

Labor immigration and long-run welfare in a growth model with heterogenous agents and endogenous labor supply

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Abstract. This paper examines the consequences of labor immigration in an OLG economy in which agents have an elastic labor supply and differ with respect to degrees of altruism and rates of time preference. It focuses on three substantive questions. First, how do immigrants influence the bequest motive of altruistic natives? Second, what impact do immigrants have on the labor supply of natives? Finally, how does immigration affect the long-run welfare of both altruistic and non altruistic natives?

JEL classification: D64, D91, F22

Key words: Immigration, OLG model, Altruism, endogenous labor supply

1. Introduction

In recent decades, there has been a resurgence of immigration in many industrialized countries. The United Nations estimates that over 60 million people (i.e. 1.2% of the world's population) reside in a country where they were not born (see Borjas 1994, p. 1667). Over half of the immigrants go to the United States, Canada or Australia. Western European countries are also receiving large immigrant flows. Zimmermann (1995) examines the historical pattern of migration and the empirical dimension of Western Europe's migration problem. Nearly 11% of the population in France, 17% in Switzerland, and 9% in the United Kingdom are foreign born. In view of the economic, cultural, and

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political significance of the issues raised by immigration, it is not surprising that the impact of immigration on the host economy is now being debated heatedly in many countries. The last two decades have witnessed increasing research interest in the economics of migration.

Following Galor's seminal article (1986), the overlapping generations approach has often been used in the international labor migration literature (see, among others, Crettez et al. 1996; Galor 1992; Galor and Stark 1990; Kochhar 1992; Kondo 1989). These studies assume that agents are non altruistic. However, recent empirical evidence on saving and bequest motives reveal wide differences in preferences and behavior between individuals (see Arrondel et al. 1997). Contemporary societies seem to include at least two types of households: altruistic parents who always provide non negative intergenerational transfers to their children and parents who care about own consumption only.

This evidence suggests that it is important to consider the coexistence of altruists and non altruists in a migration model. As pointed out recently by Tcha (1995a, 1995b, 1996) and Gaumont and Mesnard (2000), altruism leads to results that differ from those obtained under the assumption of selfishness. Moreover, analyzing the relationship between migration and altruism is intricate. For example, since migrants move with their own inheritance, capital mobility is linked to labor mobility. In a partial equilibrium framework, Tcha (1995a) constructs a model of rural-urban migration with altruistic agents. Differences in dynastic utility among generations cause both a higher mobility of the young and a disagreement among young and old family members on migration decisions. This leads to conflicts between generations. In spite of these complications, Gaumont and Mesnard (2000) build a migration model with altruistic agents that examines labor migration decisions in a general equilibrium framework. However, in order to avoid generational conflicts, they consider paternalistic altruism. Contrary to dynastic altruism, parents do not take into account the utility of their child, but the level of bequest.

In this paper, we construct an OLG model in which two types of individuals coexist. The population consists of an exogenous fraction of altruistic and of nonaltruistic agents. We adopt Barro's (1974) recursive definition of altruism: parents care about their children's welfare, and possibly leave them a bequest. The bequest is required to be non negative, a constraint which may be binding. We examine the impact of immigration on an economy in which agents have an elastic labor supply and differ with respect to degrees of altruism, rates of time preference and preference for leisure.

Our study is centered around three questions. First, how do immigrants influence the bequest motive of altruistic natives? Second, what impact do immigrants have on the labor supply of natives? Finally, how does immigration affect the long-run welfare of natives?

Our results allow us to tackle the issue of quotas on migration flows. Quotas are a major policy tool used by rich countries since the early 1900s (they were introduced by Great Britain in 1905, and by the United States in 1921). High growth rates in the developed countries have raised immigration incentives and made immigration quotas increasingly important as a policy measure. Our study allows a host country to determine the immigration quota that maximizes the long-run welfare of natives.

We first show that our model possesses a unique steady state. However, there exist three types of equilibria. If the degree of altruism is too low, altruists work and do not leave a bequest to their children. If the intensity of altruism is strong enough, the stationary bequests of altruists are positive. Moreover, an altruist can choose to stop working. In fact, we show that altruists do not work if non altruistic agents are impatient and in a sufficiently large proportion in the economy. In this case, savings of non altruists are low and a large share of capital belongs to a few altruists.

In order to analyze the impact of immigration on the behavior of natives, we study the effects of a change in the structure of the population on the steady state. Depending on the rate of impatience of natives, immigrant flows can incite native altruists to bequeath or can deter them from leaving an inheritance. However, when non altruists are impatient, the size of bequests increases with the number of selfish immigrants. By making the altruists richer, these inflows of non altruists may lead native altruists to stop working. Conversely, an immigration of altruists can make the altruistic natives work.

Finally, we study the welfare implications of immigration for the natives. This analysis is crucial to determine the "right" immigration policy. Although partial, our study can explain why immigration policy varies considerably across countries. Indeed, depending on the set of socio-economic characteristics of agents living in the autarkic economy, the immigration of one type of agents can improve or worsen the long-run welfare of all natives. If altruists leave positive bequests under autarky, the immigration of impatient non altruists never worsens the welfare of natives. However, even if the bequests of impatient altruists are positive under autarky, the immigration of patient non altruists can worsen the welfare of natives.

The remainder of the paper is organized as follows. Section 2 sets up our OLG model. In Sect. 3 we establish results about the existence of a unique steady state equilibrium. We investigate the consequences of immigration on the behavior and the welfare of natives in Sects. 4 and 5, respectively. Section 6 concludes. All proofs are gathered in Appendix.

