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Applications of hybrid optimization techniques for model updating of rotor shafts

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Abstract This paper presents an approach by combining the genetic algorithm (GA) with simulated annealing (SA) algorithm for enhancing finite element (FE) model updating. The proposed algorithm has been applied to two typical rotor shafts to test the superiority of the technique. It also gives a detailed comparison of the natural frequencies and frequency response functions (FRFs) obtained from experimental modal testing, the initial FE model and FE models updated by GA, SA, and combination of GA and SA (GA–SA). The results concluded that the GA, SA, and GA–SA are powerful optimization techniques which can be successfully applied to FE model updating, but the appropriate choice of the updating parameters and objective function is of great importance in the iterative process. Generally, the natural frequencies and FRFs obtained from FE model updated by GA–SA show the best agreement with experiments than those obtained from the initial FE model and FE models updated by GA and SA independently.

Keywords Model updating · Finite element model · Optimization · Genetic algorithm · Simulated algorithm · Rotor shaft

1 Introduction

1.1 Concept of FE model updating

Finite element (FE) model-based dynamic analysis has been widely used to predict the dynamic characteristics of structures and rotating machinery. However, the results obtained from FE model often differ from the experimental results of vibration or modal testing. This disparity is due to modeling and experimental errors. The former results from uncertainties in geometry, boundary conditions, discretization error, modeling error of joints, variation of material properties, ignorance of nonlinear effect, and other simplifications, which could be the possible sources of inaccuracy presented in a FE model, while experimental errors are caused by incorrect imposition of boundary conditions, lower sensor accuracy, random excitation by noise, and signal processing errors (Levin and Lieven 1998; Modak et al. 2002). To reduce the errors between analytical and experimental results, efforts must be made in trying to reduce experimental errors or to improve the analytical model. Generally, to some extent, experimental errors can be controlled by applying high-accuracy sensors, reliable data acquisition, and well-developed signal measuring and extraction methods (Maia and e Silva 1997; Ewins 2000). On the other hand, many efforts have been made to minimize the difference between model prediction and experiment by correcting model errors. This process is termed as model updating, which is concerned with the correction of FE models by comparing records of dynamic properties from model prediction and experimental data. Obviously, for the sake of reducing the errors between analytical and experimental results, it will be more expensive to run a sophisticated experiment than to use computer simulation on FE model updating. In addition, many practical situations occur where the phenomenon of interest cannot be measured directly (Ranhois et al. 2001). For example, experiments can be carried out on a limited number of measured degrees of freedom (DOF) and also in a limited frequency range. Therefore, FE model updating has become very popular nowadays. However, model updating is after all an inverse process and contains highly nonlinear characteristics. In model updating process for practical applications, it is essential to obtain satisfactory correlation between analytical and experimental results and also to maintain physical significance of the updating parameters (Kim and Park 2004). For the above reasons, model updating is still a difficult problem to overcome.

In spite of the problems mentioned above, model updating is very useful for design, development, and application phase of a mechanical system. Many mechanical systems require dynamic response prediction, modification of dynamic

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characteristics, damage identification, and fault parameters identification. In the last 20 years, various methods for model updating have been proposed (Mottershead and Friswell 1993). Most of these studies are devoted to approaches such as the optimal matrix updating, sensitivity-based parameter estimation, eigen-structure assignment algorithms, and neural networks updating methods (Jeong and Lee 1996). These approaches can be broadly classified into two groups, namely, direct method and parametric method, depending on whether there is any adjustment on the mass and stiffness matrices either directly or indirectly. It has been shown that direct methods are not appropriate for model updating as the directly updated elements of the mass and stiffness matrices have no physical meaning, although the resulting updated matrices can exactly reproduce the measured modal data (Levin and Lieven 1998; Kim and Park 2004). For the parametric method, many approaches have been proposed as shown in reference (Friswell and Mottershead 1998). One conventional approach is to consider an objective function that quantifies the difference between the experimental and analytical data. Generally, the objective function to be minimized is usually defined as a penalty function involving the weighted sum of the differences between analytical and experimental results, such as natural frequencies or mode shapes. Although the ability to weight the different data seems versatile, the weighting factors are very difficult to be determined because the relative importance among the data is not obvious but specific for different problem. Furthermore, this method usually takes a very long time to obtain the satisfactory weights (Kim and Park 2004).

1.2 FE model updating based on optimization methods

Genetic algorithm (GA) and simulated annealing (SA) algorithms are both probabilistic search algorithms and are capable of finding globally optimum results to complicated optimization problems, which may be incorporated into model updating. Levin and Lieven (1998) employed the GA and SA independently in model updating for a beam and a flat plate wing structure. Kim and Park (2004) introduced a multi-objective optimization technique, *Parato* GA, to model updating. The emphasis of this technique was on the selection of updating parameters. Modak and Kundra (2000) proposed a model updating method to solve a constrained nonlinear optimization problem. Zimmerman (Zimmerman et al. 1999) investigated the GA-based approach for FE model topology and parameter adjustment. His contribution is in the formulation of a GA fitness function which can avoid both analytical and experimental problems associated closely with spaced modes of vibration. Although there are many papers on GA and SA applications in FE model updating, they are mainly focused on simply supported cantilever beams, plates, tubes, and space truss structures. Furthermore, GA and SA are incorporated independently into the model updating process. There are limited works on FE model updating for rotor shafts in rotating machinery systems.

The main aim of this paper is to present an approach by combining GA with SA to perform model updating of rotor shafts. Then, the results obtained from the FE model updated by GA-SA are compared with those independently updated by GA or SA. The paper has five main parts. A basic description of GA, SA, and combined GA–SA is presented in Section 2. This section provides an insight to the implementation procedure of GA and SA for a particular problem, and also a detailed GA–SA implementation procedure. The model updating procedure based on optimization technique is discussed in detail in Section 3. This section discussed the two important procedures, selection of updating parameters, and definition of objective functions in model updating. A detailed discussion and analysis on the results of two cases are explained in Section 4. In this section, some comparisons of natural frequencies and frequency response functions (FRFs) obtained from initial FE model, updated FE models, and experiments have been carried out while using different objective function in the model updating process. The final part is a conclusion, which describes the effectiveness of GA, SA, and combined GA–SA as optimization techniques for successful application in FE model updating. FE model updated by GA–SA can predict much better results matching the experiment than those models updated by GA and SA separately.

2 Basic descriptions of GA, SA, and GA–SA

Detailed tutorials on the subject of GA and SA can be seen from references (Holland 1975; Rao 1996; He et al. 2001; Pham and Karaboga 2000). In this section, some basic knowledge of these methods and the implementation procedure of the technique to a particular problem are presented.

