Journal of Population Economics (2023) 36:1799–1827
https://doi.org/10.1007/s00148-023-00938-0

ORIGINAL PAPER

Demographic change and the rate of return in pay-as-you-go pension systems

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Received: 12 March 2021 / Accepted: 20 January 2023 / Published online: 13 March 2023

Abstract

The ongoing demographic change in most developed countries consists of two coinciding independent developments that differ in structure and persistence: A slow, monotonic and (presumably) permanent longevity effect caused by an increasing life expectancy; and a more rapidly changing, non-monotonic and less permanent cohort effect caused by fluctuations in the size of cohorts. This paper shows the longevity effect has a positive impact on the rates of return households generate within a pay-as-you-go (PAYG) pension system. The cohort effect, by contrast, results in winners and losers in PAYG systems. The paper additionally shows that the type of PAYG pension system alters the results significantly. Taking the remarkable demographic change in Germany as an example, a large-scale overlapping generation model quantifies rates of return within the PAYG pension system for every cohort. The results show that the two effects combined cause return differentials of almost 1.3 percentage points between generations.

Keywords Demographic change · Pension system · OLG models

1 Introduction

In most advanced economies, pay-as-you-go (PAYG) pension systems are under enormous pressure as the proportion of older people in the overall population is rising. This increase, also known as demographic change, is often regarded as a one-dimensional phenomenon. In reality, though, demographic change is a combination of several developments coinciding. The two most important causes are that people are living longer (longevity effect) and that large "baby boom" cohorts are currently close to retirement (cohort effect). These two effects differ substantially.

Responsible editor: Gregory Ponthiere

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The longevity effect is slow and monotonic and will (probably) be sustained. By contrast, the number of births per year and net migration can move in either direction and changes more rapidly over time. The impact of past birth rates and migration on the demographic structure diminishes in the long run, however.

This paper illustrates how a longer life expectancy and fluctuating cohort sizes differentially affect PAYG pension systems. More specifically, the paper asks who achieves higher returns due to these effects within a PAYG pension system. Furthermore, the paper answers the question of how the type of a PAYG pension system alters the results. Developing a sustainable solution for PAYG pension systems requires separating the two effects.

To answer these research questions, the paper decomposes the effect of demographic change into cohort and longevity effects, taking Germany as an example. A simple theoretical model provides intuition for the underlying mechanisms. It illustrates how the two effects, in general, alter the payment structure within various kinds of PAYG pensions systems. The paper then applies a large quantitative overlapping generation (OLG) calibrated to the German population. This model shows that differences between the rates of return of cohorts within different PAYG pension systems are economically sizeable. The model also decomposes the overall rate of return differential. It quantifies how much of the differentials stem from the longevity effect, the cohort effect and a third employment effect originating from the rising labour force participation.

Contrary to the widespread view, decreasing mortality rates have an unambiguously positive impact on the rates of return generated within a PAYG pension system. Holding the cohort size at birth constant, longer life of individuals results in a population increase. In turn, the population increase expands the budget of the pension system. In a PAYG pension system (permanent), expansions result in gains that are distributed depending on the specific pension system.

Varying cohort sizes, by contrast, result in winners and losers in PAYG systems. The key difference to the rise in life expectancy is that the impact of a large cohort is transitory. So each expansionary gain to the system is offset in the long run with a loss. However, cohorts benefiting from gains are not necessarily those bearing losses.

Overall, the demographic change in Germany has a positive impact on the rates of return participants realize in the German PAYG pension system. The distribution among the cohorts is unequal, with a difference of 1.3 percentage points per year over the whole life cycle. The cohort that benefited the most is the birth cohort of 1939, with an excess return of 1.2 percentage points compared to a world without demographic change. The birth cohort of 2026 will have a slightly negative excess return of −0.1 percentage points. The longevity effect drives most of the excess return. The maximum excess return due to the longevity effect realized by birth cohort 1937 is 0.8 percentage points. The cohort effect is responsible for a maximum excess return of 0.2 percentage points for the cohort born in 1936. The 2005 cohort loses out on returns to the tune of 0.4 percentage points. So the maximum return differential between cohorts is 0.6 percentage points compared with a demographic setting without cohort fluctuations.