2. The autarkic economy

Economic activity extends over infinite discrete time. We consider a population of size N_t which consists of a fraction p of altruistic agents (0 and <math>q = 1 - p of non altruistic agents denoted with a and e, respectively. Each individual of type i, altruistic or not (i = a, e), gives birth to (1 + n) children of type i and lives for two periods. When young, he supplies a portion l_t^i of his time endowment, and earns the labor income $w_t l_t^i$. When old, he is retired. The agent perfectly foresees the rate of interest, r_{t+1} . Preferences are defined over consumptions (c_t^i) , when young, and d_{t+1}^i , when old) and leisure $(\mathcal{L}_t^i) = 1 - l_t^i$ by:

$$U^{e}(c_{t}^{e}, \mathcal{L}_{t}^{e}, d_{t+1}^{e}) = \mu \ln c_{t}^{e} + \xi \ln \mathcal{L}_{t}^{e} + \gamma \ln d_{t+1}^{e}$$

$$U^{a}(c_{t}^{a}, \mathcal{L}_{t}^{a}, d_{t+1}^{a}) = \mu' \ln c_{t}^{a} + \xi' \ln \mathcal{L}_{t}^{a} + \gamma' \ln d_{t+1}^{a}$$

where $\mu, \mu', \gamma, \gamma', \xi, \xi'$ are positive and satisfy $\mu + \xi + \gamma = \mu' + \xi' + \gamma' = 1$.

When old, non altruists consume the proceeds of their savings, $(1 + r_{t+1})s_t^e$. Hence, a non altruist born in t solves the following maximization problem:

$$\max_{c_t^e, s_t^e, \mathcal{L}_t^e, d_{t+1}^e} \mu \ln c_t^e + \xi \ln \mathcal{L}_t^e + \gamma \ln d_{t+1}^e$$

s.t.
$$w_t(1 - \mathcal{L}_t^e) = c_t^e + s_t^e$$
 (1)

$$(1+r_{t+1})s_t^e = d_{t+1}^e (2)$$

$$\mathcal{L}_{t}^{e} \in [0, 1] \tag{3}$$

Non altruists always work $(\mathcal{L}_t^e < 1)$ since they have no other source of income. Since $\mathcal{L}_t^e \in (0,1)$, the optimality conditions are given by:

$$\frac{\mu}{c_t^e} = \frac{\gamma(1 + r_{t+1})}{d_{t+1}^e} \tag{4}$$

$$\frac{\mu w_t}{c_t^e} = \frac{\xi}{\mathcal{L}_t^e} \tag{5}$$

For altruistic agents, we adopt Barro (1974)'s definition of altruism: parents care about their children welfare by weighting their children's utility in their own utility function and, possibly leave them a bequest. When young, altruists receive a bequest x_t . When old, they consume part of the proceeds of their savings and bequeath the remainder $(1+n)x_{t+1}$ to their (1+n) children. Importantly, the bequest is restricted to be non-negative. If this constraint is binding (resp: not binding), then the bequest motive is said to be inoperative (resp: operative). We denote by V_t the utility of an altruist:

$$V_t(x_t) = \max_{c_t^a, \mathcal{L}_{t+1}^a, d_{t+1}^a, x_{t+1}} \{ \mu' \ln c_t^a + \xi' \ln \mathcal{L}_t^a + \gamma' \ln d_{t+1}^a + \beta V_{t+1}(x_{t+1}) \}$$

where $V_{t+1}(x_{t+1})$ denotes the utility of a representative descendant who inherits x_{t+1} . $\beta \in (0, 1)$ is the intergenerational discount factor or degree of altruism.³

The sequence of these maximization problems can be rewritten as an infinite horizon problem:

$$\max_{\{c_{i}^{a},\mathcal{L}_{i}^{a},d_{i+1}^{a},s_{i}^{a},x_{j+1}\}_{j=t}^{j=+\infty}} \sum_{j=t}^{\infty} \beta^{j-t} (\mu' \ln c_{j}^{a} + \xi' \ln \mathcal{L}_{j}^{a} + \gamma' \ln d_{j+1}^{a})$$

s.t.
$$\forall j \ge t \quad w_j(1 - \mathcal{L}_j^a) + x_j = c_j^a + s_j^a$$
 (6)

$$(1+r_{j+1})s_j^a = d_{j+1}^a + (1+n)x_{j+1}$$
(7)

$$x_{j+1} \ge 0 \tag{8}$$

$$\mathcal{L}_i^a \in [0, 1] \tag{9}$$

 x_t given

Since an altruist can choose to live only with his inheritance, the optimization problem possesses two inequality constraints $(x_{j+1} \ge 0 \text{ and } \mathcal{L}_i^a \le 1)$.

Hence, the optimality conditions are:

$$\frac{\mu'}{c_j^a} = \frac{\gamma'(1 + r_{j+1})}{d_{j+1}^a} \tag{10}$$

$$\frac{\mu' w_j}{c_j^a} - \frac{\xi'}{\mathcal{L}_j^a} \le 0 \quad (= \text{if } \mathcal{L}_j^a < 1)$$

$$\tag{11}$$

$$-\frac{(1+n)\gamma'}{d_{j+1}^a} + \beta \frac{\mu'}{c_{j+1}^a} \le 0 \quad (= \text{if } x_{j+1} > 0)$$
 (12)

Let us now turn to the production side. Production occurs according to a Cobb-Douglas technology using two inputs, capital K_t and labor L_t : $Y_t = F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha}$, $\alpha \in (0, 1)$. Homogeneity of degree one allows us to write output per young as a function of the capital/labor ratio per young: $Y_t/L_t = F(z_t, 1) = f(z_t) = Az_t^{\alpha}$ with A > 0 and $z_t = K_t/L_t$, the intensity of capital/labor ratio.