2.1 Genetic algorithm

GA works on the principles of genetic and natural selection based on Darwin's *survival of the fitness* strategy. In natural evolution, members of population compete with each other to survive and reproduce successfully. If the genetic makeup of an individual member of a population has an advantage over its rivals, then it is more likely to breed successfully. As a result, the combination of genes that confers this advantage is likely to spread across the population. In this way, the population continuously adapts to its environment and also improves its *fitness* (Levin and Lieven 1998).

The application of GA needs six basic issues: chromosome representation, selection function, genetic operators such as mutation and crossover for reproduction function, creation of the initial population, termination criteria, and the evaluation function. In accordance, the first step in this algorithm is to define a coding process between the solutions and chromosome. In this process, two commonly encoding methods are normally used as the standard binary encoding and real-number encoding. The second step involves the creation of an initial population of the solutions. This is followed by the determination of the objective function and fitness. The three key genetic operators, selection, crossover, and mutation (Holland 1975), are applied to the old generation to generate a new generation. The process is repeated until a convergence result can be obtained. The last task is to *translate* the best chromosome into the solutions of the problem.

2.2 Simulated annealing algorithm

SA was derived from an analogy with the annealing process of material physics described by Kirkpatrick et al. (1983). It is well known that certain materials have multiple stable states with different molecular distributions and energy levels. The annealing process consists of heating the substance until it is molten, then slowly and discretely lowering the temperature until the desired condition is achieved. In this process, the substance is allowed to reach thermal equilibrium at each temperature. Eventually the temperature is lowered until the material freezes. If the temperature is lowered sufficiently slowly, the annealing process can always pick out the global minimum energy state from the unlimited number of possible states (Kim and Park 2004).

Generally, to simulate the annealing process in optimization problems, the following preparatory steps are needed. Firstly, the analogue of the physical concepts in the optimization problem itself needs to be identified, noting that the energy function corresponds to the objective function and the configurations of particles represent the configurations of the parameter values. Finding a low-energy configuration is related to finding a near optimal solution. Temperature represents the control parameter for the annealing simulation. The next step involves a fast cooling process (annealing schedule) which consists of a set of decreasing temperatures and iteration times at each temperature. The last step is to supply a method of generating and selecting new solutions. Although there are many approaches to implement SA algorithm, the most thoroughly investigated SA algorithm is the adaptive SA (ASA) published at the web site (http://www.ingber.com/). ASA is developed to statistically find the best global solution for a nonlinear constrained nonconvex cost-function over a *D*-dimensional space. This algorithm permits an annealing schedule for "temperature" decreasing exponentially in annealing time which converges faster than Cauchy annealing and Boltzmann annealing (Pham and Karaboga 2000). The introduction of re-annealing also permits adaptation to changing sensitivities in the multidimensional parameter space (Jeong and Lee 1996).

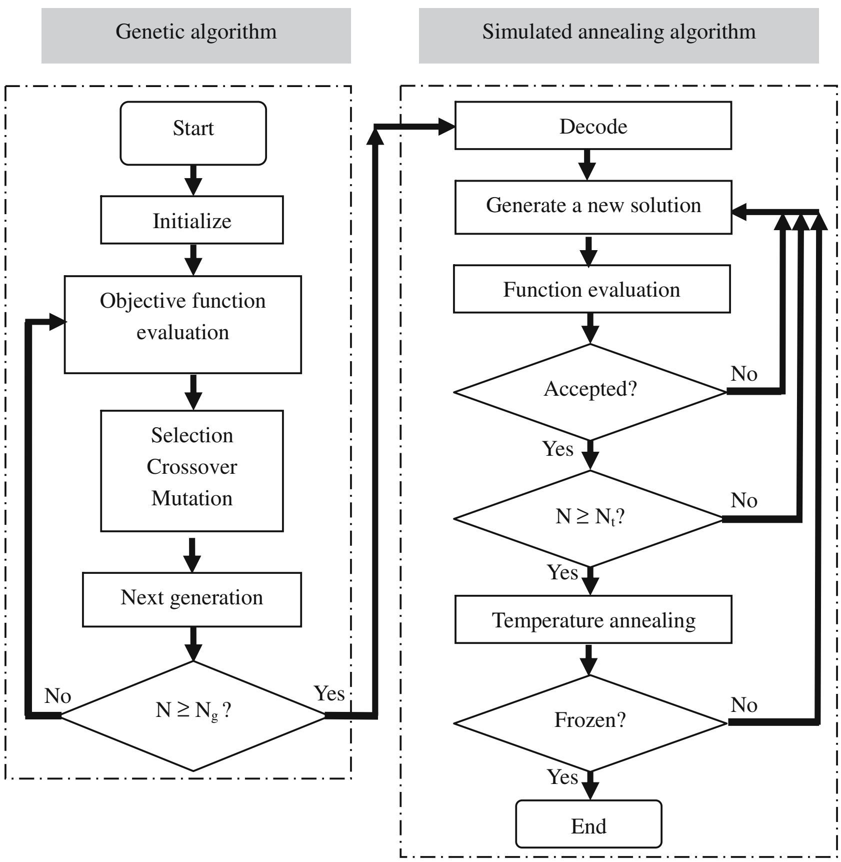

Fig. 1 The flowchart of GA–SA implementation

2.3 Combination of GA and SA algorithm—a hybrid algorithm

It has been verified that GA and SA are both probabilistic search algorithms which are capable of finding the global minimum among many local minima. However, empirically GA often lacks a hill-climbing capability and it does not work well when the objective function is a huge multimodal function or a highly coupled function such as the banana function (Rao 1996). On the other hand, SA has a statistical hill-climbing capability and the solution state will not stay at a fixed point for a long time. Therefore for a particular problem, no matter what type of the solution space of the objective function is, if GA were applied at the first step to obtain a relatively optimal solution, then this solution is considered as the initial solution of SA. With this, the final solution obtained from SA will be greatly improved than that obtained from a single GA or SA. This is because the initial global search is achieved by the GA which allows fast convergence and is independent of the initial parameters. The global search performance can then be improved by introducing the SA which has an entirely different global searching mechanism. By combining the outstanding features of GA and SA, it results in a hybrid optimization technique (GA–SA) which can reduce the probability of convergence to local minima due to the complementary global search supplied by SA. When comparing GA–SA with an independent GA and SA, it has a number of advantages. In short, GA–SA not only can overcome the demerit of GA, but also increase the probability of finding the global optimum (Jeong and Lee 1996; Kim et al. 2003a). The working process of GA–SA is graphically represented in Fig. 1. In this figure, N_g is the maximum generation number set in GA, while N_t is the number set to control the time staying at one given temperature in SA.

