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Parameter optimization of the sheet metal forming process using an iterative parallel Kriging algorithm

J. Jakumeit, M. Herdy and M. Nitsche

Abstract Different numerical optimization strategies were used to find an optimized parameter setting for the sheet metal forming process. A parameterization of a time-dependent blank-holder force was used to control the deep-drawing simulation. Besides the already well-established gradient and direct search algorithms and the response surface method the novel Kriging approach was used as an optimization strategy. Results for two analytical and two sheet metal forming test problems reveal that the new Kriging approach leads to a fast and stable convergence of the optimization process. Parallel simulation is perfectly supported by this method.

Key words Kriging, parallel simulation, process parameter optimization, sheet metal forming

1 Introduction

Nowadays simulation programs are widely used in the car industry to optimize production processes or test the behavior of the car before a prototype is built or real-life tests are performed. The sheet metal forming process is one example, where simulation is used in an early stage of the car's development to test difficult geometries and to optimize the process parameters to ensure safe and correct production. For such an optimization of process parameters an engineer has to run many simulations with different parameter constellations and the results have to be analyzed. Since such an optimization 'by hand' rarely follows a clear strategy, it remains uncertain whether the optimal process parameters and tool geometries have actually been identified. In such a situation the use of a numerical optimization program, which tests the different parameter constellations of an optimization strategy, can provide significant support for the engineer and lead to better results.

This work applies parameter optimization to the sheet metal forming process. Figure 1 shows the active surfaces of the deep-drawing tools for a sheet metal forming. These tools consist of a die, a blank-holder and a punch. The actual tools in the deep-drawing press are massive metal blocks. The shape of the active surfaces is defined by the part geometry and the so-called addendum. At the start of the deep-drawing process the blank is fixed to the die by the blank-holder. The blank-holder force required depends on the process. During the forming process the punch forces the blank into the die. In this plastic process the blank takes on the shape of the die or the punch. The formed part will usually be processed to the final part in further steps.

The first results for a numerical optimization of the sheet metal forming process have been published in Hillmann and Kubli (1999), Ghounati *et al.* (1998, 1999), Scott-Murphy *et al.* (1999). This paper focuses on a new optimization strategy, Kriging, and investigates the feasibility of the parallel simulation of sheet metal forming processes with different parameter settings in order to gain shorter computation times.

2 The optimization problem

For the numerical optimization of a sheet metal forming process the whole process has to be mapped into a mathematical optimization problem. The goal of the optimization is to find process parameters which lead to a metal sheet with the desired geometry that is without cracks or wrinkles. In order to achieve this goal, the optimization algorithm uses the sheet metal forming simulation program INDEED to calculate the influence of design param-

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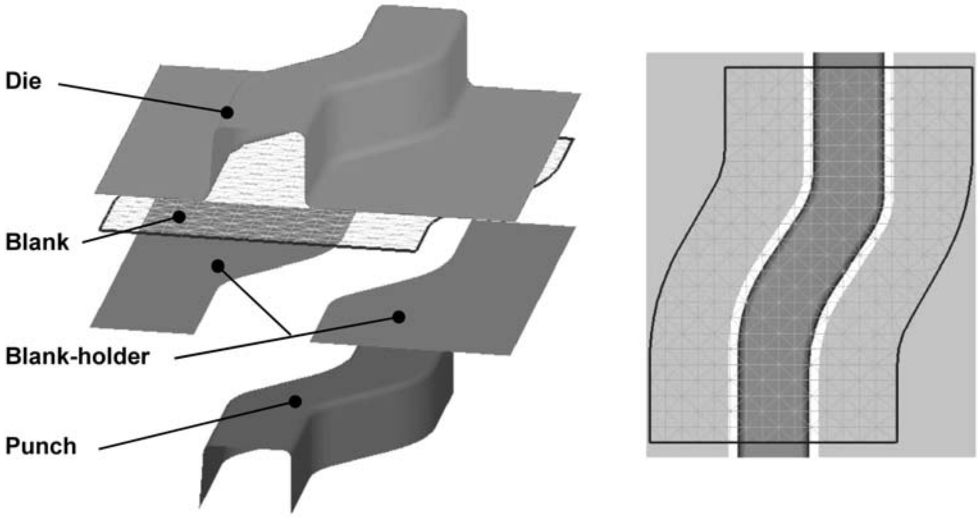

Fig. 1 Toolset for the deep drawing part "S-rail"

eters on the forming process result. After the simulation the results of the simulation are evaluated by an objective function in order to measure the quality of the simulation result. A series of test simulations was used to identify design parameters. These control the final form and the crack and wrinkle tendency of the simulation result. In the following chapters the three parts of the optimization problem are described in detail: sheet metal forming using INDEED, the evaluation of the results by a multicriteria objective function and the identification of design parameters.

2.1 Forming simulations with INDEED

The deep-drawing simulations were performed using INDEED, a special purpose program for sheet metal forming processes. It is based on the incremental Finite Element Method (FEM). In each incremental load step the balance between inner and outer loads is calculated. Developed in close cooperation with well-known German automobile companies (Volkswagen AG, DaimlerChrysler, ThyssenKrupp Automotive) INDEED has become a highly effective simulation software for car body parts. Naturally, other sheet metal parts can also be simulated. The correct treatment for the springback effect is still being developed. However, general tendencies are correctly simulated and can be used for the optimization process.

Two types of simulation are available with INDEED (Fig. 2). A *membrane element* is used for very quick analyses. It is used in the very early stage of the design process of a car body. For highly accurate results and a simulation of the springback behavior a *shell element* is included. This element has been developed especially for the deep-drawing process. In addition to excellent bending and membrane characteristics, the element formulation includes the change of thickness as an independent degree of freedom. In contrast to classic shell elements, this allows an efficient description of the contact on both sides. In addition, friction is included. The material models are another feature of INDEED. For successful numerical simulations a good approximation of the characteristics of the real material is needed. The general strain hardening which occurs during the forming process has been validated experimentally by extensive biaxial tests for many materials. The INDEED software provides the conditions for a realistic evaluation of springback.

