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INDUSTRIAL APPLICATIONS

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Computer-aided derivation of the optimal mathematical models to study gear-pair dynamic by using genetic programming

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Abstract A general problem is addressed to perform optimal identification of the dynamic system automatically, by using genetic programming algorithm (Koza 1992). The main objective of this approach is to derive optimal mathematical model (reliable and accurate) and determine optimal parameter values for generated mathematical model on the basis of measured dynamic response for selected structure that behaves dynamically. A gear-pair dynamic is studied as an example.

Keywords Genetic programming · Gear dynamic response · Mathematical model · Optimization · System identification

1 Introduction

These days, dynamic analyses of real life structures are of particular interest. In general, engineers use their knowledge and experience to identify dynamic system and simplify real structure on the basis of identification, derive mathematical models, and choose parameter values for derived model. Considering real life structures, optimal parameters value calculation procedures based on mathematical programming procedures are already well known (Prebil et al. 2002). However, procedures for optimal identification and modeling are not used very often for solving real life structure problems (Ciglarič and Kidrič 2003).

In the presented paper, genetic programming is used to perform fully automatic (computer-aided) optimal identification of the dynamic system and generate mathematical model in a symbolic form, on the basis of measured dynamic response. Moreover, optimal system parameter values are calculated to obtain good agreement between measured and calculated dynamic response. The objective function is developed as a measure for the optimal identification of the mathematical model and its parameter values. The main advantage of the proposed approach is the optimal computer-aided identification of the dynamic system and automatic derivation of mathematical model in a symbolic form that could be used for any further analysis. The system of nonlinear differential equations in a symbolic form is generated fully automatically by the developed genetic programming algorithm on the basis of measured dynamic response data and chosen set of terms that are common elements in the mathematical formulation of the Newton–Euler laws of mechanics. Presented genetic algorithm and objective function are capable of identifying a dynamic system with one or more degrees of freedom and with linear and nonlinear characteristics.

2 Genetic programming

Genetic programming is an algorithm that iterates through the following steps:

1. Generation of the initial population.
2. Individual member evaluation by using objective function.
3. Generating new members of a population for the next generation on the basis of evaluation.
4. Building a new population for the next generation.

Second, third, and fourth steps have to loop, until criterion imposed by objective function is fulfilled.

Initial population is defined with the finite number of members of the population. Members of the population are generated coincidentally on the basis of selected list of the base functions and the list of variables and time-dependent functions. Each member of the population is a mathematical expression, which is represented symbolically as a tree structure. A tree structure is a structure with the nodes of which are filled with base functions and the endpoints are

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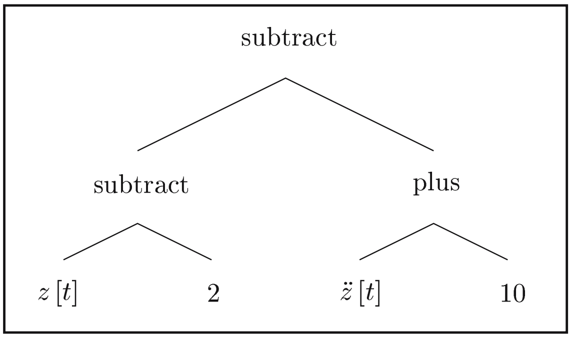

Fig. 1 Parent 1

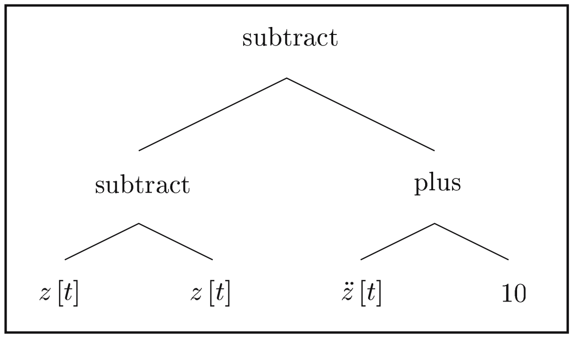

Fig. 3 Child 1

filled with variables and time-dependent functions. For better understanding, let us define a list of base functions:

$$funcs = \{plus, subtract, times, divide\} \tag{1}$$

and a list of variables and time-dependent functions:

$$terms = \{z(t), \dot{z}(t), \ddot{z}(t), Rnd[int], Rnd[real]\}. \tag{2}$$

The base function, (1), represents symbolical names for well-known mathematical operations, such as addition, subtraction, multiplication, and division. Furthermore, $Rnd[int]$ and $Rnd[real]$ represent random integer and random real number from chosen interval. Typical members of a population, generated on the basis of the list of base functions, (1), and the list of terms, (2), are presented in Figs. 1 and 2.