3 Model updating procedure

In model updating based on optimization techniques, the first step is to define the objective or cost function and the variables, i.e., updating parameters. The next step is to establish a generation strategy of initial solutions and new solutions. Each solution needs to be evaluated with a specific stopping condition. The optimization or iteration is terminated when the condition of optimum is achieved.

Figure 2 shows a general flowchart used in model updating based on the optimization technique. The model updating procedure proposed in this paper is similar to Fig. 2 after expanding the procedures in the broken line rectangle to the detailed procedures as Fig. 1. The following subsections will explain the main parts in the model updating procedures shown in Fig. 2.

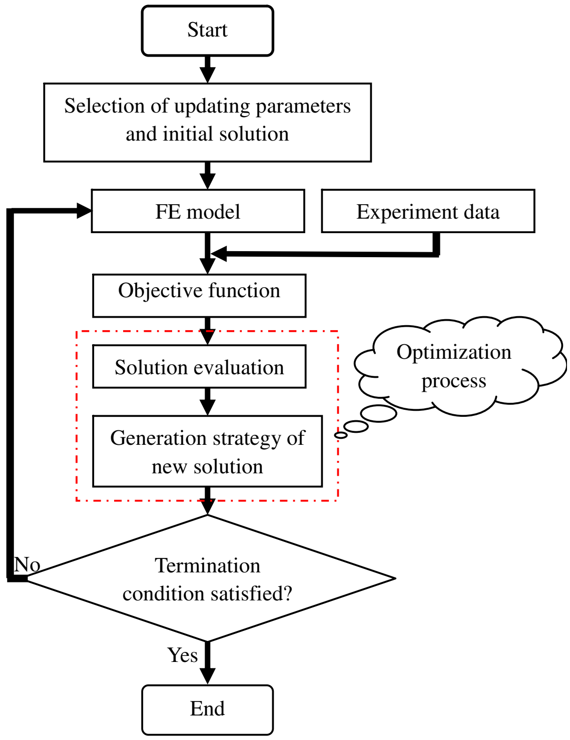

Fig. 2 The flowchart of model updating procedure using optimization techniques

3.1 Selection of updating parameters

One of the most important and difficult issues in the process of FE model updating is the selection of updating parameters. Inadequate updating parameters will cause the updated model unsatisfactory or unrealistic, whereas too many updating parameters might cause an ill-conditioned numerical problem. Hence, the number of updating parameters should be carefully selected. Generally, updating parameters should be selected with the aim of correcting the modeling errors or the uncertainty in the model. Furthermore, the selected parameters should be sensitive to the calculated and experimental natural frequencies, mode shapes, and dynamic response. There are many methods for selecting updating parameters, such as sensitivity analysis (Zhang et al. 2000; Bohle and Fritzen 2003) and error localization algorithm (Bohle and Fritzen 2003). In practical applications, updating parameters are often selected as global, substructural, local material or geometric parameters. Many researches are focused on the updating of material properties because these parameters are often unknown or partially known. The knowledge of geometric parameters of the structure can be easily obtained, although they might be simplified in the model (Jeong and Lee 1996; Ewins 2000; Modak and Kundra 2000; Modak et al. 2002). One of the strategies to ensure that only meaningful corrections are made after the updating process is to select the updating parameters on the basis of engineering judgment on the possible locations of modeling errors or material properties variation in a structure (Modak et al. 2002). As rules of thumb, for the case of a rotor with or without mass disks, the parameters to be considered include material properties, such as Young's modulus E, mass density ρ, Poisson ratio ν, shear coefficient γ, and effective stiffness diameters

d_{es} of the shaft stations with mass disks. In this study, the concept of effective stiffness diameters of the shaft stations needs to be explained in detail. Generally, for a uniform diameter shaft, the stiffness of each shaft station depends on the material properties, shaft length, and the polar area moment of inertia. In this situation, the mass and stiffness matrices in FE model can be calculated with the geometric diameters and standard material properties. However, for shaft sections with various diameters or assembled with mass disks, the stiffness matrix in FE model cannot be calculated directly with the geometric diameters and standard material properties. This has been verified to be caused by the assembled interference of the shrunk shaft and diameters variation (Kim et al. 2003b). Thus, the potential updating parameters which are taken into account for the difference between analytical and experimental results is shown as following:

$$\mathbf{p} = \{E, \rho, \nu, \gamma, d_{ei}\}^T . \tag{1}$$

Other parameters, such as mass distribution of the cylinder, have been tried in the past but only those mentioned above turned out to be the most effective and consequently we use these parameters in our current research. To avoid producing physically meaningless updated results, the selected parameters are assumed to be bounded in some prescribed regions which are determined according to the uncertainties that exist in the parameters.

3.2 Objective function

It is important to decide the form of objective function in practical applications of model updating. An obvious first choice is the sum-squared difference between the natural frequencies, mode shapes, or FRFs obtained from experiment and analytical FE model. In this paper, two forms of objective functions are defined. The first one is using the sum-squared difference of natural frequencies obtained from analytical model and experiment, which is expressed in (2). The second form is defined in (3), which is using the sum-squared difference between the amplitudes of the experimental and analytical FRFs (in natural logarithm) and summed over each available frequency point.

$$f_1(\mathbf{p}) = \sum (\omega_{Ai} - \omega_{Xi})^2, \ (i = 1,\ 2,\ \ldots,\ m) \tag{2}$$

$$f_2(\mathbf{p}) = \sum \left(\log\left(\left\| {}_A\alpha_{jk}(\omega_i)\right\|_2\right) - \log\left(\left\| {}_X\alpha_{jk}(\omega_i)\right\|_2\right)\right), \quad (i = 1,\ 2,\ \ldots,\ n) \tag{3}$$

where $\mathbf{p}$ represents the updating parameters, E, ρ, ν, γ, and $\mathbf{d}_{es}$. $\mathbf{d}_{es}$ is a vector that indicates the effective stiffness diameters of the shaft stations with mass disk. ω_{Ai} and ω_{Xi} are analytical and experimental natural frequencies, respectively. ${}_A\alpha_{jk}(\omega_i)$ and ${}_X\alpha_{jk}(\omega_i)$ are amplitudes of the analytical and experimental FRFs, respectively, at frequency ω_i. m and n represent the order of natural frequencies and the number of frequency components, respectively, considered in the FRF, while j and k are degrees of freedom at which the impacting force is applied and the response is measured, respectively.