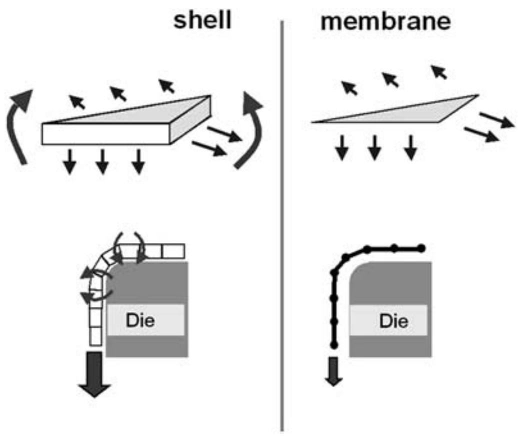

Fig. 2 The element types of INDEED

2.2 Multicriteria objective function

The definition of an objective function, which evaluates a simulation result with all its different properties with one single value, can be very difficult for technical applications. The quality of the function definition may have a significant influence on the time needed to solve the problem. In the case of a sheet metal forming process, four goals have to be met:

G1. the final sheet should not have cracks
G2. the final sheet should not have wrinkles
G3. the sheet should be stretched enough to gain a stable form, but thinning should not be excessive.
G4. after the opening of the tools, the sheet is likely to change its form slightly due to internal forces, the so-called springback effect. The final form after springback should be the desired one.

The first three requirements can be interpreted as constraints. But for a conservative design of the sheet metal forming process it is not enough to avoid cracks or wrinkles in the simulation result. Process parameters have to be found which reduce the crack and wrinkle tendency as much as possible. Therefore, all four requirements are handled as goals of the optimization process and are part of the objective function. For each goal an individual objective function is defined. The total objective function is calculated by a weighted sum of the four individual functions.

The forming limit diagram (FLD) is analyzed (Hillmann and Kubli 1999; Hora *et al.* 1996) for the first two objectives $G1 = G_{crack}$ and $G2 = G_{wrinkles}$. In the FLD for each element of the FE-sheet representation the major strain (h_{major}) is plotted against the minor strain(h_{minor}) (see Fig. 3). If an element comes close to or lies above the form limiting curve (FLC) a crack has to be expected in this area of the sheet. Similarly, a risk of wrinkles can be assumed if an element lies below the thickening line $h_{major} = -h_{minor}$, i.e. if there is more pressure then tension in a certain area. G1 is just the number of elements which lie inside the crack region, i.e. which have values of h_{major} and h_{minor} which lead to cracks or indicate a crack tendency.

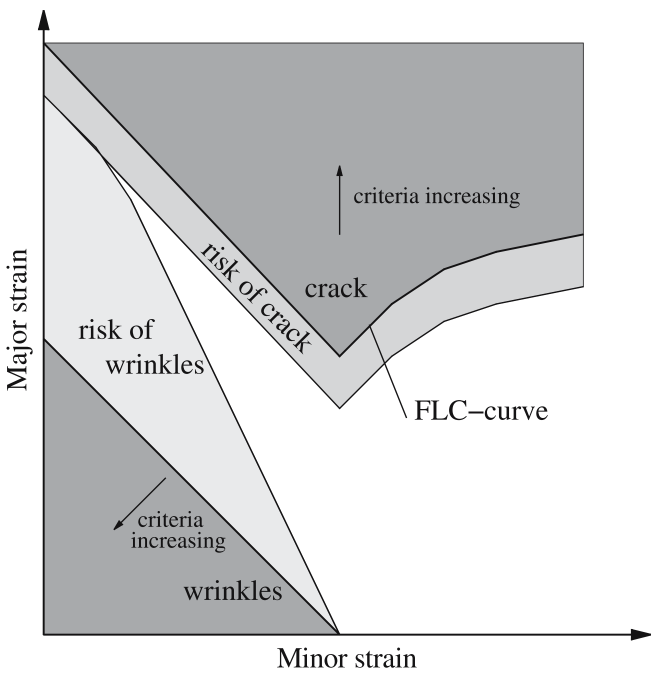

Fig. 3 Schematic FLD-diagram with critical regions for crack, risk of crack, risk of wrinkles and wrinkles

Fig. 4 Points used to measure the distance between the desired geometry and the simulation result in the SRAIL test case

A tendency to wrinkle is assumed if the element lies below the line $h_{major} = -2 \times h_{minor}$. Similarly to G1, G2 is given by the number of elements in the wrinkle region of Fig. 3 with values of h_{major} and h_{minor} that indicate a wrinkle tendency.

The third criteria, $G3 = G_{thinning}$, can be directly taken from the thinning data, given by the sheet forming simulation with INDEED. The thinning is used as the value for G3.

For the fourth goal, $G4 = G_{springback}$, the difference between the desired geometry and the one simulated by the design parameter set is measured at several points on the sheet. Figure 4 shows the nine points used to evaluate the shape of the SRAIL test case (see Example 2 below). G4 is given by the difference between the desired and realized position in the direction of the punch movement, given in mm.

The total objective function is then given by the weighted sum of the individual objective functions:

$$G_{total} = w_{crack}\,G_{crack} + w_{wrinkle}\,G_{wrinkle} + w_{thinning}\,G_{thinning} + w_{springback}\,G_{springback} \quad (1)$$

In general this weighting (1) of the individual objective functions is useful if the values of the functions have significant differences (i.e. G1 is on average 100 times larger than G2) or if the optimization should focus on one specific objective. For the two application cases presented here all weights were 1 and the total objective function was just the sum of the individual values. An optimization algorithm that can detect Pareto-optimal solutions for the four criteria was not applied.