By using criteria defined in an objective function, we indicate suitability of a particular member for placing it to the population of the next generation. We also assure that members of each generation produce better solutions which through iteration of genetic programming procedure converge to optimal solution.

Therefore, objective function represents the one and only criterion, which indicates the successfulness of a generation or its individual member. Objective function has therefore an essential influence on solution quality.

Generating new members of a population for the next generation is done with genetic operators:

1. Crossover,
2. Mutation,
3. Perturbation, and
4. Reproduction.

Genetic operation crossover deals with two selected members of a population. In general, those two members of a population are called parents. A new member of a population is generated with the replacement of the coincidental part of the tree structure of the first parent with the coincidental part of the tree structure of the second parent. Figures 3 and 4 show children of parents represented in Figs. 1 and 2.

Genetic operation mutation replaces a part of the tree structure of an individual member of a population with a new, coincidentally generated tree structure. In many cases, the major improvement of a member of a population can be obtained with small changes. Perturbation is a genetic operator, which is capable of making such small changes.

This operator replaces coincidentally chosen variable of a tree with a new value or a new variable. Figures 5 and 6 show possible results of the mutation of the child 1 and perturbation of the child 2.

Reproduction is the simplest genetic operator, which creates a new member of a population as a copy of existent one.

An individual genetic operator does not have the same influence on the population (Koza 1992). Each of the genetic operators appears with a certain probability. In general, a genetic operator crossover has the largest influence on the population.

Building a population for the next generation is reaching a decision, which members of a generation will be replaced with new members. There are some different strategies that could be used to accomplish this task. Elitist strategy is used

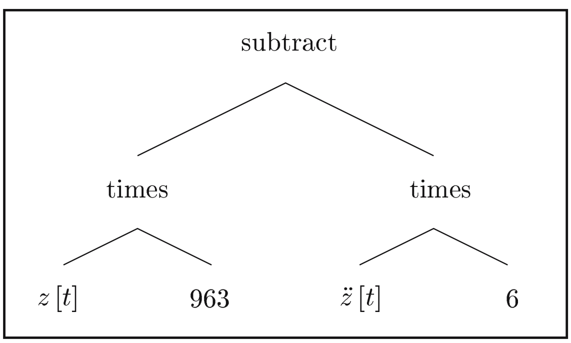

Fig. 2 Parent 2

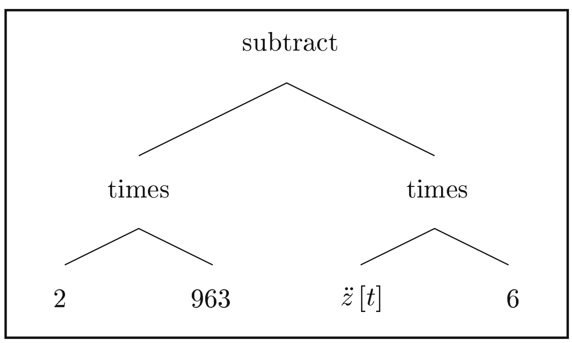

Fig. 4 Child 2

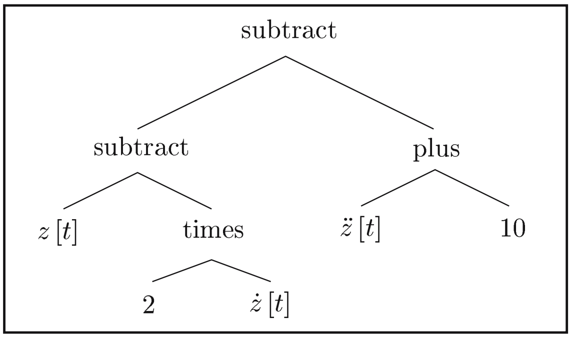

Fig. 5 Mutated child 1

in our model. With elitist strategy, the members, who are placed to the next generation, have the best calculated value of fitness function.

