Struct Multidisc Optim 30, 66–75 (2005)
DOI 10.1007/s00158-004-0386-z

Structural optimization as a harmony of design, fabrication and economy

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Abstract The main requirements of load-carrying engineering structures are safety, producibility and economy. The role and importance of fabrication constraints and costs is shown in the case of a compression tubular strut, a tubular truss of parallel chords and a welded stiffened square plate. The evaluation of optimal versions of these structures shows that, neglecting the fabrication constraints and costs, the design cannot result in up-to-date solutions.

Key words fabrication constraints, minimum-cost design, residual welding deformations, tubular structures, welded structures

Requirements of engineering structures

Optimization means a search for better solutions, which better fulfil the requirements. For a modern load-carrying engineering structure the main requirements are as follows: load-carrying capacity (safety), producibility and economy. These requirements can be fulfilled by a structural synthesis system, in which a cost function is minimized considering constraints on design and producibility.

This system can be symbolized by a simple spatial structure as shown in Fig. 1. This symbol also shows two important aspects: (1) when a requirement is missing, the system does not work well, does not give better solutions; (2) a tight cooperation (harmony) should be realized among these main aspects, since they affect each other significantly. Structural optimization is a general system, which can synthesize all the important engineering aspects.

Received: 30 August 2001
Published online: 15 February 2005

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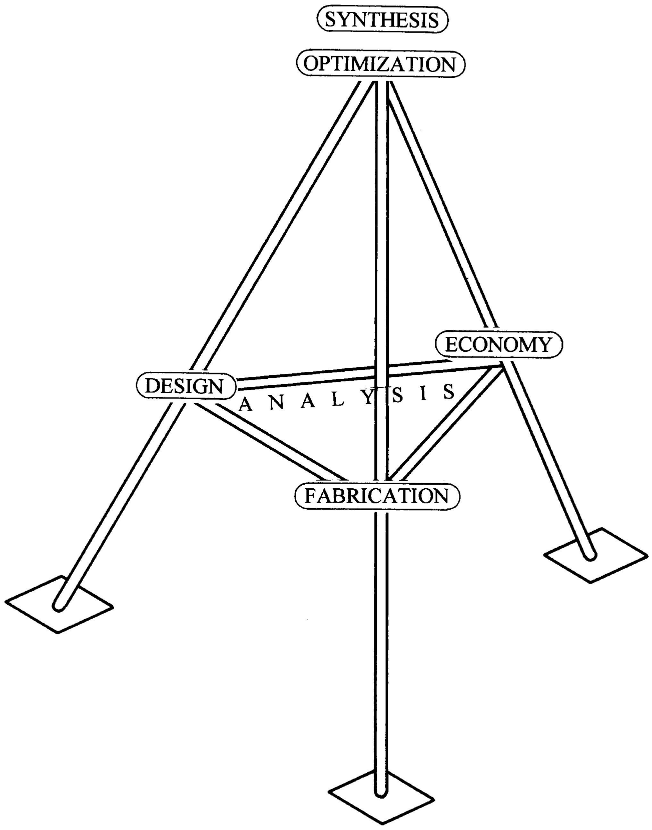

Fig. 1 A structural optimization system

At the analytical level the structural characteristics of the investigated type should be analysed as follows: loads, materials, geometry, boundary conditions, profiles, topology, fabrication, joints, transport, erection, maintenance, costs. Those variables should be selected, changing which best solutions can be achieved. A cost function and constraints on design and fabrication should be mathematically formulated as a function of these variables.

At the level of synthesis the cost function should be minimized using effective mathematical methods for the constrained function minimization. Comparing the optimum solutions, designers can select the most suitable one. This comparison can result in significant mass and cost savings in the design stage.

Structural solutions can be evaluated with respect to the three main requirements mentioned above. The following three examples can serve as illustrations of this evaluation.

Example 1: a circular hollow section (CHS) strut of constant cross-section, loaded in compression

Two solutions are compared: the Euler solution does not take into account the initial imperfection, which is unavoidable in fabrication, the Eurocode 3 solution calculates with initial crookedness and residual stresses due to welding or cold-forming. Both solutions consider the minimum cross-sectional area as the objective (cost) function.

The Euler solution

The critical flexural buckling stress is

$$\sigma_E = \pi^2 E/\lambda^2\,; \quad \lambda = KL/r\,; \quad r = \sqrt{I_x/A} \tag{1}$$

where E is the elastic modulus, λ is the slenderness, K is the end restraint factor (for pinned ends $K=1$), I_x is the moment of inertia, A is the cross-sectional area, and r is the radius of gyration. In the calculation, the values of yield stress $f_y = 355$ MPa, $a = (50/(8\,\pi))^{1/2} = 1.4105$, $K=1$ and a limiting slenderness of $\delta = D/t = 50$ are used.

