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Stochastic optimization of acoustic response – a numerical and experimental comparison

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Abstract The objective of the work presented is to compare results from numerical optimization with experimental data and to highlight and discuss the differences between two fundamentally different optimization methods. The problem domain is minimization of acoustic emission and the structure used in the work is a closed cylinder with forced vibration of one end. The optimization method used in this paper is simulated annealing (SA), a stochastic method. The results are compared with those from a gradient-based method used on the same structure in an earlier paper (see Tinnsten 2000).

Key words acoustic optimization, stochastic optimization, optimization, BEM, acoustics

1 Introduction

It is possible to change the characteristics of the sound emanating from a vibrating structure by changing structural design variables such as geometric dimensions, shell thickness, material parameters, and, for fiber reinforced material, fiber orientation. Of course, changes to one or more of these variables will result in changes to other structural characteristics. If we consider a mechanical structure, two very important quantities to have control over are stiffness and strength, but there are also other quantities that it is important to know and to be able to control, for example sound emission, productivity, functionality, environmental effects and so on. To find the best design, i.e. the one that satisfies all the demands is a question of optimization. This often requires a multidisciplinary approach i.e. analytical tools from different disciplines must be used in concert.

A typical problem formulation could be to minimize the structural weight whilst the sound intensity in certain domains and the maximum stress in the structure do not exceed some given value. A thorough discussion of the formulation of optimization problems involving acoustic response is given by Christensen *et al.* (1998a,b). The search for the optimum solution can be performed in many different ways. A common approach is to use mathematical programming techniques. These techniques were used for an earlier analysis of the structure under investigation in the present work (see Tinnsten 2000). Here, a gradient-based method called MMA (see Svanberg 1987, 1993) was used to achieve optimization.

Another method, which is conceptually quite different from the mathematical programming techniques, is to optimize using some form of natural selection process. One such technique is the simulated annealing, SA, a stochastic method based on the simulation of metal (or solids) annealing (see Kirkpatrick *et al.* 1983; Corana *et al.* 1987).

Annealing is the physical process of heating up a solid and then cooling it down slowly. The slow and controlled cooling of the solid ensures proper solidification with a highly ordered crystalline state. At high temperature the atoms in the heated material have high energies and more freedom to arrange themselves. Annealing results in a material with an atom arrangement that corresponds to the lowest internal energy.

There are many other optimization methods, such as genetic algorithms, neural networks and so on, which are based on natural selection of solutions to achieve an optimum. In this paper, simulated annealing is used as the optimization algorithm and results from this compared with experimental results and with results from the gradient-

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based (first-order in this case) mathematical programming technique MMA that has been used with great success for a variety of problems (see Esping 1995; Tinnsten 2000; Carlsson 2000).

In order to calculate acoustic-related quantities, such as sound intensity in an open or closed domain, a code based on the boundary element method (BEM) has been developed (see Tinnsten 1994; Tinnsten *et al.* 1998). The BEM code is linked together with a modified version of the finite element code FEMP (see Nilsson and Oldenburg 1983) and the simulated annealing optimization algorithm (see Goffe *et al.* 1994) to achieve an acoustic optimization program, which incorporates structural design changes in an automatic fashion.

There are many optional ways with which to change the sound field emanating from a vibrating structure. Discrete masses, with respect to weight and location, can equally well be used as optimization variables (Constants *et al.* 1998; St. Pierre and Koopmann 1995; Christensen *et al.* 1998a,b). In shell structures the thickness can serve as variable (Belegundu *et al.* 1994; Lamancusa and Eschenauer 1994; Tinnsten 2000). Optimization involving fiber-reinforced materials offers several additional variables. For instance, the fiber direction can be fixed and the volume fraction between fiber and matrix varied (Lamancusa and Eschenauer 1994) or the volume fraction can be constant and the fiber direction chosen as a variable (Tinnsten *et al.* 1998). In this paper, the thickness variation in one boundary surface of a shell structure is used as the variable.