To evaluate the welfare consequences of different pension systems and the interplay with the population scenario, I compute consumption equivalent variations (CEV). The CEVs are the percentage change in consumption over the life cycle required as compensation for individuals to be indifferent between the benchmark German pension system and alternative pension systems, i.e. fixed contribution and fixed benefits. For most cohorts, a pension system with fixed benefits would be better than the current German pension system. On the other hand, only a few cohorts would be better off under a pension system with fixed contributions instead of the current system.

This study is related to papers that investigate the role of demographic change in pension systems and its scope for reforms using microsimulation models, including Werding (2013) and Börsch-Supan et al. (2016). Fenge and Peglow (2018) identify the isolated effects of mortality, fertility and migration developments on the dynamics of the German pension system. These papers use budgetary linkages between the pension system and the rest of the economy to forecast pension variables and investigate the sustainability of specific PAYG pension systems. In contrast, this study shows how demographic change affects the economic benefits of PAYG pension systems from a generational perspective. This study also investigates this generational perspective under different forms of PAYG systems.

In this sense, this study relates to a strand of the literature that uses small-scale general equilibrium OLG models to derive analytical results for the effect of changes in demographic variables in PAYG systems. Gori and Fanti (2008) show how crucial the capital share in production is to determine whether prolonged longevity increases or decreases the pension level in PAYG systems. Fanti and Gori (2012) analyse how a change in fertility affects pensions. They find a falling fertility rate need not necessarily cause a reduction in the pension level in the long run. Building on this, Cipriani (2014) shows in a two-period OLG framework with exogenous fertility as well as endogenous fertility that increased longevity implies a reduction of pension payouts for those who survive until retirement. In a three-period OLG model, Cipriani (2018) shows that when retirement decisions are endogenous, the effect of longevity on payouts in PAYG pension systems is in general ambiguous. Dedry et al. (2017) show also in a two-period OLG framework that both the type of ageing, i.e. declining fertility or increasing longevity, and the type of pension system are important for capital accumulation and welfare.

This study adds to the literature by deviating from these papers in three ways. First, the applied model excludes any indirect effect that stems from behavioural responses of households or general equilibrium effects. Even though these endogenous reactions are important in a full economic analysis, they fundamentally rely on the specific model assumption, e.g. the degree of capital market integration. By ignoring general equilibrium and behavioural effects, this study determines the direct effects of the demographic change that would otherwise be disguised. This study also assumes that individuals cannot optimize their participation in the state-run PAYG pension system.[1]

[1] This is the case in most countries, cf. OECD (2018).

Second, a multi-period OLG model like in this paper is better suited to fully capture the demographic change observed in the data. The complex population structure of the model allows for a more detailed discussion of quantitative results. This is not equally feasible in the two- or three-period OLG framework of the above-mentioned studies used to obtain analytical results.[2]

Third, as with the first strand of literature, the discussed papers focus on the effect of changes in demographics on pension benefit levels or contribution rates at specific points in time. This paper takes a more general approach and calculates the rate of return each cohort makes within the PAYG system. This rate of return better reflects the life cycle of cohorts and it also takes the changing demographics into account, e.g. decreasing mortality risk.

This study describes the effects of different aspects of ageing on the rates of return within a PAYG pension system. It focuses on the interplay between demographic change, balanced intratemporal pension budgets and lifetime cash flows of individuals. Given the current structure of such a system, this paper evaluates the redistributional effect of the demographic change within PAYG pension systems.

The remainder of this analysis is organized as follows. In Section 2, the paper presents the empirical details on the demographic change in Germany. It contains a decomposition of the general demographic change into its components. In Section 3, a simple analytical framework illustrates the effects these demographic developments pose for different types of PAYG pension systems. In Section 4, a quantitative OLG model is presented that incorporates these effects in a more complex setting. The calibration of the quantitative model is shown in Section 5. Quantitative results for the rate of return within PAYG systems are then presented in Section 7 and welfare implications in Section 8. Section 9 discusses the limitations of the analysis and Section 9 concludes the paper.