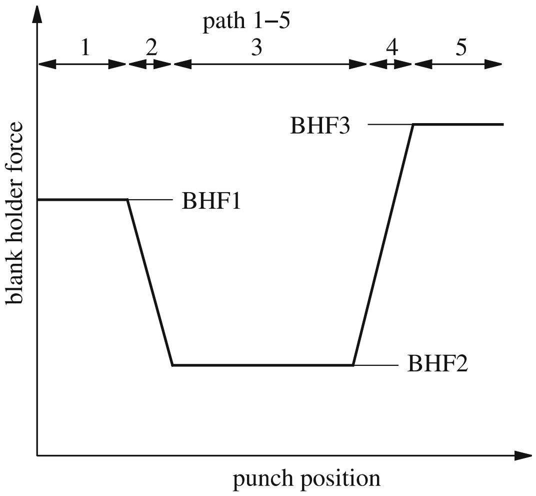

Fig. 5 Blank-holder force as a function of the drawing depth described by seven parameters (three forces and four step lengths. The length of the third step is given by the total drawing depth)

2.3
Design parameters

The simulation of the sheet metal forming process is controlled by many parameters. In this paper we focus on the optimization of process parameters such as the strength of a draw-beat and the time-dependent blank-holder force and not on the optimization of an optimal geometry for the forming tools. Hillmann and Kubli (1999) used a time-dependent variation of the blank-holder force to control the simulation. A parameter study performed on the hat-shaped profile (see Sect. 4.1) verified that a parameterization of the time dependence by three plateaus with constant force and two ramps between the plateaus is sufficient to control crack and wrinkle tendency as well as the final form of the sheet. Since the total length of the drawing process is given, seven parameters are necessary to describe the course (three forces and four step lengths).

3
Comparison of different optimization strategies

3.1
Optimization strategies

There exists a wide literature on optimization strategies that are able to solve the nonlinear optimization problem defined in the previous section (Press *et al.* 1999; Nocedal and Wright 1999; Gill *et al.* 1981; Torczon 2000; Lewis and Torczon 2000; Myers and Montgomery 1995; Simpson 1998; Watraon 1984; Welch and Sacks 1991; Tang 1993). The most popular strategies are conjugated gradient, Quasi Newton and sequential quadratic programming methods, which use information from the first or second derivative to move through the parameter space towards an optimal solution (Press *et al.* 1999; Nocedal and Wright 1999; Gill *et al.* 1981). If the derivatives are not directly accessible and must be numerically calculated, direct-search algorithms are often more robust and quicker. They use only a direct comparison between the objective values to determine a parameter set for the next iteration. Examples of these algorithms are the downhill-simplex algorithms, Powell's method of conjugated directions and pattern search methods (Press *et al.* 1999; Torczon 2000; Lewis and Torczon 2000).

An important characteristic of the optimization of the sheet metal forming process is that a single simulation needs a computation time in the order of hours or even days on a modern workstation. Therefore, the number of simulations should be small and parallel simulation of the forming process for different parameter sets should be used. Optimization strategies based on surrogate models are especially suitable for optimization under such conditions. Figure 6 shows a flowchart of such an optimization strategy using surrogate models. In a first step, Design of Experiment (DoE) methods are used to determine parameter sets X, for which a simulation has to be performed. In the second step, objective values $f(X)$ at these points are used to establish a surrogate model s as an approximation to the actual objective function f. The third step is the search for the optimum of the surrogate model. Since this model is an analytic function this optimization can be solved very fast by the computer. If the new optimum found is close to the last optimum (convergence) and the estimation of the surrogate model is close to the result obtained by a complete simulation, the algorithm is stopped. Otherwise the surrogate model is improved in a further iteration by adding additional simulation results at new points and reducing the modeled region.

In the response surface methodology (RSM), s is given by polynomials, which are fitted to the simulation results using a least squares fit (Myers and Montgomery 1995; Simpson 1998). In Kriging, the surrogate model consists of a sum of many local carrier functions (very often Gauss functions) and one global function, which might be a polynomial as in RSM but generally is simply a constant. For each simulated point x from X a local car-

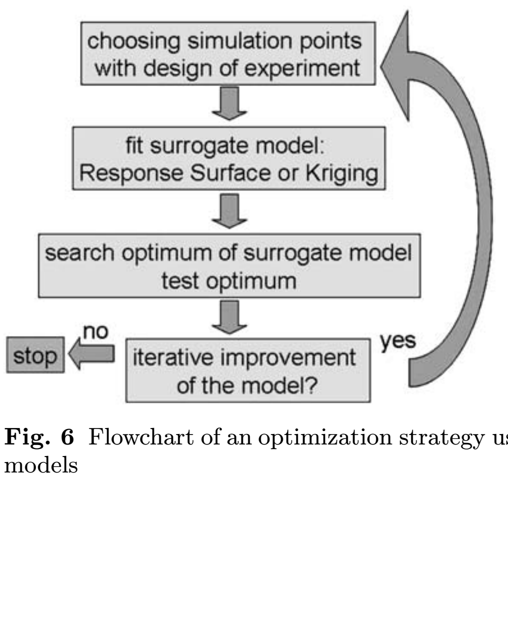

Fig. 6 Flowchart of an optimization strategy using surrogate models

rier function is used to bridge the difference between the global function and the simulation results. The width of the local carrier functions is tailored to get the most likely surrogate model to the simulation points. A more detailed description of the Kriging algorithm can be found in Simpson (1998), Watraon (1984), Welch and Sacks (1991). An advantage of the Kriging method compared to RSM is that Kriging can use any number of simulation points to create a surrogate model, while in RSM the number of necessary simulations is given by the number of unknowns in the polynomial. This number can be rather high in a high-dimensional search space, i.e. for a higher number of design parameters *d*. A principle difference between RSM and Kriging is that RSM is a global model while Kriging is local. Here, global means that each simulated point used to define the polynomial influences the complete surrogate model even at remote points in search space. In contrast, due to the use of local carrier functions in Kriging, simulated points influence the surrogate model only in a nearby region, making Kriging a local model.

