantages, but as will be seen in the numerical examples, the exact non-smooth reformulation turns out to be slightly more efficient at least for the problems considered in this paper.

In the approach of Facchinei et al. (1999), Signorini's contact conditions, for node A, are replaced by the following smooth approximation:

$$\Phi_1^A = \Lambda_\mu \left(P_N^A, r \left(g^A - \boldsymbol{C}_N^A \boldsymbol{d} \right) \right) = 0, \quad (14)$$

where $r > 0$ is any constant and the function Λ_μ was introduced in (2). The value of μ could possibly be

decreasing during the optimization iterations. We have also found that a proper value of $r > 0$ is important for the numerical performance. This is only a small modification of what was used in Hilding (2000). All Φ_1^A are collected in the column vector $\boldsymbol{\Phi}_1 = \boldsymbol{\Phi}_1(\boldsymbol{x})$, where $\boldsymbol{x} = (\boldsymbol{d}, \boldsymbol{P}_N)$.

The exact but non-smooth reformulation of Signorini's contact conditions that is related to the augmented Lagrangian approach is as follows:

$$\Phi_2^A = -P_N^A + \left(P_N^A + r \left(\boldsymbol{C}_N^A \boldsymbol{d} - g^A \right) \right)_+ = 0. \quad (15)$$

All Φ_2^A are collected in the column vector $\boldsymbol{\Phi}_2 = \boldsymbol{\Phi}_2(\boldsymbol{x})$.

In conclusion, the Karush-Kuhn-Tucker equations in (5) and (6) can now be replaced by the following two equation systems:

$$\boldsymbol{H}_i = \boldsymbol{H}_i(\rho, \boldsymbol{x}) = \left\{ \begin{array}{c} \boldsymbol{K}(\rho)\boldsymbol{d} + \boldsymbol{C}_N^T \boldsymbol{P}_N - \boldsymbol{F} \\ \boldsymbol{\Phi}_i(\boldsymbol{x}) \end{array} \right\} = \mathbf{0}, \quad (16)$$

where $i = 1$ or $i = 2$, and the optimization problem in (13) can be rephrased as

$$\begin{cases} \min\limits_{\rho, \boldsymbol{x}} \boldsymbol{F}^T \boldsymbol{d} \\ \text{s.t.} \begin{cases} \boldsymbol{H}_i(\rho, \boldsymbol{x}) = \mathbf{0} \\ V(\rho) - V_0 \leq 0 \\ \boldsymbol{\epsilon} \leq \rho \leq \mathbf{1}. \end{cases} \end{cases} \quad (17)$$

In the following, no summation convention is applied on index i, which should be set to 1 or 2.

When (17) is solved by using SLP, it is first rewritten such that it is formulated in the design variables only. This step reads

$$\begin{cases} \min\limits_{\rho} c = c(\rho) = \boldsymbol{R}^T \boldsymbol{x}(\rho) \\ \text{s.t.} \begin{cases} V(\rho) - V_0 \leq 0 \\ \boldsymbol{\epsilon} \leq \rho \leq \mathbf{1}, \end{cases} \end{cases} \quad (18)$$

where $\boldsymbol{R} = (\boldsymbol{F}, \mathbf{0})$ and $\boldsymbol{x} = \boldsymbol{x}(\rho)$ is implicitly defined by the state equations in (16). Explicitly, for a given density distribution $\rho = \hat{\rho}$, the corresponding solution $\hat{\boldsymbol{x}} = \boldsymbol{x}(\hat{\rho})$ is obtained by using a Newton algorithm with an Armijo line-search, see Strömberg (1997).

The search direction in the Newton algorithm is given by

$$\boldsymbol{z} = \{ \boldsymbol{z}_d, \boldsymbol{z}_N \} = - (\boldsymbol{J}_i(\boldsymbol{x}))^{-1} \boldsymbol{H}_i(\hat{\rho}, \boldsymbol{x}), \quad (19)$$

where $\boldsymbol{J}_1 = \boldsymbol{J}_1(\boldsymbol{x}) = \nabla_x \boldsymbol{H}_1(\hat{\rho}, \boldsymbol{x})$ is a Jacobian matrix for the smooth reformulation. In particular,

$$\frac{\partial \Lambda_\mu}{\partial \xi} = 1 - \frac{(\xi - \eta)}{\sqrt{(\xi - \eta)^2 + 4\mu^2}},$$

$$\frac{\partial \Lambda_\mu}{\partial \eta} = 1 + \frac{(\xi - \eta)}{\sqrt{(\xi - \eta)^2 + 4\mu^2}}. \quad (20)$$