3 Objective function

Considering genetic programming, objective function is defined as a criterion, which recognizes individual member structure and calculates its measure that evaluates member as potentially optimal and therefore suitable for the next generation.

Each member of the population is evaluated by two criteria:

1. Evaluating its structure and
2. Calculating value of the objective function.

In comparison to classical, nonlinear programming methods, objective function in genetic programming is much more complex. Objective function should be capable of evaluating the structure of each mathematical expression that represents particular member in a population in such a way that mathematical expression complies with Newton–Euler laws of mechanics. Also, each member that fulfilled the first criterion should be evaluated by calculated value of the objective function that represents chosen measure for optimality.

In this research, dynamic systems have to be identified and dynamic system parameters should be chosen to obtain good agreement between measured and calculated dynamic response. As dynamic systems are mathematically represented in the form of nonlinear differential equation, the list of terms, (2), is not only variables but also time-dependent functions and their derivatives as well. Mathematical models in the form of nonlinear differential equations cannot exist in any form because the laws of mechanics bound the structures of mathematical models for dynamic systems. The major problem is therefore how to define and mathematically formulate a criterion to identify each member structure and which measure should be used to evaluated member as potentially optimal. Also, it is essential to know how to combine these two elements into the objective function.

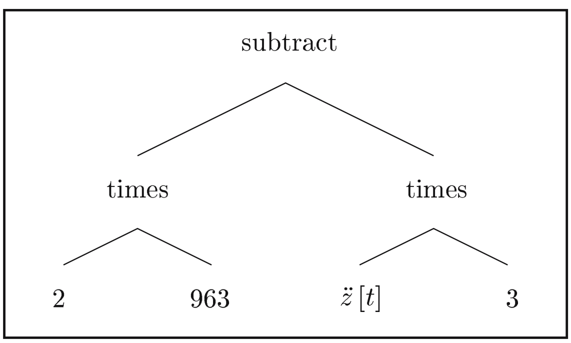

Fig. 6 Perturbated child 2

Observing and analyzing mathematical expressions produced by Newton–Euler laws of mechanics, one could see that dynamic equation for particular degree of freedom essentially represents mapping of time-dependent function and its derivatives. Such mathematical expression always appears in the following form:

$$M[\ddot{z}(t)] + \sum_{i=1}^{n1} C_i[\dot{z}(t)] + \sum_{j=1}^{n2} K_j[z(t)] = \sum_{k=1}^{n3} F_k(t), \tag{3}$$

$$J[\ddot{\varphi}(t)] + \sum_{i=1}^{n1} C_i^{\varphi}[\dot{\varphi}(t)] + \sum_{j=1}^{n2} K_j^{\varphi}[\varphi(t)] = \sum_{k=1}^{n3} F_k^{\varphi}(t). \tag{4}$$

In (3) and (4), C_i, K_j and F_k represent the functions that could be locally noncontinuous.

Considering the adequacy of a structure of (3), some conditions have to be fulfilled. The first condition is that individual member of a population has to fulfill $\ddot{z}(t)$ a term existence. Term represents system center of gravity acceleration with defined degree of freedom. Once the number of degree of freedoms is fixed, such term has to exist in (3). Nonexistence of such term means that there is no dynamical system. What is more, we consider only such dynamical systems where $\ddot{z}(t)$ does not exist in any other form than a linear combination of $\ddot{z}(t)$ and time-independent constants. In such way, only dynamical systems with constant inertia properties are considered. Considering terms $C_i[\dot{z}(t)], i = 1, \dots, n$ and $K_j[z(t)]$, $j = 1, \dots, m$ in (3), additional restrains have been imposed in such a way that mapping:

$$C_i : \dot{z}(t) \to C_i[\dot{z}(t)],\ i = 1, \dots, n \tag{5}$$

could not depend on expression $z(t)$ and mapping:

$$K_j : z(t) \to K_j[z(t)],\ j = 1, \dots, m \tag{6}$$

could not depend on expression $\dot{z}(t)$. This means that we consider only dynamical systems that could be identified with nonlinear differential equations with constant coefficients. The similar explanation is valid for (4).

