J Popul Econ (2000) 13: 173–188

Journal of
Population
Economics

A modified hurdle model for completed fertility

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Received: 6 July 1998/Accepted: 14 January 1999

Abstract. In this paper it is argued that models for completed fertility have to take into consideration that childless couples and couples with an only child are qualitatively different from couples with two or more children. Indeed, these differences may be the cause of the underdispersion that characterizes completed fertility data. An empirical illustration using Portuguese data suggests that accounting for the qualitative difference between having zero, one, or more children leads to considerable improvements over a model of the type generally used to describe this sort of data.

JEL classification: C25, J13

Key words: Count data, generalized poisson, hurdle models, underdispersion

1. Introduction

This paper is concerned with the econometric analysis of completed fertility data. Specifically, the variate of interest is the number of births to a woman, past her childbearing age, who is in her first marriage or de facto marriage. In contradistinction to what happens with most types of microeconomic data, completed fertility data are generally characterized by the presence of under-

All correspondence to João Santos Silva. This paper was prepared while João Santos Silva had a consultancy position at Instituto Nacional de Estatística, whose support is gratefully acknowledged; we are specially grateful to M.J. Carrilho for making this work possible. We are indebted to J.M. Andrade e Silva and M. Dumangane for helpful discussions and suggestions. We also thank E. Arranhado, J. Peixoto, P.T. Pereira, J. Ramalho and two anonymous referees for helpful comments on a previous version of this paper. The usual disclaimer applies. *Responsible editor:* Rainer Winkelmann.

dispersion, rather than overdispersion (see for example, Winkelmann and Zimmermann 1994; and Wang and Famoye 1997). Therefore, the generalizations of the Poisson regression which have dominated the econometrics literature in the last decade or so are not useful to model this kind of data (Hausman et al. 1984; Cameron and Trivedi 1986).

The fact that fertility data raise uncommon modelling problems led to the use and development of more flexible statistical methods based on different generalizations of the Poisson distribution, allowing both for over and underdispersion. Examples of this are the generalized Poisson model of Consul and Jain (1973), the generalized event count model of King (1989), its extension proposed by Winkelmann and Zimmermann (1991), and more recently the gamma count regression model proposed by Winkelmann (1995). These models have been used in the analysis of fertility data by Winkelmann and Zimmermann (1992, 1994), Winkelmann (1995) and Wang and Famoye (1997).

Besides the typical underdispersion, completed fertility data have other characteristics that make it quite special. The total number of children a couple has is the result of a sequential process in which parents decide, conditionally on the current number of children, whether or not to have a new child (see Barmby and Cigno 1990). This point is important because the reasons that lead a couple to have their first child are likely to be different from the reasons that may lead them to have further children (see Schoen et al. 1997). Moreover, couples with one child may desire to have a second, just to avoid having an only child (see Falbo 1992).

This qualitative difference between having zero, one, or more children has serious implications for the way completed fertility data should be modelled. In fact, besides causing the underdispersion that characterizes completed fertility data, these qualitative differences will also affect the functional form of the conditional expectation. Therefore, if this is the case, underdispersion is just a symptom of a much more serious problem that cannot be solved by replacing the Poisson regression by a model with a flexible mean to variance ratio.

The remainder of the paper is organized as follows. In the next section, the implications for econometric modelling of the existence of qualitative differences between having zero, one, or more children are analysed. In Sect. 3, a model to account for these features of the data is suggested. The usefulness of the proposed model is illustrated in Sect. 4 using Portuguese data. Finally, Sect. 5 concludes the paper.

2. The econometrics of childlessness and only children

Modern count data models for fertility data are based on generalizations of the Poisson distribution that allow the presence both of under and overdispersion (see Winkelmann and Zimmermann 1992, 1994; Winkelmann 1995; and Wang and Famoye 1997). Although these models capture the distinctive underdispersion that characterizes fertility data, they fail to account for other potentially important features of the data. Specifically, these studies neglect the fact that there are strong reasons to believe that the zero and positive counts are generated by different mechanisms.

It is obvious that some couples do not have more children, not by choice, but as a result of sterility problems. These problems are likely to be the main reason for a couple to remain childless, having little or no impact on the number of children other couples have. Moreover, the neoclassical theory of fertility emphasizes that an important determinant of the number of children a couple has is the trade-off between their quantity and their quality (see Becker and Lewis 1973). However, this argument cannot be used to justify the reduction of the number of children to zero. On the other hand, Schoen et al. (1997) point out that in western societies the main motivation for a couple to have children at all may have more to do with social reasons than with strictly economic factors.[1] Therefore, standard economic theory may provide a good guidance for the number of children parents have, while being of little help to explain why couples have children at all. This suggests that in modelling completed fertility data, some kind of hurdle model (Mullahy 1986) has to be used to take into consideration that the zeros and positive counts are generated by different mechanisms.