For CHS, using the notation $\delta = D/t = (d-t)/t$, where D is the mean diameter and d is the outside diameter, t is the thickness, the following formulae are valid (Farkas and Jármai 1997)

$$I_x = \frac{\pi D^3 t}{8} = \frac{\pi D^4}{8\delta}\,; \quad A = \frac{\pi D^2}{\delta}\,; \quad r = \frac{D}{\sqrt{8}} = a\sqrt{A}\,;$$

$$a = \sqrt{\frac{\delta}{8\pi}} \tag{2}$$

Using (2), the slenderness can be expressed by A as follows:

$$\lambda^2 = \frac{L^2}{r^2} = \frac{L^2}{a^2 A} = \frac{10^4}{a^2}\frac{1}{10^4 A/L^2} = \frac{5027}{10^4 A/L^2} \tag{3}$$

The flexural buckling constraint can be expressed as

$$\frac{N}{A} \le \chi f_y\,; \quad \chi = \frac{1}{\bar{\lambda}^2} \quad \text{for} \quad \bar{\lambda} \ge 1$$

$$\chi = 1 \quad \text{for} \quad \bar{\lambda} \le 1 \tag{4}$$

where

$$\bar{\lambda} = \lambda/\lambda_E\,; \quad \lambda_E = \pi\,(E/f_y)^{1/2} = 76.4091 \tag{5}$$

From

$$\frac{10^4 N/L^2}{10^4 A/L^2} \le \frac{f_y}{\bar{\lambda}^2} = \frac{f_y \lambda_E^2}{\lambda^2} \tag{6}$$

using (4) one obtains

$$\frac{10^4 A}{L^2} = \frac{1}{76.4091}\sqrt{\frac{5027}{355}}\sqrt{\frac{10^4 N}{L^2}} = 0.049247\sqrt{\frac{10^4 N}{L^2}} \tag{7}$$

valid for $\lambda \ge \lambda_E$. For $\lambda \le \lambda_E$ taking $\chi = 1$ in (4) we get

$$\frac{10^4 A}{L^2} \ge \frac{10^4 N}{L^2 f_y} \tag{8}$$

The Eurocode 3 (1992) (EC3) solution

The differential equation for a centrally compressed strut with pinned ends and a sinusoidal initial crookedness (Fig. 2)

$$a = a_0 \sin(\pi z/L) \tag{9}$$

is

$$\frac{\mathrm{d}^2 y}{\mathrm{d}z^2} = -\frac{M}{EI_x} = -\frac{N(a+y)}{EI_x} \tag{10}$$

N is the compressive force. Searching the solution in the form of

$$y = y_0 \sin(\pi z/L) \tag{11}$$

we get

$$y_0 = \frac{a_0}{F_E/N - 1} \tag{12}$$

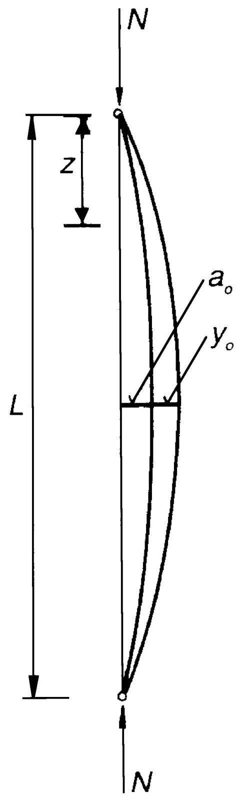

Fig. 2 An initially crooked compression strut and its second-order deformation

The overall buckling formula can be derived on the basis of the check for eccentric compression

$$\frac{N}{A}+\frac{N(a_0+y_0)}{W_x}\leq f_y \tag{13}$$

The European overall buckling curves have been determined for various sections based on statistical evaluation of many test results.

The EC3 formula can be derived from (13) considering one parameter which expresses the effect of initial imperfection and residual (welding or cold-forming) stresses as follows.

With notations of

$$\sigma = N/A\,;\quad \sigma_E = F_E/A\,;\quad \eta_b = a_0 A/W_x$$

Equation (13) can be written in the form

$$(f_y-\sigma)\,(\sigma_E-\sigma)=\eta_b\sigma\sigma_E \tag{14}$$

This equation can be transformed using the following relationships

$$\sigma/f_y=\chi\,;\quad \sigma_E/f_y=\pi^2E/\left(f_y\lambda^2\right)=1/\bar{\lambda}^2 \tag{15}$$

to obtain

$$(1-\chi)\left(\frac{1}{\bar{\lambda}^2}-\chi\right)=\frac{\chi\eta_b}{\bar{\lambda}^2} \tag{16}$$

This leads to the following quadratic equation

$$\chi^2-\left(1+\frac{\eta_b}{\bar{\lambda}^2}+\frac{1}{\bar{\lambda}^2}\right)\chi+\frac{1}{\bar{\lambda}^2}=0 \tag{17}$$

The solution of (17) is

$$\chi=\frac{\phi-\sqrt{\phi^2-\bar{\lambda}^2}}{\bar{\lambda}^2}=\frac{1}{\phi+\sqrt{\phi^2-\bar{\lambda}^2}} \tag{18}$$

where

$$\phi=0.5\left(1+\eta_b+\bar{\lambda}^2\right)\ \text{and}\ \eta_b=\alpha\left(\bar{\lambda}-0.2\right)$$

For $\bar{\lambda}\leq 0.2$ it is $\chi=1$.

$\alpha=0.34$ is the imperfection factor for cold-formed CHS struts.

The strut should be checked for

$$N\leq \chi A f_y/\gamma_{M1} \tag{19}$$

where $\gamma_{M1}=1.1$ is the safety factor for buckling.

Numerical example

$N=1100\,\text{kN}$, $L=6\,\text{m}$, $10^4N/L^2=305.56\,\text{N/mm}^2$,

(a) Euler solution

$$\frac{10^4}{L^2}=0.049247\left(\frac{10^4N}{L^2}\right)^{1/2}=0.8608\,;$$

$$A=0.8606\times 3600=3099\,\text{mm}^2\,;$$

$$D=\sqrt{A\delta/\pi}=222.1\,;\quad r=1.4105(A)^{1/2}=78.53\,\text{mm}\,.$$

For these values the profile of 244.5×5 mm is selected using data of prEN 10219-2 (1996), $A=3760\,\text{mm}^2$, $r=84.7$ mm.