The next step of the particular member evaluation is a numerical calculation of objective function for each member, which passes the conditions about structure. Numerical value of objective function for members with inadequate structure is not calculated numerically at all and obtains high numerical value by definition. For our particular problem which considers gear-pair dynamics the part of objective function, considered for numerical evaluation are the embedded two criteria. One criterion is related to time domain, while the other is related to frequency domain. Objective function is therefore represented as linear combination:

$$f = w_1 \cdot f_{time\,domain} + w_2 \cdot f_{frequency\,domain}, \quad (7)$$

where w_1 and w_2 are chosen weight factors.

The time domain part of objective function is defined in such a way to minimize the difference between measured and calculated driving shaft torsion deformation. The following formulation has been found suitable to minimize the difference:

$$f_{time\,domain} = \sqrt{\sum[\varphi_1(t_i) - \varphi_2(t_i)]^2};\ i \in [1, 2, \ldots, n], \quad (8)$$

where $\varphi_1(t_i)$ represents $i-$th measured driving shaft torsion deformation and $\varphi_2(t_i)$ represents $i-$th calculated driving shaft torsion deformation.

The frequency domain part of objective function is defined in such a way to minimize the difference between measured and calculated driving shaft frequency-dependent torsion deformation. The following formulation has been found suitable to minimize the difference:

$$f_{frequency\,domain} = \sum_{j=1}^{m} \left|v_j^1 - v_j^2\right|;\ j \in [1, 2, \ldots, m]. \quad (9)$$

In (9), v_j^1, $j \in [1, 2, \ldots, m]$ represents the sequence of first $m-$th measured driving shaft torsion deformation amplitudes expressed in frequency domain, while v_j^2, $j \in [1, 2, \ldots, m]$ represents the same values calculated with generated mathematical model.

4 Experimental results

Experimental results of gear-pair dynamics are essential because mathematical models are generated on the basis of measured dynamic response. Measuring device is shown in Fig. 7. Device consists of servomotor as a driving unit, driving shaft equipped with sensors, and reduction gear and servo break used to set the proper driving torque value. Driving shaft has been equipped with Mohilo–Steiger torque sensor. The system has been used to measure time-dependent driving torque values.

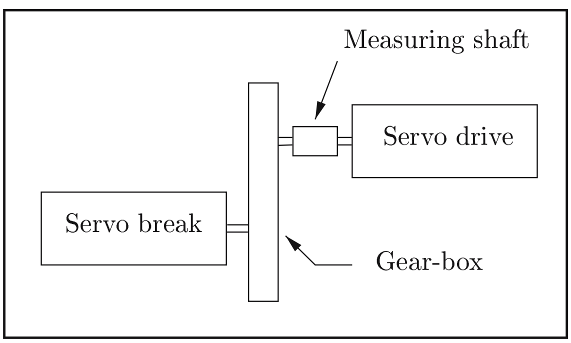

Fig. 7 Scheme of measurement equipment

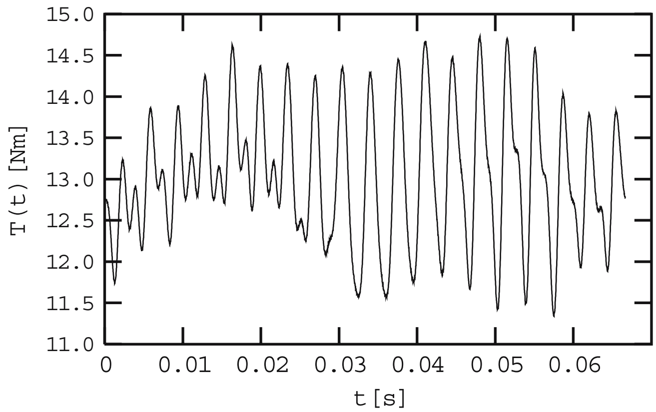

Fig. 8 Generic diagram of driving torque

Time-dependent driving torque values have been measured at driving shaft angular velocity $n = 900\,rpm$ and nominal driving torque $T = 13\,Nm$. Values have been measured within time interval of $1\,s$.