An interesting characteristic of hurdle models is that they allow the probability of observing a zero to be independent of the mean number of counts. Therefore, in these models, the mean number of children can vary, while the probability of a couple having no children remains constant. This is interesting because there is evidence that in developed countries the average number of children per couple has been declining sharply but the percentage of couples that are childless by the end of their childbearing years has remained relatively stable. Specifically, the data presented by Muñoz-Perez (1987) show that, in Portugal, the reduction of the average number of children per couple was accompanied by a reduction of the frequency of childless couples.

Besides the split between zero and positive counts, it is also important to note that the positive observations may not be generated by an homogeneous process. In particular, it is well known that many couples avoid having an only child, either as the result of social prejudice, or simply because the education of an only child is considered to be more demanding in terms of time and effort (see Blake 1981; Falbo and Polit 1986; and Falbo 1992). Bernheim, Shleifer and Summers (1985) give further economic reasons for a couple to want more than a single child. This suggests that the branch of the hurdle model describing the positive counts has to take into account the special nature of families with a single child.

Although many couples may avoid having an only child, it is clear that the process generating the ones is not entirely different from the process generating the other positive observations. In particular, a couple with a child is unlikely to suffer from sterility problems and may have decided to have a single child in order to concentrate the available resources on its upbringing. Therefore, it would not be adequate to model the special nature of the observations equal to one using a second hurdle model for the positive observations. What is needed is some way to incorporate into the model a measure of how much couples dislike having only children.

The existence of different processes generating the zeros, and the prejudice against only children, are likely to cause the underdispersion that is typical of completed fertility data. For example, assume that for social reasons every couple wants to have at least two children, while for economic reasons they want to minimize the number of children. In this scenario, ignoring sterility problems and unplanned children, every couple would end up having two children and obviously the data would be underdispersed.

However, if this is the cause of underdispersion, just going from the standard Poisson distribution to a distribution with a flexible mean to variance ratio does not solve the problem. In fact, the qualitative differences between having zero, one, or more children will affect not only the conditional variance, but also the functional form of the conditional mean. Generally speaking, neglecting the special nature of the zeros and ones in this sort of data is likely to produce serious misspecification and lead to results that are of little use. In particular, the parameters that are estimated are a mixture of the parameters of the different processes and have no clear interpretation. Moreover, the first moment of the conditional distribution is misspecified and no pseudo likelihood result can be invoked to justify the use of these models (Gourieroux et al. 1984). Even as a descriptive tool, these models are likely to be inadequate, as it is illustrated in Winkelmann (1995, page 472, Fig. 5).

3. A probabilistic model

Hurdle models of the kind described by Mullahy (1986) are now standard, and have been used with great success in several areas (see Pohlmeier and Ulrich 1995; and Gurmu and Trivedi 1996). The main characteristic of a hurdle model is that it combines a binary model for the choice between zero and positive counts with a count data model for the positive integers.

Let Y be the number of children a couple ever had, and denote by $P(y|x)$ the probability of observing a couple with $Y = y$ children, conditional on a set of covariates x. In a hurdle model, the conditional probability mass function can be expressed as

$$P(y|x) = \begin{cases} P(0|x) & \text{for } y = 0 \\ [1 - P(0|x)]P(y|x, y > 0) & \text{for } y = 1, 2, \ldots \end{cases} \tag{1}$$

where $P(0|x)$ is the probability of childlessness and $P(y|x, y > 0)$ is the probability of observing $Y = y$, given x and $y > 0$. The specific way in which $P(0|x)$ and $P(y|x, y > 0)$ depend on the regressors is an empirical matter and need not be specified at this point.

Because $P(y|x, y > 0)$ is defined only over the positive integers, it is usually specified as a truncated distribution (see Mullahy 1986; Pohlmeier and Ulrich 1995; and Gurmu and Trivedi 1996). However that is not strictly necessary. In fact, if the reason to use a hurdle model is the belief that the zero and positive counts are generated by different mechanisms, ignoring the zeros does not exclude from the sample any of the observations coming from the process generating the positive counts. In the case of fertility data, considering only the positive observations all the couples with children are retained in the sample. Therefore, the distribution of the number of children parents have is not truncated by excluding from the sample childless couples.