2 Demographic change in germany

Demographic change is prevalent in all developed economies. For decomposing the overall demographic developments into longevity and cohort effects, the paper uses Germany as an example. It is especially suitable for the following analysis as Germany due to World War I and World War II exhibits a distinctly large baby boomer cohort that was born in the 1960s.

The main data source for the demographic process in Germany is the German Federal Office of Statistics (GFOS). For the period between 1960 and 2018, the actual German data for the age distribution and mortality rates are used. For the period between 2019 and 2060, the projections of the recent *14th Coordinated Population*

[2]There is a vast literature using large-scale OLG models to investigate welfare effects. Important examples are Auerbach and Kotlikoff (1987), Imrohoroglu et al. (1995), De Nardi et al. (1999), Attanasio et al. (2007), Krueger and Ludwig (2007), and Golosov et al. (2013), Kitao (2018). Papers that model the pension system in great detail are for example Ludwig et al. (2009), Börsch-Supan and Ludwig (2009), Ludwig and Reiter (2010), and Buyse et al. (2013) and Vogel et al. (2017). However, none of these studies focuses on the disaggregated demographic effects on the economic benefits.

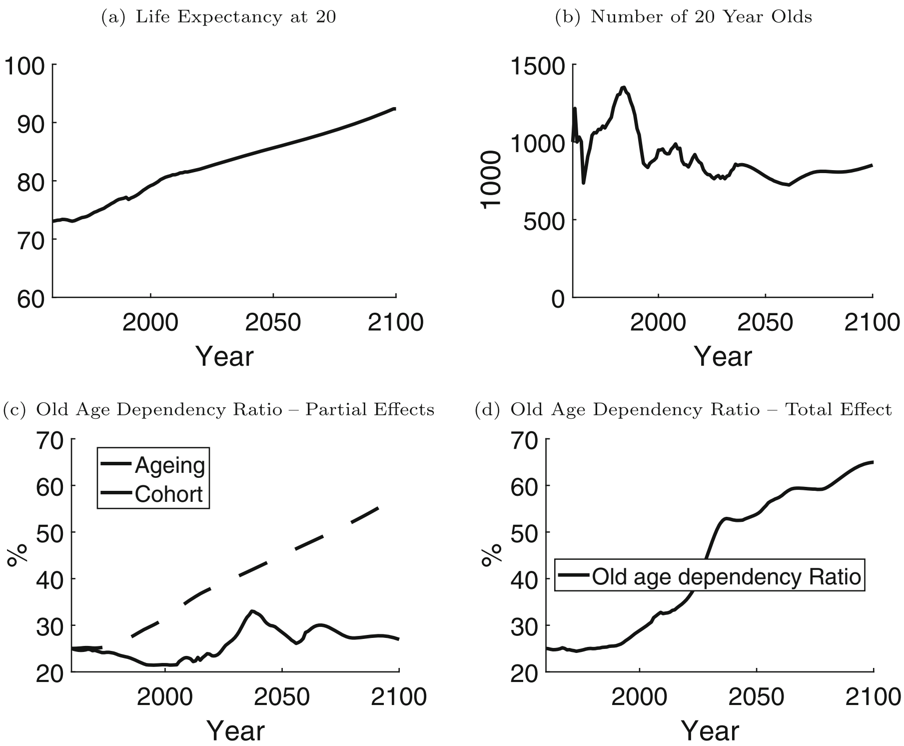


Fig. 1 Demographic change in Germany Panel (a) shows (averaged over men and women) the life expectancy of 20-year-olds. Panel (b) shows the number of 20-year-olds. Panel (c) shows the old-age dependency ratio, defined as persons older than 65 years to persons ages 20 to 65 years, for longevity (ageing) effect and cohort effect, separately. Panel (d) shows the overall old-age dependency ratio

Projection are used.[3] After the end of this projection in 2060, the mortality probabilities by age until 2100 are linearly extrapolated and kept constant afterwards. Migration and fertility rates are kept constant at the projected level of 2060.

The alteration of the age composition of the population (demographic change) currently being observed in Germany is essentially made up of two different, coinciding effects: a longevity effect and a cohort effect.

