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INDUSTRIAL APPLICATION

Multidisciplinary design optimization of a small solid propellant launch vehicle using system sensitivity analysis

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Abstract Multidisciplinary design optimization approaches have significant effects on aerospace vehicle design methodology. In designing next generation of space launch systems, MDO processes will face new and greater challenges. This study develops a system sensitivity analysis method to optimize multidisciplinary design of a two-stage small solid propellant launch vehicle. Suitable design variables, technological, and functional constraints are considered. Appropriate combinations of disciplines such as propulsion, weight, geometry, and trajectory simulation are used. A generalized sensitivity equation is developed and solved. These results are basis for optimization. Comparison of the developed approach with gradient optimization methods reveals that developed approach requires less computation time.

Keywords Design · Optimization · Multidisciplinary · System · Sensitivity · Analysis

1 Introduction

Many disciplines such as aerodynamics, propulsion, structure, and control are involved in the design of aerospace vehicles. Many design decisions of all related disciplines depend on one another. Usually a design engineer in one particular discipline makes decisions without paying any attention to the requirements of other disciplines. This causes contradictory results from different design teams; therefore, multiple revisions throughout the design process are resulted.

Generally, excessive iterations would be required for an optimum result. These excessive iterations will clearly increase the cost and time of the design process. Since late 80s, engineers decided to use multidisciplinary design optimization (MDO) approaches to attack this problem. With MDO, quantitative measures replace subjective ones; mutual discipline effects are considered (Blair et al. 2001; Geethaikrishnan 2003).

MDO reduces the time required to execute the design process. In the modern era, it is not only required to design a system which fits a customer's needs, but it is also required to deliver an optimized system. An optimized system is a system with the lowest cost, greatest efficiency, and greatest effectiveness. These requirements have driven designers to use a multidisciplinary approach; they have also required the use of numerical optimization methods. Thus, by reducing design time and cost, an optimized design is achievable. By using MDO methods, designers may quickly and efficiently conduct alternative design points over a wide range of parameters (Rowell and Korte 2003).

Classical optimization methods face some limitations in solving multidisciplinary problems such as size and the coupling of disciplines. Therefore, these methods encountering serious difficulties in solving strongly coupled problems; they also face problems when there are many interdisciplinary parameters. Thus, new methods have been developed that can solve these problems. The results of these efforts are methods such as

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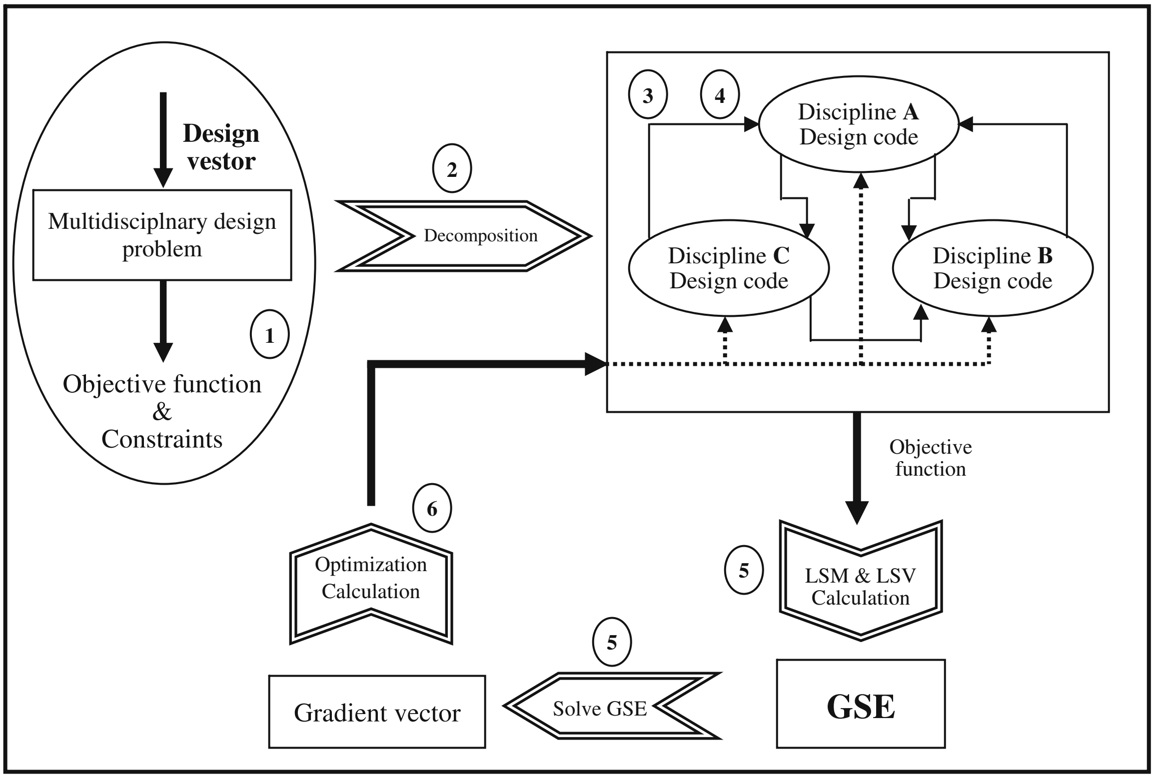