In the objective function, the experimental data are obtained from modal testing in the laboratory. The tested shaft is supported using two steel piano strings. Two high-accuracy accelerometers, amplifiers, were employed to acquire the data, and modal analysis is completed by using the Medallion software. In the modal testing, 5 times average on the measured FRFs data was used to reduce the random error. Figure 3 shows the experimental setup for shaft modal analysis in our research. In the figure, the x and y indicate the horizontal and vertical directions, respectively. In the modal testing, the impacting force was applied either at the x-direction or y-direction, and the acceleration response was measured at the x-direction or y-direction accordingly. Theoretically, if the impacting force is applied at one node in the x-direction, no signals can be measured by the accelerometer at another node in the y-direction. In practice, the response signal can be also obtained although it might not be the actual response excited by the impacting force at the respective direction. Therefore, the direction of the measured acceleration response should be maintained in accordance with the direction of the impacting force.

In GA, the objective function, f_1 or f_2, is used to evaluate the fitness of each individual in the population. The best individual achieved through an evolution corresponding to the highest fitness is the expected parameters, $\mathbf{p}$. While in SA, f_1 or f_2 is considered as the cost function to be minimized.

4 Application and results

FE model-based vibration analysis and fault diagnosis have been widely studied in the research area of rotor dynamics.

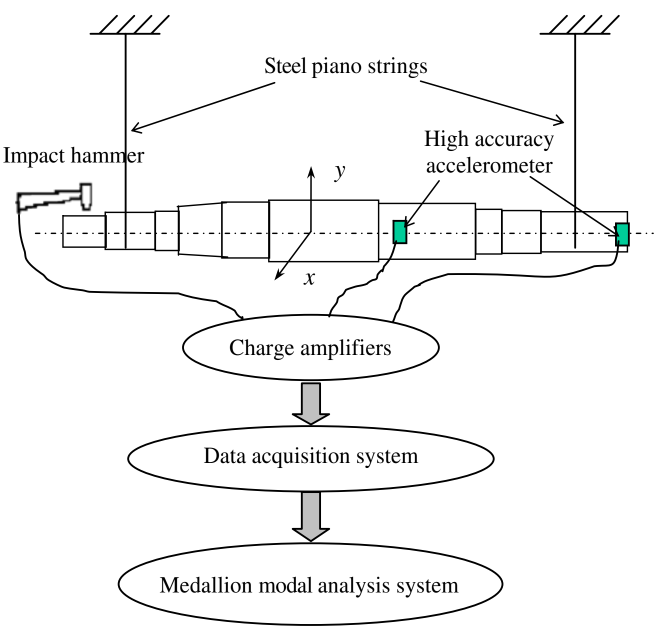

Fig. 3 An example set-up for modal testing

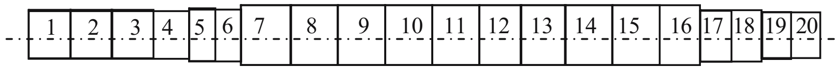

Fig. 4 FE model of the shaft

In the process of establishing an FE model for a rotating system, one of the most important works is to establish the submodel of rotor shafts consisted in the system. Generally, the submodels of rotor shafts are often carried out based on the standard material properties, standard geometric dimensions, and simplified boundary conditions of the rotor shaft. Unfortunately, many material properties (E, ρ, ν, and γ) cannot be determined exactly for a specific shaft. In addition, the stiffness diameter of a general shaft station is assumed equal to the geometrical diameter (mass diameter). For some special shaft stations, such as shaft stations assembled with a mass disk by means of shrunk method, they display higher stiffness characteristics than that calculated with the geometrical diameter. In this situation, the diameters used for the calculation of stiffness matrix in FE model are not equal to the mass diameters for some shaft stations. Consequently, the effective stiffness diameters of some special shaft stations are included in the updating parameters in this paper so as to cover the possible unknown parameters mentioned above and also supply a reliable FE model for the further research on vibration analysis and fault diagnosis. Two cases of shaft conditions are used in this paper. Case 1 consists of a typical and simple shaft without any mass disk component and is used as an example for material properties updating. Case 2 consists of a shrunk shaft in the middle with a cylindrical mass assembled by means of shrinkage fitting method. Because the interference fitting will generally stiffen the shaft station assembled with cylinder, this case is employed in this study to explore the stiffening effects on FE model, i.e., effective stiffness diameters updating.

4.1 Case 1: shaft model without a shrunk fitted mass disk

The shaft considered in this study is modeled in Fig. 4. The figure shows the shaft without any mass disk which is made of 45C steel. Detailed geometric parameters of this shaft are listed in Table 1. Figure 5 shows one experimental FRF between nodes 3 and 8 at the x-direction. The FRF is calculated by the rational polynomial method supplied in Medallion software, from which the first three natural frequencies can be extracted. The last column of Table 2 lists the first three natural frequencies obtained from experiment, and the natural frequencies calculated by the initial FE model (FE_{in}) are listed in the second column of Table 2. It should be noted that the FE_{in} model is based on the practical geometric parameters and standard material properties of the shaft. The initial value of these property parameters are listed in the third column of Table 4. It is obvious that there are some differences between the natural frequencies obtained from experiment and FE_{in} model. Especially in the higher natural frequencies, for the third natural frequency, the absolute difference is about 24 Hz.

In Table 2, the values in parentheses are the relative differences of natural frequencies obtained from FE_{in} model, FE model updated by GA (FE_{GA} model), and experiment. The relative difference is defined as:

$$\text{Relative difference } (\%) = (f_i - f_{mi}) / f_{mi} \times 100\% , \quad (4)$$

where f_i is the natural frequency calculated by FE_{in} model or FE_{GA} model, and f_{mi} is the experimental natural frequency. Note that the other higher natural frequencies cannot be measured due to the limited frequency range in modal analysis.