3.2 Analytical test

Different analytical test problems were used to compare the various optimization strategies, influence of different DoE designs, RSM or Kriging surrogate models before they were applied to the computation-time-expensive sheet forming simulation. Since the CPU time of a sheet metal forming simulation is much higher than the time needed by the optimization strategy, the CPU time for the whole optimization will be determined by the number of simulations. Therefore, this number is used to monitor the convergence. For this engineering application it is not the number of simulations needed to reach the optimum that is a measure of the quality of the algorithm, but rather the ability to reach an acceptable improvement with as few simulations (i.e. objective function evaluations) as possible.

It is not possible to describe all these tests and results in this publication, which focuses on industrial application. Below four different optimization strategies are compared for the Rosenbrock and the Fletcher–Powell function as analytical test problems. Similar tests gave the following findings for different surrogate models:

- The fastest convergence could be found using Latin hypercube sampling (Tang 1993). This design seems to give a rather good sampling of the design space with a low number of points compared to standard DoE designs as partial or full factorial design or d-optimal design.
- For the RSM base surrogate model a quadratic polynomial outperformed a linear approximation. Higher order polynomials were not tested since the number of necessary simulations to build one surrogate model would make such a model computational very expensive. An iterative improvement of the surrogate model would be impossible with a reasonable amount of computation time
- For the Kriging surrogate model Gauss functions were used as local carrier functions. Our investigations showed that the optimization was more robust if the width of the Gauss function was the same for all directions of the search space and not optimized for each direction separately.

The Rosenbrock function

$$f(x) = \sum_{i=1}^{n-1} 100\,(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \tag{2}$$

is characterized by one long curved valley, where many gradient methods show a very slow convergence (see Fig. 7(a)). The Fletcher–Powell function has many randomly distributed minima (see Fig. 7(b)):

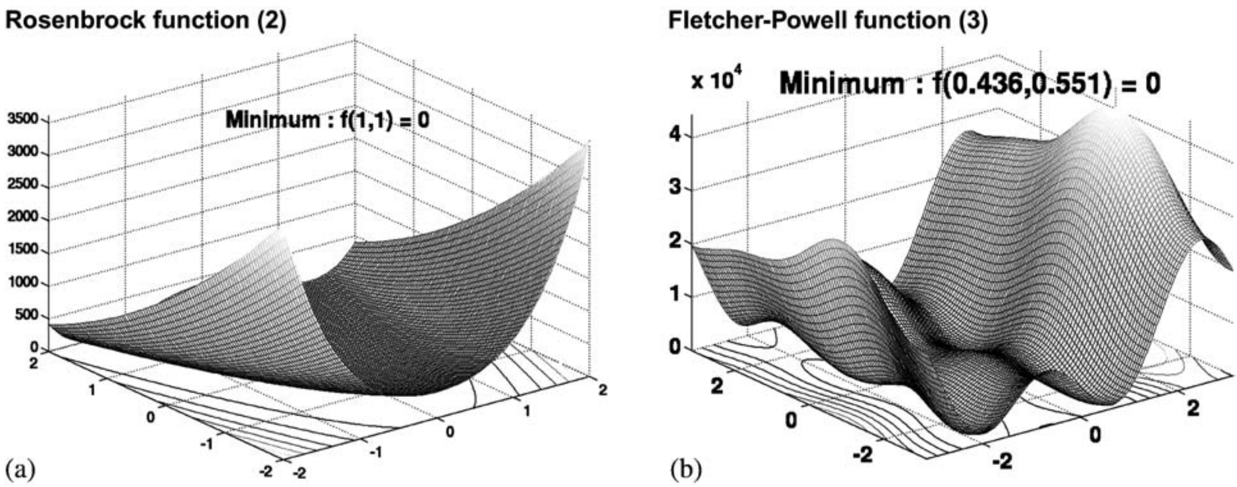

Fig. 7 Rosenbrock and Fletcher–Powell functions in two dimensions

$$f(x) = \sum_{i=1}^{n} (\mathrm{A}_i - \mathrm{B}_i)^2 \,, \quad \mathrm{A}_i = \sum_{i=1}^{n} (\mathrm{a}_{ij} \sin \alpha_j + \mathrm{b}_{ij} \cos \alpha_j) \,,$$

$$\mathrm{B}_i = \sum_{i=1}^{n} (\mathrm{a}_{ij} \sin x_j + \mathrm{b}_{ij} \cos x_j) \,, \quad -\pi \le x_i \le \pi \,, \tag{3}$$

$\mathrm{a}_{ij}, \mathrm{b}_{ij} \in [-100, 100]$, $\alpha_j \in [-\pi, \pi]$ can be chosen randomly, but are taken from Tang (1993).

Both functions are defined for any number of parameters d, i.e. the dimension d of the problem can be scaled. In this comparison we took the conjugated gradient method (CG) for a gradient algorithm, downhill simplex (DS) for a direct method, RSM with a quadratic polynomial as a surrogate model and Kriging with Gauss functions and a constant as a global function. CG and DS were taken from Numerical Recipes (Press *et al.* 1999). Figure 8 compares the convergence of the four strategies for both objective functions in an eight-dimensional search space. Kriging shows the fastest convergence for both problems. DS is slightly slower. The CG approach has problems with the multi-minima Fletcher–Powell problem and RSM shows problems with the Rosenbrock function.