The hurdle model described by (1) is appropriate to capture the special nature of the zeros in completed fertility data. In order to allow this model to account for a possible tendency of couples to avoid only children, the specification of $P(y|x, y > 0)$ has to be carefully considered. Let $\pi(y|x)$ denote a probability mass function with support on the positive integers. Then, the different nature of observations for which $y = 1$ can be captured by specifying

$$P(y|x, y > 0) = \begin{cases} \dfrac{\theta\pi(1|x)}{1 - (1-\theta)\pi(1|x)} & \text{for } y = 1 \\[2ex] \dfrac{\pi(y|x)}{1 - (1-\theta)\pi(1|x)} & \text{for } y = 2, 3, \ldots \end{cases} \tag{2}$$

where $\theta \geq 0$ is a parameter that measures how much the proportion of only children deviates from what is predicted by $\pi(y|x)$. The model described by (2) is akin to the one proposed by Yoneda (1962) (see also Johnson, Kotz and Kemp 1992) and allows $P(1|x, y > 0)$ to take arbitrary values between 0 and 1, with the right tail probabilities being proportional to $\pi(y|x)$. Naturally, θ may also depend on x.

It is worth noting that (2) can be expressed as

$$P(y|x, y > 0) = \begin{cases} \varphi + (1-\varphi)\pi(1|x) & \text{for } y = 1 \\ (1-\varphi)\pi(y|x) & \text{for } y = 2, 3, \ldots \end{cases}$$

with $\varphi = \dfrac{(\theta - 1)\pi(1|x)}{1 - (1-\theta)\pi(1|x)}$. Therefore, (2) can be formulated as a model with an inflated count of the type described by Mullahy (1986) and Lambert (1992). However, these models are specially interesting when they can be interpreted as describing a finite mixture, that is, when φ is positive. In the present case interest is focused on values of $0 < \theta < 1$, to which correspond negative values of φ. Therefore, viewing (2) as a member of this class of models does not lead to any insightful interpretation.

The modified hurdle model (MHM) defined by (1) and (2) is very flexible. In particular, with the appropriate specification of $P(0|x)$ and $\pi(y|x)$ and setting $\theta = 1$, this model encompasses the Poisson distribution and its generalizations that allow for under or overdispersion. However, the proposed model adds flexibility to these distributions by allowing the fine tuning of the estimated probabilities of observing childless couples and couples with only children.

Because this model introduces new parameters, it can easily become overparametrized. However, if voluntary childlessness is rare among couples (see Muñoz-Perez 1995; and Schoen et al. 1997) and if the tendency to avoid only children is widespread in the society, it is likely that both $P(0|x)$ and θ will depend only on a small number of covariates. Therefore, a parsimonious specification of these functions will help to avoid the dangers of overparametrization.

As in any hurdle model (Mullahy 1986), the log-likelihood function for the MHM is the sum of two parametrically independent log-likelihoods, and the parameters of $P(0|x)$ and $P(y|x, y > 0)$ can be separately estimated. It is also interesting to notice that the structure of this model is such that

$$P(y|x, y > 1) = \frac{\pi(y|x)}{1 - \pi(1|x)}. \tag{3}$$

Therefore, using only the observations for couples with two or more children, it is possible to estimate the parameters of $\pi(y|x)$ without the need to specify θ. This is interesting for two reasons. On the one hand, it makes the estimation of these parameters robust to the misspecification of θ. On the other hand, it suggests a specification test based on the comparison of the estimates obtained by the two methods. It must be noted, however, that a standard Hausman (1978) test is not applicable here since the estimator based on the truncated sample may not be less efficient than the estimator that uses the full sample. This is because of the different number of parameters estimated in the two cases. Alternatively, the split-sample version of the Hausman test suggested in Browning and Meghir (1991) can be used.

4. An empirical illustration

4.1. The data

In this section, data for women on their first marriage, or de facto marriage, taken from the 1997 Portuguese Fertility and Family Survey are used to illustrate the empirical application of the proposed model. This survey was carried out by Instituto Nacional de Estatística and is based on a version of the questionnaire proposed by the Fertility and Family Surveys project of the United Nations Economic Commission for Europe.

Despite being very much up-to-date, the data contained in this survey are not entirely adequate for the analysis of the determinants of completed fertility. In particular, only women between 15 and 49 years of age participated in this survey, and it contains no information at all on the couples' income.

The age range of the women that participated in the survey means that only a relatively small portion of the observations can be used for the analysis of completed fertility. In order to obtain an acceptable sample size, while still being reasonably close to considering only women with completed fertility histories, the analysis was limited to the women which were at least 40 years old at the time of interview. Of course, this procedure excludes from the sample every woman under the age of 40 that has completed her reproductive activity. However, since this survey does not contain reliable information allowing the identification of these individuals, it was decided to left them out of the sample. Naturally, this implies that the results reported here should be viewed as specific to this age group, and can be very different from those for younger women that have completed their reproductive activity.