2 Problem definition

In order to, as far as possible, eliminate problems associated with geometrical complexity, a simple geometry is chosen for the comparison between numerical and experimental results. The optimization analysis was performed on a structure in the form of a cylinder with top and bottom plates (see Fig. 1). This is the same structure and has the same geometrical dimensions as that used in an earlier investigation (see Tinnsten 2000).

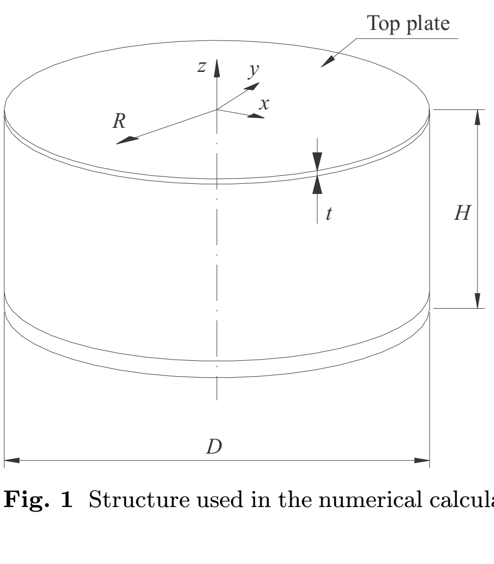

Fig. 1 Structure used in the numerical calculations

For the numerical calculations, diameter $D = 200\,\text{mm}$ and height $H = 100\,\text{mm}$, were used. The initial thickness t was the same over the entire surface and taken as 3 mm. The structure consists of two materials; the cylindrical wall was of steel, and the top and bottom plate were of aluminum with Young's modulus $E = 70\,\text{GPa}$, Poisson's ratio $\nu = 0.3$, and density $\rho = 2750\,\text{kg/m}^3$. The cylindrical wall and the bottom plate are much stiffer than the top plate and are therefore modelled as rigid in the numerical analysis. The top plate is excited with a harmonic force applied perpendicular to the surface at its centre. The optimization problem is formulated as:

$$\begin{aligned} &\text{minimize} && I(x) \\ &\text{such that} && w(x) - \overline{w} \le 0\,, \\ & && \underline{x}_j \le x_j \le \overline{x}_j\,; \quad j = 1, J\,. \end{aligned} \tag{1}$$

That is, minimize the sound intensity $I(x)$ perpendicular to the top surface in such a way that the structural weight $w(x)$ does not exceed the upper limit $\overline{w}$, where x_j are the design variables with lower limit $\underline{x}_j$ and upper limit $\overline{x}_j$.

Simulated annealing is a nongradient (zeroth-order) stochastic optimization technique based on random evaluation of the objective function in such a way that transitions away from a local minimum are possible. Although the method usually requires a large number of function evaluations to find the optimum design, it will find the global optimum with a high probability even for problems with numerous local minima (see Corana *et al.* 1987). Starting from an initial point, the algorithm takes a step, for each variable in turn, and then evaluates the function. When minimizing a function, any downhill step may be accepted and the process repeats from this new point. Uphill moves may be accepted; the decision whether to do this being made by the Metropolis criteria (see Metropolis *et al.* 1953). The criteria uses T (the temperature according to the analogy with annealing of metal) and the size of the uphill move in a probabilistic manner. The larger the value of T and the smaller the increase of the objective function is, the more likely it is that move will be accepted. As the optimization process proceeds, the length of the step and the "temperature" decline and the algorithm closes in on the global optimum. The simulated annealing method solves unconstrained problems, that is problems with no behavior constraints (side constraints are of course allowed). When behaviour constraints are present an equivalent unconstrained objective function must be formulated. This can be achieved by using the concept of penalty functions and the transformed problem formulation can be stated as follows:

$$\text{minimize} \quad I(x) + P \cdot \Psi[w(x) - \overline{w})] \,,$$

$$\underline{x} \le x_j \le \overline{x}_j \,; \quad j = 1, J \,, \tag{2}$$

where P is the penalty parameter and Ψ the penalty function defined as:

$$\Psi(Z) = \langle Z \rangle^2 \,, \tag{3}$$

$$\langle Z \rangle = \begin{cases} Z & \text{if} \quad Z > 0 \\ 0 & \text{if} \quad Z \le 0 \end{cases} \,. \tag{4}$$

