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Design optimization of hybrid material structures

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Abstract Design optimization of hybrid material components and structures often leads to multi-criteria and mixed continuous-discrete optimization problems. Moreover, for achieving readily implementable design solutions, it is highly important to consider interactions in structural and material behavior (e.g., thermo-elastic mismatches) together with thermo-mechanical and manufacturing aspects simultaneously. For the latter, response surface approximations are utilized which are also based on quantification of qualitative knowledge via fuzzy set rules. From a series of practical applications solved by Evolutionary Strategy Algorithms general conclusions for hybrid material component optimization and design are derived.

Keywords Hybrid materials · Thermo-elasticity · Manufacturing constraints · Response surfaces · Fuzzy rules

1 Introduction

Hybrid material components and structures are composed of different layered or otherwise combined materials, e.g., produced via extrusion or bonding. Reasons for such material combinations are the exploitation of their different favorable properties, which, in their combination, serve for fulfilling a broad scope of quite different requirements as shown in Kim (1998). For example, while fiber-reinforced plastics have high strength and stiffness and low coefficients of thermal expansion, their combination with certain metal alloys further improves also electrical or heat conduction (Alderliesten et al. 2003) together with impact resistance (Thuis 2004) and handling robustness.

Lieping et al. (2006) investigated typical hybrid material structural members for civil engineering applications. The focus was mainly on the limited types of mechanical loads only. Kovács et al. (2004) considered also manufacturing cost and provided optimized designs with a trade-off between load carrying capability and cost.

Another important aspect is the manufacturing of the separate subcomponents and their joining. Hufenbach et al. (2005) describe the influence of shrinkage and moisture absorption on the performance and shape of composites. For metal parts, other interrelationships have to be considered, e.g., minimum wall thicknesses achievable in extrusion molding according to Kammer (1995).

From a design optimization point of view, the types of materials to be selected and to be combined also become design optimization parameters in addition to those related to the geometry and topology of the structure. The system equations describing the relationship between the design parameters and relevant responses such as displacements, stresses, or temperature distribution have to cover also eventual mismatch effects, e.g., related to different thermal expansion and heat conduction properties.

Together with rules and constraints from manufacturing processes, it is this completeness of the optimization model (i.e., in the objectives, constraints, and thus, also the system

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equations) which is highly relevant in practical applications. Such aspects, together with algorithmic considerations and different practical applications, are discussed in this paper.

First, a representative technical example will be discussed, followed by the formal problem statement and the discussion of selected solution strategies. From a further set of practical applications of optimizing hybrid material components, some general conclusions for proper optimal design will be drawn.

2 Carbon fiber reinforced aluminum bridge girder as a representative example

As a structural example, a hollow aluminum plate reinforced by carbon fiber reinforced plastic (CFRP) as shown in Fig. 1 is considered. This hybrid material component is to be used as the top girder and vehicle track of a quickly erectable bridge structure. This top girder is made out of an extruded Al alloy hollow plate being reinforced by longitudinal CFRP stiffeners bonded into the different chambers of this hollow plate. This reinforcement shall increase both stiffness and load carrying capability. In addition to bridge traffic loads resulting into global compression and local shear and bending loads of the girder, hot (+80°C) and cold (−50°C) temperature cases are to be also taken into account.

The design optimization problem for this top girder then qualitatively is:

- Objective: minimize total girder weight (of Al and CFRP parts)
- Constraints: strength, stiffness, buckling stability under different load cases, extrusion manufacturing constraints as function of material properties, and geometry
- Design variables: thicknesses in each chamber for the aluminum base material and of each of the CFRP stiffeners, the number and geometry of chambers, the type of aluminum base material to be chosen out of four possible options
- System equations: section forces taken from global finite element model, use of Timoshenko plate theory for local shear and bending. Because of the material combination, temperature loads and stresses due to thermo-elastic material incompatibility are to be considered as well.