Capital fully depreciates after one period. Since markets are perfectly competitive, each factor is paid its marginal product.

$$w_t = f(z_t) - z_t f'(z_t) = A(1 - \alpha)z_t^{\alpha}$$
 and $1 + r_t = f'(z_t) = A\alpha z_t^{\alpha - 1}$ (13)

The capital stock of period t + 1 is financed by the savings of the generation born in t. Hence, in reduced form $(l_t = L_t/N_t)$ and $k_t = K_t/N_t$ we have:

$$l_t = pl_t^a + ql_t^e$$
 and $(1+n)k_t = ps_{t-1}^a + qs_{t-1}^e$ (14)

3. The autarkic steady state

We now confine the analysis to the steady state of the autarkic economy.

Definition 1. A steady-state competitive equilibrium with perfect foresight of the OLG model with heterogenous agents and endogenous labor supply is a vector $(c^e, d^e, s^e, \mathcal{L}^e, c^a, d^a, s^a, \mathcal{L}^a, k, l, w, r, x)$ that satisfies (1) to (14).

Stationary levels of consumption and savings of non altruists depend on the wage and interest rates whereas the behavior of altruists depends on whether or not they choose to work or/and bequeath. Indeed, merging Eqs. (1), (2), (4), (5), (6), (7), (10) and (12) we have:⁴

Since the labor supply of non altruists is constant, changes in the aggregate labor supply stem from the behavior of altruists.

Consider the following degrees of altruism $\bar{\beta}$, $\hat{\beta}$ and the proportion \bar{p} :

With these values we can distinguish three kinds of steady state: a steady state where bequests are constrained; a steady state where the bequest motive is operative and altruists work; a steady state where bequests are positive and altruists do not work. We now establish conditions for the existence of each type of steady state.

Theorem 1. Existence and uniqueness of the steady state.

The model possesses a unique steady state. Bequests are positive if and only if $\beta > \hat{\beta}$. Altruists do not work if and only if $\beta > \bar{\beta}$ and $p \leq \bar{p}$.

Bequests are positive if the intensity of altruism is sufficiently strong. Hence, we extend the standard result⁵ of Weil (1987) to a model with heterogenous agents and endogenous labor supply.

In fact, altruists decide to leave a bequest to their offspring if the capital/labor ratio $\tilde{z} = \left(\frac{A\alpha\beta}{1+n}\right)^{1/(1-\alpha)}$ of the modified golden rule is higher than the capital/labor ratio $z_0 = \left(\frac{(p\gamma' + q\gamma)A(1-\alpha)}{(1+n)(p(\mu' + \gamma') + q(\mu + \gamma))}\right)^{1/(1-\alpha)}$ of the economy without bequest motive.

We notice that the critical value of the degree of altruism above which altruists leave a bequest depends on the proportion p.

When bequests are positive, altruists may not work. Indeed, the economy is at the modified golden rule steady state which depends on the degree of altruism, but not on the proportion of altruists. Since the interest factor is equal to $(1+n)/\beta$, investing βx is sufficient to leave (1+n)x to one's children. The difference, $x-\beta x$, between the bequest received and the actualized value of bequest handing down is defined as the rent⁶ (or patrimony return) of the altruist and is denoted by κ .

It is necessary that the proportion of non altruists is large enough so that an heir chooses not to work. Indeed, when $\beta > \hat{\beta}$, bequests are a decreasing function of p. Hence, below the critical value \bar{p} , the size of bequests incites altruists not to work.

To choose not to work, an altruist must have a sufficiently large degree of altruism because the bequest motive must be operative. However, this degree of altruism must not be too high. Indeed, agents can be too altruistic to benefit from a rent. For instance, if an agent is fully altruistic (i.e. $\beta = 1$), he transmits the received bequest in its entirety ($\kappa = \bar{p} = 0$).

To clarify our interpretation, consider the degrees of impatience to consume of non altruists $\delta = \mu/\gamma$ and of altruists $\delta' = \mu'/\gamma'$. An agent is said to be impatient (patient) if his degree of impatience to consume⁷ is higher (lower) than $\delta = (\alpha^{-1} - 1)\beta^{-1} - 1$.

Since $\bar{\beta}$ does not depend on the set of socio-economic characteristics of altruists (i.e μ' , ξ' , γ' and β), the condition $\beta > \bar{\beta}$ is equivalent to $\delta < \bar{\delta}$. Hence, it is necessary that non altruists are impatient and in a large proportion so that altruists choose not to work. In this case, savings of non altruists are lower and a large share of capital belongs to a few altruists. Since production is provided by numerous non altruists, heirs choose not to work.

4. The effects of immigration

So far, we have assumed that the proportion p of altruists was given. In this section we study the long-run consequences of a change in the structure of the population. Indeed, we focus on the impact of immigration on the behavior of natives in the steady state. The immigration of altruists (resp: non altruists) leads to an increase (resp: a decrease) in p.

The fact that migrants' tastes are similar to natives' tastes is a strong assumption but is justified in the long run. Indeed, according to Becker (1996), the preferences of agents can be "extended" in order to account for the formation of personal and/or social capital. For Becker (1996, p. 19): "Initial stocks of personal and social capital, along with technologies and government policies, do help determine economic outcomes. But the economy also changes tastes and preferences by changing personal and social capital." In the personal capital case, an agent's past experience influences his current tastes. Under the assumption of rational expectations, agents forecast the change in their future preferences to compute their optimal choices (see, the discussion in Pashardes 1986). In the social capital case, the history of the society or the group to which an agent belongs influences his future tastes. Studies on the assimilation of immigrants also back our assumption (see, Durkin 1998; Dustmann 1996; and Michel et al. 1998).

Inflows have no impact on the labor supply of native non altruists since their labor supply does not depend on p. This simplification⁸ allows us to concentrate more specifically on altruists. Indeed, we establish the effects of immigration on the size of bequests and the labor supply of native altruists. An economy without bequest can turn to an economy with positive bequests (or vice versa) as a result of immigration. In order to study possible transitions, we can distinguish three cases according to individuals'degrees of patience.