Furthermore, the Jacobian $\boldsymbol{J}_2 = \boldsymbol{J}_2(\boldsymbol{x})$, used in the non-smooth reformulation, is defined by the following relationship:

$$\boldsymbol{J}_2(\boldsymbol{x})\boldsymbol{z} = \begin{Bmatrix} \boldsymbol{K}\boldsymbol{z}_d + \boldsymbol{C}_N^T\boldsymbol{z}_N \\ -\left\{z_N^A\right\}_{A\in\Gamma_1} \\ \left\{r\boldsymbol{C}_N^A\boldsymbol{z}_d\right\}_{A\in\Gamma_2} \end{Bmatrix} \tag{21}$$

and the following index sets:

$$\Gamma_1 = \left\{ A : P_N^A + r(\boldsymbol{C}_N^A\boldsymbol{d} - g^A) \leq 0 \right\},$$
$$\Gamma_2 = \left\{ A : P_N^A + r(\boldsymbol{C}_N^A\boldsymbol{d} - g^A) > 0 \right\}. \tag{22}$$

Note that non-smooth points are treated by picking one out of two possible directional derivatives, see Strömberg (1997) for more details regarding this implementation.

The sensitivity of the compliance,

$$\frac{\partial c}{\partial \rho_e} = \boldsymbol{R}^T\frac{\partial \boldsymbol{x}}{\partial \rho_e}, \tag{23}$$

is determined by introducing the following two adjoint equations:

$$\boldsymbol{J}_i(\hat{\boldsymbol{x}})^T\boldsymbol{\Upsilon}_i = \boldsymbol{R}. \tag{24}$$

The transpose of (24) in (23) yields

$$\frac{\partial c}{\partial \rho_e} = \boldsymbol{\Upsilon}_i^T\boldsymbol{J}_i\frac{\partial \boldsymbol{x}}{\partial \rho_e}. \tag{25}$$

Furthermore, taking the derivative of (16) yields

$$\boldsymbol{J}_i\frac{\partial \boldsymbol{x}}{\partial \rho_e} = -\frac{\partial \boldsymbol{H}_i}{\partial \rho_e}, \tag{26}$$

where, in particular,

$$\frac{\partial \boldsymbol{K}}{\partial \rho^e} = n\rho_e^{n-1}\boldsymbol{k}_e. \tag{27}$$

By putting (26) into (25), we obtain

$$s_e^i = \frac{\partial c}{\partial \rho_e} = -\boldsymbol{\Upsilon}_i^T\frac{\partial \boldsymbol{H}_i}{\partial \rho_e}. \tag{28}$$

Finally, before formulating the approximating LP-problem, we make a filtering of the sensitivities s_e^i to avoid well-known difficulties of mesh-sensitivity and checkerboards. We use Sigmund's direct filtering of sensitivities which consist of the following formula:

$$\hat{s}_e^i = \sum_{f=1}^{n_{el}} \delta_f\rho_f s_f^i \bigg/ \rho_e\sum_{f=1}^{n_{el}} \delta_f , \tag{29}$$

where

$$\delta_f = (r_{\min} - \text{dist}(e, f))_+ . \tag{30}$$

Here $\text{dist}(e, f)$ denotes the distance between the centers of element e and f, and $r_{\min}$ is a parameter. All $\hat{s}_e^i$ are collected in the column vector $\hat{\boldsymbol{s}}_i$.

By using $\hat{\boldsymbol{s}}_i$ to linearize (18) according to the procedure discussed above, we obtain the following two LP-problems ($i = 1$ or $i = 2$) at an iteration point $\boldsymbol{\rho} = \hat{\boldsymbol{\rho}}$:

$$\begin{cases} \min_{\boldsymbol{\rho}} \hat{\boldsymbol{s}}_i^T\boldsymbol{\rho} \\ \text{s.t.} \begin{cases} V(\boldsymbol{\rho}) - V_0 \leq 0 \\ \hat{\boldsymbol{\rho}} + \boldsymbol{\rho}^l \leq \boldsymbol{\rho} \leq \hat{\boldsymbol{\rho}} + \boldsymbol{\rho}^u, \end{cases} \end{cases} \tag{31}$$

where $\boldsymbol{\rho}^l$ and $\boldsymbol{\rho}^u$ define lower and upper move limits, respectively. In the numerical examples we let $\rho_e^l = -0.025$ and $\rho_e^u = 0.025$. Furthermore, we also check that the global limits are satisfied. For instance, if $\hat{\rho}_e + 0.025 > 1$, then $\rho_e^u = 1 - \hat{\rho}_e$ instead of $\rho_e^u = 0.025$.