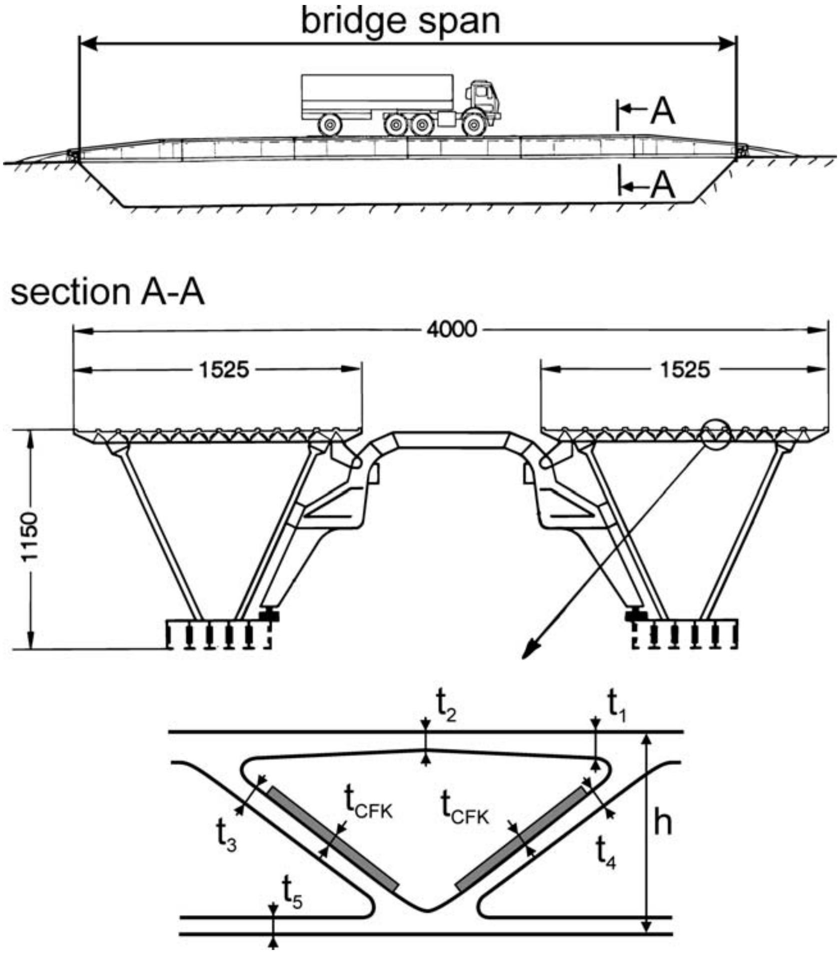

Fig. 1 Extruded hollow plate with bonded CFRP reinforcement

For strength evaluation, failure criteria for metals and fiber composites as well as for interface bonding have been applied according to Tsai (1988) and Puck and Schuermann (2002).

This design optimization problem contains the following challenging features:

1. The number of the hollow plate's chambers and the type of aluminum alloy are discrete design variables, which, together with continuous geometric variables, result into a mixed continuous-discrete problem.
2. There is a significant influence of the type of aluminum base material (i.e., its yield and plasticity limit) on strength on the one side and on manufacturing aspects (ease of plastification during extrusion molding, and thus, achievable minimum thicknesses) on the other. This leads to a considerable interaction between structural mechanics and manufacturing aspects.
3. Due to thermal loads in addition to mechanical vehicle loads, thermal mismatch and resulting stresses between aluminum and CFRP affect the optimal design.

As will be shown, the second and third features and their interaction might lead to lower mass designs for low strength alloys compared to the use of high strength alloys!

Achievable manufacturing geometries such minimal thicknesses in extrusion depend on the overall girder geometry and the type of Al alloy (Kammer 1995). Minimal extrudable thicknesses $t_{\min}$ increase with increasing girder height h because the circumscribed diameter of the profile increases. In addition, the yield strength σ_y of the Al alloy base material influences $t_{\min}$. This relationship graphically outlined in Fig. 2 is included in the optimization model as well via constraints of type $t_i \geq t_{i\,\min}(h, \sigma)$. Therefore, using the yield strength as a further (continuous) optimization variable will lead to an optimal material distribution within the hollow plate, i.e., w.r.t. strength and higher thicknesses on the one side and low thicknesses wherever possible on the other. It is this interaction which makes sense for using yield strength (and with that related also ultimate strength) as a design variable because this interaction avoids strength to go to infinity. After having determined the optimal continuous yield limit together with the other design variables, the actual discrete aluminum alloy coming closest to this value then has been selected for the actual design.

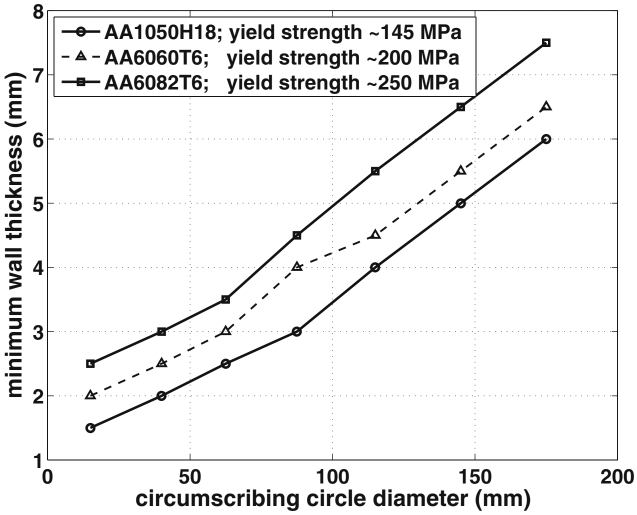

Fig. 2 Smallest achievable wall thickness in extrusion molded Al plate as function of circumscribing circle diameter (function of girder height and width) and yield strength of Al alloy

Main results of this hybrid material structure optimization problem are qualitatively shown in Fig. 3. Minimum girder masses are given without and with CFRP reinforcement. It is obvious that CFRP reduces weight especially for girders in bridges to be used for high traffic loads and/or for large bridge spans.

It is interesting to note that the optimal fraction of CFRP typically ranges between 20 and 30% of the total cross-section al area. This only limited fraction of the "better" material CFRP vs Al alloy is due to thermal loading and material mismatch resulting into thermal stresses. Although an increase of CFRP from a low fraction helps to take more loads, high percentages of CFRP would induce higher thermal stresses in the Al base material, and thus, would lead again to an increased overall weight. Moreover, bridges to be used for lower to medium loads or spans should be made out of lower to medium yield strength Al alloy. This is because in those parts of the girder's cross-section where only low thicknesses are needed to satisfy strength and buckling constraints, these low thicknesses can be produced more reliably with lower (yield) strength alloys. For high traffic load or span requirements with dominating strength constraints, a high yield (and ultimate) strength alloy is in order.

