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An investigation of structural optimization in crashworthiness design using a stochastic approach

A comparison of stochastic optimization and the response surface methodology

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Abstract In this paper the response surface methodology (RSM) and stochastic optimization (SO) are compared with regard to their efficiency and applicability in crashworthiness design. Optimization of simple analytic expressions and optimization of a front rail structure are the applications used to assess the respective qualities of both methods. A low detail vehicle structure is optimized to demonstrate the applicability of the methods in engineering practice. The investigations reveal that RSM is better compared to SO for fewer than 10–15 design variables. The convergence behaviour of SO improves compared to RSM when the number of design variables is increased. A novel zooming method is proposed which improves the convergence behaviour. A combination of both the RSM and the SO is efficient, stochastic optimization could be used in order to determine appropriate starting points for an RSM optimization, which continues the optimization. Two examples are investigated using this combined method.

Key words finite element analysis, response surface methodology, stochastic optimization, structural optimization

1 Introduction

The use of structural optimization has increased rapidly during recent years, mainly due to faster computers, better algorithms and more frequent use of finite element (FE) simulations. Optimization is a useful tool to improve design in a well-structured manner. Structural optimization often uses gradients of the objective and constraints to find a search direction towards the optimal solution. For dynamic problems, like impact problems, the responses are often noisy and it is hard and expensive to find these gradients.

Stochastic optimization (SO) does not create these gradients therefore the amount of samples needed does not depend on the number of parameters or variables but simply on the precision of the desired statistical description. As opposed to traditional gradient-based optimization methods or the response surface methodology (RSM), the aim of SO is not to find the absolute optimum solution, but to develop sufficiently improved design.

In SO different background variables, e.g. initial velocity or different material properties, can vary stochastically during the optimization procedure. Thus, the optimum solution found includes natural changes from these background variables. A major difference between SO and traditional optimization methods is that SO does not need artificially frozen conditions. It is rather a reproduction of the real model considering uncertainties of manufacturing tolerances and material properties. However, we want to compare the convergence speed of SO and RSM, therefore all our design parameters in this paper are deterministic parameters and no background variables are used.

Today, RSM is the preferred optimization method in vehicle crashworthiness design. Several attempts have been made to use optimization methods in crashworthiness design problems. Etman *et al.* (1996) and Etman (1997) were among the first using RSM in structural optimization. Yamazaki and Han (1998) crashed tubes into a rigid wall and compared the optimized results with real physical tests. Roux *et al.* (1998) determined an optimal number of experimental points such the surface approximation error was reduced a lot. Schramm and Thomas (1998, 1999) and Schramm (2001, 2002) have applied RSM in a vehicle design context. Marklund and Nilsson (2001) were among the first to use RSM for an industrial application, they minimized the mass of a B-pillar of a vehicle. Sobieszczanski-Sobieski *et al.* (2000) used

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RSM to minimize the mass of a vehicle when the roof crush performance was coupled to its noise, vibration and harshness (NVH) as constraints. Yang (2001, 2002) have made large industrial applications of optimization and multidisciplinary optimization using RSM. Redhe (2001), Redhe and Nilsson (2002a,c) and Redhe *et al.* (2002b) have studied different aspects of RSM in crashworthiness applications and carried out some work on space mapping compared to RSM. Craig *et al.* (2002) applied RSM to multidisciplinary optimization where he separated the design variables for the different disciplines. Finally Forsberg (2002) has studied accuracy aspects of RSM using linear polynomials and Kriging.

As opposed to RSM, the number of evaluations per iteration for SO is user defined, typically 10–20 evaluations per iteration. Thus, one can expect far fewer necessary evaluations using SO for improving a system with a large number of design variables. Marczyk (1999) shows a procedure for how to use stochastic design improvement in a simulation-based design context. Dudeck *et al.* (2003) uses SO to optimize a vehicle with respect to crash and noise, vibration and harshness (NVH). Yang *et al.* (2003) has used the commercial program ST-ORM to optimize the crash performance of a vehicle. Their conclusions was that it was easy to use SO and it can improve the design even for many design variables. However, it does not guarantee the production of an optimal solution; other conventional methods gave better optimum solutions. The stochastic optimization method belongs to the group of 0-th order methods. Some other methods using a 0-th order approach are for example the pattern search method, see Torczon (1997), and the downhill simplex method, see Nelder and Mead (1965). These methods are based on other experiment selection strategies than stochastic simulation.

This paper aims to compare the RSM with SO and determine an upper/lower limit to the number of design variables for which the convergence speed of each method is too low compared to the other method. A novel zooming in combination with the SO method is presented to improve the convergence speed. Finally, a combined method using both SO and RSM is presented and exemplified with a larger vehicle crash optimization problem.

All optimization problems are solved using both RSM with linear polynomials and SO. To solve the optimization problems using RSM, the optimization package LS-OPT, see Stander *et al.* (2002a), was used. Solving the problems using the SO we used our own MATLAB code.

2 Successive optimization methods

2.1 Successive response surface approach

The response surface methodology is a method for constructing global approximations of the objective and constraint functions based on functional evaluations at various points in the design space. The strength of the method is in applications where gradient-based methods fail, i.e. when design sensitivities are difficult or impossible to evaluate, global optimization, exploration of design spaces and in multidisciplinary optimization.

The design domain is the space spanned by the design variables, i.e. $\{x_1, x_2, \dots, x_i\}$. The design domain can be further narrowed by introducing limits on the design variables separate from the global limits. This creates a sub-domain called the region of interest, see Fig. 1, where the approximations are calculated. When the optimum is found, the region of interest is moved in the indicated direction during the next iteration and the optimization continues, see Stander and Craig (2002b) for an automatic panning and zooming scheme. The selection of approximation functions to represent the actual behaviour is essential. For a general quadratic polynomial surface approximation the function will be,

$$y_i = \beta_0 + \sum_{j=1}^{n} \beta_j x_{ij} + \sum_{j=1}^{n} \sum_{k=j}^{n} \beta_{jk} x_{ij} x_{ik} + \varepsilon_i$$

$$i = 1, 2, \dots, N \qquad (1)$$

where β_j are the constants to be determined, x_i are the design points in the region of interest and ε_i includes both bias errors and random errors.

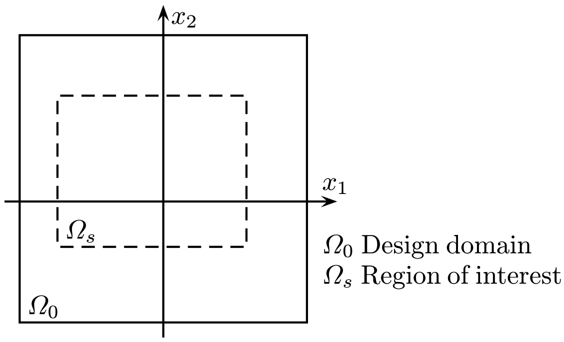

Fig. 1 Example of the design domain and the region of interest (two design variables x_1, x_2)

The minimum number of function evaluations (N_{min}) is equal to the number of unknown constants β_j. The minimum number of simulations for different approximations is stated as a function of the number of design variables in Table 1.