Although this lower bound on the women's age is not uncommon (see Winkelmann and Zimmermann 1992, 1994), it is certainly the case that some of the women included in the sample have not completed their reproductive activity. To check the sensitivity of the results presented in this section to the lower bound imposed on the women's age, the analysis was repeated considering only women that were at least 42 years old at the time of the interview. Despite the obvious numerical differences resulting from losing about 20% of the sample, the results obtained considering only women above 42 are qualitatively similar to those presented in the paper. (These results are available from the authors upon request.)

Having no information on the couples' income makes it difficult to analyse the economic determinants of fertility, as described by the neoclassical theory (Becker and Lewis 1973, Becker 1981). Also, the lack of information on the income of the couples' parents precludes the analysis of the relative income hypothesis put forward by Easterlin (1987). However, this data set contains information on the couple's education, work status and place of residence, which can be thought of as indirect measures of the resources available to the couple and of the costs of raising a child. Besides these variables, the data set also contains information on other characteristics of the couple that are likely to influence its preferences. In particular, the survey gives indication about the woman's religion, the number of siblings she had, her age at first sexual intercourse and the age at marriage.

The dependent variable CHILDREN is the number of children ever born to the respondent. The complete description of the covariates used in this study is as follows: AGEW, woman's age; AGEH, husband's age; WORKW, dummy variable indicating if the woman is employed; WORKH, dummy variable indicating if the husband is employed; EDUW0–EDUW3, dummy variables indicating if the woman completed no school grade (reference), has some basic education but did not finish secondary school, completed secondary school (12 years), or has a degree; EDUH0–EDUH3, dummy variables indicating if the husband completed no school grade (reference), has some basic education but did not finish secondary school, completed secondary school (12 years), or has a degree; CATHOLIC, dummy variable indicating if the woman is catholic; SIBLINGS, the number of siblings the woman has; AGE1ST, woman's age at first sexual intercourse; AGEM, woman's age when started cohabiting; URB0–URB2; dummy variables indicating that the couple lives in a urban (reference), semi-urban, or rural area; and REGION0–REGION3, dummy variables indicating if the couple lives in Lisbon or surrounding area (reference), north of the river Tagus, south of the Tagus, or in the islands of Azores or Madeira.

It is important to note that some of these covariates may have had different values during the woman's childbearing years. Therefore, their role in a model of this type must be viewed with care. Moreover, some of the regressors used in this study can be endogenous. For example, it is likely that the number of children and the work status of a woman are simultaneously determined. Consequently, the model presented here should be interpreted as describing the conditional distribution of the number of children ever born to a woman, given the values of the covariates, and not as a model of the couples' fertility decisions. In particular, from this study it is not possible to draw any precise conclusion relating to causal links between the covariates and the dependent variable, although in some cases the model can give hints with respect to these.

Retaining only the observations corresponding to women aged 40 or over, in their first (de facto) marriage, for which there is information on all the covariates described above, a sample of 1093 observations is obtained. The relative frequencies for CHILDREN are displayed in Table 1 and the descriptive statistics for this variable, as well as for all covariates described above, are contained in Table 2.

Table 1. Relative frequencies of CHILDREN

Counts	0	1	2	3	4	5	6
Frequencies	0.0339	0.2141	0.4767	0.1720	0.0522	0.0265	0.0110
Counts	7	8	9	10	11	12	13
Frequencies	0.0037	0.0046	0.0009	0.0028	0.0009	0.0000	0.0009

Table 2. Descriptive statistics

Variable	Mean	Std. Dev.	Min.	Max.
CHILDREN	2.2104	1.3279	0	13
AGEW	44.4703	2.8539	40	49
AGEH	47.6944	5.4576	28	85
WORKW	0.5700	0.4953	0	1
WORKH	0.8618	0.3452	0	1
EDUW0	0.0732	0.2606	0	1
EDUW1	0.8097	0.3927	0	1
EDUW2	0.0357	0.1856	0	1
EDUW3	0.0814	0.2736	0	1
EDUH0	0.0576	0.2332	0	1
EDUH1	0.8371	0.3694	0	1
EDUH2	0.0430	0.2030	0	1
EDUH3	0.0622	0.2417	0	1
CATHOLIC	0.9579	0.2009	0	1
SIBLINGS	4.0183	3.1323	0	18
AGEM	22.0869	4.3108	15	44
AGE1ST	20.9076	3.6933	11	43
URB0	0.5855	0.4929	0	1
URB1	0.2232	0.4166	0	1
URB2	0.1912	0.3934	0	1
REGION0	0.1775	0.3823	0	1
REGION1	0.3907	0.4881	0	1
REGION2	0.2763	0.4474	0	1
REGION3	0.1555	0.3626	0	1