Another type of manufacturing aspects concerns the manufacturing effort. Therefore, at least rough cost estimations are to be made as function of relevant design parameters, which are approximating the estimated process time for the manufacturing steps of each part and additional effort such as tooling cost.

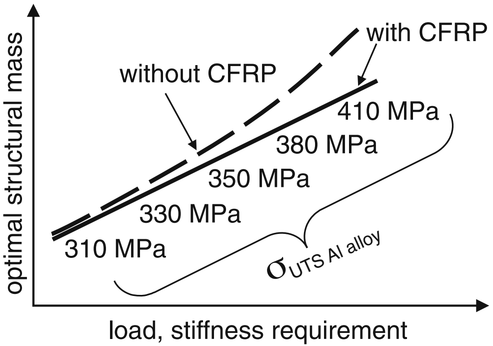

Fig. 3 Minimum girder mass without and with CFRP enforcement (qualitatively)

For the estimation of the manufacturing effort of structures, not only the primary forming process is crucial, but also the further manufacturing steps. High fidelity models for such relations are rarely available especially in early development and design optimization stages. Therefore, the experience of extrusion-molding engineers should be used to establish the necessary knowledge base for constructing approximate and parameterized models. This aspect is discussed further below and is then highlighted with some technical examples.

3 Representative mathematical problem statement

Such material and structural design optimization problems are nonlinear and eventually multi-criteria optimization problems of type

$$\text{minimize } f_i(\{x\},\{y\}), \quad i = 1, 2, \ldots \tag{1}$$

$$g_j(\{x\},\{y\}) \geq 0, \quad j, 1, 2, \ldots \tag{2}$$

$$s_k(\{x\},\{y\}) = 0, \quad k = 1, 2, \ldots \tag{3}$$

where f_i and g_j are the objective functions (mass, displacement) and constraint functions, e.g., on stresses, displacements, eigenfrequencies, or temperatures, respectively. The design variables $\{x\}$ may be continuous (i.e., geometry) or discrete (i.e., types of materials), and for given variables $\{x\}$, the response variables $\{y\}$ (displacements, stresses, temperatures, etc.) are to be determined from the system equations s_k. Depending on the type of problem, these system equations might be composed from those of structural mechanics (ranging, e.g., from laminate plate theory to large finite element models) and, e.g., those related to heat conduction. Usually, these two types of system equations can be treated separately, i.e., first the temperature distribution over the structure is determined followed by thermo-elastic structural analysis. So while these disciplines are physically uncoupled (or coupling goes in one direction only), there might be a strong design coupling. This means that variations of design variables $\{x\}$ may cause variations in structural quantities *and* in temperature distribution.

For multiple objectives, the solution of (1) is the Pareto-optimal set where loosely speaking, one of the objectives can be reduced only by increase of one of the others. As this is often achievable by an infinite number of designs, the preferred optimal compromise out of this Pareto set has to be selected by a decision process. This is the reason why “minimize” is put in quotation marks above.

As mentioned, especially in early development phases, not all relevant (high fidelity) system equations might be fully available in time, which might especially hold for example for manufacturing aspects. To assure the necessary completeness of the problem statement, approximate models based, e.g., on response surfaces derived from qualitative knowledge should be used instead, as outlined in the following.

4 Meta-modeling and algorithmic aspects

In early design stages, the lack of high fidelity models and considerably test results often prevent formulating an optimization problem properly. Low fidelity models can be used to overcome these difficulties. With such models, the most important design parameters can be identified via optimization at the beginning of a product development and, therefore, reduce the risk to major design changes in later product development phases. Hajela (2002) utilized fuzzy rule-based systems with one input parameter as low fidelity models to integrate expert knowledge at least in an approximative approach. This approach is extended to multiple input variables and a computer-based knowledge acquisition tool.

4.1 Fuzzy rule-based modeling of qualitative knowledge

Fuzzy logic (Zadeh 1965) provides the basics for fuzzy modeling, and it was introduced as a method of formally describing linguistic information. So-called fuzzy rule-based systems (FRBS) have the ability to model complex behavior. The most important task is to build the knowledge base, which includes the j rules describing the relationship between inputs and outputs. The problem considered here is a system with n inputs [x_1,..., x_n] (i.e., relevant design variables) and one output y (the relevant response quantity such as manufacturing effort). These rules Rj have the following structure:

$$R_j : \text{if } x_{j1} \text{ is } A_{j1} \text{ and } x_{j2} \text{ is } A_{j2} \text{ and} \ldots \text{and } x_{jn} \text{ is } A_{jn} \text{ then } y_j \text{ is } B_j. \tag{4}$$

where x_{j1},..., $x_{jn} \in [x_1,..., x_n]$ and A_{ji}, B_j are fuzzy sets on the respective domains of the variables. The degree to which an input or output belongs to a fuzzy set is defined by a membership value between zero and one. A membership function associated with a given fuzzy set maps a value to its appropriate membership value. Gaussian, triangle, trapezoidal, and monotonically in-/decreasing membership functions are used.