The problem in (31) is solved by the interior point method (Mehrotra 1992) that is available in Matlab. The optimal solution to the problem in (31) defines a new design point $\hat{\boldsymbol{\rho}}$ where we define a new LP-problem by following the procedure above. In this way a sequence of LP-problems are generated and the sequence

Fig. 3 The design domain, loads and boundary conditions are shown to the left. The graph to the right presents the convergence in compliance for the first load case

(0,-1E4)   (0,1E4)

(1E4,0)   (-1E4,0)

Two first loads   All loads
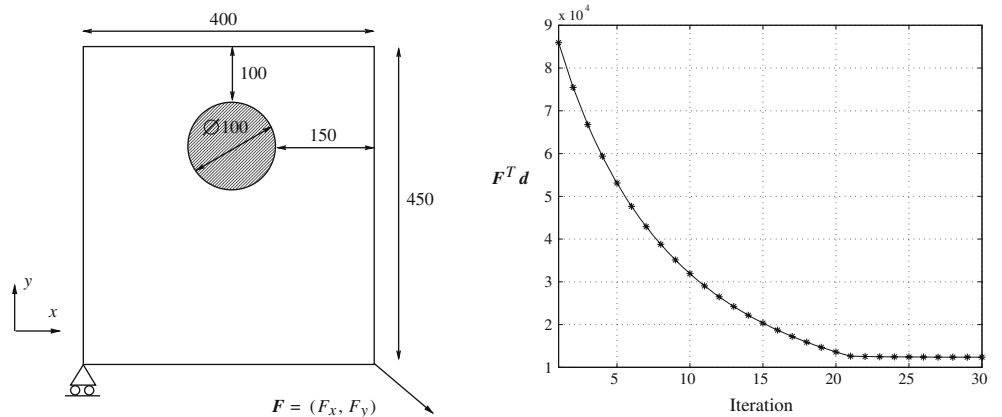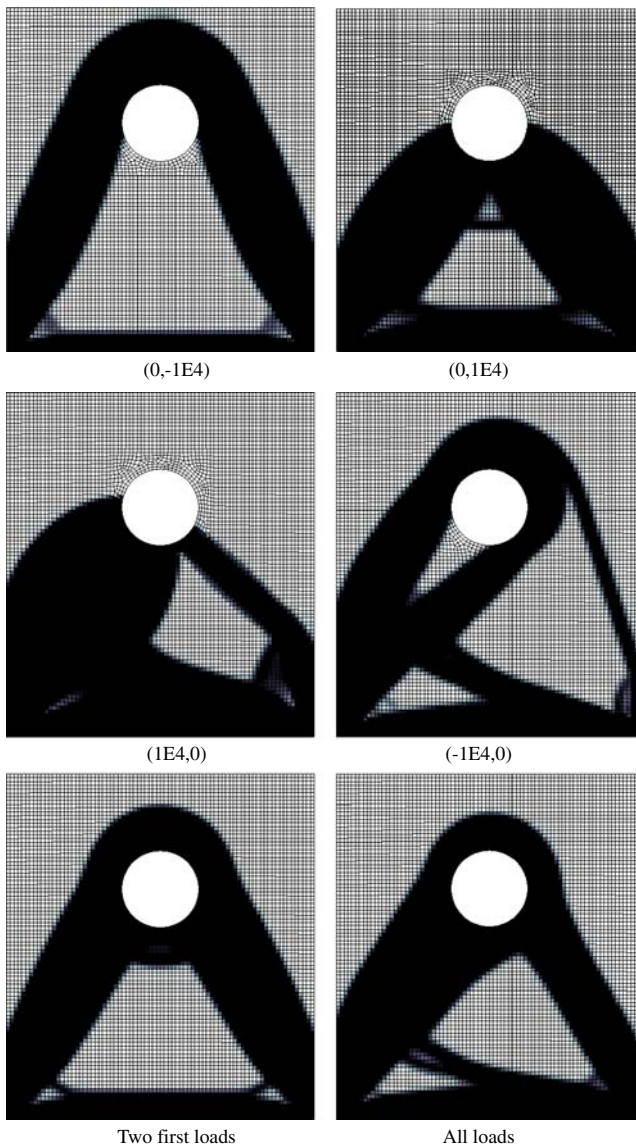
Fig. 4 Different optimal designs for different load cases. Here, $g = 0$