4.2. Model specification

For the MHM proposed in Sect. 3 to be fully specified, it is necessary to define $P(0|x)$, $\pi(y|x)$ and θ. Although $P(0|x)$ and $\pi(y|x)$ can be independently specified, here they are defined in such a way that if their parameters are equal and $\theta = 1$, the MHM reduces to one of the count data models recently used to model fertility data. In particular, the specification employed here has as a special case the so-called restricted generalized Poisson regression (RGPR) used by Wang and Famoye (1997) (see also Consul 1989; and Famoye 1993). The restricted generalized Poisson was chosen as the basic model for its simplicity and flexibility.

The probability mass function for the RGPR model is given by (see Wang and Famoye 1997)

$$P(y|x) = \left(\frac{\exp(x\beta)}{1+\alpha\exp(x\beta)}\right)^{y}\frac{(1+\alpha y)^{y-1}}{y!}\exp\left(-\frac{\exp(x\beta)(1+\alpha y)}{1+\alpha\exp(x\beta)}\right), \tag{4}$$

where $\alpha > \max\left\{\frac{-1}{\exp(x\beta)}, \frac{-1}{y}\right\}$ is a parameter and β is a vector of parameters. From here, $P(0|x)$ can be defined as

$$P(0|x) = \exp\left(-\frac{\exp(x\gamma)}{1+\alpha^{0}\exp(x\gamma)}\right), \tag{5}$$

where α^0 is a parameter and γ is a vector of parameters. Notice that $P(0|x)$ is bounded from below by $\exp\left(-\frac{1}{\alpha^0}\right)$ and therefore the probability of having children has an upper bound of $1-\exp\left(-\frac{1}{\alpha^0}\right)$. This means that (5) is particularly adequate to model the probability of having children since sterility problems will force some couples to remain involuntarily childless, regardless of their social and economic characteristics.

If $\pi(y|x)$ is defined as a truncated RGPR of the form

$$\pi(y|x) = \left(\frac{\exp(x\beta)}{1+\alpha\exp(x\beta)}\right)^{y} \frac{(1+\alpha y)^{y-1}}{\left[1-\exp\left(-\frac{\exp(x\beta)}{1+\alpha\exp(x\beta)}\right)\right] y!} \times \exp\left(-\frac{\exp(x\beta)(1+\alpha y)}{1+\alpha\exp(x\beta)}\right),$$

it is clear that the model defined by (1) and (2) reduces to (4) when $\alpha^0 = \alpha$, $\gamma = \beta$ and $\theta = 1$.

For the model to be complete, it is only necessary to specify the way in which θ depends on the regressors. Since θ is a positive parameter, it is convenient to specify $\theta = \exp(x\delta)$, which has $\theta = 1$ as a special case when $\delta = 0$.

Naturally, there are many other ways in which the MHM could be specified. However, a comparative analysis of different specifications of this model is beyond the scope of the present work.

4.3. Estimation results

With the data set used in this study, estimation of $P(0|x)$ is a difficult task. In fact, from the 1093 observations available, only 37 (3.39%) correspond to childless couples. With such an unbalanced sample, it is very hard to identify the effect of the regressors in a binary model. However, for sake of completeness, a simple model for $P(0|x)$ is presented.

The estimation of a model for $P(0|x)$ using all the regressors showed that the only covariate with a coefficient that is significantly different from zero at the usual 5% level is AgeM. However, a model with just this regressor is rejected when tested against the model with all the covariates. In view of these results, the binary model defined by (5) was estimated, having Catholic, AgeM and Region3 as regressors. These parameter estimates, together with the corresponding standard and misspecification robust t statistics (White 1982), are reported in Table 3. It must be emphasised that, due to the problems mentioned above, these results should be viewed with some caution.

Although both Catholic and Region3 are not individually significant at the usual significance levels, they are jointly significant. In fact, the score test statistic[2] for the exclusion of these covariates is 17.317, to which corresponds a p-value of 0.02%. As expected, these results suggest that catholic women are less likely to remain childless (see Poston 1990). Similarly, the model indicates that couples in the islands of Azores and Madeira are more prone to have

Table 3. Estimates for $P(0|x)$

Variable	Estimated parameters	t statistics (Newton)	t statistics (White)
Constant	5.5573	2.8256	2.4411
Catholic	0.8289	1.3953	1.2327
AgeM	−0.1657	−2.5595	−2.2102
Region3	1.1673	1.6819	1.7388
α^0	0.1925	3.4053	3.0310
Log-likelihood		−129.91	
Sample size		1093	

children. Naturally, these results are merely indicative as a much larger sample would be needed to precisely gauge the effect of these regressors on the probability of being childless.