The longevity effect is reflected in an ever longer life expectancy. Life expectancy has risen in all developed countries with minor interruptions over the past 150 years. In 1960, life expectancy in Germany at the age of 20 was 73 years (Fig. 1(a)). Since then, it has increased by 9 years to nearly 82 years. The GFOS predicts that the average lifespan will continue to increase in future, too. Based on their projection, it will have gone up by a further 10 years by 2100. This further increase in life expectancy

[3] The 14th coordinated population projection includes various scenarios for the future trends of fertility, migration and mortality. The chosen assumptions are in the medium range of all scenarios (W2-L2-G2), cf. Statistisches Bundesamt (2019).

stems almost entirely from the declining mortality of people older than 65 years. Consequently, longer life expectancy translates into a growing number of people who are older than 65 years. Already, the probability of dying at a younger age is practically zero. The number of persons under the age of 65 years remains virtually unchanged by the longevity effect.

A varying birth rate primarily drives the cohort effect. Figure 1(b) shows the number of 20-year-olds in Germany over time. In the middle of the last century, there was a pronounced fluctuation in the cohort strength in Germany. The sharp drop at the beginning of the 1960s originates partly from a lower birth rate during World War II. The fact that the cohorts of childbearing age at this time were themselves reduced by low birth rates during World War I enhanced this effect. After this sharp decline, the number of people born exploded in the mid-1960s (baby boomers). At the beginning of the 1970s, the birth rate again plummeted sharply and stayed low for the next decades. In the baseline variant of its current population projection exercise, the GFOS assumes a broadly unchanged birth rate of 1.55 in the future. Both the sharp increase and the decline of cohort strength 50 years ago have led to a hump within the age distribution of the German population. At the beginning of their life cycle, relatively large cohorts make the overall population on average younger. This effect, however, reverses when the cohorts reach a more advanced age. Then, the baby boomer cohort disproportionally increases the average age of the overall population. So this hump plays an important role in understanding the currently observed demographic pressure.

Besides birth rates, cohort size is affected by migration. In recent years, there has been considerable net immigration in Germany. Over the past 10 years, this has amounted to an annual average of around 400,000 persons.[4] In the cited population projection, net migration falls to 206,000 persons per year by 2026 (corresponds largely to the long-run median). Afterwards, net migration remains constant. Migration is thus currently counteracting the effect of the low birth rate. With a net migration of 200,000 people per year and a birth rate of 1.55 children per woman, the German population will converge to 80 million people.

All three demographic factors thus have an impact on the ratio of older to younger people in a population. A commonly used metric for this ratio is the old-age dependency ratio (OADR). This is defined as the ratio of the population group aged over 65 years to that aged under 65 years. Figure 1(c) shows how longevity and cohort effect affect the OADR.

To separate the longevity effect, a population distribution is simulated where cohort sizes at age 20 years are identical over time. Therefore, the differences in the population age structure result solely from the decline in mortality rates. The longevity effect (blue dashed line) causes a monotonic increase in the OADR. This increase is also permanent.

[4] What is crucial for pension systems is the extent to which migration alters the number and structure of its contributor base and at a later date, the number and structure of pension recipients. Three things are of central importance: the age of those immigrating and emigrating, integration into the labour market and the impact on future demographic developments.

The variations in cohort size have a different effect (black solid line).[5] To separate the cohort effect, the population is simulated by taking the cohort size over time from the data and keeping the mortality rates constant (at the 1960 level).[6] The cohort effect on the old-age dependency ratio is, first, not monotonic and, second, only temporary. In the very long term, this effect vanishes entirely.[7]

Both effects impact the old-age dependency ratio of Germany (Fig. 1(d)). In 1960, the OADR was 25%. In other words, for every person of 65 years and above, there were roughly four persons of working age. This ratio remained virtually stable until 1990. The reason for that is that the longevity effect and the cohort effect offset each other. The longevity effect exerted upward pressure where the cohort effect itself lowered the OADR. After 1990, the OADR started to sluggishly increase and reached 35% in 2018. With the retirement of the baby boomer cohort, the sign of both effects align and the old-age dependency ratio sharply rises to 53% by 2037. At this point in time, for every person of statutory retirement age and above, there are then fewer than two persons younger than 65 years. Although life expectancy continues to rise, the baby boomer cohorts gradually die out. The pressure from the cohort effect vanishes. This briefly stabilizes the OADR between 2040 and 2050. However, the OADR then again starts to increase to values above 60% and further as long as the life expectancy rises.