Martinez et al. (2001) suggest the following procedure to transform expert knowledge into a mathematical model:

step 1 Identify the input and output variables and their respective domains.

step 2 Define the fuzzy membership functions for each input and output variable to cover the respective domains.
step 3 Transform the description of the system behavior into fuzzy rules stating the relations between the variables.
step 4 Evaluation of the model.

A graphical user interface for the "card sorting" technique based on Wagner and Zubey (2005) allows to gather the relevant model parameters (=design variables in optimization) and organize them into a hierarchy (steps 1 and 2). From that information, a structured interview is derived to create the rules of the fuzzy model for the most important parameters (step 3). After the rule base is established, the FRBS can generate the estimated output parameter.

As an example, the estimated extrusion time for a newly developed manufacturing process should be modeled. The extrusion of long fiber reinforced hollow aluminum profiles is investigated by Kleiner et al. (2006). Examples for profile cross-sections are given in Fig. 4.

Only a few different cross-sections and reinforcing parameters are investigated at the moment, and, therefore, test data and simulation data (Schikorra and Kleiner 2005) cannot provide enough parametric results for the response surface approximation method or a Kriging model. The extrusion time is therefore estimated with fuzzy rule-based systems.

For constant extrusion molding parameters, the manufacturing time for a profile increases with increasing length and cross-section of the profile. The billet refill must also be taken into account as downtime; this occurs more often for long and big profiles. The following part of the rule base (25 rules) was generated from expert knowledge:

- If length of profile is very short and the cross-section area is very small, then the estimated manufacturing time is very small.
- If length of profile is very short and the cross-section area is small, then the estimated manufacturing time is very small.
- If length of profile is very short and the cross-section area is medium, then the estimated manufacturing time is small.
- etc.

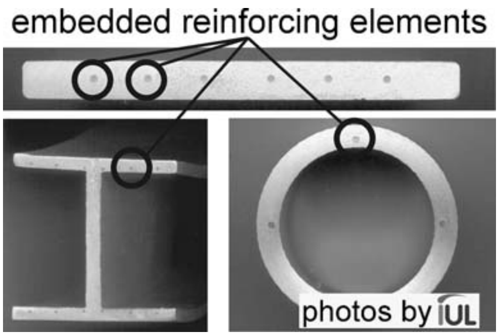

Fig. 4 Different profile cross sections with embedded reinforcing elements

The resulting model output "estimated time" describing this relationship is shown in Fig. 5.

To keep the rule base manageable, a tradeoff between number of input parameters and the number of membership function for each input parameter has to be done. Three to five linguistic membership functions for each parameter provided good modeling results for the given examples.

4.2 Genetic algorithm GAME

The mix of different types of models and constraints leads to problems where also gradients for response quantities with respect to design variables are difficult to obtain. Such problems and the need to handle multiple objectives and discrete optimization parameters has led to select as an optimizer the evolutionary and genetic algorithm GAME (genetic algorithm for multi-criteria engineering)—please refer to Langer (2005) and Puelhofer et al. (2004) for detailed information.

The main stop criterion is the maximum number of generations. Additionally, the number of different ranks in one generation or the change of the hyper volume defined by the Pareto-optimal solutions can be used as convergence criteria.

The major drawback of genetic algorithms like GAME, namely the high number of evaluations of the optimization model and system equations, is compensated to some extent by the relatively ease of using parallel processing for system evaluations. In addition, the relatively high robustness and applicability for discrete variables is favorable. In

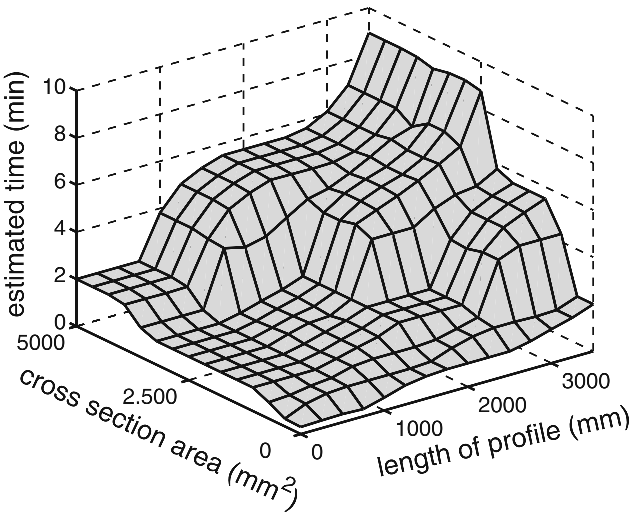

Fig. 5 Output of a manufacturing time estimation model for extrusion molding

case the design problem has a reasonable number of discrete variables, a built-in sequential quadratic programming (SQP) optimizer utilizes response surface approximations in parallel to the genetic process. This can improve the convergence speed drastically as shown in Langer (2005). If the quality of the approximation models is high, the mathematical optimality criteria of the SQP optimizer indicate that a (local) optimal design has been found.

For each of the examples, different algorithm parameter sets (number of generations, population size, etc.) were applied to check the robustness of the optimization process itself. Provided that convergence was given, these results were very similar, and the one presented are for the best algorithm settings.