Table 1 Minimum number of simulations, N_{min}, as function of the number of variables, n

Assumption	N_{min}
linear	$n+1$
elliptic	$2n+1$
quadratic	$(n+1)(n+2)/2$

In this paper the D-optimal criterion is used to determine the experimental plan, see Myers and Montgomery (1995) for further reading.

In order to minimize the error ε_i a least square approach is used to find the estimates of β_j. These values of β_j are denoted $\mathbf{b}$. Thus,

$$\mathbf{b} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}\,. \tag{2}$$

For all applications in this paper, only linear polynomial are used as response surfaces.

2.2 Stochastic optimization

Stochastic analysis is based on Monte Carlo simulation (MCS). The result of an MCS is a response cloud. It is important to distinguish between the objective response and constraint responses. Stochastic optimization, also called stochastic design improvement (SDI), aims to transport the entire objective response cloud towards the target value in order to improve the design. However, it is not always possible to move a constellation of points to an arbitrary location in the design space due to physical limitations or boundary conditions. At each MCS constraint, response clouds can be created as well. An improved design is therefore only valid if all constraint values meet the boundary conditions to which they are subject. If the objective response cloud is close enough to the desired location and all constraints are fulfilled, a valid and improved design is found.

A response cloud consists of a user-defined number of points. These points result from different sets of randomly chosen design and background variables. In all examples in this paper no background variables are used.

$$y_i = f(x_i) \qquad i = 1, 2, \ldots, N \tag{3}$$

where N is the number of evaluations.

The first step is the definition of a set of design variables which generally follow a stochastic distribution; in this paper a uniform distribution is used. Then, two different types of limits have to be implemented. At first, engineering limits are required within which the design variables are allowed to be varied. Secondly, so called sampling limits have to be defined. These sampling limits must not exceed the engineering limits and can be considered as analogies of the region of interest used in RSM. They define the width of the uniform distributions of all design variables. In this way, a subregion of the design space (limited by the engineering limits) is created. Sampling limits can be described as follows

$$x_{l,k} < \mu_k < x_{r,k} \qquad k = 1, 2, \ldots, n \tag{4}$$

where $x_{l,k}$ and $x_{r,k}$ define the sampling limits and n is the number of design variables.

A Monte Carlo simulation is conducted considering the distributions of design and background variables. This simulation leads to a first response cloud with N response values of y_i. Experiment j with the minimum objective value, result and input variables x_j is then selected. At this stage, the uniform distributions of all design variables are redefined. Their mean values are shifted to the input variable values x_j of experiment j.

$$\mu_k = x_{j,k} \qquad k = 1, 2, \ldots, n \tag{5}$$

Thus, the initial model is replaced by the best experiment j. After this redefinition, a new MCS is carried out in order to find a response value even closer to the optimum value. This iterative procedure is continued until convergence is reached. A working schedule of SO can be found in Fig. 2.

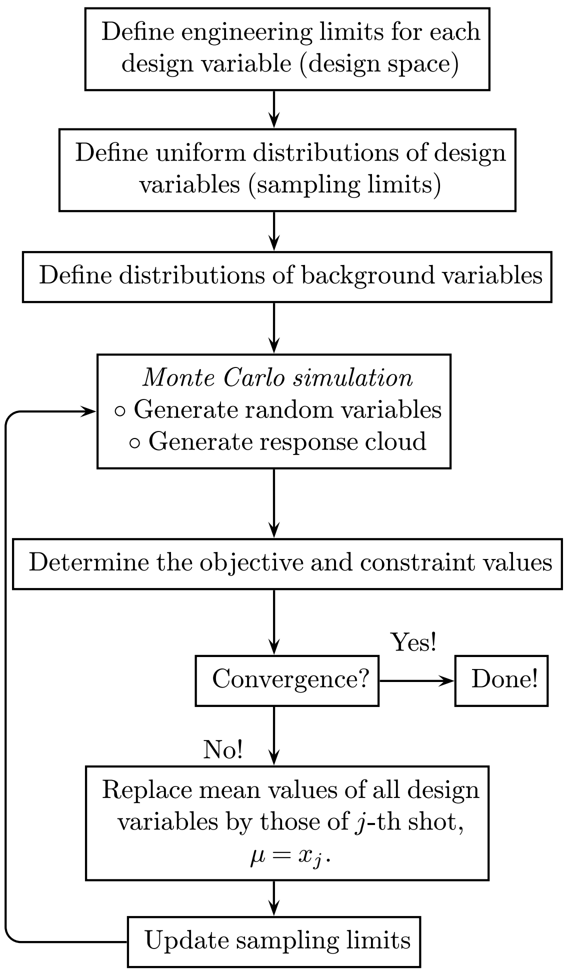

Fig. 2 Working schedule of stochastic optimization

It is important to note that the transport of response clouds is not arbitrarily fast. The "velocity" of transport of such a constellation of points is limited to approximately half of the diameter of the response cloud per iteration. Since the response clouds are generally not round in shape, this is only an estimated value.

Furthermore, all sampling limits have to be defined carefully. If the width of the uniform distribution of the design variables is too small, the design variables cannot achieve the engineering limits within a given number of iterations. Therefore, either the number of iterations or the distances of the sampling limits need to be increased.

One question is under which conditions the design improvement converges to the optimum value. Convergence means that in each subsequent iteration a smaller objective value can be found. Thus, it is necessary to estimate the probability of finding a value closer to the optimum value in the next iteration step. In the subsequent iteration there are N function values y_i, of which at least one should be smaller than the last iteration function value. The distribution of the y_i around the last iteration function value is supposed to be approximately symmetric. The ST-ORM User's Manual (2000) shows that the probability of finding at least one y_i smaller than the y from the last iteration for a monotonic decreasing function with only feasible design points is given by

$$p(N) = 1 - \left(\frac{1}{2}\right)^N \tag{6}$$

Thus, the probability of advancing towards the optimum value in the next iteration step depends on the number of experiments in each iteration. Table 2 gives the probability of convergence subject to the supposed conditions. In order to guarantee convergence it is sufficient to choose about 8 to 32 experiments per iteration. This convergence rate cannot be reached for practical problems due to infeasible design points and non-monotonic decreasing functions; this convergence behaviour can only be a theoretical upper limit.

Table 2 Probability of convergence for stochastic optimization

N	$p(N)$
1	0.5
2	0.75
4	0.9375
8	0.9960938
16	0.9999847
32	0.9999999

2.3 Zooming methods for the region of interest

Response surface methodology

The response surface optimization in LS-OPT uses a region of interest (RoI), which is a subspace of the complete design space to determine an approximative optimum. The initial RoI is given by the user and can only shrink during the iterations. The shrinking depends on three factors ($\gamma_{pan}, \gamma_{osc}$ and η) and the design variable history. If the new potential optimum is located on the boundary of the RoI, there will be no shrinking of the updated RoI (if $\gamma_{pan} = 1$). If the potential optimum point oscillates inside the RoI, it will shrink by a factor γ_{osc}. Finally, if the potential optimum is found inside RoI, the updated RoI will shrink with a factor η. If a combination of all the above happens, a combination of all factors will shrink the RoI. All equations to calculate the new RoI, i.e. (7) to (11), follow Stander and Craig (2002b).