Despite the limitations of the sample, it is possible to find a statistically significant negative effect of a late marriage on the probability of having children. This result is not at all surprising and it is in line with, for example, the findings of Kiernan (1989).

As for the effects of the other regressors, the score test statistic comparing the model in Table 3 to a model including all the covariates (21 parameters) has a value of 16.508, to which corresponds a p-value of 41.81%. Therefore, it can be concluded that, at conventional significance levels, the effect of the excluded regressors can safely be ignored.

Due to the low number of childless couples in this sample, it is not possible to draw firm conclusions from this analysis. However, the fact that most regressors have no impact on the probability of being childless and the nature of the regressors included in this model are consistent with the idea that, independently of their social and economic characteristics, most couples want at least one child.

As pointed above, α^0 determines the upper bound for the probability of having children. However, because for women in this age group AgeM cannot be below 14, in this particular model the upper bound on the probability of having children depends both on α^0 and on γ. In 1978 the minimum legal age for marriage in Portugal was set to 16, for both men and women. Before that it was 16 for men and 14 for women. Given the results in Table 3, the estimated value for this upper bound is 0.994, but obviously this is not a precise estimate.

Turning to the model for the positive counts, the analysis started with the estimation of a model in which both $\pi(y|x)$ and θ were allowed to depend on all the regressors. These results showed that, as expected, most covariates have no significant effect on θ. In view of this, it was decided to eliminate from the specification of θ all the variables whose coefficient in the unrestricted model is not statistically significant at the usual 5% level. The results obtained with the restricted model are presented in Table 4. Because individual t statistics were used to decide upon the exclusion of a set of regressors, a joint test of the significance of the excluded variables should be performed. The statistic for the score test of the restricted model presented in Table 4 against the fully parametrized model has a value of 25.637, to which corresponds a p-value of

8.13%. Therefore, at the pre-defined 5% level, the restricted specification can be viewed as a valid simplification of the fully parametrized model.

The fact that θ depends only on a very small number of regressors shows that the tendency of couples to avoid single children is indeed widespread in the Portuguese society. Not surprisingly, this tendency is strongest among couples in which the woman is catholic. On the other hand, this tendency is much attenuated for couples in which the husband is not employed. These are mostly couples in which the husband is in early retirement and therefore it is likely that their income is below its expected level. If that is the case, this result might be viewed as confirmation of the effect of relative income on fertility decisions (see Easterlin 1987; and Macunovich 1998).

The analysis of the effects of the regressors on $\pi(y|x)$ also leads to some interesting findings. To start with, given that all the women in this sample are between 40 and 49 years old, it is somewhat surprising to find such a significant effect of the variable AgeW. Considering the value and the significance of the parameter associated with AgeM, these results suggest that what is relevant for this part of the model is the duration of the marriage, that is, the difference between AgeW and AgeM. In fact, the score test statistic for the hypothesis that the sum of the coefficients on AgeW and AgeM is zero is 2.396, to which corresponds a p-value of 12.17%. Another covariate with an important impact, and a similar interpretation, is Age1st. Most of the other woman's characteristics (work-status, education and number of siblings) also have significant coefficients.

Although being significant (at 5%) only when the robust t statistics are used, the coefficient of Catholic is particularly interesting for its negative sign. In fact, this regressor also has a significant negative impact on θ, suggesting that catholic women are much less prone to have only children. However, that effect is partially offset by the negative coefficient this regressor has on $\pi(y|x)$. These conflicting effects may suggest that, because this regressor enters the model in two different ways, it is difficult to separately identify its effects on the two parts of the model. A similar situation occurs with WorkH, whose parameter has the same sign on both parts of the model, although being significant only on θ. However, as it will be shown latter, there is evidence to suggest that these conflicting effects are real, and not just the result of an identification problem.

In what concerns the other husband characteristics, AgeH has a significant negative impact on $\pi(y|x)$, but the husband education dummies do not have significant t statistics.

Finally, the degree of urbanization does not appear as significant, but the regional dummies indicate that families tend to be larger in the north of the country and specially in the islands of Azores and Madeira. This is in accordance with the findings of Bandeira (1996).