To update the initial FE model that can predict natural frequencies more closely to the experimental natural frequencies, GA is applied to the model updating process. Because this shaft is a simple structure, almost all geometric parameters can be obtained directly and easily. In the model updating process, the updating parameters are selected from the

Table 1 Shaft parameters, length, and diameters of each element

Element number	Shaft length (mm)	Shaft diameters (mm)	Element number	Shaft length (mm)	Shaft diameters (mm)
1	14.7	15.5	11	16.3	19.5
2	14.7	15.5	12	16.64	19.5
3	14.7	15.5	13	16.64	19.5
4	14.7	15.5	14	16.64	19.5
5	9.5	16.9	15	16.64	19.5
6	9.5	16.9	16	16.64	19.5
7	17.0	19.5	17	11.25	16.9
8	17.0	19.5	18	11.25	16.9
9	16.9	19.5	19	11.5	12.8
10	16.3	19.5	20	11.5	12.8

Material properties: $E = 210$ GN/m^2, $\rho = 7{,}800$ kg/m^3, $\nu = 0.3$, $\gamma = 0.9$
Total length: $L = 290$ mm

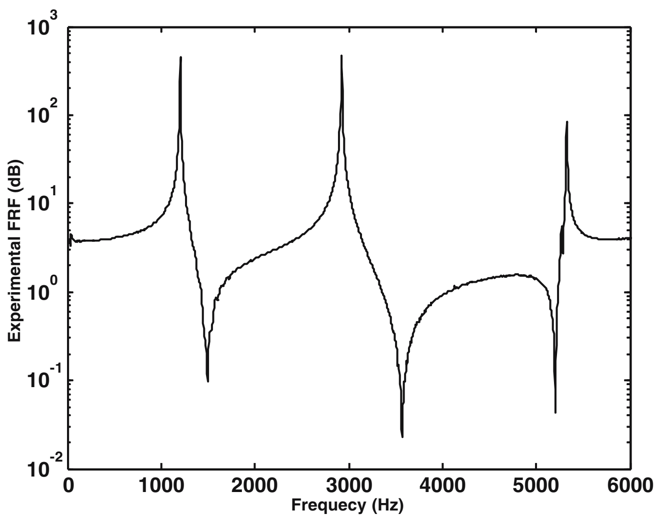

Fig. 5 The experimental FRF between nodes 3 and 8 in the x-direction

Table 2 Natural frequencies of the initial FE model, updated model, and experiment

Mode number	Natural frequencies (Hz)		
	Initial model	Updated model	Experiment
1	1,211 (0.67%)	1,205 (0.17%)	1,203
2	2,939 (0.48%)	2,925 (0.0%)	2,925
3	5,348 (0.43%)	5,325 (0.0%)	5,325
4	8,394	8,370	–
5	11,876	11,858	–
6	15,783	15,777	–
7	20,211	20,229	–
8	25,054	25,105	–
9	30,157	30,241	–
10	35,610	35,738	–

Table 3 Optimization algorithm-related parameters

Number of generation in GA	20
Number of population in GA	150
Crossover probability, P_c	60%
Mutation probability, P_m	5%

material property parameters (E, ρ, ν, and γ) and the objective function is defined as $f_1(\mathbf{p})$. Table 3 gives the value of GA-related parameters. Table 4 gives the initial value, lower and upper limits, and the final value of the updating parameters. Here, the ranges of the updating parameters are determined empirically. The first ten natural frequencies were calculated from the updated FE_{GA} model and are shown in Table 2. It clearly shows, except for a small difference existing in the first natural frequencies, that there are no differences in the second and third natural frequencies obtained from experiment and FE_{GA} model. When comparing the natural frequencies obtained from FE_{in} model, FE_{GA} model and experiment,

Table 4 Updated results of parameters

Parameter	Lower and upper limits	Initial value	Updated value
Young's modulus, p_1 (GPa)	[190–215]	210	211.05
Material density, p_2 (kg/m^3)	[7600–7950]	7,800	7,902
Poisson ratio, p_3	[0.2–0.49]	0.3	0.200
Shear coefficient, p_4	[0.5–0.9]	0.9	0.8969

it can be seen that the results obtained from FE_{GA} model are more close to the experiment than those obtained from FE_{in} model.

With the updated FE_{GA} model, FRF analysis is carried out so as to verify the updating technique. Figure 6 shows the comparison of FRF between nodes 1 and 8 in the x-direction. The result shows that the FRF obtained from FE_{GA} model matches the experimental FRF much better than that obtained from FE_{in} model, especially in the higher frequency range. To understand whether the updating parameters have the same influence on FRF between any other two degrees in the shaft model, comparisons among other FRFs were carried out in our research. In this test, ten FRFs between different DOFs were measured and compared. The results show that all the FRFs obtained from FE_{GA} models exhibit better agreement with experiments than those from FE_{in} model. However, for some FRFs, such as the FRF between nodes 3 and 8 shown in Fig. 7, there are still some differences in the frequency range from 3,200 to 5,000 Hz. Big difference at the antiresonance frequency in the frequency range can be seen which

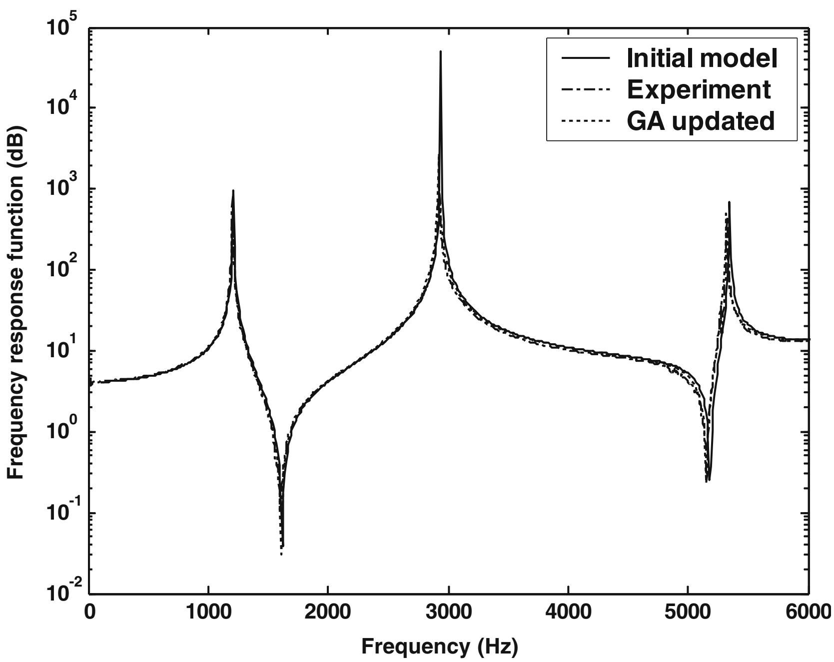

Fig. 6 Comparison of FRFs obtained from initial model, GA-updated model, and experiment between nodes 1 and 8 in the x-direction