The parameter T is very important if SA is to be used successfully since it influence the step length over which the algorithm searches for optima. A small initial T can give a step length that may be to small and thus not enough of the function is evaluated. The parameter P (penalty parameter) in the transformed problem formulation is also important since a small value of P might cause violation of the constraint. The optimization process was carried out using the points discussed above for the case where the top plate edge was clamped. The objective function, the sound intensity perpendicular to the top plate, was computed at a single point above the top plate located at (7, 7, 100 mm) in the coordinate system given in Fig. 1. The structure was discretized in a symmetric manner with constant triangular elements as shown in Fig. 2.

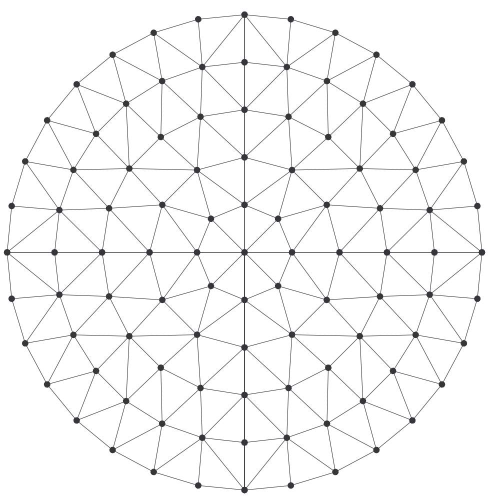

Fig. 2 Discretization of the top plate

As can be seen in Fig. 2, the nodes are located at 6 different radii on the top plate; $R = 0$, 20, 40, 60, 80, and 100 mm. For manufacturing purposes the plate thickness must be constant at any given radius, but can vary over different radii. The design variables are thus the thicknesses at the different radii, i.e. variable one is the thickness at the centre, variable two at radius $R = 20$ mm, variable three at radius $R = 40$ mm, and so on. This means that the optimization problem has six variables ($J = 6$).

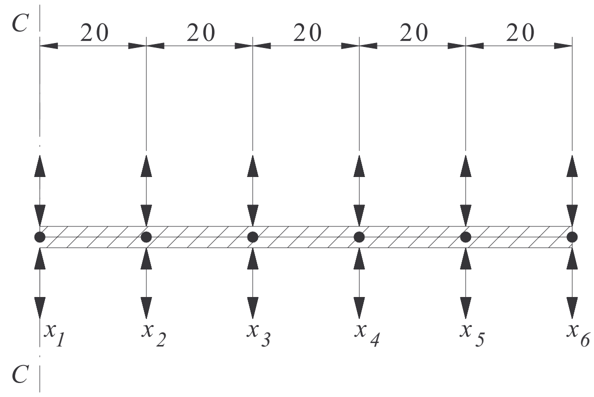

Fig. 3 Axisymmetric figure of top plate with design variables. Dimensions in [mm]

Figure 3 shows an axisymmetric picture of the top plate together with the design variable $x_1 - x_6$.

3
Experimental setup

In order to allow comparison with the numerical calculations, experiments on the structure were performed. The experimental setup is sketched in Fig. 4.

The experimental setup consisted of: an impedance head (1) B&K type 8001 measuring force and acceleration; a vibrator (2) model 200 (Ling Dynamic System); a steel cylinder (3) with outer diameter 204 mm, height 350 mm, and thickness of material 9.5 mm; an aluminum top plate (4), with initial thickness of 3 mm, and optimum thickness according to Table 1 and Fig. 5; foam rubber (5) with a thickness of 70 mm; a soft insulating mat (6), with