Fig. 1 Design Algorithm using SSA

parametric methods based on Design of Experiments (DoE), stochastic methods (such as genetic algorithms) and System Sensitivity Analysis (SSA) (Hammond 1999, 2001; Kodiyalam 2001) (Fig. 1).

In the SSA method, finite differences or small changes in design parameters are used; gradient computation at the system level to redesign the whole system is not considered. The SSA method retains the benefits and advantages of gradient methods while eliminating their disadvantages. For example it is possible to use the SSA method when the integration of the commercial disciplinary codes is not possible.

In recent years, Small Solid Propellant Launch Vehicles (SSPLVs) have become the main option of launching satellites up to two tons to low Earth orbits due to their low cost and high operational effectiveness (Isakowitz et al. 2004).

In this paper, the formulated SSA approach optimizes the conceptual design of a SSPLV. Propulsion, weight, geometry and trajectory codes are developed; this development is in a suitably integrated structure. Comparisons with other MDO formulations demonstrate the superiority of the developed method.

2 System sensitivity analysis

Equation (1) shows Generalized Sensitivity Equation (GSE) for the three disciplines formulation. It includes the Local Sensitivity Vector (LSV), Local Sensitivity

Fig. 2 Generic configuration of a two-stage solid propellant launch vehicle

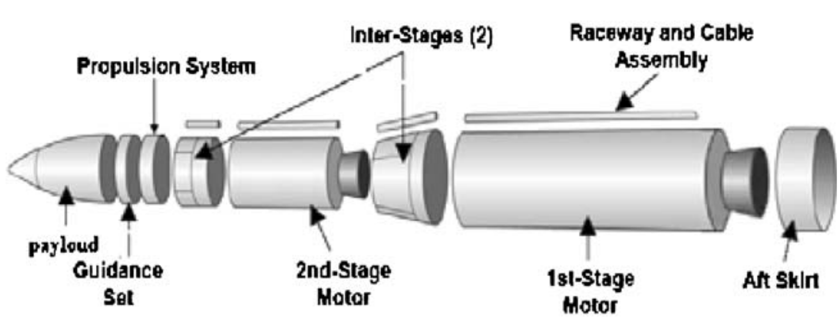

Table 1 Design variables and their feasible bounds

Design variable		Description	Lower band	Higher band
Geometric	X1(m)	1st stage diameter	1.3	2.5
	X2(m)	2nd stage diameter	1.3	2.5
Propulsion	X3(kN)	1st stage average thrust	1500	1700
	X4(kN)	2nd stage average thrust	150	300
	X5(sec)	1st stage burning time	60	90
	X6(sec)	2nd stage burning time	100	170
Trajectory	X7(deg)	1st stage maximum angle of attack	0	8