In this updating scheme, x_k and x_{k-1} are equal to the size of the current and the previous designs, respectively, and δ_k is the current size of the RoI. The lower index k is defined as the iteration index and in this section the upper index is equal to the respective design variable. An oscillation indicator is defined as,

$$c^i = d_k^i \cdot d_{k-1}^i \tag{7}$$

where

$$d_k^i = 2 \cdot \Delta x_k^i / \delta_k^i\,; \quad \Delta x_k^i = x_k^i - x_{k-1}^i\,; \quad d_k^i \in [-1, 1] \tag{8}$$

The oscillation indicator is normalized as

$$\hat{c}^i = \sqrt{|c^i|}\,\mathrm{sign}\left(c^i\right) \tag{9}$$

Next, the shrinking parameter γ is defined as

$$\gamma^i = \frac{\gamma_{pan}\left(1 + \hat{c}^i\right) + \gamma_{osc}\left(1 - \hat{c}^i\right)}{2} \tag{10}$$

Finally, the new RoI is defined as,

$$\delta_{k+1}^i = \lambda^i \delta_k^i\,, \text{ where } \quad \lambda^i = \eta + |d_k^i|\left(\gamma^i - \eta\right) \tag{11}$$

The factors $\gamma_{pan}, \gamma_{osc}$ and η are given by the user. Typical values are $\gamma_{pan} = 1.0$, $\gamma_{osc} = 0.6$ and $\eta = 0.6$.

Stochastic optimization

The RSM zooming method cannot be adapted to SO as the stochastic distribution of the design variables does not generally give a suboptimal solution on the boundary of the sub-domain, and due to the low number of evaluations, not all corners of the sub-domain are evaluated. This might give a suboptimal solution that seems to oscillate. This is only due to the fact that there is no design point on this side of the centre, therefore the same zooming/panning method as RSM cannot be used for SO.

We introduce a novel zooming of the RoI, which we call the stochastic optimization zooming method (SOZM) and the basic idea is as follows: The initial size of the RoI remains constant as long as all subsequent iterations produce lower objective values. In the case that the optimization stops converging, i.e. the current iteration produces a worse result than the previous one, this indicates the necessity for zooming. In general, the topology of the objective function is unknown. Thus, there is no evidence indicating which zooming factor should be used. One has to be aware that each optimization problem has a unique zooming factor for maximum efficiency. Consequently, an assumption has to be made for the zooming factor (between 0 and 1). However, a rather large zooming factor close to the maximum value of 1 seems to be useless, because the probability of finding a better point would only be increased marginally.

Most optimization problems are subjected to constraints. As a consequence, the result of the best experiment of each iteration is restricted to meet all constraints, otherwise the centre of the region of interest of the next iteration would violate the boundary conditions. Thus, most of the subsequent evaluations would also violate the constraints and the optimization process would produce a lot of inadmissible results. In the case that no better design can be found close to a constraint, the RoI has to be scaled down. Figure 3 shows the proposed working schedule of SOZM.

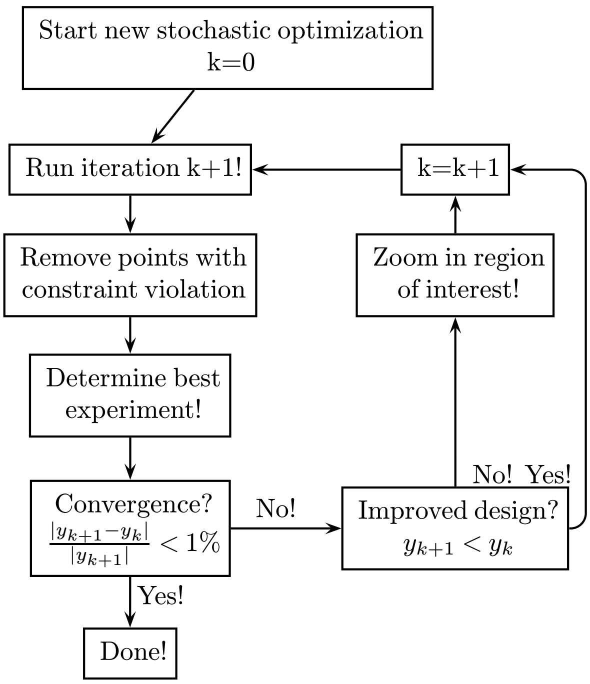

Fig. 3 Proposed working schedule of stochastic optimization zooming method and check on constraint violations

2.4 Combination of RSM and SO

In many cases traditional RSM produces results that violate the constraints early in the optimization process. Normally, this is caused by inaccurate response surfaces and a lot of iterations are needed in order to come close to a valid design. Consequently, it is impossible to abort the optimization after a few iterations in the case that only a slightly improved design is desired, or computing capacity is limited. A major quality of SOZM is that all improved designs meet the constraints, provided that at least one design point is valid. Thus, the first few iterations already produce improved and valid designs. The optimization process can be stopped, if the design improvement is sufficient. However, in cases where SOZM closely approaches one of the constraints, the method generally loses its efficiency.

It is therefore proposed to switch the optimization method to RSM in cases where further design improvements are demanded. The current best design is used as the starting point for RSM.

2.5 Stopping criteria

To determine if the optimization method has converged, we calculate the percentage change in the objective function value in the last two iterations. If this change is less than 1%, the optimization routine is terminated. The stopping criteria for RSM as well as for SO is calculated as,

$$\frac{|f_{k+1} - f_k|}{|f_{k+1}|} < 1\% \tag{12}$$

3 Analytical test examples

In this section the RSM is compared to SO with respect to accuracy and efficiency. Three different analytic functions are defined for this investigation. A quadratic polynomial function, the so-called generalized Rosenbrock function and the 12-corner polytope is used. In this investigation, the functions are optimized for several different numbers of design variables varying from 2 to 50.

It is important to note that the dimension of the analytic functions affects the number of necessary experiments per iteration using RSM. The D-optimality criterion and 50% more than the minimum required number of points are used for constructing linear response surfaces. The number of experiments per iteration for SO is defined to be 20.