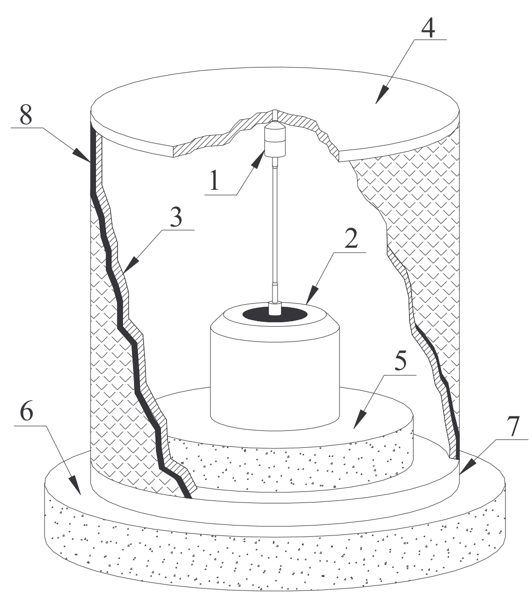

Fig. 4 Experimental setup. 1: Impedance head. 2: Vibrator. 3: Steel cylinder. 4: Aluminum top plate. 5: Foam rubber. 6: Soft insulating mat. 7: Thick aluminum bottom plate. 8: Heavy coach work mat

a thickness of approximately 50 mm; an aluminum bottom plate (7), with thickness 10 mm; and a heavy coach work mat (8), with thickness 4 mm, glued to the steel cylinder. The remaining empty space in the steel cylinder was filled with a lightweight, woolly damping material. The impedance head was joined to the center of the top plate mechanically (screwed) and calibrated using a B&K 4291. The force and acceleration signals were amplified with B&K 2635 before entering the dual channel FFT analyzer B&K 2032. The intensity was measured perpendicular to the top plate at a specified point. The intensity probe used was a B&K 3519, calibrated using a B&K 3541. The signal from the intensity probe was amplified with a B&K 2804 before entering the 16-channel measurement system (LMS CADA-X with a HP Paragon front end).

Table 1 Values of objective function, variables and weight

Variables on objective function, variables and weight		
variables	initial	optimal
x_1 [mm]	3.0	9.3
x_2 [mm]	3.0	10.0
x_3 [mm]	3.0	2.1
x_4 [mm]	3.0	2.0
x_5 [mm]	3.0	2.0
x_6 [mm]	3.0	2.0
weight, w [kg]	0.259	0.245
weight reduction Δw [%]		5.5
objective function intensity, I [dB]	109.7	91.0
intensity reduction ΔI [dB]		18.7

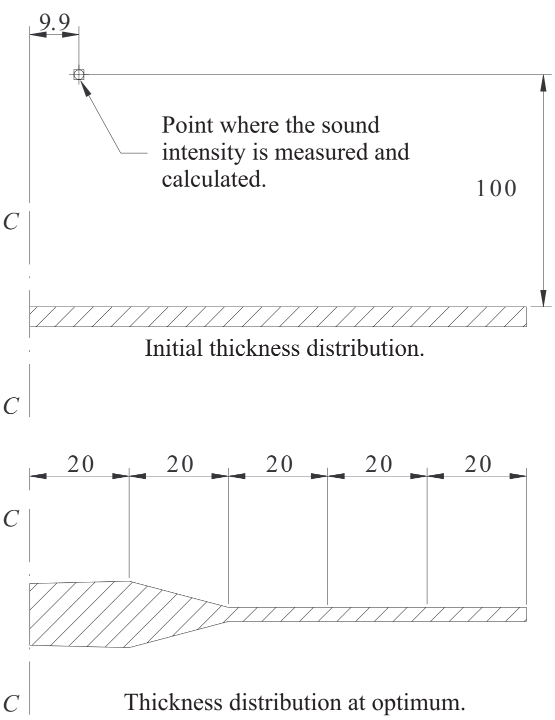

Fig. 5 Axisymmetric figure of top plate showing the thickness profile following the optimization process. All dimensions in [mm]

In the numerical analysis the edge of the top plate was assumed to be clamped and to simulate this in the experimental setup, the edge was glued with epoxy cement to the end face of the top of the steel cylinder. The intensity measurements were performed with two boundary conditions: one with initial thickness (3 mm) and one with optimal thickness of the top plate, according to the numerical result given in Table 1 achieved by simulated annealing.