The considered gear pair consists of pinion with 19 teeth and gear with 54 teeth. The module of gears has been $m = 4mm$, since the addendum modification coefficients have been $x_1 = 0.228$ and $x_2 = -0.228$. The helix angle has been $\beta = 0^0$ and the thickness of gears has been $b = 25mm$. Therefore, we considered a gear pair with gear ratio of 2.8421. Gear ratio is such that within 15 revolutions that occur within the time interval of $1\,s$, there is always a different teeth pair in the gear meshing. Therefore, gear-pair driving torque is influenced due to geometrical and material imperfections of the gear pair. To neglect the influence of geometrical and material imperfection of gear pair, well-known statistical methods (Chase and Parkinson 1991; Cvetko et al. 1998) have been used to generate generic time-dependent driving torque, as shown in Fig. 8. Generic driving torque has been calculated on the base of 15 successive measurements (Belšak 2004) of driving torque, each measured on time interval corresponding to one revolution of the pinion. Generic driving torque at specific time represents the mean value of measured values. On the basis of generic time-dependent driving torque and known geometrical and material properties of driving shaft, it was possible to calculate driving shaft torsion deformations, as shown in Fig. 9. A driving shaft time-dependent torsion deformation therefore represents gear-pair dynamic response which should also be calculated with a generated mathematical model.

5 Identified mathematical model and numerical results

To generate gear-pair mathematical model, we have used the following list of base functions:

$$funcs = \{plus, subtract, times\}, \quad (10)$$

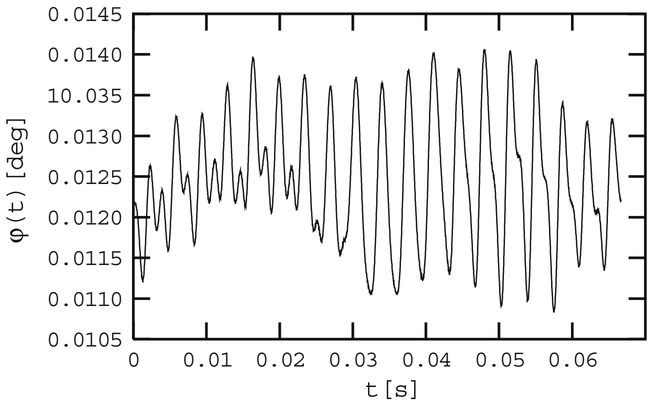

Fig. 9 Generic diagram of driving shaft torsion deformations

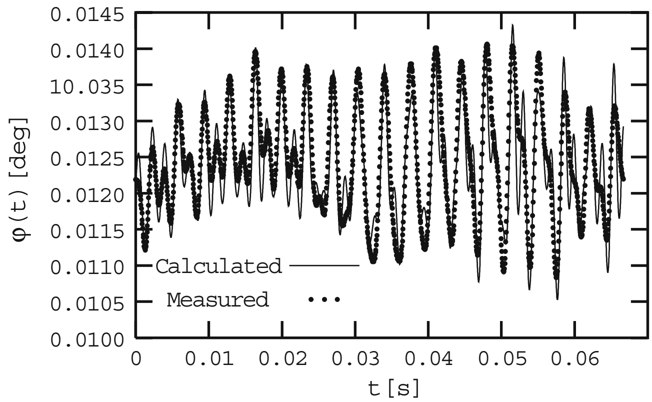

Fig. 10 Measured and calculated driving shaft torsion deformation

and a list of variables and time-dependent functions:

$$terms = \begin{Bmatrix} \varphi[t],\ \ddot{\varphi}[t],\ T[t], \\ k_1[Rnd[Re\{0.1, 20\}], Rnd[Re\{800, 1000\}]], \\ k_2[Rnd[Re\{0.1, 20\}], Rnd[Re\{800, 1000\}]], \\ J,\ N_1,\ r_1,\ t \end{Bmatrix}. \quad (11)$$

In (11), $\varphi[t]$ represents time-dependent torsion deformation due to driving shaft oscillations, $\ddot{\varphi}[t]$ its second time derivative, and $T[t]$ the generic driving torque that has been calculated on the base of 15 successive measurements (Belšak 2004). Furthermore, k_1 and k_2 are two elements of resultant gear mesh stiffness, which could be numerically represented as a combination of real numbers from two selected intervals. Intervals have been set on the base of experience we have had with modeling of such dynamic systems. Finally, J represents the moment of inertia of driving gear, N_1 is the number of teeth of driving gear, r_1 is the kinematics radius of driving gear, and t represents time. The latest four terms are numerically unbounded but should be nonnegative numbers.