3.1 Quadratic polynomial

The quadratic polynomial function is given by

$$f(\mathbf{x}) = \left[\sum_{i=1}^{n} x_i^2\right] + 1\,. \tag{13}$$

Apparently, the minimum value $f_a^\star(\mathbf{x}_\mathbf{a}^\star)$ can be found at the point of origin

$$f_a^\star(\mathbf{x}_\mathbf{a}^\star) = 1\,, \qquad \mathbf{x}_{\mathbf{a},\mathbf{i}}^\star = 0\,, \qquad i = 1, 2, \ldots, n$$

where the index a indicates the exact analytic solution and n stands for the number of design variables. The optimization problem is formulated as follows

$$\min\ (f(\mathbf{x}))$$

$$\text{s.t.} \qquad -4 \le x_i \le 4 \quad i = 1, 2, \ldots, n$$

where the values of -4 and 4 define the boundaries of the design space. The starting point is set to,

$$x_i^{SP} = 1.5\,, \qquad i = 1, 2, \ldots, n \tag{14}$$

and the initial RoI for all RSM optimizations is defined as follows

$$-1.0 \le x_i \le 4.0\,, \qquad i = 1, 2, \ldots, n \tag{15}$$

Originally, SO was planned to use the same RoI, but the first results obtained by these optimizations already reveal that the size of the RoI is considerably too large. Stochastic optimization could not produce significantly improved function values. The explanation of this problem is that the region of interest is relatively large and the minimum value of the analytic function is already situated within the region of interest. The probability of finding a better point in the subsequent iteration is rather small due to the fact that the RoI cannot shift towards the absolute minimum point. In fact, the RoI remains at almost the same position. This effect is made worse by increasing the number of design variables, which is the same as an enlargement of the design space without increasing the number of experiments. Consequently, a lot of fortune or a lot of iterations are needed to find a "lucky" point which is situated very close to the optimum point. For this reason, the size of the RoI is reduced to 10% of the design space and is therefore given by

$$1.1 \le x_i \le 1.9\,, \qquad i = 1, 2, \ldots, n \tag{16}$$

Optimization results

Table 3 gives a summary of the obtained results of the quadratic polynomial function optimization. The table shows the number of design variables, the number of performed iterations and the overall cost, i.e. the total number of evaluated experiments.

The analysis of the results produced by the optimization of the quadratic polynomial function delivers insight into the convergence behaviour of RSM and SO. In general, RSM provides extremely accurate minimum values $f^\star_{RSM}$ independent of the number of design variables. The total number of experiments increases with the number of design variables.

Stochastic optimization shows a different behaviour. For the 2-dimensional problem a number of 220 evaluations is performed in order to produce an accurate solution. This is much more than the 40 experiments needed by RSM. Only a few more experiments (260) are performed with SO to solve the 10-dimensional problem, while RSM approximately quadruples the number of experiments to 170. Thus, one could expect that the 25-dimensional problem solved by RSM should perform significantly more experiments compared to SO, but this is not the case.

Table 3 Results of the quadratic polynomial function optimization for RSM and SO

n	RSM # it.	RSM cost	$f^\star_{RSM}$	SO # it.	SO cost	$f^\star_{SO}$
2	8	40	1.009	11	220	1.0026
10	10	170	1.009	13	260	1.4057
25	12	468	1.011	24	480	4.8372
50	12	900	1.021	26	520	21.2344

Both methods converge after almost the same number of experiments, 468 and 480, respectively. Obviously, this phenomenon is caused by the larger RoI; 25 design variables instead of 10 are simulated. Since the number of experiments per iteration remains constant for SO, the probability of finding a better point in the next iteration decreases significantly due to the enlarged region of interest. Several repetitions of the 25-dimensional optimization lead to almost identical results. Thus, this cannot be a coincidental result.

In comparison, SO cannot guarantee the same result quality as RSM. In particular for a larger number of design variables the minimum values $f^\star_{SO}$ are much worse compared to the respective RSM solutions.

3.2 Extended Rosenbrock

The generalized Rosenbrock function is given by

$$f(\mathbf{x}) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right] \tag{17}$$

The analytic minimum value $f_a^\star(\mathbf{x}_\mathbf{a}^\star)$ is equal to zero and can be found at the point defined by

$$f_a^\star(\mathbf{x}_\mathbf{a}^\star) = 0\,, \qquad x_{a,i}^\star = 1.0\,, \quad i = 1, 2, \ldots, n \tag{18}$$

where the index a indicates the exact analytic solution and n stands for the number of design variables. For this Rosenbrock function the design space depends per definition on the number of design variables. The optimization problem is formulated as

$$\min\ f(\mathbf{x})$$

$$\text{s.t.} \qquad -n \le x_i \le n \qquad i = 1, 2, \ldots, n$$

where the values of $-n$ and n define the boundaries of the design space. Consequently, starting points and regions

Table 4 Starting points and regions of interest of the RSM Rosenbrock optimizations

n	Starting p. $(x_1,\ldots,x_n)$	Region of interest RSM
2	$x_i = 0.8$	$x_i^{SP} - 1.2 \leq x_i \leq x_i^{SP} + 1.2$
10	$x_i = 4.8$	$x_i^{SP} - 6.0 \leq x_i \leq x_i^{SP} + 6.0$
25	$x_i = 10.0$	$x_i^{SP} - 15.0 \leq x_i \leq x_i^{SP} + 15.0$
50	$x_i = 24.0$	$x_i^{SP} - 30.0 \leq x_i \leq x_i^{SP} + 30.0$

Table 5 Starting points and regions of interest of the stochastic Rosenbrock optimizations

n	Starting p. $(x_1,\ldots,x_n)$	Region of interest SO
2	$x_i = 0.8$	$x_i^{SP} - 0.2 \leq x_i \leq x_i^{SP} + 0.2$
10	$x_i = 4.8$	$x_i^{SP} - 1.0 \leq x_i \leq x_i^{SP} + 1.0$
25	$x_i = 10.0$	$x_i^{SP} - 2.5 \leq x_i \leq x_i^{SP} + 2.5$
50	$x_i = 24.0$	$x_i^{SP} - 5.0 \leq x_i \leq x_i^{SP} + 5.0$

of interest have to be adapted to the number of design variables. Tables 4 and 5 summarize the values of starting points and regions of interest for all optimization problems. Apparently, one has to take into account that the same problem of the too large RoI occurs here as well. For this reason, the sizes of the regions of interest for SO are reduced in the same way.

Optimization results

The Rosenbrock function shows a more complicated shape with larger gradients than the quadratic polynomial function.

Significantly more iterations are performed compared to the square function optimization. The results of the 2-dimensional problem reveal that RSM is more expensive and produces a worse result than SO. Thus, for a larger number of design variables an even better convergence behaviour of SO could be expected. In fact, all further investigations with an increased number of design variables indicate a contrary behaviour. Although SO converges after fewer experiments than RSM, the quality of the produced results is extremely worse. In spite of increasing the number of iterations and ignoring the convergence criterion, the objective value cannot be reduced significantly.

Table 6 Results of the Rosenbrock function optimization for RSM and SO

n	RSM # it.	RSM cost	$f^{\star}_{RSM}$	SO # it.	SO cost	$f^{\star}_{SO}$
2	14	70	2.159	2	40	0.0072
10	77	1309	8.616	17	340	436.9
25	52	2028	276.81	16	320	8.0×10^5
50	49	3675	1821.2	13	260	6.1×10^8

Optimization results using different region of interest

In Table 7, four different regions of interest are defined by the width of their uniform distributions. Each design variable is restricted to follow exactly the same uniform distribution limited by the boundary values a and b, respectively. The mean value of each distribution is equivalent to the starting point of the optimization problem at hand.