Matrix (LSM), and Sensitivity Derivative Vector (SDV) (Olds 1992).

$$\begin{bmatrix} I & -\frac{\partial Y_1}{\partial Y_2} & -\frac{\partial Y_1}{\partial Y_3} \\ -\frac{\partial Y_2}{\partial Y_1} & I & -\frac{\partial Y_2}{\partial Y_3} \\ -\frac{\partial Y_3}{\partial Y_1} & -\frac{\partial Y_3}{\partial Y_2} & I \end{bmatrix} \begin{bmatrix} \frac{dY_1}{dX} \\ \frac{dY_2}{dX} \\ \frac{dY_3}{dX} \end{bmatrix} = \begin{bmatrix} \frac{\partial Y_1}{\partial X} \\ \frac{\partial Y_2}{\partial X} \\ \frac{\partial Y_3}{\partial X} \end{bmatrix} \tag{1}$$

The SSA method considers the system as a series of decomposed sub problems (discipline) which may result from a fully coupled problem. These sub-problems usually include routine design disciplines and have their own design codes. The design codes are black boxes, and they can exchange inputs and outputs with other black boxes. This allows the designer to use existing design codes with different computational platforms without any limitations or need to change.

Assume required sensitivity data are available, and solve the GSE equation. Then, calculate the differential effects of design variables for the whole system. For continuous system improvement, the system sensitivity derivative is used intuitively or numerically. Generally, the SSA method is composed of the following steps.

1. Define the problem—selection of design variables, constraints and the objective function.
2. Decompose the existing problem into disciplinary sub-problems with a specific data flow.
3. Form the GSE, and choose a starting point and a suitable optimization algorithm.
4. Use each discipline code and finite difference analysis to provide a (LSV) and a (LSM).
5. Form the GSE using LSV and LSM, and solve it to achieve a gradient vector.
6. Use the gradient vector to achieve new design variables.
7. Reach an optimized point by repeating steps 4 to 6 while satisfying convergence conditions.

3 Problem definition

The goal of this research is to formulate the SSA method for multidisciplinary conceptual design optimization of an SSPLV. The SSPLVs usually have a generic configuration. The generic configuration of these launch vehicles and its sections is shown in Fig. 2. It would be launched from 34° latitude and to the east. Its mission is to directly inject a 500 kg mass into a circular orbit with 350 km height and 34° inclination.

3.1 Objective function

There can be different objective functions for Launch Vehicle (LV) optimization problems. For example, one could minimize cost, maximize payload for a fixed launch weight, maximize injection accuracy in orbit, and minimize launch weight for placing a specific

Table 2 Constraints of the SSPLV problem

Constraint	Description	Constraint	Description
C1	Tolerance of terminal velocity	C7	Maximum motor specific impulse
C2	Tolerance of terminal height	C8	Maximum Loading of motors density
C3	Tolerance of terminal injection angle	C9	Minimum structural fraction
C4	Maximum dynamic pressure	C10	Separation height
C5	1st stage diameter limit (bigger than or equal to the 2nd stage diameter)	C11	Range safty criteria
C6	Launch vehicle length-to-diameter ratio limits	C12	1st stage maximum allowable angular velocity

Fig. 3 Design structure matrix of the SSPLV design problem

Rocket motors burning time, thrust, Nozzle exit area and exit pressure		Stages propellant mass, chamber pressure	Rocket motor dimensions	**Propulsion**
Launch vehicle dimensions	Rocket motor dimensions, fairing		**Geometry**	Rocket motor dimensions
Stages mass, components structural mass		**Weight**		
Aerodynamic coefficients	**Aerodynamic**			
Trajectory	Height, Mach No.			

payload in a particular orbit. Here, the objective function is minimizing SSPLV launch mass (weight) to inject a specific payload into orbit.