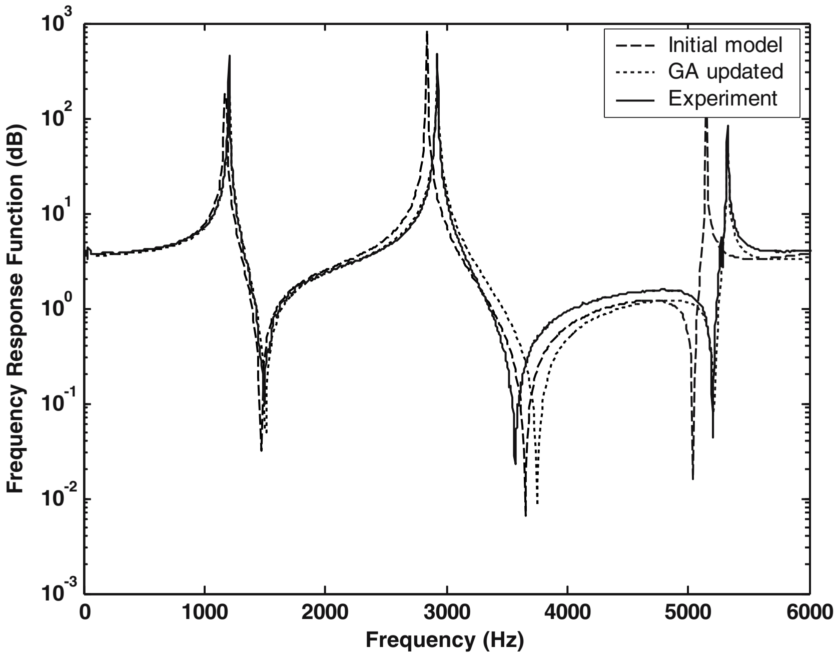

Fig. 7 Comparison of FRFs obtained from initial model, GA-updated model, and experiment between nodes 3 and 8 in the x-direction

must be caused by some other modeling errors in this figure. Again, huge difference can be seen at the higher frequency range even if f_2 is used as the objective function in the updating process. The results cannot be improved further but the CPU time needed to accomplish one run is 20 times than that needed while using f_1 as objective function.

4.2 Case 2: shaft model with a shrunk fitted mass disk

A shaft assembly with a mass cylinder attached to it by using shrinkage fitting method is considered in this study (Kim et al. 2003b). Figures 8 and 9 show the structure and initial FE model of the considered shrunk shaft, respectively. Table 5 gives the dimensions of each of the shaft stations and material property parameters of the shrunk shaft.

In this research, the concept of effective stiffness diameters is employed to consider the stiffness effects on shaft stiffness matrix caused by shrinkage fitting methods. Although there are some other empirical formulas for calculating the effective stiffness diameters of special shaft stations, the results are usually not satisfactory. In this case, the effective stiffness diameters are considered as the updating parameters, $\mathbf{d}_{es}$, that is, $d_6 \sim d_{11}$, as shown in Fig. 9. Assuming symmetric characteristics of the shaft, then let $d_6 = d_{11}$, $d_7 = d_{10}$, and $d_8 = d_9$. The natural frequencies obtained from FE_{in} model and experiments are listed in Table 7. Here, it should be noted that the effective stiffness diameters of the shaft stations with cylinder in the initial FE model are considered as 0.065 m empirically (Kim et al. 2003b). The results show that in the higher frequency range, the difference is relatively big. The third natural frequency has an absolute frequency difference of about 556 Hz which must be caused by the incorrect value of the effective stiffness diameters given in the initial FE model. Moreover, a big difference exists in the FRFs obtained from FE_{in} model and experiment shown in Fig. 10, especially in the frequency range from 6,000 to 8,000 Hz. To minimize the difference of the natural frequencies, the parameters, d_6 to d_{11}, need to be updated with optimization methods.

Initially, GA-based model updating method was applied using f_1 as the objective function. Table 6 gives the parameters related to GA, SA, and GA–SA, and also the range of the updating parameters. Because the outer diameters of the shaft stations with a shrinkage cylinder are 0.045 m and the outer diameter of cylinder is 0.075 m, the effective stiffness diameters can be valued in the range of [0.045–0.075]. The natural frequencies predicted by FE_{GA} model are listed

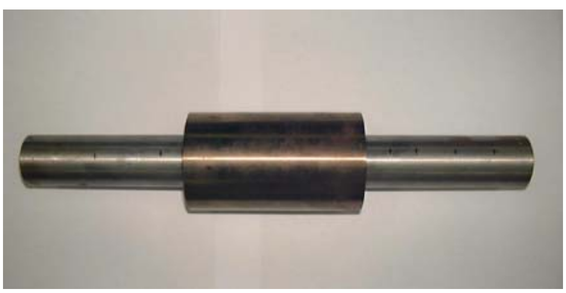

Fig. 8 Structure of the shrinkage fitted shaft

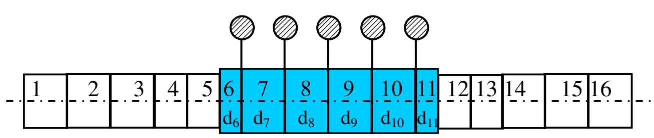

Fig. 9 FE model of the shrinkage fitted shaft

Table 5 Shaft parameters, length, and diameters of each element

Element number	Shaft length (mm)	Shaft diameters (mm)	Element number	Shaft length (mm)	Shaft diameters (mm)
1	30	40	9	26	45
2	30	40	10	26	45
3	30	40	11	13	45
4	20	40	12	20	40
5	20	40	13	20	40
6	13	45	14	30	40
7	26	45	15	30	40
8	26	45	16	30	40

Material properties: $E = 205$ G N/m^2, $\rho = 7835$ kg/m^3, $\nu = 0.3$, $\gamma = 0.9$
Total length: $L = 370$ mm

in the second column of Table 7. The final values of updating parameters are given in Table 8. When comparing the natural frequencies obtained from $\mathrm{FE_{GA}}$ model and experiments, a great improvement was achieved on the difference of the third natural frequency. However, differences of the first and fourth natural frequencies increase slightly. Meanwhile, the FRF comparisons from different FE models are made in Fig. 10. It is obvious that the FRF obtained from $\mathrm{FE_{GA}}$ model shows a much better agreement with the experimental FRF than that obtained from $\mathrm{FE_{in}}$ model. However, there are still some differences in the higher frequency range.