The resulting convergence behaviour of each optimization process is depicted in Fig. 4. The fifth curve is used as a basis for comparison and represents the convergence behaviour of the previously performed RSM optimization. Curve (1) needs a lot of iterations to approach a minimum value as a consequence of the relatively small RoI. In each iteration the centre of the RoI cannot be shifted more than half of the width of each uniform distribution. For this reason, it is advisable to use a rather large RoI, otherwise it is too expensive to approach a better solution.

Curves (2) and (3) indicate an improved convergence behaviour in the beginning, but then they show a rather noisy behaviour. Furthermore, both objective function values are not reduced substantially compared to the RSM optimization. Apparently, the theory of SO, i.e. more than 99.9% probability of convergence, does not agree with the real convergence behaviour of these two curves.

For the first iterations of curve (4) one can observe a comparatively fast convergence behaviour, but later the same noisy behaviour occurs. Finally, it is important to note that none of the SO processes provides a better result than the RSM optimization. It is a matter of fact that at least in the beginning of the optimization process a large RoI converges much faster than a small one. Thus, a large RoI is well suited to approach a local or global minimum with only a few iterations, but only as long as the objective function has rather large gradients. As soon as the region of interest intersects more than once with the objective function, the probability of improvement

Table 7 Definition of four different regions of interest

ID	Size	Mean	a	b
(1)	0.2	10	9.9	10.1
(2)	1.0	10	9.5	10.5
(3)	2.0	10	9.0	11.0
(4)	4.0	10	8.0	12.0

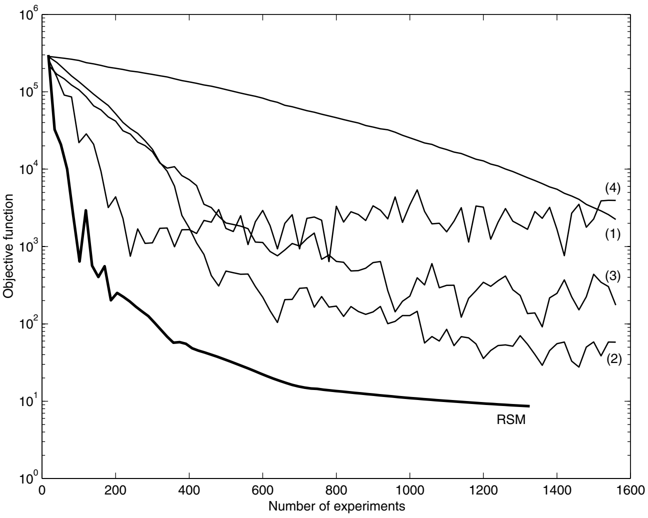

Fig. 4 Convergence behaviour depending on different regions of interest

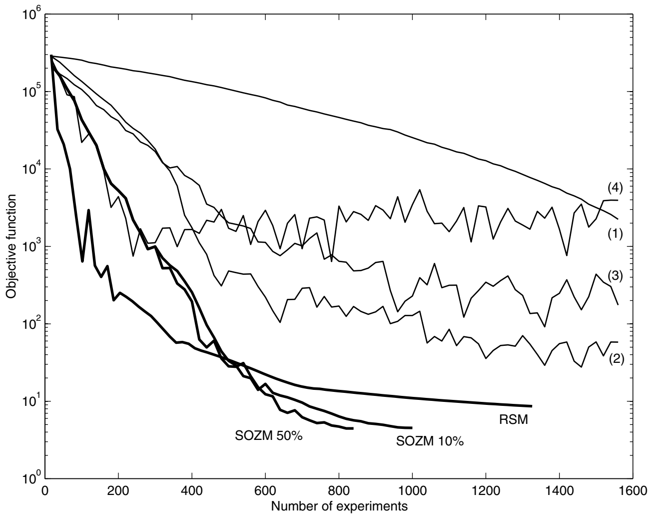

Fig. 5 Convergence behaviour of SOZM for the 10-dimensional Rosenbrock problem

decreases significantly. The functions have small gradients close to their local or global minima. Consequently, SO loses its efficiency when approaching to a minimum as soon as this minimum lies within the RoI. Basically, there are two different alternatives to improve the general convergence behaviour:

1. Increasing the number of experiments per iteration in order to achieve a higher probability of finding a better solution.
2. Automatic scaling of the size of the RoI.

Some additional tests have shown that increasing the number of experiments per iteration is very expensive and does not improve the quality of the results substantially. In particular for a large number of design variables doubling and not even tripling the number of experiments per iteration guarantees better results. The second alternative seems to be a promising approach due to the constant number of experiments.

Optimization results using SOZM

To improve the convergence speed and find the true optimum solution the SOZM, see Sect. 2.3, is used to optimization the extended Rosenbrock function. An assumption has to be made for the zooming factor (between 0 and 1). However, a rather large zooming factor close to the maximum value of 1 seems to be useless, because the probability of finding a better point would only be increased marginally. For this reason, two different zooming factors of 0.1 and 0.5, respectively, have been used for the optimization. Both the resulting convergence plots are depicted in Fig. 5. Apparently, the proposed zooming method shows an improved convergence behaviour compared to the initial optimizations. The zooming method is even better compared to the RSM optimization and achieves convergence after 42 and 50 iterations, respectively. Table 8 gives a brief summary of the results of RSM and SOZM.

Table 8 Results of 10-dimensional Rosenbrock optimization using RSM and SOZM

	RSM	SOZM $f = 0.1$	SOZM $f = 0.5$
$f^\star$	8.616	4.535	4.454
Iterations	77	50	42
Experiments	1309	1000	840
Zoomings	–	2	6

The main difference between the two optimizations with different zooming factors is the number of zoomings itself. The optimization process using a zooming factor of 0.5 requires a total of six zoomings, whereas the other optimization with a factor of 0.1 only zooms in twice. This means that a zooming factor of 10% produces more iterations in order to approach the minimum value and, therefore, this optimization process is less efficient. Additionally, the probability of ending up in a local minimum is slightly higher, in particular in cases with a noisy response. Thus, the zooming factor of 0.5 seems to be a reasonable value.

3.3 12-corner polytope

In this section the combined method presented in Sect. 2.4 used to solve the optimization problem.

The problem maximizes the area of the polytope subject to a constraint on the circumference. Objective and constraint functions are defined as follows

$$f(r_j, v_j) = -\frac{1}{2}\sum_{j=1}^{10} r_j r_{j+1} \sin v_j \qquad (19)$$

$$c(r_j, v_j) = r_1 + r_{11} + \sum_{j=1}^{10} \left(r_j^2 + r_{j+1}^2 - 2r_j r_{j+1} \cos v_j\right)^{1/2} \tag{20}$$

In consideration of the design space limits, the optimization problem can be formulated as follows

$$\begin{aligned} &\min f(r_j, v_j) \\ &\text{s.t.} \quad c \le 60 \\ &\qquad 1 \le r_j \le 30\,, \quad j = 1, \ldots, 11 \\ &\qquad 1 \le v_j \le 45\,, \quad j = 1, \ldots, 10 \end{aligned} \tag{21}$$

The starting point is $r_j = 11$ and $v_j = 18$ for all $j's$ resulting in a constraint value of $c_{sp}(r_j, v_j) = 56.416$. The initial region of interest of all design variables is defined to be 10 for RSM as well as for SOZM. The default number of 33 experiments per iteration is used for RSM and a number of 20 experiments per iteration is used for SOZM.