3.2 Design variables

Design variables should be selected to represent an overall description of the design, and they also significantly affect the objective function and constraint values. This choice is a big challenge for the designer, and it is an important factor in the ability to achieve a real optimized design and computation time. Choosing suitable design variables heavily depends on designer creativity and experience. Selected design variables for the current problem are two geometrical parameters, four propulsion variables and a trajectory variable. Table 1 shows these parameters and their limits. These variables intensively affect the selected objective function and contain multidisciplinary characteristics. The results of sensitivity analysis confirm this selection.

3.3 Constraints

Many constraints should be satisfied in the design of LVs. This study considers several constraints as shown in Table 2. These are mission (C1 to C3), structure (C4 to C6, C9,C12), technology (C7,C8), reliability (C10), and safety (C11). The constraints would be satisfied at the discipline level (except C1 and C5) constrains.

3.4 Design problem decomposition

In the SSA method, computational cost is very important when interdisciplinary variables increase and the mutual effects of discipline (bandwidths) rise. Thus, number of required iterations to produce partial derivatives increase heavily. For the current problem, the design structure matrix and interdisciplinary relationships can be considered as shown in Fig. 3.

This structure has a great deal of interdisciplinary data exchange. In addition, there is a strong link between aerodynamics and Trajectory disciplines. The trajectory code should give the flight height and Mach number, and should receive aerodynamics coefficients at each time step. This significantly increases computation costs. Therefore, the structure of the problem is modified.

The first possible solution to revise the structure is to unbind the aerodynamics and trajectory codes. To do this, the aerodynamic code generates look up table of aerodynamic coefficients with respect to launch vehicle geometry and its flight conditions. Select a generic SSPLV geometry; thus, merge the aerodynamics and geometry disciplines.

On the other hand, the main part of the SSPLV weight belongs to the Solid Rocket Motors (SRMs). The SRMs include propellant and structural masses.

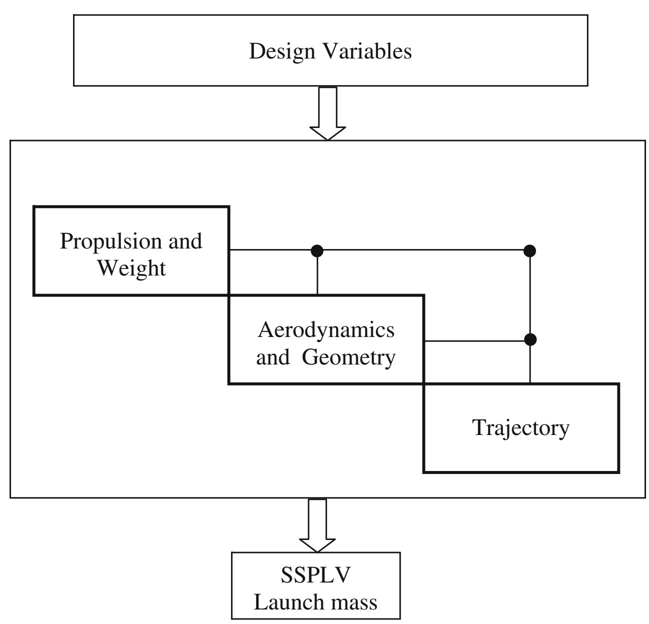

Fig. 4 Modified design structure matrix of SSPLV

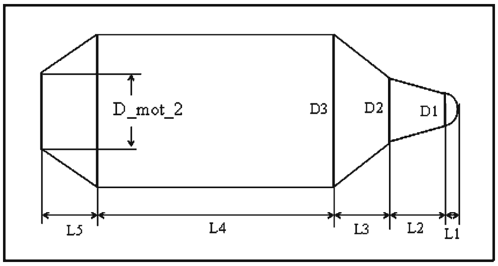

Fig. 5 General scheme of the SSPLV fairing

The weights of other components of SSPLV with respect to the SRMs are relatively small, and consider them in the ratio of SRM weight. Therefore, the propulsion and weight codes are also merged as a single code. The modified and compact form of the design structure matrix is demonstrated in Fig. 4.