As mentioned above, due to the large computation time needed for a simulation, a parallel simulation of different parameter settings should be used in the optimization strategies as often as possible. In the case of the DS algorithm only the first simplex with $d+1$ points can be simulated in parallel, but during the iteration the algorithm needs only one new simulation result in each iteration. In the CG method the numerical approximation of the gradient can use d simulation results in parallel. The line search by Brent's Method again cannot use a parallel simulation. For the methods with a surrogate model the simulation of the points determined by design of experiment methods can be done in parallel. The best speed up is achieved when the number of points is equal to the number of nodes times an integer. Since the number of points in Kriging is free an ideal speed up can always be achieved with this strategy. Figure 9 shows the convergence for the four algorithms using eight nodes in parallel. Here, Kriging is the fastest algorithm since DS cannot make use of the eight nodes. The CG approach remains as

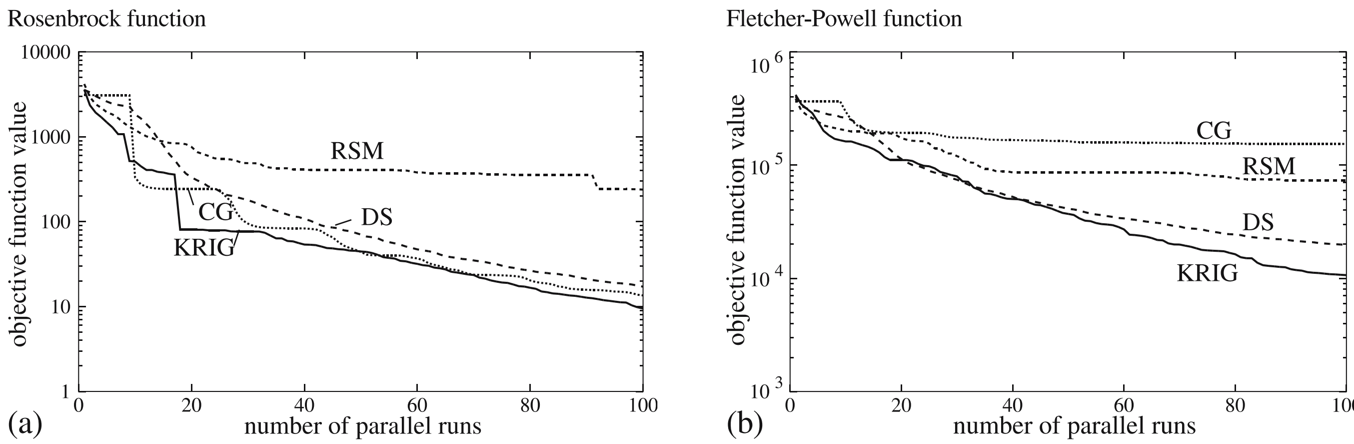

Fig. 8 Convergence of conjugated gradient (CG), downhill simplex (DS), response surface method (RSM) and Kriging for (a) Rosenbrock function and (b) Fletcher–Powell function

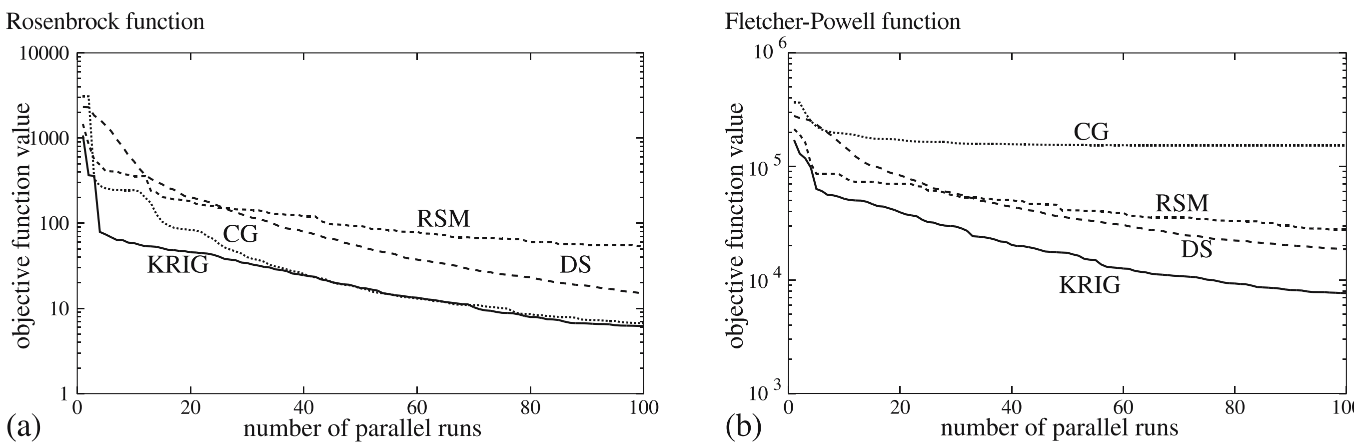

Fig. 9 Convergence of conjugated gradient (CG), downhill simplex (DS), response surface method (RSM) and Kriging for the (a) Rosenbrock function and (b) Fletcher–Powell function running on a parallel machine with eight nodes

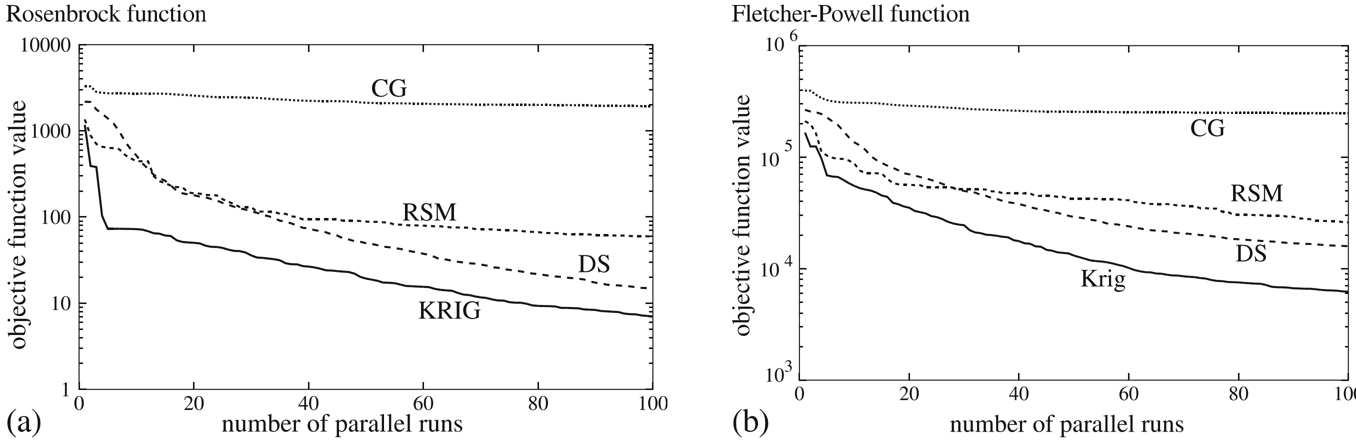

Fig. 10 Convergence of conjugated gradient (CG), downhill simplex (DS), response surface method (RSM) and Kriging for the (a) Rosenbrock function and (b) Fletcher–Powell function with 10% degree of random noise on eight nodes