Check of the constraint: $\sigma=293<355$ MPa.

(b) EC3 solution

Using a computer method, we get a profile of 273.0×6 mm, $r=94.4$ mm, $A=5030\,\text{mm}^2$. Check: $219<251$ MPa.

Evaluation: the Euler solution is "not the better" one, it is an unsafe solution, since it does not take into account the fabrication aspect (initial crookedness and residual cold-forming stresses). The EC3 solution is a "better" one, since it considers all the engineering aspects. This solution requires 25% larger cross-sectional area than that of the Euler solution.

Example 2: a tubular truss with parallel chords

The span length of a simply supported CHS truss (Fig. 3) is $L=12a=12\times 1.875=22.5$ m and the original height is $H=1.6$ m, i.e $L/H=14.0625$ (Krampen 2001).

It is advantageous to use in calculations the ratio of $\omega=H/a$, in this case it is $1.6/1.875=0.85333$.

It will be shown that, changing the H/a ratio from 0.85333 to 1.3, i.e. the height from 1.6 to 2.4375 or the ratio of L/H from 14.0625 to 9.23, significant mass and cost savings can be achieved.

Member groups, having the same cross-section, are as follows: 1. all chords, the governing member for tension is GG′, for compression EJK (Fig. 3); 2. bracing members at supports AB and BC, the governing member for compression is BC; 3. all other bracing members, the governing member for compression is DE.

Member forces in function of H/a

Loads: dead load $0.6\,\text{kN/m}^2$, truss spacing $L_a=6.25$ m, safety factor 1.35; service load $0.75\,\text{kN/m}^2$, safety factor 1.50; two concentrated forces at joints J from dead load 150 kN, safety factor 1.35.

Loading vertical forces at upper nodes (symmetric) (Fig. 3):

$F'_A=22.676$ kN; $F_C=F_E=F_K=2F'_A=45.351$ kN; $F_J=202.5$ kN. The support reaction force is $F_A=338.55$ kN.

The uniformly distributed load is $p=1.35\times 0.6\times 6.25+1.5\times 0.75\times 6.25=12.0938$ kN/m.

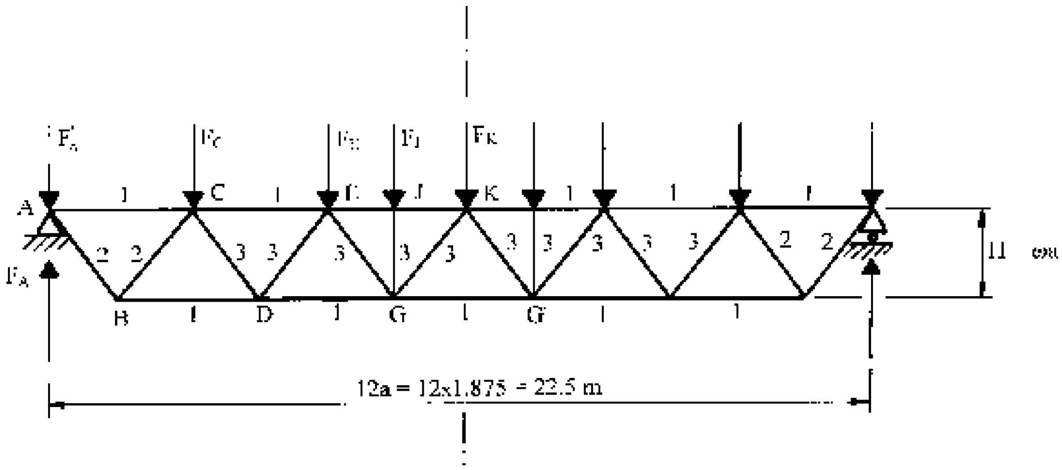

Fig. 3 The investigated simply supported CHS truss with parallel chords

The bending moment on node K is

$$M_K = pL^2/8 + 1.35 \times 150 \times 5 \times 1.875 = 2663.75\,\text{kNm}\,.$$

Forces in the governing members are as follows:

$$N_{GG'} = M_K/H = M_K/(\omega a) \tag{20}$$

(tension)

The bending moment on node G is

$$M_G = (F_A - F'_A)5a - 3aF_C - aF_E = 2621.15\,\text{kNm}$$

$$N_{JK} = M_G/H = M_G/(\omega a) \tag{21}$$

(compression)

$$N_{BC} = (F_A - F'_A)\left(\omega^2 + 1\right)^{1/2}/\omega \tag{22}$$

(compression)

$$N_{DE} = (F_A - F'_A - F_C)\left(\omega^2 + 1\right)^{1/2}/\omega \tag{23}$$

(compression)

Check of profiles for $\omega = 0.85333$ and 1.3

Check of profiles selected for governing members according to prEN 10210-2 (1996) are summarized in Tables 1 and 2.

Check for tension:

$$\frac{N}{A} \le \frac{f_y}{\gamma_{M1}} = \frac{355}{1.1} = 323\,\text{MPa} \tag{24}$$

Check for compression of hot-rolled sections:

$$\frac{N}{A} \le \chi \frac{f_y}{\gamma_{M1}}\,; \quad \chi = \frac{1}{\phi + \left(\phi^2 - \bar{\lambda}^2\right)^{1/2}}\,; \tag{25}$$

where

$$\phi = 0.5\left\lfloor 1 + 0.21\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2 \right\rfloor; \quad \bar{\lambda} = \frac{0.9L_i}{\lambda_E r_i}\,;$$

$$\lambda_E = \pi\left(\frac{E}{f_y}\right)^{1/2} = 76.4\,;$$

r is the radius of gyration.