3.5 Disciplinary codes

3.5.1 Aerodynamics and geometry

Overall diameters of SSPLV stages and fairing parameters (Fig. 5) are selected by geometry module. It set typical configuration of SSPLV. The aerodynamic coefficients necessary for the trajectory assessment are computed through semi-empirical methods that have been developed to allow rapid evaluation of the vehicle performance in the frame of conceptual design. These methods allow assessing the aerodynamic characteristics. In particular, asses the axial and normal coefficients of SSPLV with respect to angle-of-attack and Mach number.

A hammerhead fairing geometry for the payload bay was considered. It has a four conic fairing section and contains a separation system mechanism, orbital maneuver propulsion system, guidance navigation and control system, satellite, and two insulated fairing caps. Design variables include the geometric dimensions shown in Fig. 5.

3.5.2 Propulsion and weight code

The code allows a large choice of parameters in order to design the vehicle stages (Ebrahimi 2005). It computes SRM stages mass (case, insulation, skirts and frames, propellant) with high details, and SRM performance (specific impulse, total impulse, actual thrust, actual burning time). The Inputs of the code are required thrust, diameter and burning time of SRMs, technological inputs (case and insulator materials, propellant characteristic,...) and architecture (nozzle shape, interstage (skirt), thrust vector control type,...). The weights of the other structural parts define on a coefficient of SRM structural mass. Usually, this coefficient is determined using statistical data (Mirshams 2005).

3.5.3 Trajectory

To analyze the ascent flight-path, perform a three degree-of-freedom trajectory analysis with the Traject program (Roshanian 2005). The three degree of freedom equations of motion are numerically integrated from an initial to a terminal set of state conditions. Within the present investigation, treat the vehicle as a point-mass; model earth rotation and earth oblateness. Use the 1976 standard atmosphere (no winds). Terminal constraints on altitude, velocity, and flight-path angle, as well as maximum flight dynamic pressure, range safety and stages separation height limits are enforced.

Table 3 Design variables at initial and optimum points

Parameter		Initial value	Optimum value
Propellant mass (Kg)	1st stage motor	42574	36103
Average thrust (KN)		1600	1599
Burning time (s)		75	63.8
Length (m)		12.2	6
Diameter (m)		1.9	2.5
Structure mass (Kg)		5545	4401
Propellant mass (Kg)	2nd stage motor	8255	8693
Average thrust (KN)		190	205
Burning time (s)		130	128.6
Length (m)		3.8	5.4
Diameter (m)		1.5	1.3
Structure mass (Kg)		1088	884
Launch vehicle total launch mass (Kg)		57926	50572

4 Solving the problem using the SSA method

Considering Fig. 4 and applying the procedure mentioned in Section 2, the resulting GSE is as follows:

$$\begin{bmatrix} I & 0 & 0 & -\partial AB/\partial CA & 0 \\ 0 & I & 0 & -\partial AC/\partial CA & 0 \\ -\partial BC/\partial AB & 0 & I & 0 & 0 \\ 0 & -\partial CA/\partial AC & -\partial CA/BC & I & 0 \\ 0 & 0 & 0 & -\partial M/\partial CA & I \end{bmatrix} \begin{bmatrix} dAB/dX \\ dAC/dX \\ dBC/dX \\ dCA/dX \\ dM/dX \end{bmatrix} = \begin{bmatrix} \partial AB/\partial X \\ \partial AC/\partial X \\ \partial BC/\partial X \\ \partial CA/\partial X \\ \partial M/\partial X \end{bmatrix} \tag{2}$$

As was mentioned, every element of the GSE matrix is itself a matrix. If we replace the relevant matrices, the sensitivity analysis matrix will be 22 by 22 elements, and SDV and LSV will be 22 by 6 elements. Initial design point is chosen by using SSPLV statistical data (Isakowitz et al. 2004). (see Table 3)

The Davidson-Fletcher-Powell (DFP) method is used to find the search direction. This is one of the most effective quasi-Newtonian methods. A second order interpolation method is selected for a linear search algorithm (Vanderplaats 1989; Venkataraman 2002).