fast as Kriging for the Rosenbrock function but has difficulties with the Fletcher–Powell function.

Another important aspect of the optimization of engineering problems such as sheet metal forming is that the objective function can be quite noisy. In the case of the sheet metal forming small changes in the parameter can yield a wrinkle and thereby change a good result to a rather bad one. To check the robustness of the algorithms a 10% degree of random noise was added to the analytical test functions by multiplying $f(x)$ with $0.1 \times rnd()$, where $rnd()$ gives a random number between 0 and 1. Figure 10 shows the convergence of the four strategies for an optimization of the noise problems on eight nodes. The CG algorithm does not converge because the random noise leads to the calculation of incorrect gradients. A comparison with Fig. 7 reveals that the other three methods are not affected by the noise in the objective function.

4
Two examples

The four optimization strategies: CG and DS taken from Press *et al.* (1999), the RSM with quadratic polynomial as surrogate model and Latin hypercube sampling as DoE design and the Kriging approach with Gauss functions of equal width in each dimension as local carries and Latin hypercube sampling as DoE design were tested with the two sheet metal forming test cases from the Numisheet 93 (Benchmark Problems: B3) and Numisheet 96 (Benchmark Problems and results: B-2) conference: the hat-shaped profile and the SRAIL. The sheet metal forming simulation was done by INDEED on a PC cluster with 16 nodes. The optimization algorithm runs on a separate PC and controls the simulations simply using Network Filesystem (NFS). Since the data exchange is limited to the change of the input file, the start of the simulation and the collection of the results, the use of a parallel environment like Message Passing Interface (MPI) was not necessary.

4.1
Hat-shaped test case

The hat-shaped profile (see Fig. 11) is a rather small test case originally employed at the Numisheet 93 (Benchmark Problems: B3) conference to compare different simulation algorithms. A flat stripe of metal is formed to a hat-shaped profile bar. Only a quarter of the bar is simulated because of symmetry conditions. The test case has 2500 elements and needs about 1 hour CPU time on a HP A9000 workstation. This test case shows a strong springback effect and is therefore ideal to test the possibility of controlling this effect during the optimization with adequate parameters (see Sect. 2.3). To find these control parameters, first simulation studies have been done with different constant blank-holder forces to verify earlier results from the literature Ghouati *et al.* (1998). The influence of the blank-holder force on the springback was clearly demonstrated.

The next studies were done with different profile curves for the blank-holder load. The curves were pa-

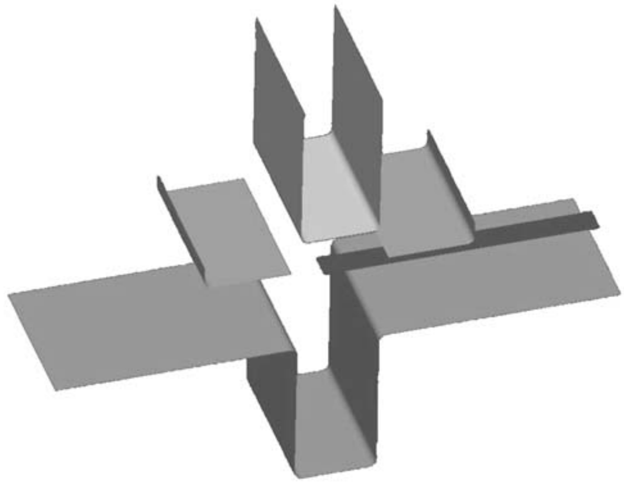

Fig. 11 Toolset for the hat-shaped test case

rameterized by three plateaus with constant force and two ramps between the plateaus as described in Sect. 2.3. Since the total length of the drawing process is given, seven parameters are necessary to describe a curve (three forces, two plateau lengths and two ramp lengths). Figure 12 shows the typical results from a parameter study. The tests demonstrated that the optimization goals, mentioned above, can be controlled by such a time-dependent variation of the blank-holder force.

The result using different optimization strategies shows that the seven design parameter (four lengths and three forces) are able to control the forming so that the re-