It can be seen that, for a higher truss, smaller profiles can be used.

Check of joints for eccentricity

K-joints

Prescription of the joint eccentricity according to Wardenier *et al.* (1991) is

$$e \le 0.25 d_1 \tag{26}$$

Table 1 Check of profiles in the case of $\omega = 0.85333$

Member group	Stress	Profile	A mm^2	Member force kN	Check MPa
1	tension	193.7×10	5770	1665	$288 < 323$
1	compression	193.7×10	5770	1638	$284 < 313$
2	compression	139.7×5	2120	487	$230 < 286$
3	compression	114.3×5	1720	417	$242 < 266$

Table 2 Check of profiles in the case of $\omega = 1.3$

Member group	Stress	Profile	A mm^2	Member force kN	Check MPa
1	tension	139.7×10	4070	1093	$266 < 323$
1	compression	139.7×10	4070	1075	$264 < 300$
2	compression	114.3×6	2040	398	$195 < 227$
3	compression	114.3×5	1720	341	$198 < 229$

From

$$\tan\theta = \frac{e + d_1/2}{g_2 + d_2/(2\sin\theta)}; \quad \sin\theta = \frac{\omega}{(\omega^2+1)^{1/2}}; \tag{27}$$

assuming that the half gap is $g_2 = 0.05d_2$, one obtains the following constraint

$$\frac{d_2}{2}\left(\omega^2+1\right)^{1/2} + d_1(0.05\omega - 0.75) \le 0 \tag{28}$$

For $\omega = 0.85333$, $d_1 = 193.7$, $d_2 = 139.7$ mm the constraint is fulfilled, because $-45.19 < 0$.

For $\omega = 1.3$, $d_1 = 139.7$, $d_2 = 114.3$ mm the constraint is also fulfilled, because $-1.96 < 0$.

Joints G and G′ with two braces and a column

Gap joint can be used, when

$$e = \frac{g}{2}\omega + \frac{d_3}{2}\left(\omega^2+1\right)^{1/2} - \frac{d_1}{2} \le 0.25d_1 \tag{29}$$

where $\frac{g}{2} = \frac{d_3}{2} + 2t_3$.

For $\omega = 0.85333$, $d_1 = 193.7$, $d_3 = 114.3$ mm the constraint is fulfilled, because $35.58 < 48.42$ mm.

For $\omega = 1.3$, $d_1 = 139.7$, $d_2 = 114.3$ mm the constraint is not fulfilled, because $111.18 > 34.925$ mm, thus, overlap joints should be used. This fact will be considered in the cost calculation by a factor of 1.2 for cutting and welding times of the two columns.

Check of chord plastification

K-joints

According to Wardenier *et al.* (1991) the criterion for chord plastification is

$$N \le N^* = \frac{f_y t_1^2}{\sin\theta}\left(1.8 + 10.2\frac{d_2}{d_1}\right) f(\gamma, g') f(n'); \tag{30}$$

where

$$\sin\theta = \frac{\omega}{(\omega^2+1)^{1/2}};$$

$$f(\gamma, g') = \gamma^{0.2}\left[1 + \frac{0.024\gamma^{1.2}}{\exp(0.5g' - 1.33) + 1}\right];$$

$$\gamma = \frac{d_1}{2t_1}; \quad g' = \frac{0.1d_1}{t_1};$$

$$f(n') = 1 + 0.3n'(1-n'); \quad n' = -\frac{\sigma}{f_y}$$

(the minus sign means compression); $\sigma = N/A$.

Calculations for joint C are summarized in Table 3.

X-joints

According to Wardenier *et al.* (1991) the criterion for chord plastification is expressed by

$$N \le N^* = \frac{f_y t_1^2}{\sin\theta}\frac{5.2}{1 - 0.81\frac{d_2}{d_1}} f(n') \tag{31}$$

Calculations for joints A and J are summarized in Tables 4 and 5.

It can be seen that the criteria for chord plastification are fulfilled for both truss heights.

Comparison of masses and costs

The volume of the structure in mm^3 is as follows:

$$\frac{V}{10^3} = 41.25A_1 + 7.5A_2\left(\omega^2+1\right)^{1/2} +$$

$$15A_3\left(\omega^2+1\right)^{1/2} + 3.75A_3\omega \tag{32}$$

Calculating with a steel density of $\rho = 7.85 \times 10^{-6}$ kg/mm^3 one obtains the mass for $\omega = 0.85333$ as $m = 2342$ kg and for $\omega = 1.3$ $m = 1913$ kg, thus, the saving in mass is 22%.