discretized by using fully integrated isoparametric elements, Young's modulus is 2.1E5 and Poisson's ratio is 0.3. The number of elements for the lug is 6949 and r_{min} is set to 9.4. It is assumed that 50 percent of the design domain can be filled by material. Four different load cases are considered $\boldsymbol{F} =$(0,-1E4), (0,1E4), (1E4,0) and (-1E4,0), respectively. Optimal designs are also found for the weighted compliance of the two first load cases as well as for all load cases. Equal weights are used. The weighted compliance problem with equal weights reads

$$\begin{cases} \min_{\boldsymbol{\rho},\boldsymbol{d}_j} \sum_{j=1}^{N_l} \boldsymbol{F}_j^T \boldsymbol{d}_j \\ \text{s.t.} \begin{cases} \begin{cases} \min_{\boldsymbol{d}_j} \frac{1}{2}\boldsymbol{d}_j^T \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{d}_j - \boldsymbol{F}_j^T \boldsymbol{d}_j & \forall j \in \{1, .., N_l\} \\ \text{s.t. } \boldsymbol{C}_N \boldsymbol{d}_j \leq \boldsymbol{g} \end{cases} \\ V(\boldsymbol{\rho}) - V_0 \leq 0 \\ \boldsymbol{\epsilon} \leq \boldsymbol{\rho} \leq \boldsymbol{1}, \end{cases} \end{cases}$$

(32)

where N_l is the number of load cases. We have here utilized that the kinematic constraints are identical for all load cases.

The optimal designs for $\boldsymbol{g} = \boldsymbol{0}$ are presented in Fig. 4. Convergence in compliance is presented for the first load case in Fig. 3. This plot shows the typical convergence behavior for the examples presented in this paper. Figure 4 shows how the load influences the optimal design. It is obvious that the optimal design is not only depending on the direction of the load but is also very sensitive to the sign of the load. This is of course explained by the unilateral character of the state equations. The solutions are obtained by using a laptop with 2.00 GHz Intel dual core processor and 1.96 GB of RAM. The CPU-times for the three first problems are approximately 50–60 s using 30 iterations. The

continues until a solution of (31) is also believed to solve the problem in (13).
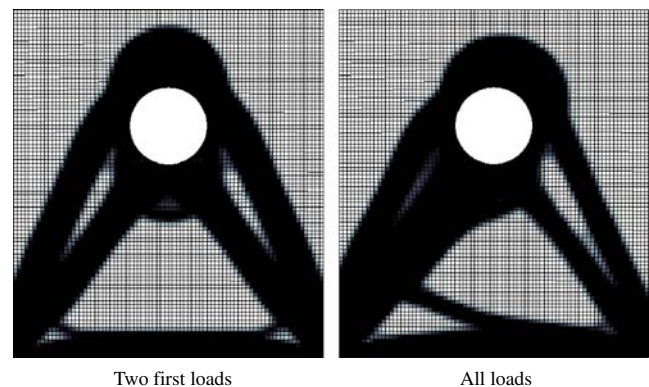
5 Numerical examples

The method presented above is implemented in Topo2D/3D which is a toolbox developed by using Matlab and Intel Fortran. The method is most efficient and robust. This is demonstrated here by first presenting the solutions for a lug unilaterally constrained by a rigid pin as shown in Fig. 3. The figure shows the design domain, loads and boundary conditions of the examples. All units are SI-units. The geometry is

Two first loads   All loads

Fig. 5 Different optimal designs when a clearance of $g^A = 1.0$ is included at all contact nodes