5 Practical hybrid material components optimization problems

As could be seen from the introductory example of Section 2, thermo-mechanical design optimization problems are those with a considerable interaction of thermal (heat conduction/temperature distribution) and mechanical (displacements, stresses) behavior influenced by the design variables. This interaction often is considered as uncoupled, where fixed temperature fields which are not influenced by the design variables are mapped onto the structural model. In other cases, this interaction might be strong, where varying design variables also lead to significant variations of temperature distributions to be updated during the optimization process. In addition, such effects could vary with time. For example, through thickness, transient temperature distributions due to thermal shock loads result into transient stresses. These stresses might be significantly different from those of stationary temperature loads. Proper identification of critical time steps or load cases is in order, which also might then have to be updated during the iteration process. In the following, different aspects, including also the consideration of manufacturing effort in addition to structural aspects, are discussed with several practical examples.

5.1 Design optimization of hybrid material panels for satellite reflect arrays

A reflect array is a flat panel antenna (or an arrangement of flat panels) which represents an alternative to conventional parabolic antenna reflectors. Several layers of well-positioned and sized copper patches within these panels, together with specially designed antenna feed horns, introduce a phase shift of the reflected electromagnetic waves in such a way that the reflection characteristics mimic those of a parabolic antenna. In Fig. 6, a testing configuration of a reflect array antenna is shown, while Fig. 7 outlines the typical layout of functional layers and honeycomb cores.

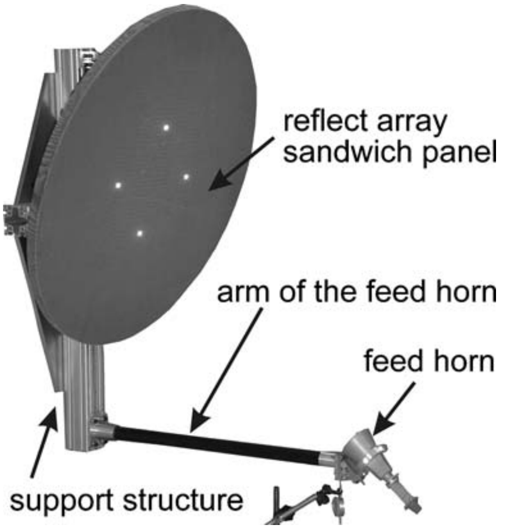

Fig. 6 Test configuration of a reflect array sandwich panel

The structural design is basically a multiple sandwich design. The decisive elements for the functionality are the reflecting layers of copper patches on Kapton® foil integrated into the upper part of the sandwich reflector. The supporting substructure consists of several layers of Kevlar® fabric plies and Nomex® honeycomb cores together with CFRP layers. A detailed description can be found in Baier et al. (2005b).

The optimization task for the design of this antenna has been to design minimum mass panels with minimum deformation (so two goals) when subjected to three thermal load cases and three moisture load cases. The thermal load cases consider the stress free state at curing temperature (around +150°C), the service temperature down to −160°C, and a temperature gradient through the plate due to space environment of ±50°. This becomes relevant because the copper layers induce significant thermal deformation which has to be compensated by proper material combinations in the sandwich structural design. The moisture load cases simulate the contraction of the Kevlar®/epoxy composites due to moisture loss after manufacturing. Because the

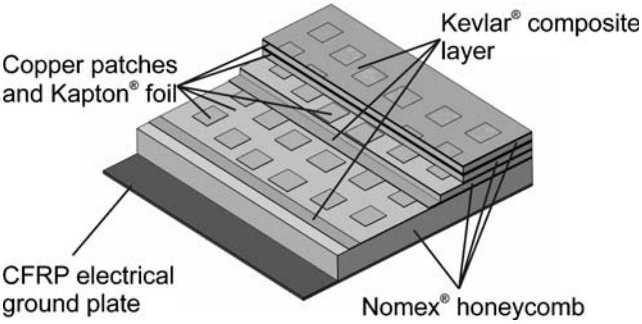

Fig. 7 Layout of reflect array sandwich panel with material combination

sandwich structure is not symmetric, a linear moisture loss gradient (top 3%–bottom 2%; top 2%–bottom 3%) leads to two load cases and a worst case scenario (top 3%–bottom 3%). The antenna has to satisfy constraints on resonance frequency (first eigenfrequency >5 Hz), heat conduction, as well as on electrical loss. This is the reason for Kevlar® fibers, together with cyanate ester matrix as the main structural materials together with CFRP, reflecting layer at the backside.

The design variables chosen for optimization are listed in Table 1.

Therefore, this task, again, is a multi-objective problem with a set of mixed discrete-continuous design variables.

The finite element model consists of shell elements for the CFRP ground plate, solid elements for the honeycomb between the ground plate, and the first Kevlar® layer and layered solid elements for the top assembly of the copper patches, the Kapton® foil, the honeycombs, and the Kevlar® layers.

The computation time for all load cases required 2 min, and so the optimization with 120 designs in each of the 30 generations (so ~3,600 function evaluations) would have lasted 120 h on a single computer. Using 20 processors in parallel, optimization time was reduced to an overnight effort.