Optimization results

The convergence plots of objective and constraint functions can be found in Figs. 6 and 7, respectively. RSM produces results far below the mathematical optimum solution of $f(r_j, v_j) = -279.9$ in the first few iterations. The constraint values are above the threshold value of $c(r_j, v_j) = 60$ and therefore these results are not valid. Approximately 400 experiments are performed in order to find a rather good solution, but the constraint is still slightly violated. Strictly speaking, RSM fails to localize a valid solution, at least for the first 400 experiments.

As opposed to RSM, SOZM shows a quite stable convergence behaviour with exclusively valid solutions. Horizontal parts of the convergence plot indicate a zooming of 50%, i.e. no better solution could be found. After exactly 200 experiments the third zooming is performed. At the same time, the constraint plot approaches the threshold value very closely. This indicates that the optimization starts converging along the constraint and the optimization becomes less efficient. This can also be seen in the convergence plot of the objective function. After experiment number 200, the gradient of the convergence plot is significantly smaller than before.

Instead of continuing SOZM, the result of the tenth iteration is used as the starting point for a RSM optimization. Unfortunately, this optimization violates the constraint at the beginning of the optimization. However, the constraint violation is much smaller than the violation of the first RSM optimization. Finally, the convergence behaviour of both RSM optimizations is very similar. There are no significant differences after about 400 experiments.

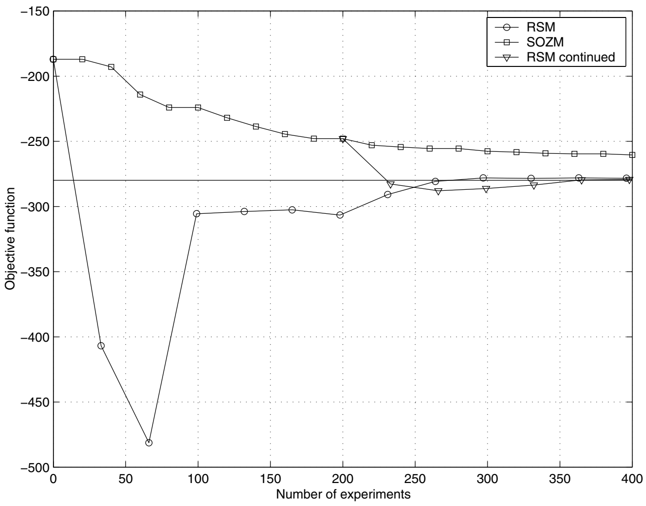

Fig. 6 Convergence plot of the objective function

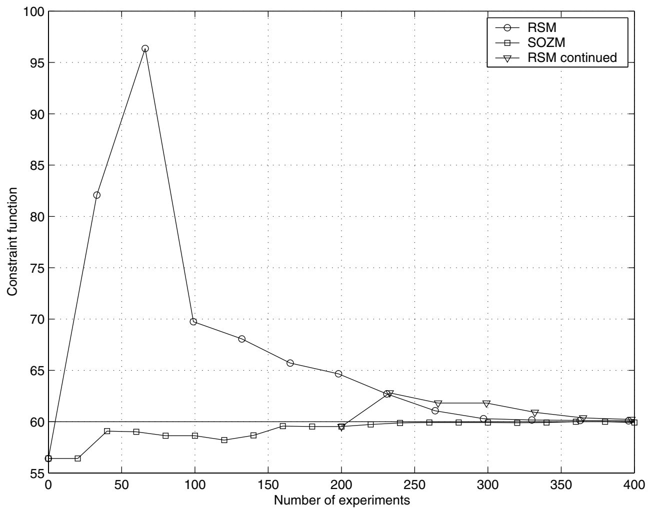

Fig. 7 Convergence plot of the constraint function

4 Crashworthiness application

4.1 Front rail structure

Figure 8 shows the front rail structure that will be optimized using SOZM and RSM. The front rail structure will be parameterized for three different numbers of design variables, namely 2, 11 and 20.

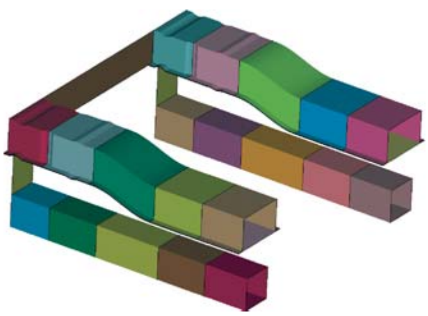

Fig. 8 Front rail structure

4.1.1
Problem description

The front rail structure of a fictitious vehicle model is subjected to a rigid wall impact. The maximum acceleration value is defined as the objective to be minimized in order to improve the crashworthiness of the vehicle. Further responses are mass and maximum intrusion after impact. Both responses are limited by predefined boundary conditions. The experimental setup is depicted in Fig. 9, with initial velocity $v_0 = 15.64\,\mathrm{ms}^{-1}$ and mass $M = 275\,\mathrm{kg}$.

The front rail structure is parameterized for a maximum number of 20 design variables, i.e. 16 thicknesses and 4 different yield stresses. For the optimizations with 2 and 11 design variables some of the variables are supposed to be constant. Thus, the optimization problem is formulated as

$$\begin{array}{lcccr} \min |a_{max}| & & & & \\ \text{subject to:} & & \text{Mass} & \leq & 17.5\,\mathrm{kg} \\ & & \text{Intrusion} & \leq & 0.3\,\mathrm{m} \\ 1\,\mathrm{mm} & \leq & t_n & \leq & 3\,\mathrm{mm} \\ 304\,\mathrm{MPa} & \leq & \sigma_m & \leq & 456\,\mathrm{MPa} \end{array}$$

All optimizations are implemented with an initial region of interest size of 1 mm for each design variable. Twelve experiments per iteration are evaluated in SOZM and the default numbers of experiments are used for RSM optimizations, namely 5, 17 and 32.

4.1.2
Optimization results

The discussion of the optimization results is divided into three sections with respect to the different number of design variables.

2-dimensional optimization

The design variables are the sheet thicknesses of the upper and the lower beams, respectively. All yield stress values are supposed to be constant (380 MPa).