3 Demographic challenges for PAYG systems

Developments in the old-age dependency ratio are of central importance to the financial situation of a PAYG pension system. While the longevity and the cohort effect affect the old-age dependency ratio in different ways, both seriously challenge the sustainability of PAYG pension systems. In the following section, a simple OLG model is used to illustrate how the discussed demographic effects vary (implicit) rates of return within a PAYG system. Additionally, it shows that these return differentials alter with the type of PAYG system.

3.1 A simple model

In a PAYG pension system, the implicit rate of return corresponds to the growth rate of the total wage bill. If the demographic structure remains constant, this return corresponds to productivity growth. Under the assumption of constant productivity growth, the returns within the PAYG pension system would be the same for all

[5] In the remainder of this paper, the effects of birth rate and migration are consolidated as the cohort effect.

[6] This separation is not perfect because the magnitude of the cohort effect depends on the mortality rates. If the life expectancy is higher, the cohort size matters over a longer period of time for the OADR.

[7] The size of a cohort affects the demographic structure beyond its lifespan. A large cohort has many descendants in absolute as well as relative terms. Assuming that each cohort were to have children in just 1 year of its life, a high one-off birth rate would continue indefinitely. However, since reproduction occurs over multiple years, cohort sizes even out again over time.

cohorts. Given uniform population growth, the return will rise/fall with the population growth rate. The simple model assumes that there is neither wage growth nor a (permanent) increase in newborns.[8]

Cohorts live J periods of equal length. At the beginning of their lives, cohorts work and contribute to the pension system. At age R, they retire and receive from then on a pension. The budget equation for the PAYG pension system is as follows:

$$\phi_t * w_t * L_t = \gamma_t * w_t * P_t \tag{1}$$

where ϕ_t denotes the contribution rate, w_t the wage rate, L_t the number of workers, γ_t the pension level, P_t the number of retirees. Equation (1) can be rearranged into

$$\frac{\phi_t}{\gamma_t} = \frac{P_t}{L_t} = \mathcal{Q}_t \tag{2}$$

where $\mathcal{Q}_t$ is the old-age dependency ratio. The simple analysis distinguishes between two types of PAYG systems, a fixed (or defined) contribution (FC) system and a fixed (or defined) benefit (FB) system.[9] In a fixed contribution PAYG system, the contribution rate is fixed, $\phi_t = \phi$. The (endogenous) pension level evens out the pension budget:

$$\gamma_t = \frac{\phi}{\mathcal{Q}_t}. \tag{3}$$

In a fixed benefit PAYG system, the pension level is fixed, $\gamma_t = \gamma$. The pension system budget now evens out the (endogenous) contribution rate

$$\phi_t = \gamma * \mathcal{Q}_t. \tag{4}$$

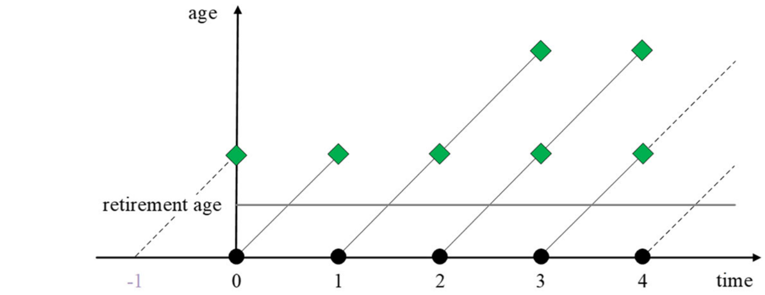
The rate of return a cohort born in period t realizes within the PAYG system is equal to the internal rate of return, i_t, of the inpayments/outpayments over its lifetime

$$\sum_{j=1}^{J} \frac{\left(\gamma_{t+j-1}\mathbf{1}_{\{j\geq R\}} - \phi_{t+j-1}(1-\mathbf{1}_{\{j\geq R\}})\right) w_{t+j-1}}{(1+i_t)^{j-1}} = 0. \tag{5}$$