Figure 8 gives typical results of two optimization runs either with or without consideration of the effects of moisture loss by GAME. The circles and squares describe individuals generated by GAME plotted in the plane of the two main objectives "mass per reflector area" and "deformation" (given in RMS—root mean square deviations over the whole panel). The circles are optimal designs if only the thermal load cases are considered; the squares are optimal designs for all six load cases including moisture loss. From this, the actual design can be selected depending on the preference of either low deformation (RMS values) or low mass/area. This Pareto frontier also shows that a good compromise between both objectives can be achieved with an area-related mass from 2 to 2.2 kg/m^2 and a RMS deformation around 0.5 mm. Smaller deformations would have to be bought by a significant mass increase.

Table 1 Design parameter for reflect array sandwich panel

Parameter name	Number	Type	Range
Honeycomb height	1	Continuous	5–50 mm
Fiber volume content	8	Continuous	40–63%
Ply material	1	Discrete	Kevlar®/epoxy M40/epoxy
Fiber orientation	8	Discrete	0/90°, ±45°
Copper patch thickness	3	Discrete	5/9/18 µm
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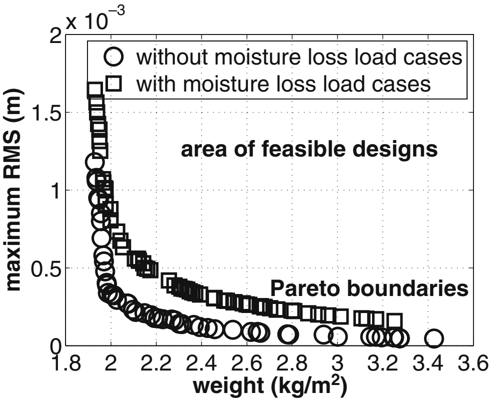

Fig. 8 Pareto front of mass vs RMS shape deviation due to thermal loads for reflect array panel

5.2 Optimization of a hybrid material stiffened satellite equipment plate

Platforms for arranging equipment, e.g., for opto-mechanical functions are relevant for science or earth observation instruments on satellites as described in Puelhofer et al. (2004). Such a platform is shown in Fig. 9 where the different components are to be arranged such that functional, geometrical, and also thermal constraints are satisfied. At the same time, the panel has to be designed such that it is dimensionally stable (very low expansion) under thermal and dynamic loads, and it also has to satisfy (dynamic) stiffness and strength requirements. A detailed description can be found in Langer et al. (2002). The 17 model parameters are described in Table 2.

The instruments' weights are m1=3 kg, m2=8 kg, and m3=5 kg, respectively. Each instrument is mounted by four attachment joints at the corners of each instrument. The platform is loaded by four load cases: static acceleration in three directions (ax=10 g; ay=−10 g; az=20 g); modal analysis—calculation of the first ten eigenfrequencies; sinusoidal vibration test; "pseudo temperature" load case, analytically calculated.

Before a finite element model (shells with point masses and connection beams) is generated, four geometric con-

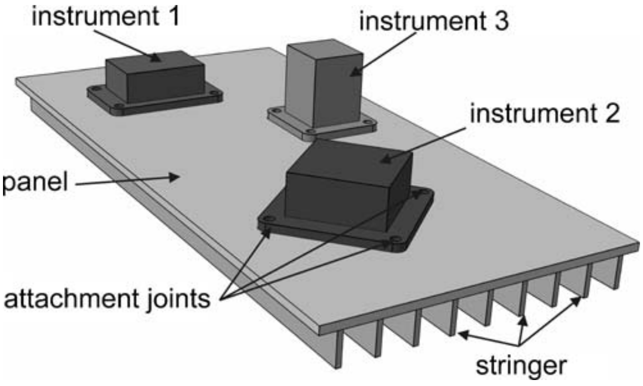

Fig. 9 Stiffened equipment panel together with equipment boxes

Table 2 Design parameter stiffened satellite equipment plate

Parameter name	Number	Type	Range
Panel length	1	Continuous	500–1,500 mm
Panel width	1	Continuous	500–1,500 mm
Panel thickness	1	Continuous	1–100 mm
Instr. Location X	3	Continuous	0.0–1.0×panel width
Instr. Location Y	3	Continuous	0.0–1.0×panel length
Instr. rotation	3	Continuous	0–90°
Stringer height	1	Continuous	50–350 mm
Stringer thickness	1	Continuous	1–100 mm
Stringer number	1	Discrete	0–10
Panel material	1	Discrete	Aluminum, steel, titan, quasiisotropic CFRP lam
Stringer material	1	Discrete	Aluminum, steel, titan, quasiisotropic CFRP lam
	17	Mixed	

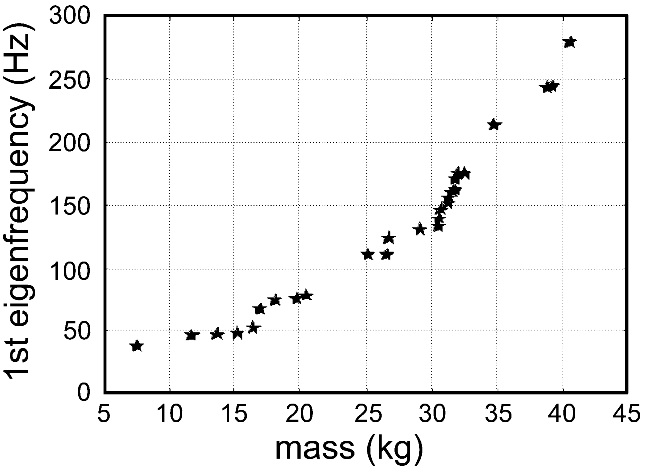

Fig. 11 Pareto front of mass vs first resonance frequency for equipment panel

straints have to be verified in a ProE-CAD model: The distance between the stringers have to be below a certain value (0.05 m), the screwed joints between instruments and panel must have a certain distance from the panel edges (>0.05 m), no instrument overlapping is allowed, and the screwed joints must have a certain distance from the stringers to fit properly. These constraints lead to highly disjoint feasible design spaces.