Theorem 2. Impact of immigration on native altruists.

- i) If all agents are patient then an immigration has no impact on the behavior of native altruists.
- ii) If the non altruists are patient and the altruists impatient then the immigration of altruists (resp: non altruists) leads to a decrease (resp: an increase) in the labor supply of native altruists and an increase (resp: a decrease) in their bequests.
 - iii) If the non altruists are impatient, then the immigration of altruists (resp:

non altruists) leads to an increase (resp: a decrease) in the labor supply of native altruists and a decrease (resp: an increase) in their bequests.

The more patient the agents are, the more they save. Hence, in an economy consisting of patient individuals the aggregate savings are so large that the capital/labor ratio z_0 is higher than the capital/labor ratio of the modified golden rule \tilde{z} . Then, the bequest motive of altruists is inoperative and their labor supply is constant. Hence, immigration does not affect their behavior.

When bequests are positive, the economy is at the modified golden rule steady state. Hence, so long as immigration does not affect the bequest motive of altruists, the wage w and the factor of interest R remain unchanged. Therefore, variations of the labor supply of altruists stem from variations in the size of bequests (or in the rent). So, the labor supply of altruists is negatively related 9 to the level of bequests.

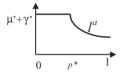
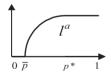




Fig. 1. Case where $\delta < \bar{\delta} < \delta'$

Consider now that the non altruists are patient but not the altruists.

On the basis of their degree of impatience, non altruists save a larger proportion of their income than altruists. As long as their share in the population is high enough $(p < p^* = [\gamma(\bar{\delta} - \delta)]/[\gamma(\bar{\delta} - \delta) - \gamma'(\delta' - \bar{\delta})])$, the contribution of non altruists to aggregate savings is such that z_0 is larger than \tilde{z} . When the non altruists are proportionally fewer $(q < 1 - p^*)$ we have $z_0 < \tilde{z}$ and the bequest motive is operative. Hence, an inflow of altruists (resp: non altruists) can incite (resp: deter) the native altruists to bequeath. When bequests are positive, an inflow of altruists leads the altruists to bequeath more (Fig. 1). To leave the economy at the modified golden rule steady state after the arrival of altruists, native altruists must work less.



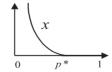
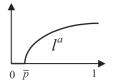


Fig. 2. Case where $\delta' < \bar{\delta} < \delta$

Assume now that the non altruists are impatient.

When the altruists are patient (Fig. 2) and numerous $(p > p^*)$ their savings ps^a are such that $z_0 > \tilde{z}$. Hence, the altruists do not bequeath. Conversely, if non altruists are sufficiently numerous $(q > 1 - p^*)$ or if altruists are too impatient Fig. 3) we have $z_0 < \tilde{z}$ and the bequest motive is operative. Unlike the previous configuration, an immigration of altruists (resp: non altruists) can deter (resp: incite) native altruists from leaving a bequest.



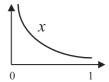


Fig. 3. Case where Min $(\delta, \delta') > \bar{\delta}$

Then, the less impatient non altruists are, the richer the altruists. Hence, bequests are increasing in q. This increase implies a decrease in the labor supply of altruists. When the proportion of non altruists is very important $(q>1-\bar{p})$ the bequest is so large that native altruists stop working. Interestingly, by making the native altruists richer, inflows of non altruists lead altruists to eventually stop working. Conversely, an immigration of altruists can make the altruists to work.

We can now summarize the findings of this section. Inflows can render operative or inoperative the bequest motive of native altruists but only the immigration of altruists (resp: non altruists) can render positive (resp: nil) the labor supply of native altruists.

5. Immigration and long-run welfare

In this section, we establish the long-run welfare consequences of immigration on both altruistic and non altruistic natives.

The steady-state welfare of a non altruist is:

$$\bar{u} = \mu \ln c^e + \xi \ln \mathcal{L}^e + \gamma \ln d^e$$

The long-run welfare of an altruist is equal to:

$$\bar{v} = (\mu' \ln c^a + \xi' \ln \mathcal{L}^a + \gamma' \ln d^a)/(1 - \beta)$$

We first consider inflows which do not change the bequest motive of altruists. The effects of these small flows can be useful to determine if the host country must resort to immigration quotas.

We first study the positive beguest case.

Proposition 1. Consider inflows keeping bequests positive.

The immigration of altruists (resp: non altruists):

- i) has no effect on the welfare of native non altruists.
- *ii)* worsens (resp: improves) the welfare of native altruists if the altruists are patient and the non altruists impatient.
- iii) improves (resp: worsens) the welfare of native altruists if the altruists are impatient or the non altruists patient.

When the proportion of altruists varies locally, the modified golden rule capital stock as well as the welfare of non altruists remain unchanged, whereas the long-run welfare of altruists decreases or increases depending on whether or not $\partial x/\partial p$ is positive. Indeed, since c^a , \mathcal{L}^a , and d^a are increasing functions of bequests x, $\partial \bar{v}/\partial p$ has the sign of $\partial x/\partial p$. According to Theorem 2, bequests are an increasing function of p if and only if the non altruists are impatient and the altruists patient.

Now we focus on the no bequest case.

Proposition 2. Consider inflows keeping bequests nil.

The immigration of altruists (resp: non altruists):

i) worsens (resp: improves) the welfare of native non altruists if $(\delta + 2 - \alpha^{-1})(\delta - \delta')$ is negative and improves (resp: worsens) it if $(\delta + 2 - \alpha^{-1})(\delta - \delta')$ is positive.

ii) worsens (resp: improves) the welfare of native altruists if $(\delta' + 2 - \alpha^{-1})(\delta - \delta')$ is negative and improves (resp: worsens) it if $(\delta' + 2 - \alpha^{-1}) \cdot (\delta - \delta')$ is positive.