If the population remains constant and $R = \frac{1}{2}J$, there are exactly as many workers as there are retirees. In this case, the old-age dependency ratio is $\mathcal{Q} = 1$ and the contribution rate corresponds to the pension level, $\phi = \gamma$. The qualitative results of the simple model are independent of the pension level in the initial steady state. In this numerical example, the pension level and the contribution rate in the initial state

[8] It is also assumed that the number of workers has no impact on wages. This is the case, for example in a small open economy.

[9] In OECD countries, both types exist, cf. OECD (2018). Public PAYG schemes follow fixed benefit rules in 17 OECD countries, e.g. Japan, Spain and Switzerland. Fixed contribution schemes exist in five OECD countries, e.g. Italy, Poland and Sweden.



∑ population	2	2	2	3	3
∑ retirees	1	1	1	2	2
∑ workers	1	1	1	1	1
Old-age dependency ratio	1	1	1	2	2

Fig. 2 **Longer life span and retirement age** Diagonal lines symbolize the life cycles of agents through age and time. Longer lines mean longer lives. Black dots denote years in working life. Green diamonds denote years in retirement. The table shows summary statistics of the population

($t = 0$) are set to 30%. As there is no population or productivity growth, the rate of return in the PAYG system is 0% in the initial and the final steady-state.

3.2 Longevity effect

An increasing life expectancy ceteris paribus increases the payout period of pensions. In a PAYG pension system, this leads to a distortion of the rates of return within the system. The contribution rate during the working life of a cohort is linked via the intratemporal budget constraint of the pension system to a then-existing shorter benefit period.

The Lexis diagram in Fig. 2 illustrates how increasing life spans affect the size and structure of a population. In each period one cohort enters the population. Cohorts born in or before $t = 0$ live for two periods. All cohorts born in or after $t = 1$ have a longer life span of three periods. This increase in life span maps into an increased population. In $t = 3$ the size of the population increases from two to three cohorts that simultaneously live in the economy. This population increase will induce an expansionary effect in the pension system. Similar to an introduction of a PAYG pension system, any expansion of PAYG systems generates gains for the system. As the increase in life spans is (by assumption) permanent, these gains will never be offset by losses. As these gains can be distributed among the pension system participants, a longer life span increases the rate of return within PAYG systems for some cohorts without decreasing it for others.[10]

[10]These expansionary gains do not vanish by adjusting the retirement age. When the retirement age is indexed to life expectancy, the longevity effect has no long-run impact on the contribution rate or the pension level. The adjustment of the retirement age, however, does not prevent fluctuations in the pension variables in the short run. These fluctuations map into different rates of return of cohorts. This shows that an adjustment of the retirement age does have distributional effects. However, it does not offset the expansionary gains of the longevity effect.

Table 1 Longevity effect with constant retirement age (Scenario 1)

Period		$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 4$
Old-age dependency ratio		1	1	1	2	2
Contribution rate	FC	30%	30%	30%	30%	30%
	FB	30%	30%	30%	60%	60%
Pension level	FC	30%	30%	30%	15%	15%
	FB	30%	30%	30%	30%	30%
Rate of return of	FC	0%	36%	0%	0%	0%
Cohort born in t	FB	0%	0%	61%	0%	0%

In the numerical example, the number of retirees increases in $t = 3$ to two. As the number of workers remains unchanged, the old-age dependency ratio increases to 2. Depending on the specific pension system either the pension level has to drop (FC) or the contribution rate has to increase (FB) (see Table 1). The pension level and the contribution rate deviate from their initial values for the entire future.

In terms of the rate of return within the pension system, it is for all cohorts either positive or zero. In an FC system, the cohort born in $t = 1$ profits from it. Their rates of return within the system are 36%. All cohorts born later have again a return of zero (equal to the wage growth of the model). In an FB system, the cohort born in $t = 2$ profits from the longevity effect. The rates of return are even higher than in an FC system. The reason for this is that in an FB system, the size of the PAYG system increases even more than in an FC system. The gains are proportionate to the expansion and therefore higher in an FB system.