Several mechanical constraints have to be fulfilled: The stresses in the panel, as well as in the stringers due to the static load case 1, must be below their elastic limits, the bearing and shear stresses in the panel at each attachment joint calculated from the instrument's acceleration at load cases 1 and 3 must be below their elastic limits, the instrument temperatures must be within their allowed ranges, and the first eigenfrequency is restricted between 35 Hz<1. EF<50 Hz [otherwise, the stiffened panel gets as stiff (and heavy) as possible in the optimization].

As objectives, minimum mass and high vibration resonance frequency are to be achieved. Because of the continuous geometry parameters on the one side and the discrete number and positioning of stiffeners and type of materials to be selected on the other side, this then leads to a combined continuous-discrete optimization problem. A population size of 200 individuals with 300 children has been chosen and the evolution has been run over 20 generations. The computation time is about 5 min per individual; overall, a complete run took 4,000 analyses. The parallel systems equation computation on a cluster again drastically reduces the overall computation time.

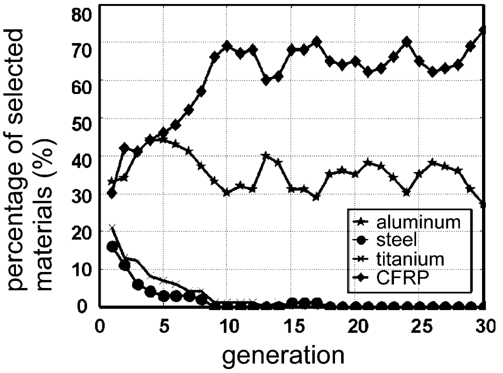

Fig. 10 Typical material selection process during optimization iteration steps

The iteration process and results obtained via GAME algorithm are outlined in Figs. 10 and 11, respectively. The material selection part of the design optimization process shows that steel and titanium are eliminated during the iteration steps, while an aluminum alloy and CFRP are kept in depending on the type and limits of constraints. Figure 11 gives the Pareto-curve of mass vs first resonance frequency and allows the designer and decision maker to select the most appropriate design out of this "set of optimal compromises". It should be noted that each design point in this plane of objective functions satisfies the constraints mentioned above, with the different related continuous and

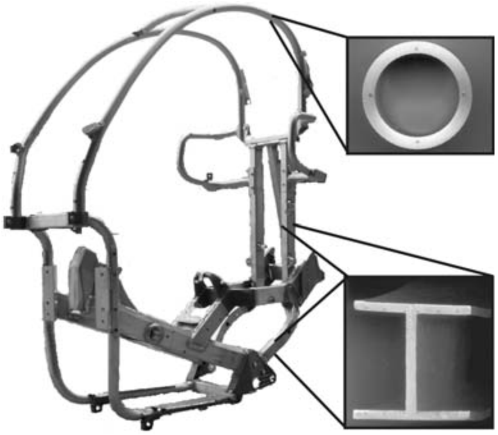

Fig. 12 Typical space frame of motorcycle with reinforced aluminum profiles

Table 3 Design parameter for reinforced aluminum framework beam

Parameter name	Number	Type	Range
I height	1	Continuous	10–80 mm
Flange width	1	Continuous	5–80 mm
Web thickness	1	Continuous	0.4–20.0 mm
Flange thickness	1	Continuous	0.4–20.0 mm
Diameter reinforcing element	1	Continuous	0.5–2 mm
No. of reinforcing elements	1	Discrete	0–15
Material combination	1	Discrete	Al/St, Al/C–fibers, Mg/St, Mg/C fibers
	7	Mixed	

discrete design parameter vectors stored in a database available for the decision process on the optimal goal compromise. A similar task was solved for the positioning of equipment boxes in the payload compartment of a satellite. Center of gravity demands and structural mechanic aspects where optimized; see also Puelhofer et al. (2004).

5.3 Reinforced aluminum framework beams with structural and manufacturing constraints

Reinforced extruded profiles (see Section 4, Fig. 4) made out of aluminum or magnesium base material are investigated for automotive space frames (Kleiner et al. 2006), e.g., Fig. 12. Reinforcement could be steel ropes or carbon fiber bundles together with proper surface treatment. One I-section beam of this structure is optimized for low mass and high stiffness with an analytical model (see also Baier and Huber 2005a). Seven design variables determine the cross-section of the beam, the number and diameter of the reinforcing elements, and the material combination used (Table 3).

The designs are constrained by: allowed manufacturing effort, maximum circumscribing diameter, minimum distance between reinforcements, minimum distance between reinforcement and surface, minimum wall thickness (the geometric relation between the global geometry, the material, and the minimum wall thickness is modeled as shown in Fig. 2), factor of safety >1.1, and two geometric constraints on the I-shape.