We use here an improved cost function, which includes costs of material (profiles) K_M, cutting of strut ends K_C, assembly K_A, welding K_W and painting K_P (Farkas and Jármai 2001a)

Table 3 Checks of chord plastification for joint C

ω	$d_1 \times t_1$ mm	d_2 mm	N_{CE} kN	$f(\gamma, g')$	$f(n')$	Check kN
0.85333	193.7×10	139.7	1057	1.9145	0.7652	$487 < 734$
1.3	139.7×10	114.3	694	1.7132	0.7867	$398 < 796$

Table 4 Check of chord plastification for joint A

ω	$d_1 \times t_1$ mm	d_2 mm	N_{AC} kN	$f(n')$	Check kN
0.85333	197.3×10	139.7	370	0.9360	$487 < 640$
1.3	139.7×10	114.3	243	0.9411	$398 < 844$

Table 5 Check of chord plastification for joint J

ω	d_1 mm	d_3 mm	N_{EJK} kN	$f(n')$	Check kN
0.85333	193.7	114.3	1638	0.5682	$202.5 \sim 200.9$
1.3	139.7	114.3	1075	0.6105	$202.5 < 334.1$

$$K_M = 41.25 k_{M1} + 7.5\left(\omega^2+1\right)^{1/2} k_{M2} + 15\left(\omega^2+1\right)^{1/2} k_{M3} + 3.75\omega k_{M3} \quad (33)$$

According to the Price List (1995) the following profile prices are considered:

for $\omega = 0.85333 \quad k_{M1} = 51.16\,, \quad k_{M2} = 16.88\,,$

$k_{M3} = 14.27\,\$/\mathrm{m}\,, \quad K_M = 2688\,\$\,;$

for $\omega = 1.3 \quad k_{M1} = 36.14\,, \quad k_{M2} = 16.88\,,$

$k_{M3} = 14.27\,\$/\mathrm{m}\,, \quad K_M = 2137\,\$\,.$

$$K_C = k_F \Theta_C \sum_i \frac{2\pi d_i}{\sin\theta}\left(4.54 + 0.4229 t_i^2\right); \quad (34)$$

according to Tizani *et al.* (1996), the cost factor of fabrication is taken as $k_F = 0.6667\,\$/\mathrm{min}$; the difficulty (complexity) factor is $\Theta_C = 3$; d_i in m, t_i in mm.

For $\omega = 0.85333$ for diagonals, $\sin\theta = 0.6491$ and $K_C = 431\,\$$; for $\omega = 1.3$ for diagonals, $\sin\theta = 0.7926$ and we consider a factor of 1.2 for two columns with overlap joints, $K_C = 384\,\$$.

$$K_A = k_F \Theta_A (\kappa\rho V)^{1/2}\,; \quad \Theta_A = 3\,; \quad (35)$$

the number of elements to be assembled is $\kappa = 16$.

For $\omega = 0.85333$, $K_A = 387\,\$$; for $\omega = 1.3$, $K_A = 350\,\$$.

$$K_W = k_F \Theta_W \sum_i C_{Wi} a_{Wi}^n L_{Wi}\,; \quad \Theta_W = 3.5\,; \quad (36)$$

for shielded metal arc welding (SMAW) $C_W = 0.7889 \times 10^{-3}$; $a_{Wi} = t_i$ (mm) is the weld size, $n = 2$; $L_{Wi} = \frac{2\pi d_i}{\sin\theta_i}$ is the weld length in mm.

For $\omega = 0.85333$ $K_W = 722\,\$$, for $\omega = 1.3$, considering also a factor of 1.2 for columns with overlap joint, $K_W = 657\,\$$.

$$K_P = k_P \Theta_P \sum_i \pi d_i L_i\,; \quad (37)$$

according to Tizani *et al.* (1996) $k_P = 14.4\,\$/\mathrm{m}^2$; $\Theta_P = 2$; L_i is the strut length.

For $\omega = 0.85333$, $K_P = 1085\,\$$, for $\omega = 1.3$, $K_P = 953\,\$$.

The costs are summarized in Table 6.

Table 6 Costs in \$ for two different truss heights

ω	K_M	K_C	K_A	K_W	K_P	K
0.85333	2688	431	387	722	1085	5313
1.3	2137	384	350	657	953	4481

It can be seen that, changing the truss height from 1.6 to 2.438 m, an 18.6% cost saving can be achieved.

Evaluation: the truss of height $H = 1.6$ m is “not the better” solution, it takes into account only the safety and fabrication (joints eccentricity) requirements and not the aspect of economy. On the other hand, the truss of height $H = 2.4375$ m is a “better” solution, since it fulfils all the engineering requirements and enables significant savings in mass and cost.

Example 3: a welded stiffened square plate

In this case the shrinkage of eccentric longitudinal fillet welds causes residual deflection. In order to produce a high-quality welded structure, this deflection should be limited. This limitation is formulated as a fabrication constraint. Three structural versions are optimized as follows:

1. minimum mass (without fabrication cost): only the design (stress and buckling) constraints (without distortion constraint) are considered
2. minimum cost (material and fabrication costs): only the design (stress and buckling) constraints (without distortion constraint)
3. minimum cost, with stress, buckling and distortion constraints

The cost function

Here, a relatively simple cost function is used, which includes the material and fabrication costs in the following form (Farkas and Jármai 1997, Jármai and Farkas 1999)

$$K = K_m + K_f = k_m \rho V + k_f \sum_i T_i \quad (38)$$

where k_m and k_f are the material and fabrication cost factors, respectively, ρ is the material density, V is the volume of the structure, and T_i are the fabrication times.

Equation (38) can be written in the form of

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}(T_1 + T_2 + T_3) \quad (39)$$

Time for preparation, assembly and tacking can be expressed as

$$T_1 = C_1 \Theta_d (\kappa \rho V)^{1/2} \tag{40}$$

where $C_1 = 1\,\mathrm{min/kg}^{0.5}$, Θ_d is a difficulty factor expressing the complexity of the structure (planar or spatial, constructed from simple plate elements or profiles), κ is number of structural elements to be assembled.

Welding time is $T_2 = \sum C_{2i} a_{wi}^n L_{wi}$ (41)

where a_W is the weld size, L_W is the weld length. Formulae for $C_2 a_W^n$ are developed using the COSTCOMP (1990) database for different welding technologies and weld types (Bodt 1990).