The welfare consequences of immigration are directly related to the degree of impatience of agents and does not depend on $\bar{\delta}$. The sign of $\delta - \delta'$ corresponds to the sign of $\partial z/\partial p$. Thus, in an economy with non altruistic agents who differ in their rate of time preference, inflows of the more (resp: less) impatient agents lead to an increase (resp: a decrease) in the stationary capital/labor ratio.

If degrees of impatience to consume of the two types are such that one is lower than $\alpha^{-1}-2$ and the other is greater, we can remark that small inflows of altruists (resp: non altruists) improve the long-run welfare of non-altruists (resp: altruists) and worsen that of altruists (resp: non altruists). However, when $\min(\delta, \delta') > \alpha^{-1} - 2$ or $\max(\delta, \delta') < \alpha^{-1} - 2$, small inflows have the same effects on all the natives.

Results of the previous propositions are only valid for migration flows which do not change the sign of $\hat{\beta} - \beta$. We now focus on mass migration. Clearly, for numerous parameter configurations, the effects of immigration are to lower the welfare of the non altruists (resp: altruists) and to increase that of the altruists (resp: non altruists). The goal of the next theorem is to exhibit the economies for which the immigration of one type of agents improves (or worsens) the long run welfare of all native agents; independently of the size of inflows. In order to characterize these situations we define seven cases:

Then, we can identify inflows of one type of agents which improve or worsen the long-run welfare of all natives.

Theorem 3. Pareto configurations

The immigration of altruists improves (resp: worsens) the welfare of native agents if and only if one among the conditions (i) to (iv) (resp: (v) to (vii)) is satisfied.

The immigration of non altruists improves (resp: worsens) the welfare of native agents if and only if one among the conditions (v) to (vii) (resp: (i) to (iv)) is satisfied.

In an inelastic labor supply framework, Michel and Pestieau (1998) prove that if all agents have the same logarithmic life cycle utility and if the bequest motive is operative, then a decrease in p is improving for the welfare of natives. According to this theorem (case (v)), their result still holds when the labor supply is elastic.

However, considering that altruists and non altruists have different life cycle utility allows us to show that an immigration of non altruists can be worsening the welfare of natives; even if the bequest motive of altruists is operative under autarky (case (iv)). We obtain this situation when altruists are impatient and non altruists patient. In this case, the level of bequests increases with the proportion of altruists present in the economy (see (iii) of Theorem 2).

Other configurations for which the arrival of altruists improves the welfare of natives (cases (i), (ii) and (iii)) correspond to economies where altruists behave like non altruistic agents. When the bequest motive of altruists is inoperative, it is also possible (cases (vi) and (vii)) that inflows of non altruists increase the welfare of all the natives.

According to these configurations we can remark that there does not exist a unique "right" immigration policy. These results can explain both why there is a so great diversity in immigration policies across countries, and why it is difficult to determine the best immigration policy. According to the parameters of the autarkic economy, a country should encourage¹¹ the inflow of one of the two types of individuals or dicourage all inflows. Moreover, Propositions 1, 2 and Theorem 3 allows us to determine when the immigration policy based on quotas is justified. Indeed, quotas are justified when the effects of mass inflows (see Theorem 3) are to lower the welfare of one type of agents and to increase that of the other type while small inflows are favourable to all natives (see Propositions 1 and 2).

Since the degree of selfishness of agents is not easily observable, a social planner cannot always base migration policy on this characteristic. However, there are at least three ways for host countries' governments to detect potential altruistic migrants. First, empirical studies show that the cost of an immigrant (for instance his level of welfare expenditures) differs according to his country of origin. For the United States, Borjas (1994) shows that the cultural habits and the mutual aid within the Puerto Rican community are such that the cost of a Puerto Rican immigrant is low. Such behaviors can be assimilated to altruism. Second, the savings behaviors of altruists and non altruists differ in our model. According to recent evidence by Carroll et al. (1999), an immigrant's savings depend on his ethnic origin. With the help of this study we can detect groups which are thriftier (or more altruistic). Lastly, it is well known that, with one-sided altruism a bequest-constrained household will under-invest in their child's human capital (see Drazen 1978 or Rangazas 1991). Hence, the educational attainment of an immigrant can be correlated to his altruistic motive.

6. Conclusion

We have considered an economy in which agents differ with respect to their degree of intergenerational altruism, their rate of time preference and their preference for leisure. This economy experiences a unique steady state equilibrium. However, three types of equilibria are possible according to the set of socio-economic characteristics of agents. Indeed, if their degree of altruism is too weak, altruists are constrained with respect to bequest. When the intensity of altruism is strong enough an altruist leaves an inheritance to his children and choose to work or not. It is necessary that non altruistic agents are impatient and numerous so that an altruist chooses not to work. In this case, savings of non altruistic agents are lower and a few altruists are in possession of a large share of capital.

In the rest of the paper we have studied the long-run consequences of immigration on the behavior and the welfare of natives. We have also exhibited the autarkic economies for which the immigration of one type of agents always improves or worsens the welfare of all natives. Interestingly, the immigration of impatient non altruists never worsens the welfare of natives if the bequest motive is operative under autarky.

Interesting extensions of this model include the analysis of temporary migration and the case of remittances to the country of origin. These points are still on the research agenda.

Appendix

Proof of Theorem 1

Step 1: Characterization of steady state from l^a and x.