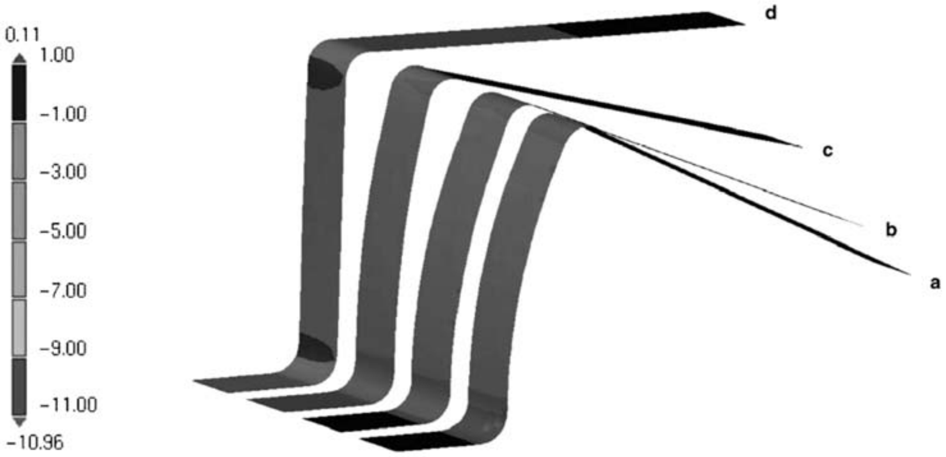

Fig. 12 Springback results and thinning results for the hat-shaped profile

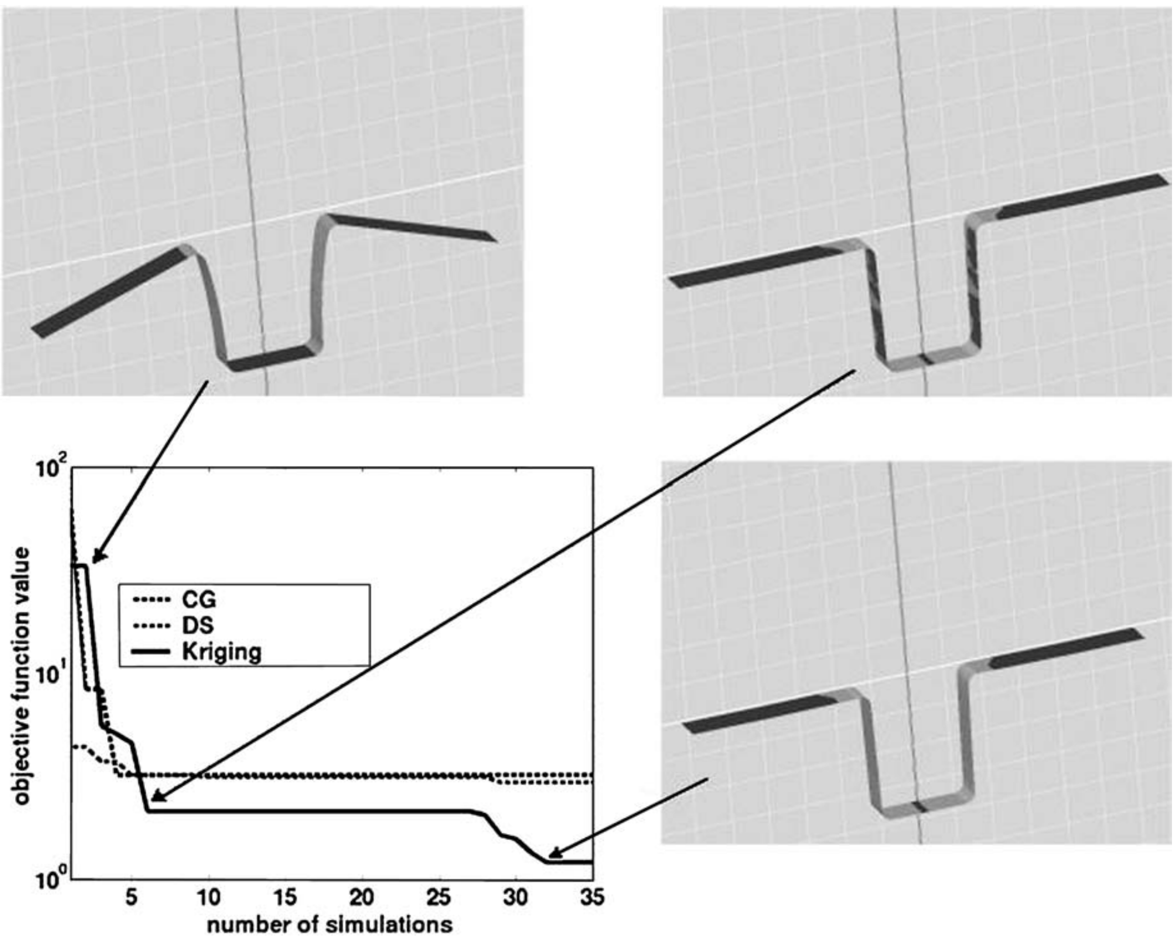

Fig. 13 Convergence of CG, DS and Kriging for the test case. Only Kriging found (within 35 simulations) a solution with a good shape and not too much thinning

sult is a bar without cracks or wrinkles and with the right shape.

Figure 13 shows the convergence history for CG, DS and Kriging. Within a reasonable amount of simulations (< 100) only Kriging was able to find a solution with a perfect geometry and acceptable thinning at the side of the bar. The small figures within Fig. 13 show the optimization process. Starting from a rather poor bar with an incorrect geometry due to a strong springback effect, Kriging quickly found a solution with a good geometry but needed about 20 simulations to optimize the thinning.

4.2 SRAIL test case

The SRAIL is a much more realistic test case (see Fig. 1) which shows all the difficulties typically found in car part geometries (Benchmark Problems and results: B-2). The test case starts with 700 elements, ends with 6000 elements and needs about 27 min CPU time on a HP A9000 workstation. The springback effect is not as pronounced as in the hat-shaped test case, while cracks and wrinkles play a more important role. Figure 14 shows the convergence for Kriging, RSM, CG and DS for serial and parallel