Perhaps the most interesting result in Table 4 is that, although in this sample the number of children exhibits the underdispersion that is characteristic of completed fertility data, the estimate of α in this model is positive. This means that all the underdispersion of the data is accounted for by taking into consideration the different nature of the couples having no children or just a single child. To confirm that this result is not just the consequence of an incorrect specification of the mean to variance ratio in the RGPR that underlays the MHM, the model in Table 4 was tested against a more general specification in which α is given by $\alpha = \alpha_0 \exp(\alpha_1 x\beta)$ (see Santos Silva 1997). The value

Table 4. Estimates for the modified Hurdle model

Variable	$\pi(y\|x)$			$\theta = \exp(x\delta)$		
	Estimated parameters	t statistics (Newton)	t statistics (White)	Estimated parameters	t statistics (Newton)	t statistics (White)
Constant	1.4428	1.8355	1.8318	−0.1330	−0.2488	−0.2536
AgeW	0.0694	3.8222	3.9465	–	–	–
AgeH	−0.0274	−2.7594	−2.7415	–	–	–
WorkW	−0.2541	−2.9015	−2.7835	–	–	–
WorkH	−0.1460	−1.0403	−0.9459	−0.7710	−2.8554	−2.7773
EduW1	−0.2907	−1.9836	−1.8491	–	–	–
EduW2	−0.7867	−2.4488	−2.1266	–	–	–
EduW3	−0.2368	−0.9133	−0.9815	–	–	–
EduH1	−0.3067	−1.8779	−1.6947	–	–	–
EduH2	−0.4222	−1.3499	−1.3706	–	–	–
EduH3	−0.1485	−0.5091	−0.5251	–	–	–
Catholic	−0.5286	−1.9293	−2.3611	−1.1589	−2.4446	−2.5988
Siblings	0.0575	4.2605	4.2383	–	–	–
AgeM	−0.0316	−1.8664	−1.6986	–	–	–
Age1st	−0.0963	−4.7292	−4.5920	–	–	–
Urb1	−0.1332	−1.1958	−1.1816	–	–	–
Urb2	0.1225	1.0134	0.9914	–	–	–
Region1	0.3976	3.0568	3.1437	–	–	–
Region2	−0.0267	−0.1930	−0.1949	–	–	–
Region3	0.9355	6.3365	6.1401	–	–	–
α	0.1561	2.6028	2.4782	–	–	–
Log-likelihood			−1343.82			
Sample size			1056			

of the score test statistic for the hypothesis $\alpha_1 = 0$ is 0.442, to which corresponds a p-value of 50.61%. Thus, this test gives no evidence against the simpler model adopted here.

As mentioned above, it can be suspected that the conflicting impacts of some covariates on the different levels of the model results from the inability to separately identify the effect of these variables on the various parts of the model. To clarify this question, the parameters of $\pi(y|x)$ were estimated using (3), which does not involve θ. These results, which are presented in Table 5, confirm that the covariates WorkH and Catholic have a negative impact on this part of the model, but that only the effect of Catholic is statistically significant.

Overall, the estimates of the parameters of $\pi(y|x)$ obtained using (3) are very close to those reported in Table 4. This is confirmed by the split-sample version of the Hausman test (Browning and Meghir 1991). Randomly splitting the sample in halves, the computed statistic for this test had a value of 14.631, to which corresponds a p-value of 84.10%.

It was pointed out before that the MHM reduces to the RGPR when $\alpha^0 = \alpha$, $\gamma = \beta$ and $\theta = 1$. Therefore, an idea of the importance of accounting for the specificity of the couples having no children or an only child can be obtained comparing the results obtained for the MHM with the results of the estimation of the RGPR model, which are also contained in Table 5.

The RGPR is clearly less satisfactory than the MHM. The first indication of that is given by a simple likelihood ratio test comparing the value of the