If x > 0, (10), (12) and (13) imply $z = k/l = [(A\alpha\beta)/(1+n)]^{1/(1-\alpha)}$. According to (c), (d), (g), (13) and (14) if x = 0 we have:

$$z = \left(\frac{A(1-\alpha)[(p\gamma'l^a)/(\mu'+\gamma')+q\gamma]}{(1+n)(pl^a+q(\mu+\gamma))}\right)^{1/(1-\alpha)}$$
(15)

Hence, from l^a and x, equations $(a), (b), \ldots, (g), (13)$ and (14) define a unique vector $(c^e, d^e, s^e, l^e, c^a, d^a, s^a, l^a, k, l, w, r, x)$ denoted by $\mathcal{A}(x, l^a)$.

Step 2: Steady state where bequest motive is operative.

When x = 0, we have $l^a > 0$ and (11) implies: $s^a = [(1 - \gamma')wl^a - \mu'w]/\xi'$. According to this equation and (q) we have: $l^a = \mu' + \gamma'$.

Hence the vector $A = \mathcal{A}(0, \mu' + \gamma')$ is the only possible steady state with constrained bequests. Since $l^a = \mu' + \gamma'$, according to (13) and (15), $1 + n = (1 + r)\hat{\beta}$.

Moreover, according to (10) and (12), we have $\beta(1+r) \leq (1+n)$.

Hence, $\beta \leq \hat{\beta}$ is a necessary condition to A is a steady state with no bequests. It is easy to check that this condition is also sufficient.

Step 3: Steady state where labor supply of altruists is nil.

If the labor supply of altruist is nil, then bequests are positive.

According to (c), (d), (g) and (14) we then show that $l^a = 0$ implies

$$x = \Upsilon = \frac{q(\gamma' + \mu')\alpha A}{q(\gamma' + \mu'\beta)} \left(\frac{\alpha A\beta}{1+n}\right)^{\alpha/(1-\alpha)} ((\mu + \gamma)\beta - \gamma(\alpha^{-1} - 1))$$

Hence, the vector $B = \mathcal{A}(\Upsilon, 0)$ is the only possible equilibrium of the model with nil labor supply of altruists.

Since $\Upsilon > 0 \Leftrightarrow \beta > \bar{\beta}$, $\beta > \bar{\beta}$ is necessary so that *B* is a steady state. When $l^a = 0$, according to (e), (13) and Υ , condition (11) is equivalent to:

$$[p(\alpha^{-1} - 1)(\gamma' + \mu'\beta)]/(q(1 - \beta)[(\mu + \gamma)\beta - \gamma(\alpha^{-1} - 1)]) - \xi' \le 0.$$

Hence, $p \le \bar{p}$ is a necessary condition so that *B* is stationary equilibrium. According to definition 1, if $\beta > \bar{\beta}$ and $p \le \bar{p}$ then *B* is a steady state.

Step 4: Steady state with positive bequests and labor supply.

When x > 0 and $l^a > 0$, (11) implies $s^a = [(1 - \gamma')wl^a - \mu'w]/\xi' + x$. According to this equation and (g) we have $l^a = \mu' + \gamma' - \xi'(1 - \beta)x/w$. Hence, (g) becomes: $s^a = \gamma'w + (\gamma' + (1 - \gamma')\beta)x$. Substituting l^a , s^a , (c) and (d) in (14) we obtain:

$$z = \frac{p(\gamma'w + (\gamma' + (1 - \gamma')\beta)x) + q\gamma w}{(1 + n)[p(\mu' + \gamma' - \xi'(1 - \beta)x/w) + q(\mu + \gamma)]} = \left(\frac{A\alpha\beta}{1 + n}\right)^{1/(1 - \alpha)}$$

Hence, according to (13) and after computations we have:

$$x = \Gamma = \frac{\alpha A[(p(\mu' + \gamma') + q(\mu + \gamma))\beta - (\alpha^{-1} - 1)(p\gamma' + q\gamma)]}{p(\gamma' + (\xi' + \mu')\beta + \xi'(\alpha^{-1} - 1)^{-1}\beta(1 - \beta))} \left(\frac{\alpha A\beta}{1 + n}\right)^{\alpha/(1 - \alpha)}$$

$$l^{a} = \Theta = \mu' + \gamma' - \frac{\xi'(1-\beta)[(p(\mu'+\gamma') + q(\mu+\gamma))\beta - (\alpha^{-1}-1)(p\gamma' + q\gamma)]}{p(\alpha^{-1}-1)(\gamma' + (\xi'+\mu')\beta + \xi'(\alpha^{-1}-1)^{-1}\beta(1-\beta))}$$

Hence, the vector $C = \mathcal{A}(\Gamma, \Theta)$ is the only possible equilibrium of the model with positive bequests and labor supply of altruists.

Moreover, $x = \Gamma > 0$ if $\beta > \hat{\beta}$ while $l^a = \Theta > 0$ if:

$$\mu' + \gamma' > \frac{\xi'(1-\beta)[(p(\mu'+\gamma') + q(\mu+\gamma))\beta - (\alpha^{-1}-1)(p\gamma' + q\gamma)]}{p(\alpha^{-1}-1)(\gamma' + (\xi'+\mu')\beta + \xi'(\alpha^{-1}-1)^{-1}\beta(1-\beta))}$$
(16)

If $\beta > \bar{\beta}$, then (16) is equivalent to $p > \bar{p}$. If $\beta \leq \bar{\beta}$, (16) is always satisfied. We easily checked that if (i) $\hat{\beta} < \beta < \bar{\beta}$ or (ii) $\beta > \max(\bar{\beta}, \hat{\beta})$ and $p > \bar{p}$, then C is a steady state.

Step 5: Existence and uniqueness of steady state.

Since $\beta > \hat{\beta}$ is necessary to have a steady state of type C, equilibria of type C and A cannot coexist.

To have a steady state of type C it is necessary that either $\beta < \bar{\beta}$, either $p > \bar{p}$. Hence, steady states of type B and C cannot coexist.