To generate the model, the following genetic operator occurring probabilities has been chosen (Koza 1992):

- Crossover—63 %,
- Mutation—27 %,
- Perturbation—9 % and
- Reproduction—1 %.

The number of members in each generation has been set to 500. The activity of genetic algorithm has been limited with two predefined criteria. The first one was a maximum number of generations, and the second one was tolerance for the objective function, (7). It was chosen that only up to ten generations could be generated during the optimization process. The maximum number of generations was limited because of high memory and CPU time consumption. The background is a numerical calculation of up to 500 differential equation generated for each generation. The other criterion used to stop genetic algorithm activity is objective function value. If objective function, (7), reaches value within predefined tolerance ε:

$$f \leq \varepsilon,\ \varepsilon = 0.01, \quad (12)$$

genetic algorithm activity is stopped. The amounts of weight factors have been set to $w_1 = 1$ and $w_2 = 1$ on the base of few numerical try-outs.

Numerical evaluation of each mathematical model has been calculated for initial conditions:

$$\varphi[0.0012973] = 0.010955$$
$$\dot{\varphi}[0.0012973] = -0.692100 \quad (13)$$

obtained from the measurements we perform. The proposed procedure generated the following mathematical model in 10^{th} generation with CPU time consumption of 1 h, 17 min, and 54 s on average personal computer:

$$J\ddot{\varphi}[t] + K_1\varphi[t] + K_2\varphi[t]^2 = T[t], \quad (14)$$

where:

$J = 0.00007918\, kgm^2$

$K_1 = 413.191\, Nm/deg$

$K_2 = 51721\, Nm/deg^2$.

Note that from physical point of view, calculated stiffness coefficients K_1 and K_2 are defined as $K_1 := r_1k_1$ and $K_2 := r_1k_2$. The dynamic response calculated by generated mathematical model, (14), and the initial conditions, (13), are shown in Fig. 10. The excellent agreement between calculated and measured values could be observed.

6 Discussion of the results

To demonstrate the effectiveness and suitability of the proposed procedure, we have had studied other known mathematical models of a gear pair that have been proposed and used by other authors. We found out that in most cases, a me-

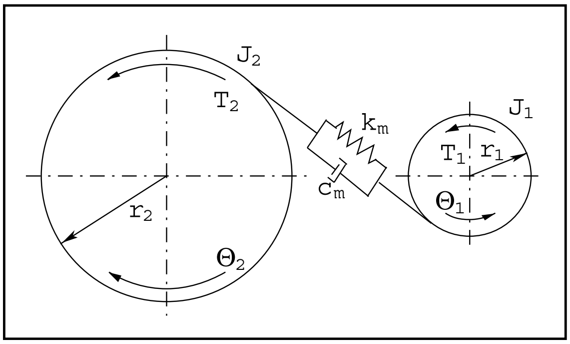

Fig. 11 Mechanical model of gear pair

chanical model shown in Fig. 11 is used as a background to develop a suitable mathematical model. A mechanical model in Fig. 11 consists of two rotating disks which represent the inertia of the gear and a spring and damper that represent resulting mesh stiffness and resulting mesh damping. The most important considered effects on gear-pair dynamics that are studied in greater details are the effect of mesh resulting stiffness and the effect of the gear manufacturing errors. To compare calculated mathematical model, (14), with some known results, we performed analysis of two typical models discussed in the works of Kuang and Lin 1997 and Ozguven and Houser 1988.

A rather complex model, which discusses the importance of resultant gear mesh stiffness and includes also other effects, has been studied by Kuang and Lin 1997. In this work, it is pointed out that resultant mesh stiffness between an engaged gear pair consists of two parts: one associated with Hertzian contact (Yang and Sun 1985) and one associated with gear tooth bending. The Hertzian stiffness is caused by local tooth surface deformation due to contact forces and could be calculated

$$k_h = \frac{\pi E b}{4(1-\nu^2)}, \tag{15}$$

where E is Young's modulus, b is the contact width of tooth face, and ν is Poisson's ratio. It is important to note that the Hertzian stiffness is only related to gear material properties and the gear tooth face width and therefore keeps constant along the path of contact.