To reduce much more the difference of FRF between experiment and updated FE models, using f_2 as the objective function, the optimization techniques involving GA, SA, and GA–SA are tried, respectively. The updated results of updating parameters are shown in Table 8. The natural frequencies obtained from FE model updated by SA ($\mathrm{FE_{SA}}$ model) and FE model updated by GA–SA ($\mathrm{FE_{GASA}}$ model) are shown in the fourth and fifth column of Table 7, respectively. It is obvious that the relative differences of natural frequencies obtained from $\mathrm{FE_{GASA}}$ model and experiment are smaller than those obtained from any other models and experiment, with the exception of the third natural frequency. The results show that $\mathrm{FE_{GASA}}$ model is the best FE model for predicting natural frequencies that can closely match the experimental results.

Figure 11 shows the comparison of FRFs obtained from $\mathrm{FE_{in}}$ model, $\mathrm{FE_{GA}}$ model, $\mathrm{FE_{SA}}$ model, $\mathrm{FE_{GASA}}$ model, and experiment. Seeing from the difference of FRFs between all models and experiment, FRF obtained from all the updated models shows much better agreement with experiment than that obtained from initial FE model, especially in the higher frequency range greater than the second natural frequency. However, when using f_2 as the objective function, FRF obtained from $\mathrm{FE_{GA}}$ model does not match the experiments better than that predicted by $\mathrm{FE_{GA}}$ model using f_1 as objective function. It seems that in this case, for GA, the selection of objective function will have little influence on the updated model. In addition, the FRF predicted by $\mathrm{FE_{SA}}$ model is very similar to that predicted by $\mathrm{FE_{GASA}}$ model, but when comparing the amplitudes of FRF at the resonant frequencies, FRF obtained from $\mathrm{FE_{GASA}}$ model shows much better agreement with experiments than that from $\mathrm{FE_{SA}}$ model. Therefore, seeing from Fig. 11, it can be concluded that $\mathrm{FE_{GASA}}$ model is the best updated FE model, especially in the frequency range below 6 kHz. However, in the high frequency range over the third natural frequency, there are still some differences of FRFs obtained from $\mathrm{FE_{GASA}}$ and experiment.

Table 6 Updating parameters and optimization algorithm related parameters

Parameter item	Parameter value/limits
p_1 ($d_6 = d_{11}$) (mm)	[45–75]
p_2 ($d_7 = d_{10}$) (mm)	[45–75]
p_3 ($d_8 = d_9$) (mm)	[45–75]
Number of generation in GA	20
Number of population in GA	150
Crossover probability, P_c	60%
Mutation probability, P_m	5%
SA-related parameters	Default in SA source code (Jones 2002)
GA–SA-related parameters	The same as that used in GA and SA

Table 7 Natural frequencies obtained form the initial FE model, updated model, and experiment

Mode number	Natural frequencies (Hz)				
	$\mathrm{FE_{in}}$ model	$\mathrm{FE_{GA}}$ model	$\mathrm{FE_{SA}}$ model	$\mathrm{FE_{GASA}}$ model	Experiment
1	1,557 (0.06%)	1,611 (3.53%)	1,588 (2.06%)	1,584 (1.8%)	1,556
2	2,915 (2.46%)	2,914 (2.43%)	2,895 (1.76%)	2,890 (1.58%)	2,845
3	6,472 (−8.7%)	7,021 (−0.9%)	7,037 (−0.68%)	7,028 (−0.8%)	7,085
4	8,906 (0.68%)	8,979 (1.5%)	8,872 (0.29%)	8,852 (0.07%)	8,846
5	11,967	1,3203	13,373	13,156	–
6	16,630	17,804	17,797	17,489	–
7	19,752	20,047	19,885	19,538	–
8	21,178	22,924	23,070	22,706	–
9	24,709	28,090	28,247	27,781	–
10	26,112	30,025	30,192	29,675	–

5 Conclusions

FE model updating based on optimization technique for two typical structures, a general shaft and a shrinkage fitted shaft assembled with a cylinder disk, has been explored in this paper. Several popular optimization techniques, GA, SA, and GA–SA, were employed in the model updating process. From the results achieved, for an initial FE model, the material

Table 8 Comparison of final updated parameters obtained by GA, SA, and GA–SA-based model updating

	p_1 (d_6=d_{11}) (mm)	p_2 (d_7=d_{10}) (mm)	p_3 (d_8=d_9) (mm)
Initial model	65.0	65.0	65.0
GA	56.0	74.6	74.9
SA	48.36	75.0	75.0
GA–SA	47.9	75.0	75.0

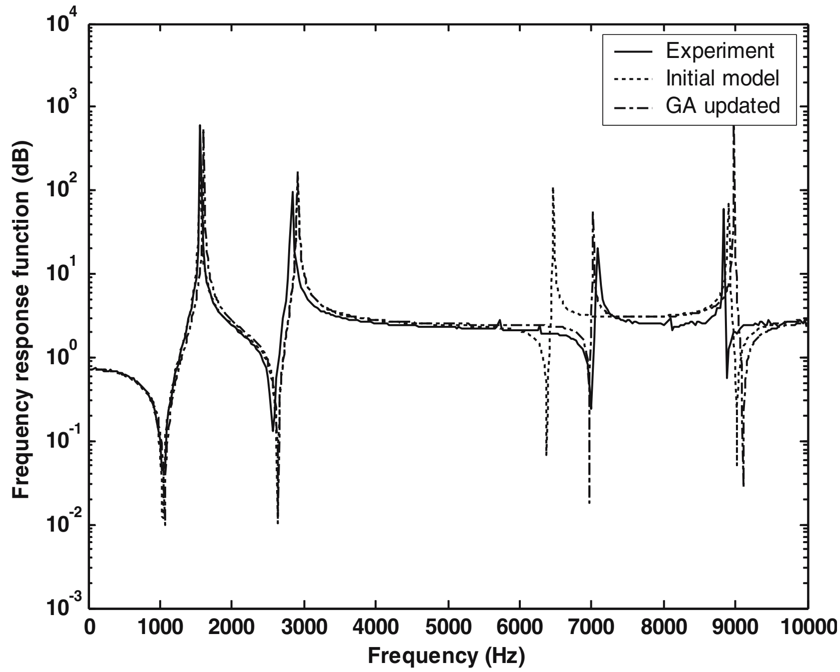

Fig. 10 Comparison of FRFs obtained from initial model, model updated by GA, and experiment

properties, such as Young's modulus, mass density, Poisson ratio, and shearing factor, effective stiffness diameters of some special shaft stations can be updated to predict natural frequencies and FRFs closely matching the experimental results. Generally, for a simple structure, such as a shaft without any mass disks, taking f_1 as the objective function in GA-based model updating process, the updated model is good enough to predict the natural frequencies and FRF that can closely match those obtained from experiment. While as for the shrinkage fitted shaft, in the model updating process based on GA, it seems that the selection of objective function, f_1 or f_2, has little influence on the updated results when comparing the FRF curve predicted by FE_{GA} model and experiment. However, from the differences among FRFs obtained from all the updated models and experiment, FRFs obtained from all the updated models show much better agreement with