The additional time for electrode changing, deslagging and chipping can be calculated as

$$T_3 = 0.3 T_2 \tag{42}$$

The final form of the cost function is

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}\left(\Theta_d \sqrt{\kappa \rho V} + 1.3 T_2\right) \tag{43}$$

The following data of cost factors are used: $k_m =$ 0.5–1.2 \$/kg, $k_f = 0$–60 \$/manhour = 0–1 \$/min. To give internationally usable results, values of $k_f/k_m = 0, 1$ and 2 kg/min are considered, the value of 0 means minimum weight design.

This cost function has been applied for several structural optimization problems, such as minimum cost design of cellular plates (Farkas and Jármai 1997), welded steel silos (Farkas and Jármai 1996) and compressed plates (Farkas and Jármai 2000; Jármai and Farkas 1999).

The above-described cost function cannot give generally valid values, but it is suitable for realistic comparisons of structural versions.

Residual welding stresses and distortions

In order to determine the residual stresses and deformations in a beam due to a thermal impulse it is necessary to know the specific strain (shrinkage) of the centroid ε_G and the beam curvature C due to the eccentricity y_G of the thermal impulse A_T. Assuming that the beam is homogenous, the cross-section and the thermal impulse is constant along the beam, and the cross-sections remain planar during the deformation, the results of a thermoelasticity calculations are as follows:

$$\varepsilon_G = A_T t / A \tag{44}$$

and

$$C = A_T t y_G / I_x \tag{45}$$

Knowing ε_G and C, it is possible to calculate the shrinkage of the beam of length L

$$\Delta L = \varepsilon_G L \tag{46}$$

and the deflection (Fig. 4)

$$f = C L^2 / 8 \tag{47}$$

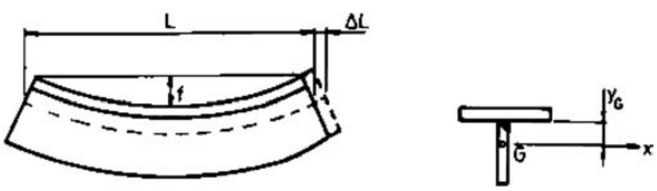

Fig. 4 Residual shrinkage and deflection of a beam due to shrinkage of a longitudinal weld

The thermal impulse due to a one-pass longitudinal weld is calculated as (Okerblom *et al.* 1963; Farkas and Jármai 1997, 1998)

$$A_T t = \frac{0.3355 Q_T \alpha_0}{c\rho} \tag{48}$$

where

$$Q_T = \eta U I_W / v_W \tag{49}$$

is the heat input in J/mm, η is the heat efficiency of the welding technology used, U is the arc voltage (V), I_W is the arc current (A), v_W is the welding speed (mm/s), α_0 is the coefficient of thermal expansion, c is the specific heat. For steels it is $\alpha_0 = 12 \times 10^{-6}$ and $c\rho = 4.77 \times 10^{-3}$, thus

$$A_T t = 0.844 \times 10^{-3} Q_T\,. \tag{50}$$

We need to calculate the welding heat input Q_T as a function of weld size a_w or cross-sectional area of the weld A_w (mm^2). We use for hand welding of fillet welds of size a_w, $Q_T = 78.8\, a_w^2$.

In the case of a double fillet weld connecting two plates of a T-section beam, when the second weld is performed after the cooling of the first one, the factor is 1.3. Thus, in the distortion constraint the formula of

$$Q_T = 1.3 \times 78.8 a_W^2 \tag{51}$$

is used.

An illustrative numerical example

A square plate with simply supported edges, loaded by uniformly distributed normal load is considered (Farkas and Jármai 2001b). The plate is stiffened by flat ribs in two directions (Fig. 5). The ribs are continuous in one direction, in the other direction they are interrupted and welded to the others by double fillet welds. The size of a fillet weld is $a_W = 0.4 t_S$, but $a_{W\,\min} = 4$ mm.

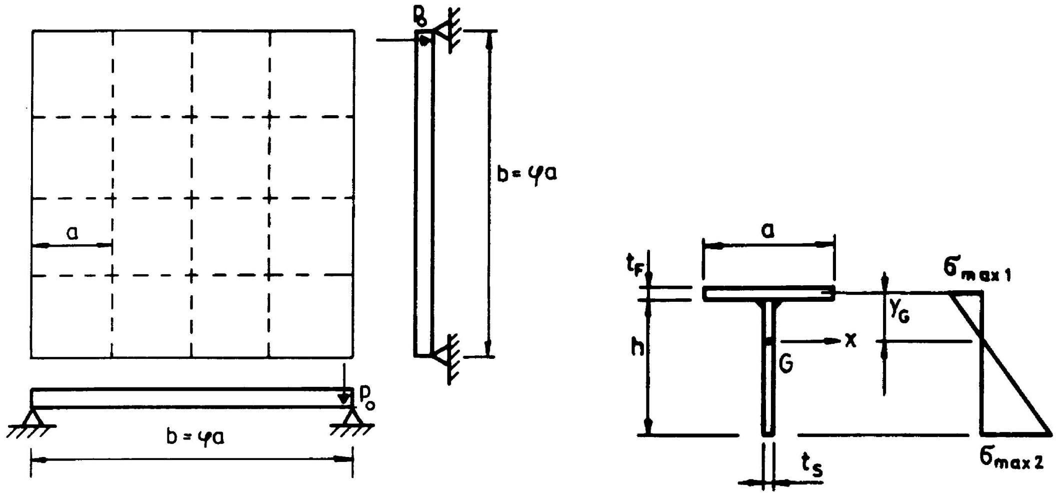

Fig. 5 A simply supported transversely uniformly loaded square plate stiffened by flat ribs

Data: $b = 6\,\mathrm{m}$, $p_0 = 5\times10^{-3}\,\mathrm{N/mm^2}$, steel of yield stress 235 MPa, the admissible stress is $\sigma_{adm} = 120\,\mathrm{MPa}$, the elastic modulus is $E = 2.1\times10^5\,\mathrm{MPa}$, $\rho = 7.85\times10^{-6}\,\mathrm{kg/mm^3}$.