3.3 Cohort effect

In a PAYG system, cohort sizes influence the returns of those cohorts. A difference to the longevity effect is that the cohort effect has in PAYG systems not just beneficiaries but also losers. Here, it is especially important to distinguish between a fixed benefit system and a fixed contribution system.

In the following numerical example (Table 2), in $t = 1$ (only), twice as many people are born than in any other period. Life expectancy is $J = 2$ and the retirement age is $R = 2$ for all cohorts. The rate of return simplifies to

$$i_t = \frac{\gamma_{t+1}}{\phi_t} - 1. \tag{6}$$

In a fixed contribution system, belonging to a large cohort (here born in $t = 1$) is a disadvantage. In the contribution period, large cohorts have the same per capita burden as all other cohorts. Because there is in $t = 1$ a large number of contribution-payers, the pension system has a higher income. The PAYG system distributes this income in the same period; the pension level doubles to 60%. This increase raises the return for the cohort born in period $t = 0$ to 100%. Cohort effects in PAYG

Table 2 Cohort effect in PAYG systems

Period		$t = 0$	$t = 1$	$t = 2$	$t = 3$
Workers		1	2	1	1
Retirees		1	1	2	1
Old-age dependency ratio		1	0.5	2	1
	FC	30%	30%	30%	30%
Contribution rate	FB	30%	15%	60%	30%
	PF	30%	30%	30%	30%
	FC	30%	60%	15%	30%
Pension level	FB	30%	30%	30%	30%
	PF	30%	30%	30%	30%
Rate of return of	FC	100%	– 50%	0%	0%
Cohort born in t	FB	0%	100%	– 50%	0%
	PF	0%	0%	0%	0%
Pension fund	PF	0	15% w L	0	0

Note: The size of the pension fund in period $t = 1$ depends on the wage rate and the number of workers

pension systems are zero-sum games. That means that if there are winners there must be losers. Because the cohort born in $t = 2$ is again of size one and the large cohort is now in retirement, the pension level must fall in period $t = 2$ to 15%. It is now even lower than its initial level. The return for the cohort born in $t = 1$ declines as a result. For all subsequent cohorts, the return is again at the initial value of zero.

A fixed benefit system is advantageous for relatively large cohorts. Based on a large number of contribution payers, the contribution rate, and hence per capita contributions, can be lower during the contribution period. Despite the large number of recipients, the pension level is guaranteed during the pension period. This ultimately leads to a high return for the cohort born in $t = 1$. Similar to the FC system, the gain of one cohort comes at a cost of another. In this system, the cohort born in $t = 2$ has to bear the brunt. This cohort has to fund greater pension expenditure through increased contributions. However, it cannot expect to receive a higher pension level.

In contrast to the longevity effect, the return differentials owing to the cohort effect can theoretically be evened out. However, the problem cannot be fully eliminated by adjusting the contribution rate, the pension level or the retirement age. The risk of belonging to a relatively large/small cohort can only be eliminated by a structural reform of the PAYG system. One approach would be no longer requiring the pension system to run a balanced budget each period. The budget equation of the pension system would then be

$$\phi_t * w_t * L_t = \gamma_t * w_t * P_t + \Delta D_t \tag{7}$$

where ΔD is the change in a potential pension fund.

The basic idea is this: as long as a relatively large cohort is working, any surpluses accrued are not paid out to current retirees (of whom there are relatively few). Instead, these pension system surpluses are saved up. When the relatively large cohort enters the pension payment period, the surplus saved up previously will finance the higher level of expenditure. Once the relatively large cohort dies, the pension fund would be used up. The pension system is thus partially funded by capital. In a PAYG system with a pension fund (PF), the return is independent of the size of the cohort. All cohorts receive a return that is equal to wage growth. The size of the cohorts only determines whether the pension fund holds assets or liabilities. In the simple model presented here, the change in the pension fund corresponds to

$$\Delta D_t = (1 - Q_t) * \phi_t * w_t * L_t. \tag{8}$$

In the example figures selected, the pension fund would amount to 15% of the total wage bill at the end of the first period. At the end of the second period, the pension fund would be used up again.