Following results published by the same authors (Kuang and Yang 1992), the effect of different gear parameters, such as module (m), tooth number (N), and the addendum modified coefficient (x), on the bending stiffness should be considered. Therefore, bending stiffness for gear 1 and gear 2 at contact points A and B could be calculated as

$$k_1^A = b \cdot \overline{k}_1^A \tag{16}$$

$$k_2^A = b \cdot \overline{k}_2^A$$

$$k_1^B = b \cdot \overline{k}_1^B \tag{17}$$

$$k_2^B = b \cdot \overline{k}_2^B,$$

where the specified gear is denoted by the subscript, and the superscript is used to indicate the contact point. In (16) and (17), $\overline{k}_i^j$ ($i = 1, 2$; $j = A, B$) represents bending stiffness per unit of gear tooth width and b is the contact width of tooth face. Based on the results calculated from finite element method (Kuang and Yang 1992) and curve fitted methods, the bending stiffness per unit of gear tooth width could be calculated as

$$\overline{k} = (A_0 + A_1 x) + (A_2 + A_3 x)\frac{(r-R)}{(1+x)\cdot m}, \tag{18}$$

where the unit stiffness $\overline{k}$ is measured in $(N/\mu m/mm)$ and the radius of pitch circle R and the module m are measured in (mm). The curve fit coefficients A_0, A_1, A_2 and A_3 could be calculated as specified in the work of Kuang and Lin 1997. Gear mesh stiffness at contact points A and B could be therefore calculated as a combination of Hertzian and bending stiffness

$$K_A = (1/k_1^A + 1/k_2^A + 1/k_h)^{-1} \tag{19}$$

$$K_B = (1/k_1^B + 1/k_2^B + 1/k_h)^{-1},$$

and resultant gear mesh stiffness as

$$K = K_A + K_B. \tag{20}$$

If only one tooth pair is in contact, then $K_B = 0$,; however for gear at high operation speeds, the resultant gear mesh stiffness could be calculated as an average from values related to single and double gear tooth contact (Shing 1994).

Derived mathematical model (14) and calculated stiffness coefficients K_1 and K_2 could be easily related to detailed mathematical model discussed in the work of Kuang and Lin 1997. Resultant gear mesh stiffness calculated with expressions from (16) to (20) is composed of from linear Hertzian stiffness and nonlinear bending stiffness. The important characteristic of derived mathematical model (14) is the combination of linear and nonlinear part of resultant gear mesh stiffness, which could be related to linear Hertzian stiffness and nonlinear bending stiffness, respectively.

The model studied by Parey and Tendon 2003 discusses the following equations of motion:

$$\begin{aligned} J_1\ddot{\Theta}_1[t] &+ r_1 c_m(r_1\dot{\Theta}_1[t] - r_2\dot{\Theta}_2[t]) - r_1 c_1 e_1'[t] \\ &- r_1 c_2 e_2'[t] + r_1 k_m\,(r_1\Theta_1[t] - r_2\Theta_2[t]) \\ &- r_1 k_1 e_1[t] - r_1 k_2 e_2[t] = T_1[t] \end{aligned} \tag{21}$$

and

$$\begin{aligned} J_2\ddot{\Theta}_2[t] &+ r_2 c_m(r_2\dot{\Theta}_2[t] - r_1\dot{\Theta}_1[t]) - r_2 c_1 e_1'[t] \\ &- r_2 c_2 e_2'[t] + r_2 k_m\,(r_2\Theta_2[t] - r_1\Theta_1[t]) \\ &- r_2 k_1 e_1[t] - r_2 k_2 e_2[t] = -T_2[t], \end{aligned} \tag{22}$$

where J_1 and J_2 are the mass moments of inertias of gears 1 and 2, respectively. Θ_1 and Θ_2 are the angular displacement of the gears 1 and 2, respectively. $\dot{\Theta}[t]$ and $\ddot{\Theta}[t]$ are the first and second derivatives of gear angular displacement $\Theta[t]$, respectively. c_m is the resultant viscous damping coefficient of the gear mesh, k_m is the resultant stiffness of the gear mesh, $T_1[t]$ and $T_2[t]$ denote the input and output torque, respectively, and $e_1[t]$ and $e_2[t]$ are the displacement excitations functions representing the relative gear errors of the meshing teeth.