To find the gradient vector of the objective function, LSM and LSV vectors should be computed and replaced in the GSE (Eq. 2). The SDV is computed, and the gradient vector of the objective function is the last row of the SDV.

All codes are considered as a black box, since it's not possible to derive an analytical derivation of existing codes, and Forward-difference approximation was used. The two percent of nominal value was selected for difference interval to have a balance between computational error and accuracy.

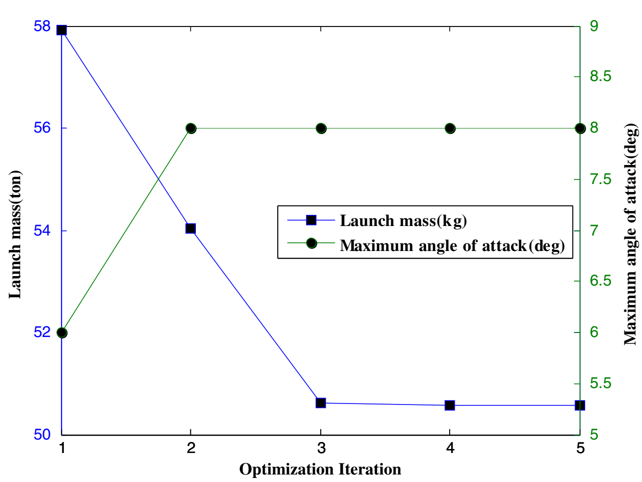

Fig. 6 Launch mass and maximum angle of attack history

Three conditions can be used to identify optimization convergence:

1. A change in the objective function value becomes lower than a specific value.
2. There are few changes in design variables.
3. Objective function gradients approach zero.

The convergence criterion satisfies after four iterations. Figure 6 shows objective function and maximum angle of attack values during the optimization procedure. The launch vehicle mass at the optimum point with respect to the initial value decreases by 12.6%. Figure 7 represent design variable histories in the optimization process.

5 Analysis of results

Figure 8 shows the optimization results of this problem in a Fully Integrated Optimization (FIO) formulation (Jodei et al. 2006). As illustrated, by using the FIO formulation to reach the global optimum point, the procedure should begin from different initial points, and also, more iterations are needed. In a FIO formulation, 12 to 14 optimization iterations (144 function calls) are needed; where as 4 iterations (13 function calls) are needed by using SSA.

As shown in Fig. 9, the times needed to solve the problem using different methods are compared:

1. Traditional method (one variable at a time)
2. Direct gradient method
3. SSA method using serial running of codes and updating the LSM at every iteration.
4. SSA method using parallel running of codes and updating the LSM every 3 iterations