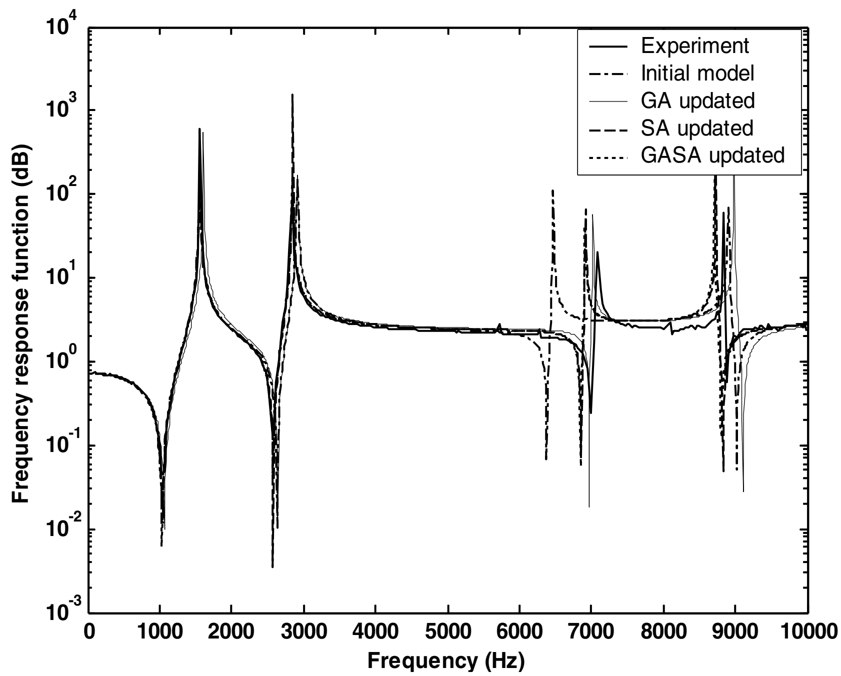

Fig. 11 Comparison of FRFs obtained from initial model, models updated by GA, SA, and GA–SA, and experiment

experiment than that obtained from initial FE model, especially in the higher frequency range. In addition, the FRF predicted by FE_{SA} model is very similar to that predicted by FE_{GASA} model. When comparing the amplitudes of FRF at the resonant frequencies, FRF obtained from FE_{GASA} model shows much better agreement with experiments than that from FE_{SA} model. It can be concluded that FE_{GASA} model is the best updating model, especially in the frequency range below 6 kHz. But in the high frequency range over the third mode, there are still some differences of FRFs obtained from FE_{GASA} and experiment.

In conclusion, the GA and SA are both powerful optimization techniques which can be successfully applied to FE model updating independently or in a combinative form. For simple structures, such as case 1, when selecting the material properties as updating parameters and f_1 as the objective function in the GA-based updating process, a good updated FE model can be obtained that can predict natural frequencies and the FRF closely matching the experimental results. But for a relatively complex structure, such as case 2, when selecting the effective stiffness diameters of some special shaft stations as updating parameters and using f_2 as the objective function, neither FE_{GA} model nor FE_{SA} model can predict natural frequencies and FRFs that match the experimental results closely. Under this condition, FE_{GASA} model will be a good choice in the model updating process.

References

Bohle K, Fritzen CP (2003) Results obtained from minimizing natural frequency and max-value errors of a plate model. Mech Syst Signal Process 17(1):55–64

Ewins DJ (2000) Modal testing: theory, practice and application. Research Studies Press, England

Friswell MI, Mottershead JE (1998) Best practice in finite element model updating. International Forum on Aeroelasticity and Structural Dynamics 2:57-1–11

He Y, Guo D, Chu F (2001) Using genetic algorithms and finite element methods to detect shaft crack for rotor-bearing system. Math Comput Simul 57:95–108

Holland JH (1975) Adaptation in natural and artificial system. The Univ. of Michigan Press, Michigan

Jeong IK, Lee JJ (1996) Adaptive simulated annealing genetic algorithm for system identification. Eng Appl Artif Intell 9(5):523–532

Jones K (2002) Finite element model updating using anti-resonant frequencies. J Sound Vib 252(4):717–727

Kim GH, Park YS (2004) An improved updating parameter selection method and finite element model update using multi-objective optimization technique. Mech Syst Signal Process 18(1):59–78

Kim YH, Yang BS, Kim YC, Lee SJ (2003a) Bearing parameters identification using hybrid optimization algorithm. Proceedings of 32nd international congress and exposition on noise control engineering, Jeju, Korea, pp 4212–4219

Kim YC, Park HJ, Park CH, Kim KW (2003b) Stiffness effect of fitting interference for a shrunk rotor. Proceeding of KSNVE Spring annual meeting, pp 319–324

Kirkpatrick S, Gelatt CD, Vecchi MP (1983) Optimizations by simulated annealing. Science 220:671–680

Levin RI, Lieven NAJ (1998) Dynamic finite element model updating using simulated annealing and genetic algorithms. Mech Syst Signal Process 12(1):91–120

Maia NM, e Silva JMM (1997) Theoretical and experimental modal analysis. Research Studies Press, England

Modak SV, Kundra TK (2000) Model updating using constrained optimization. Mech Res Commun 27:543–551

Modak SV, Kunard TK, Nakra BC (2002) Use of updated finite element model for dynamic design. Mech Syst Signal Process 16(2–3):303–322

Mottershead JE, Friswell MI (1993) Model updating in structural dynamics: a survey. J Sound Vib 162:347–375

Pham D, Karaboga D (2000) Intelligent optimization techniques. Springer Berlin Heidelberg New York

Ranhois F, Hemez M, Doebling SW (2001) Review and assessment of model updating for non-linear, transient dynamics. Mech Syst Signal Process 15(1):45–74

Rao SS (1996) Engineering optimization, theory and practice. A Wiley-interscience publication, Wiley, New York

Zhang QW, Chang CC, Chang TYP (2000) Finite element model updating for structures with parametric constraints. Earthq Eng Struct Dyn 29:927–944

Zimmerman DC, Yap K, Hasselman T (1999) Evolutionary approach for model refinement. Mech Syst Signal Process 13:609–625