Table 5. Other estimation results

Variable	Estimates for $\frac{\pi(y\|x)}{[1-\pi(1\|x)]}$			Estimates for the RGPR		
	Estimated parameters	*t* statistics (Newton)	*t* statistics (White)	Estimated parameters	*t* statistics (Newton)	*t* statistics (White)
Constant	1.9242	2.0894	2.0547	1.2452	3.6080	3.9065
AGEW	0.0647	3.0159	3.1904	0.0280	3.5007	3.6848
AGEH	−0.0306	−2.5289	−2.5027	−0.0110	−2.4439	−2.4613
WORKW	−0.2659	−2.5659	−2.3934	−0.0928	−2.4031	−2.4763
WORKH	−0.1663	−1.2148	−1.0552	0.0178	0.3285	0.3001
EDUW1	−0.3901	−2.4627	−2.3880	−0.0677	−1.0321	−0.9199
EDUW2	−0.1045	−0.2369	−0.2963	−0.1895	−1.3796	−1.6239
EDUW3	−0.6076	−1.6381	−1.8904	−0.0261	−0.2268	−0.2679
EDUH1	−0.2237	−1.2729	−1.1208	−0.1422	−1.9906	−1.4542
EDUH2	−0.5636	−1.1593	−1.3806	−0.1439	−1.0631	−1.2159
EDUH3	−0.1859	−0.4509	−0.4898	−0.0818	−0.6369	−0.6997
CATHOLIC	−0.6131	−2.3095	−2.6225	0.0077	0.0694	0.0764
SIBLINGS	0.0522	3.4382	3.3792	0.0234	4.0877	4.1351
AGEM	−0.0287	−1.3292	−1.2902	−0.0242	−3.1654	−2.9852
AGE1ST	−0.0906	−3.5080	−3.7056	−0.0328	−3.7498	−4.0003
URB1	−0.1613	−1.2327	−1.1472	−0.0266	−0.5336	−0.5116
URB2	0.2126	1.5597	1.5258	0.0772	1.4258	1.3696
REGION1	0.3187	1.8959	1.8923	0.1232	2.1885	2.7282
REGION2	−0.2049	−1.0878	−1.0417	0.0033	0.0539	0.0675
REGION3	0.9844	5.5967	5.4211	0.4131	6.6448	6.4888
α	0.1109	2.3897	2.2795	−0.0496	−10.2493	−5.6183
Log-likelihood		−813.77			−1661.80	
Sample size		822			1093	

Table 6. True and predicted relative frequencies

CHILDREN	0	1	2	3	4	5+
Sample	0.0339	0.2141	0.4767	0.1720	0.0522	0.0512
MHM	0.0338	0.2149	0.4787	0.1627	0.0607	0.0493
RGPR	0.1052	0.2405	0.2724	0.2020	0.1101	0.0699

maximum of the likelihood functions of both models (−1661.80 for the RGPR and −1343.82 − 129.91 = −1473.73 for the MHM), which strongly disfavours the simpler RGPR. The lack of fit of the RGPR can be clearly seen in Table 6, where the sample relative frequencies for CHILDREN are compared with the predictions of the two models. These results show that the RGPR grossly underpredicts the value of the mode and overpredicts the tails of the distribution. It must be noted that this behaviour is not specific of the RGPR, being shared by other models that try to model the underdispersion of the data just by using a generalization of the Poisson distribution that allows for underdispersion. As for the MHM, it obviously fits perfectly the left tail. However, what is more important is that it also describes accurately both the mode and the right tail of the distribution.

Perhaps the most important disadvantage of the RGPR is made clear by the estimates of the coefficients of the covariates WorkH and Catholic. It was noted above that these regressors had conflicting but significant impacts on the different levels of the model proposed here. However, the RGPR fails to recognize the existence of these different levels, estimating just the aggregate effect of the regressors. Therefore, it is not surprising to find that in the RGPR model both WorkH and Catholic have very small coefficients which are not statistically significant.

5. Conclusions

The recent literature on the reasons for couples to have children emphasizes that the decision to have children is essentially different from the decision of how many children to have (see, for example, Schoen et al. 1997). Moreover, social prejudice against single children may have an important role in the couples decision of how many children to have (see Falbo 1992). The main contribution of this paper is the probabilistic model for completed fertility data proposed in Sect. 3, which explicitly accounts for the qualitative differences between having zero, one, or more children. A particular specification of this model was used in Sect. 4 to analyse completed fertility data from the 1997 Portuguese Fertility and Family Survey.

The results obtained with the proposed model are quite encouraging as they are compatible with the idea that, regardless of their economic and social characteristics, most couples avoid remaining childless or even having just an only child. Moreover, the results show that some covariates may have opposite effects on the different parts of the model and that ignoring these differences may lead to erroneous conclusions. Finally, in this example the differences between the various components of the model are responsible for the underdispersion that is so characteristic of completed fertility data. Therefore, the model proposed here, more than allowing for the presence of underdispersion, allows the causes of that phenomenon to be modelled.

Endnotes

[1] There is a large literature suggesting that, due to social pressure, the decision to remain voluntarily childless is quite different from the decision to have few children. Early references in this area include Veevers (1980), Faux (1984) and Cameron, (1986).

[2] Throughout the paper, score tests statistics are computed using the Hessian estimator of the covariance matrix. Under the null, these statistics are distributed as χ^2 variates with the number of degrees of freedom equal to the number of restrictions being tested.

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