Fig. 6 The design domain, loads, boundary conditions as well as the optimal topology are shown to the left. The graph to the right presents the convergence in compliance, which is obtained after 44 s
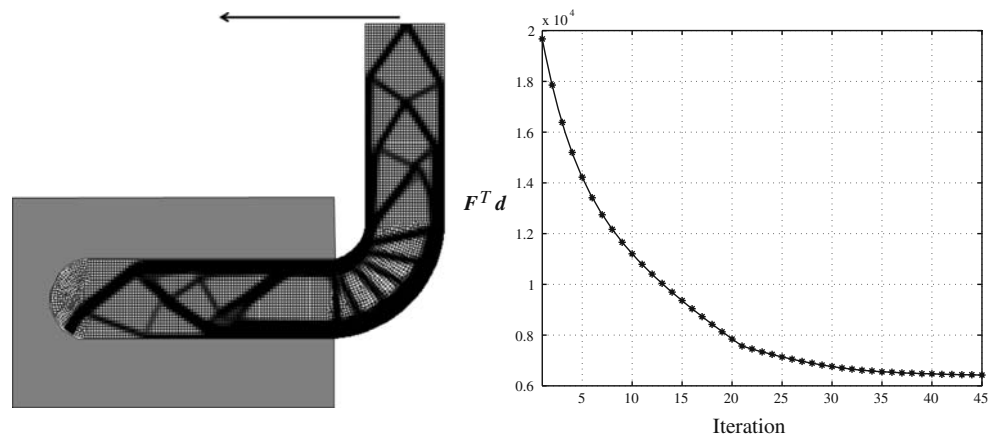
convergence is slower for the fourth problem depending on the tiny member to the right. The number of iterations is now doubled. The CPU-time for the final example with weighted compliance of four load cases is 218 s using 50 iterations.

The optimal designs might also be very sensitive to the initial contact gaps g^A. This is demonstrated in Fig. 5. The only difference compared to the previous examples is that a small clearance $g^A = 1.0$ is included at all contact nodes. The explanation for this differences in solutions is of course that the distribution in contact pressure is very sensitive to the gap clearance, see e.g. Strömberg (1999).

The next problem is motivated by a real application. A bracket of extruded aluminum should carry load in one direction but if the load direction is switched the bracket should be removed easily from the support. The problem is depicted in Fig. 6. The number of elements is 5484 and the number of contact nodes is 177. A small clearance between the elastic bracket and the rigid support is included in the formulation. The optimal design is obtained after 45 iterations and the CPU-time is 44 s when the augmented Lagrangian approach is used. On the contrary, the CPU-time is 52 s when Facchinei's smooth approximation is used since more Newton iterations are needed in this approach. Of course, the solutions obtained are numerically identical.

Finally, we also demonstrate that the method also performs well on 3D-problems. Figure 7 shows the design domain, load and boundary conditions for a 3D-problem. The optimal design, which is obtained after 30 iterations, is also depicted in this figure. The problem is very similar to the one presented in Fig. 3. The 2D geometry is identical, but here a width of 200 is also included. There is only a small difference in the boundary conditions. Instead of fixing the left corner in the y-direction, we now fix the corresponding two end-points in all directions, see the figure. Only the first load case is considered and we apply this load

Fig. 7 A 3D-problem. Elements with densities greater than 0.95 after 30 iterations are plotted by using Abaqus/CAE
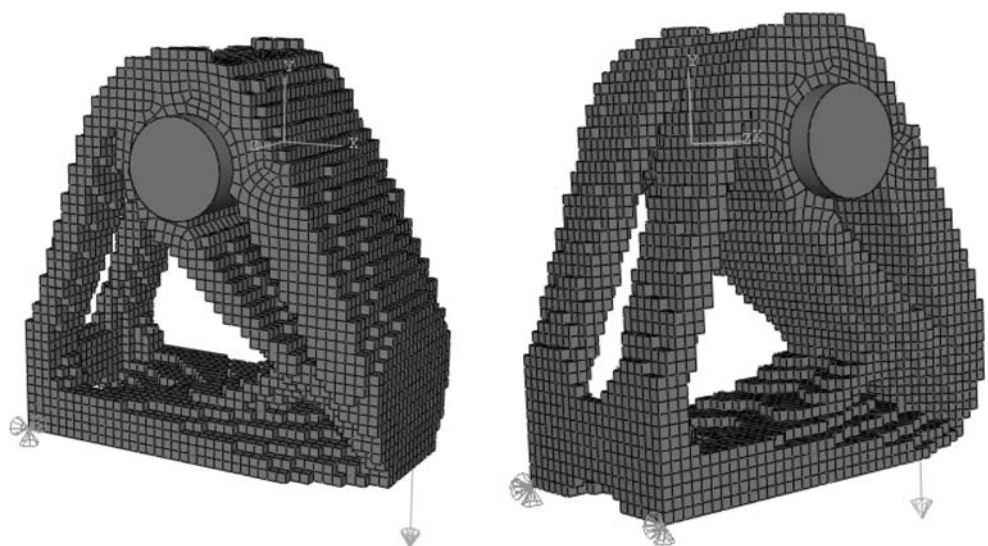
at the corresponding center-point. 8-noded fully integrated isoparametric bricks are utilized. The number of elements is 33820, r_{min}=20 and 588 contact nodes are defined between the design domain an the rigid pin.