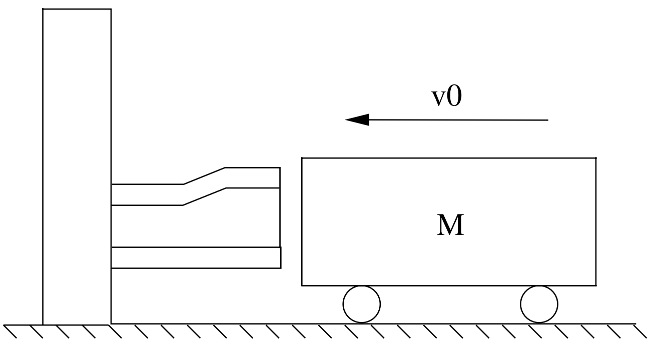

Fig. 9 Front rail structure subjected to impact

Table 9 2-dimensional front rail optimization

	SOZM	RSM
Experiments per iteration	12	5
Number of iterations	6	7
Total number of experiments	72	35
Optimum result		
Acceleration, $\left[\mathrm{m\,s^{-2}}\right]$	−1050	−1033
Mass, [kg]	12.30	12.23
Intrusion, [m]	0.291	0.297

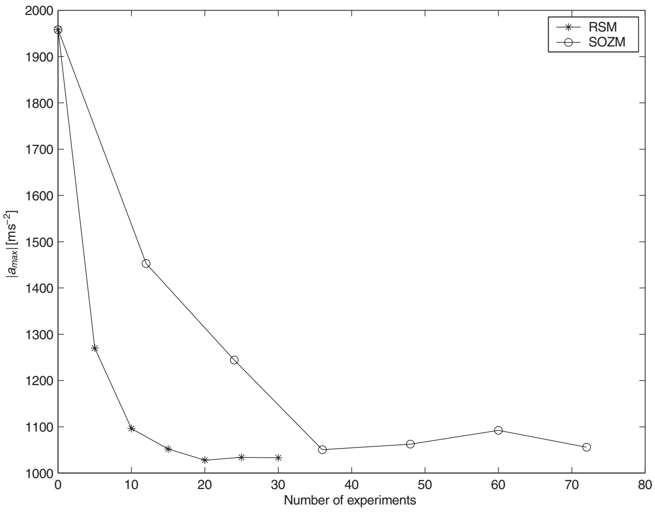

Fig. 10 Convergence plot of 2-dimensional optimization

The results stated in Table 9 and Fig. 10 confirm that RSM is definitely superior to SOZM for a small number of design variables. Both methodologies converge towards the same optimum solution, but RSM performs about half as many experiments as SOZM. In addition the efficiency of SOZM decreases because of constraint violations. Some of the experiments violate the intrusion constraint and therefore the RoI has to be scaled down.

The optimization leads to an improved front rail structure design with a maximum acceleration that is reduced to approximately 50% of the initial value and the intrusion is at threshold value of 0.3 m. The newly determined thicknesses of the upper and lower beams are $T_\mathrm{U} = 1.75$ mm and $T_\mathrm{L} = 1.00$ mm, respectively.

11-dimensional optimization

The eleven design variables are all thicknesses. The lower and upper beam parts are divided into five sections, respectively. Furthermore, the thickness of additional stiffening plates in the upper beams is one more design variable.

The results stated in Table 10 and Fig. 11 reveal that SOZM and RSM show almost identical convergence behaviours, at least in the beginning of the optimization

Table 10 11-dimensional front rail optimization

	SOZM	RSM
Experiments per iteration	12	17
Number of iterations	9	13
Total number of experiments	108	221
Optimum result		
Acceleration, $\left[\mathrm{m\,s^{-2}}\right]$	-786.0	-662.8
Mass, [kg]	15.37	12.95
Intrusion, [m]	0.303	0.303

Table 11 20-dimensional front rail optimization

	SOZM	RSM
Experiments per iteration	12	32
Number of iterations	9	14
Total number of experiments	108	448
Optimum result		
Acceleration, $\left[\mathrm{m\,s^{-2}}\right]$	-726.7	-651.1
Mass, [kg]	14.76	13.06
Intrusion, [m]	0.272	0.311

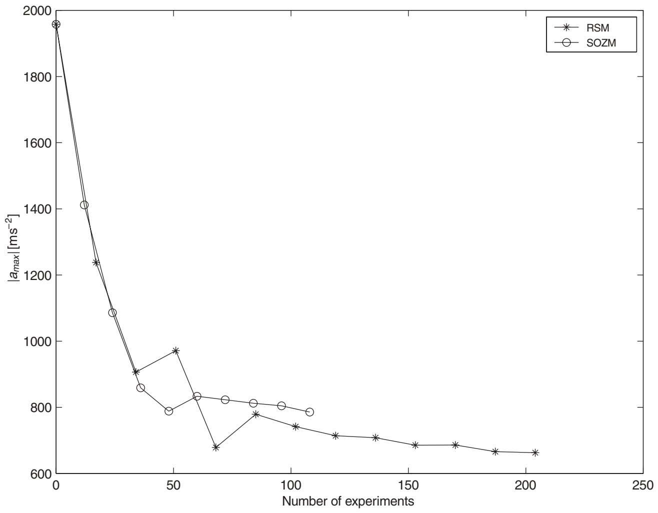

Fig. 11 Convergence plot of 11-dimensional optimization

processes. Once the first zooming of the region of interest is implemented in SOZM (iteration 5), an obvious loss of efficiency appears. A comparison of the optimum results after 108 (SOZM) and 102 (RSM) experiments, respectively, results in almost the same objective function values. Thus, none of the methods is significantly superior to the other. However, in this case the RSM solution should be preferred due to the lower weight of the front rail structure. The optimizations converge to different optimum solutions.

In some cases, RSM produces better solutions than SO at the beginning of the optimization process, but violates the constraints due to an inaccurate response surface. However, there is no guarantee that RSM produces exclusively valid designs, i.e. without any constraint violations.

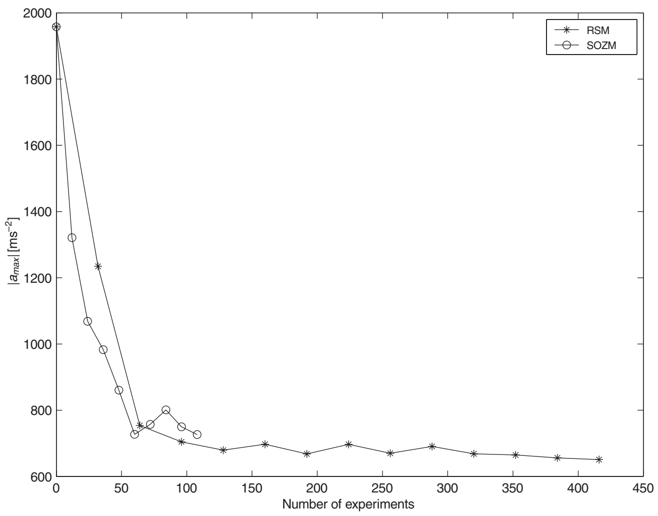

Fig. 12 Convergence plot of 20-dimensional optimization

20-dimensional optimization

The number of design variables is increased to 20 by adding variable yield stress values and five additional thickness values.

SOZM shows a better convergence behaviour in the beginning of the optimization process compared to RSM. While SOZM determines five improved and valid front rail designs in the first 60 experiments, RSM has not even completed the second iteration. For a larger number of design variables this advantage of SOZM will be even greater, because RSM needs a lot more experiments in order to complete its iterations. As soon as the first zooming is implemented in SOZM, there is a loss of efficiency as seen for the 11-dimensional optimization case. However, it is obvious that SOZM produces much better solutions in the first few iterations compared to RSM.