4 Results

4.1 Numerical results

For the purposes of optimization the cylindrical wall and the bottom plate were modelled as rigid, i.e. the only moving part in the model was the top plate. Due to the discretization, illustrated in Fig. 2, the model of the top plate is not axisymmetric. The normal velocity for the BEM element at the top plate was determined from the response analysis in the FEM code, for all other elements in the model the normal velocity was given as zero. The excitation force was harmonic, perpendicular to the top plate with an amplitude of 2.0 N, applied at the centre of the surface. In the response analysis, the edge of the top plate was modelled as clamped. Proportional damping was included in the analysis as $[C] = \alpha \times [K]$ and the damping factor α taken as 1.0×10^{-5}. The frequency of the exciting force was 700 Hz, and the intensity calculated at the point (7, 7, 100 mm). A weight increase of 10% was allowed. The initial thickness t of the top plate had the constant value of 3 mm (constant distribution) and the lower and upper limit for the plate thickness was 2 mm and 10 mm, respectively. All of the above are the same as the conditions given in work reported earlier (see Tinnsten 2000).

Following optimization, the sound intensity decreased from an initial value of 109.7 dB to 91.0 dB ($\Delta I =$ 18.7 dB). The weight decreased from initially 0.259 kg to 0.245 kg. The thickness distribution (being shown in Fig. 5), the weight decrease and the value of the objective function for the initial state and at the optimum reached are given in Table 1.

4.2 Experimental results

The excitation force was harmonic with an amplitude of 2.0 N and a frequency of 700 Hz. The intensity was measured perpendicular to the axisymmetric top plate, 100 mm above it ($z = 100$ mm in Fig. 1, and offset some 9.9 mm from the centre axis, according to Figs. 1 and 5. The intensity was measured at three points, with approximately 120 degrees separation, at a radius of 9.9 mm; the mean value of these measurements was then used for comparison with the results from the numerical analysis. The results from the measurements are presented in Table 2.

Table 2 Measured intensity at radius, $r = 9.9$ mm, according to Figs. 1 and 5

Measured instensity				
		intensity (experimental) [dB]		
point	R [mm]	initial thickness	optimal thickness	
1	9.9	111.2	87.2	
2	9.9	111.3	87.3	
3	9.9	111.3	87.2	
I_{mean}		111.3	87.2	
ΔI_{mean}				24.1

5
Discussion and conclusions

The optimization problem was to minimize the sound intensity at a specified point such that the weight of the top plate did not increase by more than 10% from its initial value. A comparison of numerical and experimental results was performed.

The comparison of measured and numerical results is presented in Table 3.

Table 3 Comparison between numerical and experimental results. [1)]Intensity calculated by numerical analysis. [2)]Measured intensity in the experiments. [3)]Difference between numerical and measured intensity

Intensity [dB]	Initial	Optimal
$^{1}I_{\mathrm{num}}$	109.7	91.0
$^{2}I_{\mathrm{exp}}$	111.3	87.2
$^{3}I_{\mathrm{num}} - I_{\mathrm{exp}}$	−1.6	3.8

Proportional damping was included in the analysis as $[C] = \alpha \times [K]$. The top plate was assumed to be lightly damped and the damping factor α taken as 1.0×10^{-5}. The damping factor had the greatest influence on the plate at it's initial thickness. Increasing the damping factor from 1.0×10^{-5} to 2.0×10^{-5} gave a decrease in intensity of 2.2 dB for the initial geometry and of 0.2 dB for optimal geometry. In the numerical analysis, the edge of the top plate was clamped. To simulate this in the experiment the edge of the top plate was glued to the end face on the top of the steel cylinder with epoxy cement. This does not, however, give an absolutely clamped boundary condition. Nevertheless, since the distribution of the epoxy cement was distributed over approximately 2–3 mm in the radial direction the boundary condition is assumed to simulate a clamped condition quite well.