It is not difficult to show that mathematical model represented with (21) and (22), and mathematical model, (14), generated by genetic programming procedure have some important similarities. In fact, the mathematical model represented with (21) and (22) could be very easily simplified as

$$J_1\ddot{\varphi}_1[t] + r_1 k_m \varphi[t] = T_1[t], \tag{23}$$

if we assume the following:

1. The stiffness of the gearing shaft is much higher than the stiffness of the driving shaft. Due to the high stiffness of gearing shaft, angular displacements of a gear are very small and could be consequently neglected. This also means that one degree of freedom could be neglected. In addition, time-dependent variable in (23), $\varphi_1[t]$, represents relative angular displacement of a pinion with respect to gear angular displacement.
2. Viscous damping of the gear mesh is very small and could therefore be neglected.
3. Moment of inertia of the gear is much higher than moment of inertia of the pinion.
4. The relative gear errors of the meshing teeth are very small and could also be neglected.

Once again, the similarity of the derived mathematical model, (14), and the mathematical model, (23), whose extraction is based on mathematical model described with (21) and (22), could not be neglected. Moreover, it should be pointed out that all four assumptions used to simplify mathematical model represented by (21) and (22) are valid for our experimental device shown in Fig. 7. The experimental device has been built up in such a way that gearing shaft is much stiffer than driving shaft, as only this approach ensure to monitor and measure dynamic response and contact forces accurately enough. In addition, all parts of considered experimental device have been produced with tight tolerances to reduce the effect of random errors on system dynamic response.

7 Conclusions

Comparison of calculated and experimental results as well as comparison of derived mathematical model, (14), with mathematical models discussed by other authors show the following:

1. It is evident from the calculated results, which are in excellent agreement with measured data, that the proposed procedure works. It is obvious that the proposed procedure is capable of generating a mathematical model and calculating optimal parameter values for a generated mathematical model as well.
2. Proposed procedure is capable of identifying the mathematical model that is closely related or even similar to the existing mathematical models published and discussed by other authors. However, this is the first time that mathematical model has been developed fully automatically by a computer and by using genetic programming procedure.
3. By using gradient-based optimization methods to minimize the difference between measured response and dynamic response calculated with (23), the best results that could be obtained show significant disagreement with measured results as could be seen in Fig. 12. Basically, these results inspire us to employ different methods and approaches, which are capable to identify not only optimal parameter values but also optimal mathematical model, and that are capable to cope with dynamic response of real life structure.
4. Incorporating mathematical models in a form of nonlinear differential equation, (14), into nonlinear programming procedure is a difficult task (Prebil et al. 2002); however, it is even more complicated to transform basic formulation in a form that could be solved with today's known numerical procedures. From this point of view, genetic programming procedure offers good possibility to avoid complicated procedures of transforming basic formulations into solvable ones.

Although composing an optimal dynamic model for multiple degree of freedom gear systems and other mechanisms is not addressed in this work, it is actually a very essential issue. In multidegree-of-freedom systems, some parameters (for example inertias of the elements) become configuration dependent. This makes the modeling problem even more complicated. The applied genetic programming strategies are already capable to address such problems; however, we need to set up some adequate experiments to obtain relevant measured data first. This is a subject of our future research work.

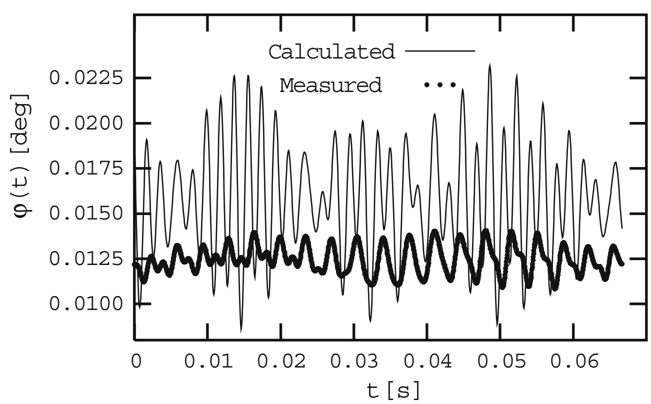

Fig. 12 Measured and calculated optimal driving shaft torsion deformation for linear model

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