In the optimization procedure we search for the optimum values of the following variables: t_F, h, t_S and φ. The number of stiffeners is $\varphi - 1$.

It is assumed that the base plate is welded with butt welds from 4 strips of dimensions 6×5 m. Than the stiffeners are welded to the base plate by double fillet welds. Finally the interrupted ribs are welded to the other ribs by double fillet welds.

For the butt welds gas metal arc welding with mixed gas (GMAW-M) technology is used, thus, the welding time depends on the thickness of the base plate as follows:

For $t_F \le 15\,\mathrm{mm}$

$$T_2' = 3b\times0.1861\times10^{-3}t_F^2\,, \tag{52}$$

for $t_F > 15\,\mathrm{mm}$

$$T_2' = 3b\times0.1433\times10^{-3}t_F^{1.9035}\,. \tag{53}$$

For longitudinal fillet welds the GMAW-M technology is used, thus, the welding time is

$$T_2'' = 4b(\varphi-1)\times0.3258\times10^{-3}a_W^2 \tag{54}$$

for transverse fillet welds the shielded metal arc welding (SMAW) technology is assumed, thus

$$T_2''' = 4h(\varphi-1)^2\times0.7889\times10^{-3}a_W^2\,. \tag{55}$$

The volume of the structure is

$$V = b^2t_F + 2(\varphi-1)bht_S \tag{56}$$

The number of assembled elements is $\kappa = 3+\varphi^2$. The cost function can be formulated as

$$\frac{K}{k_m} = \rho V + \frac{k_f}{k_m}\left[\Theta_d\,(\kappa\rho V)^{0.5} + 1.3\,(T_2'+T_2''+T_2''')\right] \tag{57}$$

where $\Theta_d = 3$.

The constraint on compressive stress in the central faceplate element is expressed by

$$\sigma_{\max} = \sigma_{\max.1} + \sigma_{f.\max} \le \sigma_{adm} \tag{58}$$

where $\sigma_{\max.1}$ is caused by the bending of the whole plate, $\sigma_{f.\max}$ is the normal stress due to the local bending of the plate elements, σ_{adm} is the admissible stress.

$$\sigma_{\max.1} = \frac{c_M p b^2}{I_x/a}y_G \tag{59}$$

The uniformly distributed normal load p also contains the self weight, approximately $p = 1.1p_0$. Since the torsional stiffness of the open section ribs is very small, the stiffened plate can be calculated as orthotropic having zero torsional stiffness. Schade (1941) has calculated for this case $c_M = 0.1102$.

According to Fig. 5 the distance of the centroidal axis y_G can be calculated as

$$y_G = \frac{h}{2}\frac{1}{1+\alpha}\,,\qquad \alpha = \frac{at_F}{ht_S} = \frac{bt_F}{\varphi ht_S} \tag{60}$$

and the moment of inertia is

$$I_x = \frac{h^3t_S}{12}\frac{1+4\alpha}{1+\alpha} \tag{61}$$

It should be noted that the admissible stress is selected so low that it is not necessary to calculate with an effective width of the face plate, thus, $a = b/\varphi$.

The local bending stress can be calculated by means of formulae valid for isotropic square plates with clamped edges (Timoshenko 1959)

$$\sigma_{f.\,\max} = \frac{5.13 \times 10^{-2} p_0 a^2}{t_F^2/6} = \frac{0.3078 p_0 b^2}{\varphi^2 t_F^2} \tag{62}$$

The constraint on the maximum tensile stress in the central ribs can be written as

$$\sigma_{\max.2} = \sigma_{\max.1}(1+2\alpha) \le \sigma_{adm} \tag{63}$$

Considering the constraint on local buckling of the central face plate element compressed from both sides (Farkas and Jármai 1997; Volmir 1967), for a plate compressed on one side the buckling factor is $k = 4$. Instead of this value we calculate with $k = 2$. For $k = 4$ the Eurocode 3 (1992) gives for the limiting slenderness

$$(a/t_F)_{\lim} = 42\varepsilon\,, \quad \varepsilon = (235/f_y)^{0.5}\,, \tag{64}$$

where f_y is the yield stress, but instead of yield stress we can calculate with the maximum stress. Thus, for $k = 2$ the buckling constraint can be written as

$$a/t_F \le 42\varepsilon_1/\sqrt{2} \approx 30\varepsilon_1\,, \quad \varepsilon_1 = (235/\sigma_{\max.1})^{0.5} \tag{65}$$

The constraint on local buckling of ribs (it is assumed that $\sigma_{\max.2}$ can also be compressive)

$$h/t_S \le 14\varepsilon_2\,, \quad \varepsilon_2 = (235/\sigma_{\max.2})^{0.5} \tag{66}$$

The constraint on shear buckling of ribs at the plate edges can be formulated as

for $\dfrac{\tau_{ub}}{\gamma_b} \le \tau_{adm}$

$$\tau = \frac{0.42pb^2}{ht_S\varphi} \le \frac{\tau_{ub}}{\gamma_b} = \frac{5.34\pi^2 E}{12(1-\nu^2)\gamma_b}\left(\frac{t_S}{h}\right)^2 \tag{67a}$$

and for $\dfrac{\tau_{ub}}{\gamma_b} \ge \tau_{adm}$

$$\tau \le \tau_{adm} \tag{67b}$$

The factor of 0.42 is considered, since the distribution of edge reactions is not uniform along the edges (Timoshenko and Woinowsky-Krieger 1959). $\tau_{adm} = \sigma_{adm}/\sqrt{3}$ is the admissible shear stress, ν is the Poisson's ratio.