The coexistence of equilibria of type A and B is impossible because if $\bar{\beta}$ < $\beta \leq \hat{\beta}$ then $p > \bar{p}$. To prove this result, assume that $\bar{\beta} < \beta \leq \hat{\beta}$. Since $\beta \leq \hat{\beta}$ we have:

$$p[(\mu + \gamma)\beta - (\alpha^{-1} - 1)\gamma + (\alpha^{-1} - 1)\gamma' - (\mu' + \gamma')\beta] \ge (\mu + \gamma)\beta - (\alpha^{-1} - 1)\gamma$$

Since $[\gamma'(\alpha^{-1} - 1)]/(\mu' + \gamma') > \beta$, this inequation is equivalent to $p \ge \hat{p}$ with:

$$\hat{p} = \frac{(1 - \beta)\xi'[(\mu + \gamma)\beta - (\alpha^{-1} - 1)\gamma]}{(1 - \beta)\xi'((\mu + \gamma)\beta - (\alpha^{-1} - 1)\gamma) + (1 - \beta)\xi'((\alpha^{-1} - 1)\gamma' - (\mu' + \gamma')\beta)}$$

Let
$$P(\beta)=(1-\beta)\xi'((\alpha^{-1}-1)\gamma'-(\mu'+\gamma')\beta)-(\gamma'+\mu'\beta)(\alpha^{-1}-1).$$
 Then $P(\beta)=A\beta^2-B\beta-C$ with $A=(\mu'+\gamma')\xi',\ B=\mu'\xi'+\xi'\gamma'\alpha^{-1}+\mu'(\alpha^{-1}-1),$ and $C=(\alpha^{-1}-1)\gamma'(\mu'+\gamma').$ Since $P(0)$ and $P(1)=(1-\alpha^{-1})\cdot(\mu'+\gamma')$ are negative and since $\lim_{\varrho\to-\infty}P(\varrho)=\lim_{\varrho\to+\infty}P(\varrho)=+\infty,$ we have $P(\beta)<0$ for all $\beta\in[0,1].$ Since $P(\beta)<0,$ we have $\hat{p}>\bar{p}.$ Hence, $p\geq\hat{p}$ implies $p>\bar{p}.$

Hence, the model experiences at most one steady state.

Since $\beta > \hat{\beta}$ is a necessary condition to obtain an equilibrium of type B it is easy to check that the model has always a steady state.

Proof of Theorem 2

With the notation of the proof of Theorem 1, we demonstrate the results of Sect. 4 by using two steps:

Step 1: Variations of Υ , Γ and T with respect to p.

 $\partial \Gamma / \partial p = -\overline{a}((\mu + \gamma)\beta - \gamma(\alpha^{-1} - 1)), \quad \partial \Upsilon / \partial p = -\overline{b}((\mu + \gamma)\beta - \gamma(\alpha^{-1} - 1))$ and $\partial \Theta/\partial p = \bar{c}((\mu + \gamma)\beta - \gamma(\alpha^{-1} - 1))$ with $\bar{a}, \bar{b}, \bar{c} > 0$. Hence, $sign\{\partial \Upsilon/\partial p\} =$ $sign\{\partial \Gamma/\partial p\} = sign\{\bar{\delta} - \delta\} \text{ and } sign\{\partial \Theta/\partial p\} = sign\{\delta - \bar{\delta}\}.$

Step 2: Analysis according to degrees of impatience.

The necessary and sufficient condition for operative bequest $\beta > \hat{\beta}$ is equivalent to $p\gamma'(\delta' - \bar{\delta}) + (1 - p)\gamma(\delta - \bar{\delta}) > 0$. Hence, if $\max(\delta, \delta') \le \bar{\delta}$, $\forall p = 0$ but if $\min(\delta, \delta') > \bar{\delta}$, $\forall p < 0$. Similarly, if $\delta < \bar{\delta} < \delta'$ we have $x > 0 \Leftrightarrow$ $p > p^*$ but if $\delta' < \bar{\delta} < \delta$, $x > 0 \Leftrightarrow p < p^*$. Since $\beta > \bar{\beta}$ is equivalent to $\delta > \bar{\delta}$, $l^a = 0$ if and only if $(p \le \bar{p} \text{ and } \delta > \bar{\delta})$.

Proof of Proposition 1

When bequests are operative, $R = (1 + n)/\beta$ and $w = A(1 - \alpha) \left(\frac{A\alpha\beta}{1 + n}\right)^{\alpha/(1 - \alpha)}$

According to (a), (b), and (d), c^e , d^e and \mathcal{L}^e depend only on w and R. Hence \bar{u} is independent on p (i.e. on migrations).

According to (e) and (f), if the type of the steady state with positive bequests is B (i.e. $l^a=0$) we have $c^a=[(1-\beta)\mu'x]/(\mu'+\gamma')$ and $d^a=[(1+n)\gamma'(\beta^{-1}-1)x]/(\mu'+\gamma')$. If the type of steady state is C, since $l^a=\mu'+\gamma'-\xi'(1-\beta)x/w$ we have $c^a=\mu'w+\mu'(1-\beta)x$ and $d^a=\gamma'w(1+r)+\gamma''=0$

 $\gamma'(1+n)(\beta^{-1}-1)x$. Hence, when the bequest motive is operative, \mathcal{L}^a , c^a and d^a are increasing function in x. Therefore $sign\{\partial \bar{v}/\partial p\} = sign\{\partial x/\partial p\}$. According to the proof of theorem 2, $\partial x/\partial p$ is positive if and only if $\delta <$ $\bar{\delta} < \delta'$.