6 Concluding remarks

In this paper a general framework for topology optimization of structures in unilateral contact with rigid supports is presented. The approach is developed for a linear elastic structure in unilateral contact. The contact is frictionless and it is formulated by using a smooth approximation of Signorini's contact conditions as well as by using the augmented Lagrangian approach. For these type of structures the compliance is minimized. The design parametrization is performed by utilizing the SIMP-model. The state problem is solved by a Newton method. The optimization problem is treated by SLP where the LP-problem is solved by an interior point method. Sigmund's filter is also utilized in this procedure in order to avoid mesh dependency. The methods are implemented in the in-house toolbox Topo2D/3D and the performance is most efficient and robust for both methods. The implementation of the augmented Lagrangian approach seems to be slightly more efficient than the approach of Facchinei.

References

Bendsøe MP, Sigmund O (2003) Topology optimization: theory, methods and applications. Springer, New York

Christensen P, Klarbring A (1999) Newton's method for frictional contact problems. In: The proceedings of European conference on computational mechanics, München, 31 August–3 September 1999

Christensen PW, Klarbring A (2009) An introduction to structural optimization. Springer, New York

Facchinei F, Jiang H, Qi L (1999) A smoothing method for mathematical programs with equilibrium constraints. Math Program 85:107–134

Fancello EA (2006) Topology optimization of minimum mass design considering local failure constraints and contact boundary conditions. Struct Multidiscipl Optim 32:229–240

Hilding D (2000) A heuristic smoothing procedure for avoiding local optima in optimization of structures subjected to unilateral constraints. Struct Multidiscipl Optim 20:29–36

Hilding D, Klarbring A, Petersson J (1999) Optimization of structures i unilateral contact. Appl Mech Rev 52:139–160

Ireman P, Klarbring A, Strömberg N (2002) Finite element algorithms for thermoelastic wear problems. Eur J Mech A Solids 36:423–440

Johansson L (2001) A Newton method for rigid body frictional impact with multiple simultaneous impact points. Comput Methods Appl Mech Eng 191:239–254

Klarbring A, Rönnqvist M (1995) Nested approach to structural optimization in nonsmooth mechanics. Struct Multidiscipl Optim 10:79–86

Klarbring A, Lundvall O, Strömberg N (2004) A flexible multibody approach for frictional contact in spur gears. J Sound Vib 278:479–499

Mangasarian OL (1976) Equivalence of the complementarity problem to a system of non-linear equations. SIAM J Appl Math 31:89–92

Mankame ND, Ananthasuresh GK (2004) Topology optimization for synthesis of contact-aided compliant mechanisms using regularized contact modeling. Comput Struct 82:1267–1290

Mehrotra S (1992) On the implementation of a primal-dual interior point method. SIAM J Optim 2:575–601

Petersson J, Patriksson M (1997) Topology optimization of sheets in contact by a subgradient method. Int J Numer Methods Eng 40:1295–1321

Sigmund O (2001) A 99 linetopology optimization code written in Matlab. Struct Multidisc Optim 21:120–127

Strömberg N (1997) An augmented Lagrangian method for fretting problems. Eur J Mech A Solids 16:573–593

Strömberg N (1999) A Newton method for three-dimensional fretting problems. Int J Solids Struct 36:2075–2090

Strömberg N (2003) A method for structural dynamic contact problems with friction and wear. Int J Numer Methods Eng 58:2371–2385

Strömberg N (2005) An implicit method for frictional contact, impact and rolling. Eur J Mech A Solids 24:1016–1029

Strömberg N (2006) Frictional contact/impact between a hyperelastic body and moving rigid obstacles. In: The proceedings of the 3rd European conference on computational mechanics, Lisbon, 5–9 June 2006