Constraint on residual distortion due to shrinkage of welds is formulated as follows.

Although the stiffeners are welded along two directions, we do not multiply the residual deflection by 2. We use a multiplying factor of 1.5 considering the fact that the longitudinal double fillet welds are intermittent due to interruption of the ribs and that the residual plastic zones of the continuous welds affect the deflection caused by intermittent welds. Thus, the distortion constraint is formulated as

$$f = 1.5Cb^2/8 \le f_{adm} \tag{68}$$

where the admissible deflection is assumed to be $f_{adm} = b/1000 = 6$ mm.

The orthotropic plate bending theory is valid only in the case when the number of stiffeners is more than 3, thus

$$\varphi \ge 4 \tag{69}$$

It can be seen that the objective function and the design constraints are highly nonlinear. For the constrained function minimization the Rosenbrock's hill-climb mathematical programming method is used (detailed description can be found in Farkas and Jármai (1997)) complemented by an additional discretization of the continuous optima considering rounded dimensions and integer numbers of stiffeners.

The optimum discrete values are given in Tables 7 and 8 in the function of φ to show the values of the cost function in the vicinity of the optima.

It can be seen that the cost difference between the investigated best and worst solutions in the case of $k_f/k_m = 2$ is $100(13975 - 9317)/9317 = 50\%$, thus it is worth using the optimization procedure.

Evaluation: three solutions are compared to each other as follows:

Table 7 Results for the case without distortion constraint. Dimensions in mm. The optima are marked by bold letters

k_f/k_m (kg/min)	t_F	h	t_S	φ	K/k_m (kg)
	7	200	11	9	3636
	7	190	10	10	3589
0	**6**	**190**	**10**	**11**	**3485**
	6	190	10	12	3664
	6	180	10	13	3730
1	**11**	**250**	**14**	**4**	**6707**
	10	220	15	5	7501
	9	210	14	6	7724
	9	200	13	7	8184
2	**11**	**250**	**14**	**4**	**9317**
	10	210	16	5	11 454
	9	210	14	6	11 521
	15	220	11	7	13 975

Table 8 Results for the case with distortion constraint. Dimensions in mm. The optima are marked by bold letters

k_f/k_m (kg/min)	t_F	h	t_S	φ	K/k_m (kg)
0	**11**	**290**	**14**	**4**	**4256**
	12	260	13	5	4665
	14	230	12	6	5256
1	**12**	**250**	**14**	**4**	**7119**
	12	260	13	5	7932
	14	230	12	6	9183
2	**12**	**250**	**14**	**4**	**9857**
	13	250	13	5	11 678
	14	230	12	6	13 109

1. minimum mass, without distortion constraint (Table 7): $t_F = 6$, $h = 190$, $t_S = 10\,\mathrm{mm}$, $\varphi = 11$, the distortion constraint is not fulfilled, since $f = 19.67 > 6.00\,\mathrm{mm}$; only the design requirement is considered, the solution is not "better";
2. minimum cost, without distortion constraint, for $k_f/k_m = 1$ (Table 7): $t_F = 11$, $h = 250$, $t_S = 14\,\mathrm{mm}$, $\varphi = 4$, the distortion constraint is not fulfilled, since $f = 6.32 > 6.00\,\mathrm{mm}$; the design and economy requirements are considered, the fabrication one is not fulfilled, the solution is not "better";
3. minimum cost, with distortion constraint, for $k_f/k_m = 1$ (Table 8): $t_F = 12$, $h = 250$, $t_S = 14\,\mathrm{mm}$, $\varphi = 4$, the distortion constraint is fulfilled, since $f = 5.82 < 6.00\,\mathrm{mm}$; this solution is "better", all the three engineering aspects are taken into account.

Conclusions

Load-carrying capacity, producibility and economy are the main requirements of modern engineering structures. These requirements can be considered in a structural optimization system, which minimizes a cost function with design and fabrication constraints.
Different structural versions can be evaluated considering these three main aspects. This structural evaluation is shown in three examples.
A compression strut of circular hollow section optimized using the Euler buckling constraint is unsafe, since this constraint does not take into account the fabrication aspects i.e. the effect of initial crookedness and residual stresses. The optimum solution using the Eurocode 3 buckling constraint is safe, but needs 25% larger cross-sectional area.
A tubular truss of parallel chords should have an optimum height (distance between chords) that minimizes the structural cost. The original height was not economical, since 18% cost savings can be achieved by changing it.
In the case of a welded stiffened square plate a fabrication constraint should be fulfilled, which expresses the limitation of residual deflection caused by the shrinkage of eccentric longitudinal welds. Neglecting this constraint the optimization cannot result in a high-quality welded structure.

Acknowledgements The research work was supported by the Hungarian Scientific Research Fund grants OTKA 29326 and Fund for the Development of Higher Education FKFP 8/2000 project.

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