In the numerical analysis the only moving part of the model is the top plate. During the experiments the sound intensity parallel and perpendicular to the steel cylinder were measured. The levels were insignificantly low and are therefore not considered to contribute to the sound intensity measured above the top plate. Another difference between the model used for the numerical analysis and the actual structure is the hole at the center of the top plate in the actual structure, which was used to screw the impedance head to the top plate. This hole was not present in the numerical analysis. With a diameter of 5 mm and the estimated decrease of plate stiffness approximately 2.5% the hole was assumed to have insignificant influence on the results. There is also a slight difference in the geometry of the top plate between the numerical analysis and the experimental study. While the top plate is not axisymmetric in the numerical analysis due to the discretization into constant elements it is in the experimental study. This difference is dependent up on manufacturing demands and may influence the response.

It is interesting to compare the numerical results from this investigation with the numerical results from the earlier paper (see Tinnsten 2000), where a gradient-based optimization method (MMA) was used. In Table 4 the numerical values obtained with the two optimization methods are presented. The initial thickness (starting values for the variables) was in both cases 3 mm. The thickness distribution at optimum for the two methods is also shown in Fig. 6.

Table 4 Values on objective function, weight and variables at optimum for the two methods, MMA and simulated annealing (SA)

Variables on objective function and variables at optimum		
variables	MMA	SA
x_1 [mm]	2.0	9.3
x_2 [mm]	10.0	10.0
x_3 [mm]	5.3	2.1
x_4 [mm]	2.0	2.0
x_5 [mm]	2.0	2.0
x_6 [mm]	2.0	2.0
weight, w [kg]	0.285	0.245
objective function intensity, I [dB]	95.0	91.0

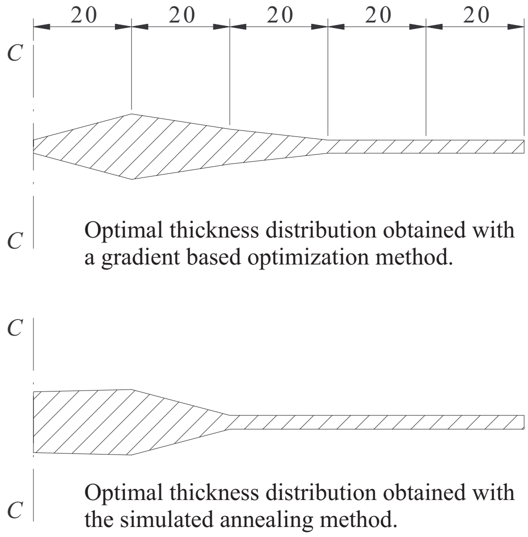

Fig. 6 Optimum thickness distribution for (a) MMA and (b) simulated annealing (SA)

The comparison shows that simulated annealing reached an optimum where the intensity (the objective function) was 4.0 dB lower and the weight was approximately 14% lower than that achieved with MMA. A possible explanation for this difference is that MMA got trapped at a local minimum. Gradient methods are, in general, more efficient than the nongradient (zeroth-order) methods which only use the function values to obtain an optimum. For problems with local minima, gradient methods can converge at one of these and it is necessary to check the solution by selecting different starting values for the design variables and then comparing the solutions. Starting MMA with the initial variable values; $x_1 = 9.3$, $x_2 = 10.0$, $x_3 = 2.1$, and $x_4 = x_5 = x_6 = 2.0$ mm (optimal variable values achieved with SA) it converged to an even better solution. The objective function remained at 91.0 dB and the weight was further reduced to 0.241 kg. The computational time for the analysis using simulated annealing was approximately twice that required for MMA. The small difference in the experimental results, 112.4 dB in the earlier paper (see Tinnsten 2000) and 111.3 dB in the present paper, obtained for the two investigations shows that the experiment was quite reproducible.

6 Future considerations

Optimization routines that use the function values (objective and constraint) and their derivatives in order to obtain an optimum design are, in general, more efficient than the nongradient (zeroth-order) methods, such as simulated annealing which use only function values to obtain an optimum. In problems with local minima, however, the gradient methods can converge at one of these and it is necessary to check the solution by selecting different starting values for the design variables and then comparing the solutions. Future work will involve further testing of optimization techniques that are able to escape from local minimum points. Another tempting idea is to mix gradient and nongradient methods in an "intelligent" manner in order to benefit the strengths of the different algorithms in the same optimization process. This could be done using distributed computers/processors and will be examined in future work.

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