The minimum wall thickness for an extruded profile without reinforcements is a function of the circumscribed diameter d_B and the matrix material. Reinforcements increase the minimum wall thickness at least about the wire/rope diameter because the reinforcing elements have to be pulled by the matrix material during the extrusion process. In addition, an approximation of the manufacturing effort (not directly cost, but a complex extrusion die will also be more expensive) is included, which consists of the following models: effort due to different wall thicknesses in one cross-section depending on the ratio of web and flange thickness for extrusion molding; effort due to the reinforcing elements depending on the number of the reinforcing elements and the circumscribed diameter of the cross-section for extrusion molding; effort due to the cutting of the profile after extrusion depending on the number of the reinforcing elements and the circumscribed diameter of the cross-section. The output of this model is a non-dimensional effort value between zero (very low manufacturing effort) and one (high manufacturing effort). This model is derived by a process as outlined above in Section 4.

As example for the fuzzy logic-based knowledge representation, the model for manufacturing effort for the extrusion process with reinforcing elements is established in the following way. First, the input parameters have to be defined. The double T profile needs feeds for the reinforcing element, and the matrix material has to flow around these feeds. Therefore, this profile is similar to hollow profiles (e.g., tube cross-section), and with an increase of the circumscribed diameter d_B, the requirements for the extrusion press increase too. The second influence is the number of reinforcing elements n, which increase the complexity of the so-called supply plate because one supply bore is necessary for each reinforcing element. Both inputs are represented with three linguistic labels and triangle membership functions. The

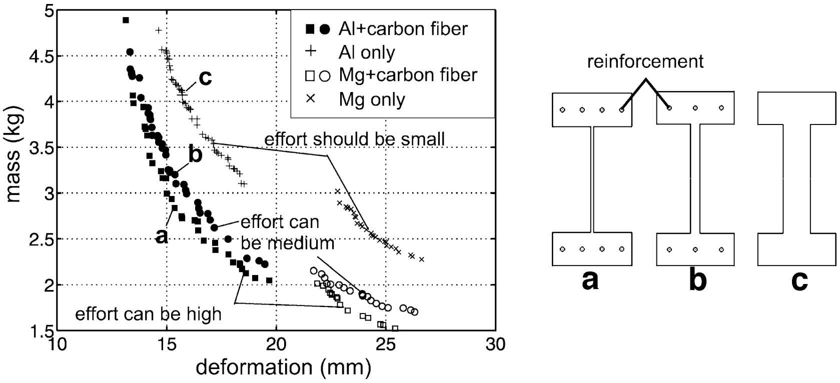

Fig. 13 Optimization results for the I-section beam: on the *left side*, Pareto fronts for different manufacturing effort restrictions resulting into different designs given on the *right side*

output of the model is represented with three labels (small, medium, high). The next step in the modeling process is the linking of the inputs to the output with rules. If no reinforcing elements are used, cost is small for all profiles. With increasing number of *n*, the cost increases especially for small profiles because of the complexity and strength requirements of the supply plate. With three labels for each input/output, all possible combinations are represented with nine rules. Two effort submodels for extrusion molding are combined with equal weights, and the overall manufacturing effort is also accumulated with equal weights for the manufacturing steps.

The optimization was performed with 35 generations, 80 individuals, and 160 children in each generation. In Fig. 13, three Pareto fronts for I-shaped profiles for varying constraints on the manufacturing effort are shown. Each point is an optimal design, which means that no design with a lower mass can be found for a given deformation. From this, the following conclusions can be drawn:

- If higher manufacturing effort can be tolerated, aluminum profiles with carbon fiber reinforcements are optimal. Parts with less then 2-kg mass can be achieved with magnesium–carbon fiber combinations. Four to six reinforcing elements are located in the flanges, and the web and flange thicknesses are quite different, which is beneficial for mass, but at a high manufacturing cost.
- A higher restriction on the manufacturing effort leads to less reinforcing elements, but a four times higher web thickness. This clearly shows the change in the optimal design if manufacturing process limitations are considered.
- A very strict restriction on the manufacturing effort can be satisfied only without reinforcements and still higher, equal and constant thickness for the web and the flange.

The three designs have a mass of 2.8 kg (a), 3.2 kg (b), and 4.1 kg (c) for an allowable deformation of 16 mm.

6 Conclusions

For design optimization of hybrid materials and structures, a series of conclusions can be drawn from different applications and results:

- Due to the structure of the optimization problem with many types of system equations (e.g., geometrical—CAD, numerical, analytical, approximations, etc.), difficult to get derivatives and discrete variables, evolutionary solution strategies are a reasonable option especially in the case of parallel processing on a computer cluster.
- In case no (high fidelity) simulation model for relevant aspects in the design optimization problem is available yet, at least approximate response surface models derived from qualitative knowledge and fuzzy rule techniques might be in order.
- In addition to beneficial effects by material composites, drawbacks, e.g., due to moisture loss and thermo-elastic mismatches, also have to be considered.
- Combined strength and manufacturing requirements might lead to an optimal selection of moderately yield strength metal parts because benefit for then lower extrudable thicknesses for lower stressed areas might overbalance required higher thicknesses needed in higher stressed areas.

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