In the beginning of the optimization process, SOZM is more efficient than RSM in terms of convergence behaviour, if the number of design variables exceeds a limit of about 15 to 20. As soon as constraints are violated or the RoI has to be scaled down, SOZM loses its excellent efficiency. Further design improvement can hardly be found without lots of subsequent experiments. The results of this chapter indicate that for optimization problems with a large number of design variables SOZM should be used.

This front rail optimization reveals that SOZM and RSM have different qualities. On the one hand, SOZM is suitable to scan the entire design space for solutions rather close to local optimum solutions, but on the other hand, RSM is able to find exact mathematical optimum solutions. Thus, the combined method will utilize their respective qualities.

4.2
Vehicle structure subjected to frontal impact

The optimization of a vehicle structure subjected to frontal impact is presented as an industrial example. The simple vehicle structure, see Fig. 13, is composed of 87 parts. In order to reduce the computing time, the engine block is represented by a rigid substructure. Furthermore, the structure is symmetric and 15 design variables are used in order to improve the crashworthiness design.

In consideration of all constraints and the limits of the design space, the optimization problem can be formulated as follows:

$$
\begin{array}{lcccl}
\min|a_{max}| & & & & \\
\text{s.t.} & & \text{Intrusion D2} & \leq & 0.095\,\text{m} \\
 & & \text{Intrusion D3} & \leq & 0.095\,\text{m} \\
 & & \text{Intrusion D4} & \leq & 0.095\,\text{m} \\
 & & \text{Average rigid wall force} & \leq & 90\,000\,\text{N} \\
 & & \text{Mass} & \leq & 45\,\text{kg} \\
1 & \leq & t_x & \leq & 3\,\text{mm}
\end{array}
$$

The intrusion is measured at three different points, i.e. D2, D3 and D4. All optimizations are implemented with an initial size of the region of interest of 1 mm for each design variable. The default number of 25 experiments per iteration is used for the RSM optimization. In SOZM, 12 experiments per iteration are evaluated.

Optimization results

All relevant results are stated in Table 12 and Fig. 14 shows the convergence plots of all three different optimization processes, i.e. RSM, SOZM and the combined optimization method.

After six SO iterations no improved design can be found and therefore the first zooming would be implemented. The current best design point ($|a_{max}| = 318.3\,\text{ms}^{-2}$, 72 experiments) is used as the starting point

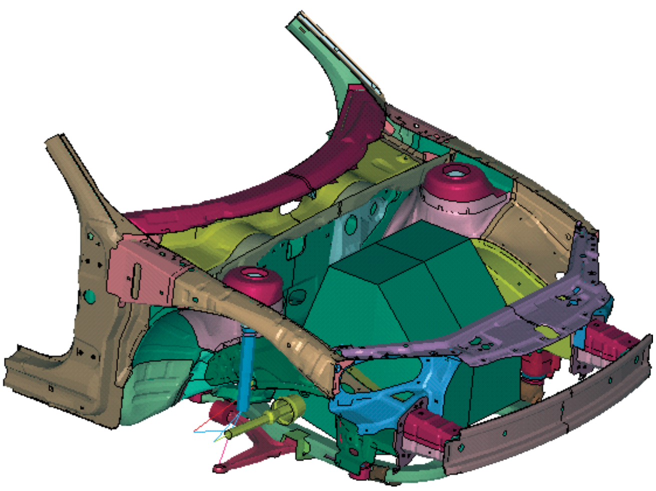

Fig. 13 Vehicle structure (symmetric)

Table 12 Results of combined vehicle structure optimization

	RSM	SOZM	COMB
Experiments per iteration	25	12	12/25
Number of iterations	11	14	6/7
Total number of experiments	275	168	247
Optimum result			
Acceleration, $\left[\text{m}\,\text{s}^{-2}\right]$	−312.8	−284.3	−282.0
Intrusion D2, [m]	0.0823	0.0771	0.0917
Intrusion D3, [m]	0.0592	0.0615	0.0858
Intrusion D4, [m]	0.0921	0.0873	0.0901
Rigid wall force, [N]	86900	87080	85580
Mass, [kg]	42.7	43.45	41.75

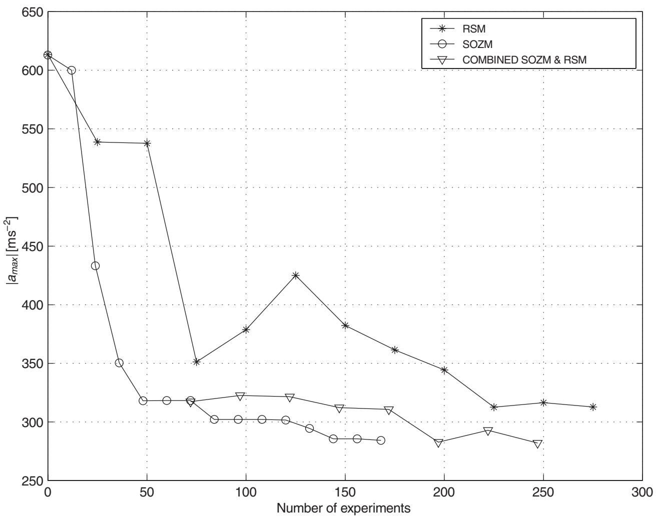

Fig. 14 Convergence plot of combined optimization method

for RSM. In order to avoid major constraint violations, the size of the RoI is reduced to 20% of the initial size. In general, it is very difficult to find optimization parameters leading to maximum efficiency. Thus, this reduction is based rather on experience than on predefined and established rules. Nevertheless, the first few RSM results slightly violate the constraints and no better design can be located compared to SOZM. Only after the fourth RSM iteration somewhat better results are found, but their determination is rather expensive.

Finally, although this does not concern the comparison of the combined optimization method and SOZM, it is important to note that both methods are better compared than RSM with respect to efficiency and result quality in this example.

5
Conclusions

The investigations in this paper reveal that SO and RSM shows rather different qualities. On the one hand, RSM is suitable for finding the mathematical optimum solution of the global or at least of a local minimum. On the other hand, for these optimization problems SO often produces a better convergence behaviour in the beginning of the optimization process for a large number of design variables. It has been shown that SO should not be used for optimization problems with less than 10–15 design variables; RSM is superior to SO for these examples. However, the more design variables the problem has, the more efficient SO becomes.

The convergence behaviour of all SO depends to a large extent on the size of the RoI. The results of this investigation indicate that SOZM has to be used instead of SO, otherwise the convergence behaviour can be unacceptable. SOZM combines the advantages of a large RoI with regard to the shifting capabilities and a small RoI with regard to the capability of finding accurate solutions. Thus, the optimizations should start with a relatively large RoI due to the fact that the RoI is scaled down automatically.

Finally, the combination of both methods is a promising approach. In particular, if only a slightly improved design is required or the computing capacity is limited, SOZM produces valid and improved designs within the first few iterations. In the case that a further design improvement is desired, the SOZM result can be used as a starting point for an RSM optimization.

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