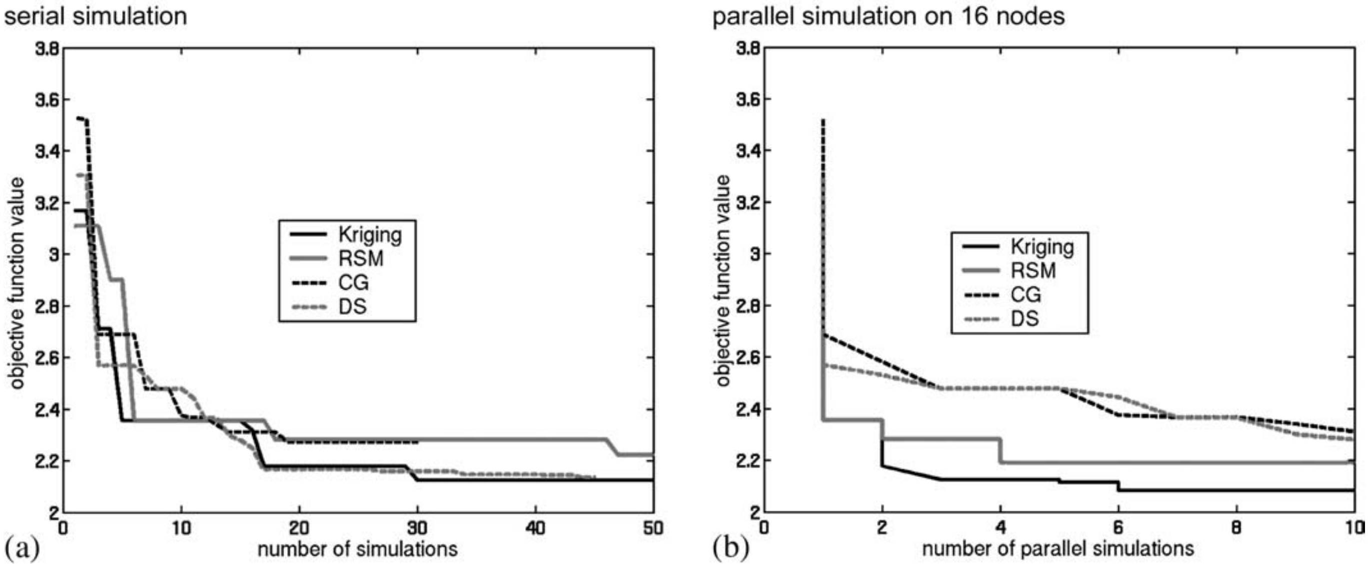

Fig. 14 Convergence behavior of Kriging, RSM, CG and DS for the SRAIL test case for serial and parallel simulation

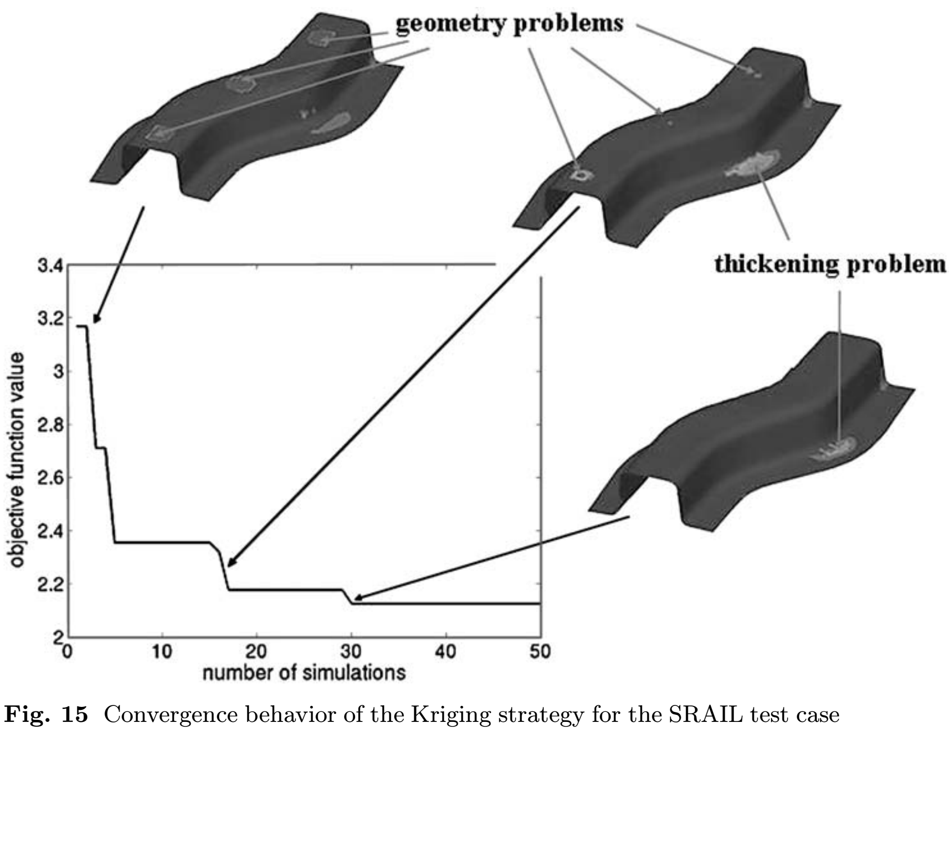

Fig. 15 Convergence behavior of the Kriging strategy for the SRAIL test case

simulations on 16 nodes. For the parallel simulation all simulations started in parallel are counted as one parallel run, since they can be executed within the same time period as one serial simulation.

On a serial machine, all four optimization strategies show a similar convergence behavior but Kriging and DS are slightly faster than the others. On the parallel machine, RSM and Kriging hold the advantage compared to DS and CG due to their better speed up. Kriging is again the fastest strategy.

Figure 15 shows the convergence behavior during the Kriging optimization. At first, the SRAIL has an incorrect geometry as indicated by the three circles on top of the model. After about 15 simulations, design parameters are found which lead to a significantly improved geometry. But a thickening occurs at the side of the SRAIL. After 30 simulations the geometry is ideal and the thickening is reduced. A further improvement could not be realized with the seven design parameters.

5 Conclusion

Numerical optimization strategies are used to find an optimal parameter setting for the sheet metal forming process. In this first application the sheet metal forming simulation is controlled by a parameterization of the time-dependent blank-holder force. The objective is to form a sheet without cracks or wrinkles, with an acceptable thinning and the desired form of the sheet after opening the tools (springback effect). Results for two analytical problems and two realistic test cases show that optimization using surrogate models is superior compared to standard strategies like gradient or direct-search methods. Kriging in particular was found to show stable and fast convergence and a very good speed up on parallel machines.

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INDEED – The Simulation Program for Forming Technology

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