Proof of Proposition 2

According to the behavior of non altruists described in Sect. 3 we have:

$$\bar{u} = (\alpha\mu + (2\alpha - 1)\gamma)\ln z + \ln((\mu A(1-\alpha))^{\mu}\xi^{\xi}(A^{2}\gamma\alpha(1-\alpha))^{\gamma})$$

Hence, $sign\{\partial \bar{u}/\partial p\} = sign\{(\delta + 2 - \alpha^{-1})(\partial z/\partial p)\}$. Since bequests are constrained, z is defined by (15). Hence, after computations, $sign\{\partial z/\partial p\} =$ $sign\{\delta - \delta'\}.$

Similarly, when altruists behavior as non altruists we have $\bar{v} = [(\alpha \mu' +$ $(2\alpha - 1)\gamma' \ln z / (1 - \beta) + constant$. Hence, $\partial \bar{v} / \partial p$ has the sign of $(\delta' + 2 - 1)\gamma' \ln z / (1 - \beta) + constant$. α^{-1}) $(\delta - \delta')$.

Proof of Theorem 3

According to Propositions 1 and 2 we can show that:

- 1) If $Min(\delta, \delta') > \bar{\delta}$ then $\partial \bar{u}/\partial p$ is nil and $\partial \bar{v}/\partial p$ is negative. 2) If $Max(\delta, \delta') \leq \bar{\delta}$ then $sign\{\partial \bar{u}/\partial p\} = sign\{(\delta + 2 \alpha^{-1})(\delta \delta')\}$ and

2) If
$$Max(\delta, \delta') \le \delta$$
 then $sign\{\partial \bar{u}/\partial p\} = sign\{(\delta + 2 - \alpha^{-1})(\delta - \delta')\}$ and $sign\{\partial \bar{v}/\partial p\} = sign\{(\delta' + 2 - \alpha^{-1})(\delta - \delta')\}$.

3) If $\delta < \bar{\delta} < \delta'$ then $sign\{\partial \bar{u}/\partial p\} = \begin{cases} sign\{-(\delta + 2 - \alpha^{-1})\} \text{ if } p \le p^* \\ 0 \text{ if } p > p^* \end{cases}$ and $sign\{\partial \bar{v}/\partial p\} = \begin{cases} sign\{-(\delta' + 2 - \alpha^{-1})\} \text{ if } p \le p^* \\ + \text{ if } p > p^* \end{cases}$
4) If $\delta' < \bar{\delta} < \delta$ then $sign\{\partial \bar{u}/\partial p\} = \begin{cases} 0 \text{ if } p < p^* \\ sign\{\delta + 2 - \alpha^{-1}\} \text{ if } p \ge p^* \end{cases}$ and $sign\{\partial \bar{v}/\partial p\} = \begin{cases} -\text{ if } p < p^* \\ sign\{\delta' + 2 - \alpha^{-1}\} \text{ if } p \ge p^* \end{cases}$ From these configurations, and since $\bar{\delta} > \alpha^{-1} - 2$, it is easy to check that \bar{u} and \bar{v} have the same variations if and only if one of the seven situations (i) to

and \bar{v} have the same variations if and only if one of the seven situations ((i) to (vii)) of Theorem 3 is satisfied.

Endnotes

- ¹ The only papers that explicitly combine altruists "à la Barro" and life cyclers in an OLG model are those by Cukierman and Meltzer (1989), Michel and Pestieau (1998), Muller and Woodford (1988), and Nourry and Venditti (2000).
- ² $\lim_{\rho \to 0} U_{\mathscr{L}}^{j}(c^{j}, \rho, d^{j}) = +\infty$ implies $\mathscr{L}_{t}^{j} > 0$.
- ³ An alternative specification would be to write $\beta = \beta'(1+n)$, β' being the factor of pure altruism and 1 + n the population growth rate. As explained by Buiter and Carmichael (1984), these two formulations are equivalent when the number of child per family is exogenous.
- ⁴ (a), (b), (c) and (d) are obtained from (1), (2), (4) and (5) while, since $\frac{(1+n)x}{1+r} = \beta x$, (6), (7), (10) and (12) give (e), (f) and (g).
- ⁵ Although intuitive this result only holds when the Diamond (1965) model without bequest motive experiences a unique and stable steady state (See, Thibault 2000).

⁶ From (6) and (7): $w(1-\mathcal{L}^a) + \left(\frac{r-n}{1+r}\right)x = c^a + \frac{d^a}{1+r}$. The life cycle income of an altruist is composed of his labor income $w(1-\mathcal{L}^a)$ and the return $\left(\frac{r-n}{1+r}\right)x$ of his patrimony. Since $Rx = \left(\frac{1+n}{\beta}\right)x$ this return is equal to $(1-\beta)x$.

⁷ $\delta^{-1} = \gamma/\mu$ is the discount factor of consumption. Since $\frac{1}{1+(\delta-1)} = \frac{\gamma}{\mu}$, $\delta-1$ can be considered as the rate of time preference for consumption.

- as the rate of time preference for consumption.
- ⁸ In the case of non altruistic agents, the pattern of international migration has been studied by Galor (1986) (inelastic labor supply) and Kochhar (1992) (elastic labor supply).
- 9 Since $l^a(w, R, x) = \mu' + \gamma' + \frac{\xi'}{w} \left(\frac{1+n}{R} 1\right) x$ and $l^a(w, \kappa) = \mu' + \gamma' + \frac{\xi'}{w} \kappa$ we have $\frac{\partial l^a}{\partial p} = -\frac{\xi'(1-\beta)}{w} \frac{\partial x}{\partial p} = -\frac{\xi'}{w} \frac{\partial \kappa}{\partial p}$.
- When $\min(\delta, \delta') > \bar{\delta}$, we have $p^* > 1$. Hence, for all $q \in [0, 1]$ we have $q > 1 p^*$.
- ¹¹ See Borjas (1995) for a study on the economic benefits from immigration.

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