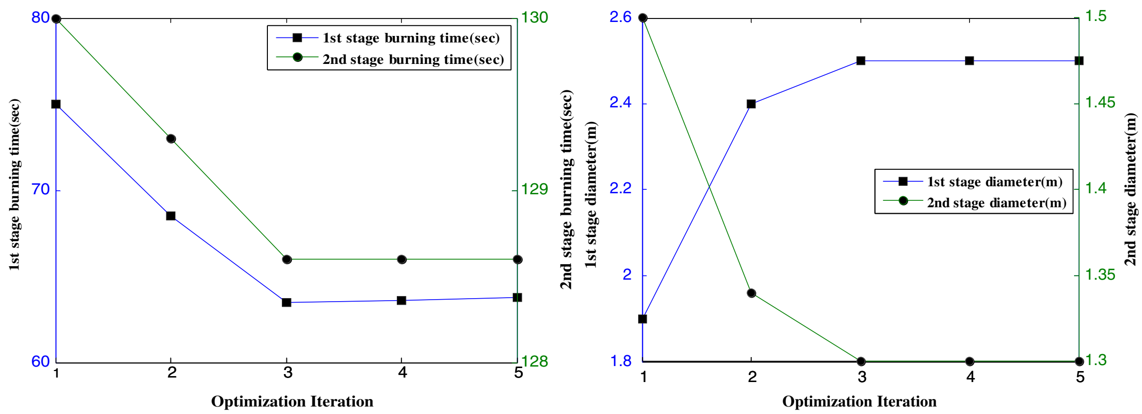

Fig. 7 History of burning times and diameters of motors

5. SSA method using parallel running of codes (each code on one machine) and updating the LSM at every iteration
6. SSA method using parallel running of codes (each code on one machine, 3 machines in total) and updating the LSM every 3 iterations
7. SSA method using parallel running of codes (each geometry, propulsion and trajectory code, 6 machines in total) and updating the LSM every iteration

Progress in the design process results in a more significant advantage for computation time savings. In preliminary design phases, high accuracy codes (for example, Fluent software in aerodynamics and Ansys in structure analysis) are used, and the running times are hours or even days. Moreover, because of the increased complexity of the design space, too much iteration is needed to gain convergence.

Another advantage of the SSA method is side information, which gives a good intuitional understanding of the design space to the designers, and enables one to answer what-if questions, such as information in the LSM, LSV and SDV. A sensitivity approach, which relies on the system sensitivity matrix, was used by McDonald to rapidly approximate the propagation of error through the complex system (McDonald 2006, 2007). Moffitt et al. use SSA to estimate the percentage of the overall uncertainty in the performance metrics based on the bias errors of the contributing analyses and the precision error of the design input variables. Based on the SSA, contributing analyses with significant contribution to the total performance uncertainty are selected and laboratory experiments are then conducted to obtain data used to increase the fidelity of these contributing analyses through calibration or regression (Moffitt et al. 2007, 2008). Robinson et al. mention that SSA should be used to quantify impact of research and technologies relevant to the system concept of interest pursuits against key capability metrics

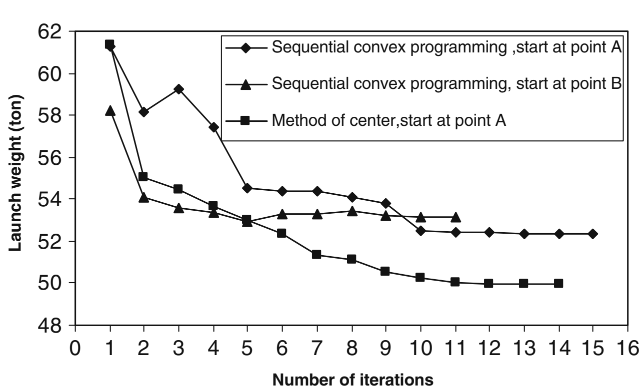

Fig. 8 Optimization results by the Fully Integrated Optimization (FIO) method

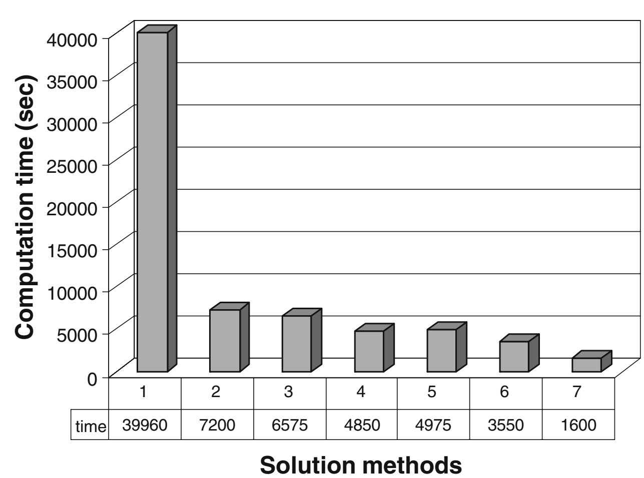

Fig. 9 Time to reach the optimum in different optimization and MDO formulations methods

to produce a set of prioritized research and technology investment options (Robinson et al. 2006).

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