4 A quantitative OLG model

The simplified problem presented in the previous section is now embedded in a quantitative model. The quantitative model contains a far more complex population structure. The rates of return determined in such a quantitative model provide more realistic estimates than the simple calculations above.

Besides the population age structure, pension systems are also affected by labour force participation (LFP) rates.[11] A change in labour force participation resembles the cohort effect. Here, it is important whether the observed increase in labour force participation is permanent or not.

Additionally, the quantitative model allows for more complex PAYG systems such as the German statutory pension system (Deutsche Rentenversicherung, or DRV) to be analysed. The DRV, which is also funded on a PAYG basis, is neither a pure FC nor a pure FB system. Its contribution rate and pension level are not fixed but have a reciprocal effect on one another. However, the DRV, too, features the above-described redistributive effects owing to the increase in life expectancy and varying cohort sizes.

[11] Pension systems depend also on the share of employment subject to social security contributions. This share may equally hinge on demographics but is not explored further in this paper. In the subsequent quantitative analysis, the share of employment subject to social security contributions is constant over time.

The following set-up is a small open economy overlapping generations model.[12] It is based on the work of Auerbach and Kotlikoff (1987) and its adoption by Schön (2020). It composes utility-maximising households, profit-maximising firms and a PAYG pension system.

4.1 The demographic model

The demographic process is taken as exogenous and represents the main driving force of the model. Several cohorts that can be of varying sizes live simultaneously in the model economy. A single cohort, c, per se is homogeneous and consists of identical households. At any point in time, t, the various cohorts are at different stages of life: households go through a life cycle in which they first work and then retire. At the end of each period, there is a given probability that households will die.[13] The older the household, the greater is this probability. Households die with certainty at age J^T. Cohorts born later have a higher life expectancy. Note that point in time, t, and age of a household, j, uniquely determine its cohort, $k = t - j + 1$.

The size of the population of age j in period t is given recursively

$$N_{j,t} = N_{j-1,t-1}\pi_{j-1,t-1} + Z_{j,t}, \tag{9}$$

where $\pi_{j,t}$ denotes the age and time-specific conditional survival rate and $Z_{j,t}$ is the net flow of people to Germany in a given period.[14]

Each year sees the entry of a new cohort. In each period newborns are determined by

$$N_{1,t} = \frac{1}{J^F}\sum_{j=1}^{J^F}\frac{N_{j,t-20}}{2} * f_{t-20}, \tag{10}$$

where J^F is the maximum age a woman is assumed to bear children f_t is the fertility rate per woman over life.

[12]The small open economy setting neglects changes in factor prices. Changing factor prices in a closed (or larger open) economy due to demographic change (labour scarcity and abundance of capital) affect the rates of return of the PAYG pension system. Ageing economies exhibit increasing wages and lower capital returns. Higher wages also increase the benefits of most pension systems and therefore alter their rate of return. Welfare effects stemming from this are analysed for example in Krueger and Ludwig (2007). This paper focuses on the effect of demographic change on the individual rates of return via the necessary balanced budget of PAYG systems. For this reason, the factor price channel is shut down by modelling a small open economy with a fixed world interest rate.

[13]The following analysis does not distinguish between men and women. The age-specific mortality rates are the average of female and male mortality rates.

[14]For computational reasons, migrants enter the economy with the same amounts of assets and earnings points that households of the same age that already live in Germany possess.

4.2 The pension system

The PAYG pension system is characterized by a contribution rate, ϕ_t, and a pension level, γ_t. The budget of the PAYG pension system is balanced at any time t,

$$\phi_t w_t \sum_{j=1}^{J_k^R-1} l_{j,t} N_{j,t} = \sum_{j=J_k^R}^{J^T} b_{j,t} p_{j,t} N_{j,t} \tag{11}$$

with

$$b_{j,t} = \begin{cases} 0 & \text{if } j < J_k^R \\ b_t = \gamma_t (1-\phi_t) w_t \frac{1}{J_k^R-1} & \text{if } j \geq J_k^R \end{cases}. \tag{12}$$

On the revenue side, w_t denotes the wage rate. Individual labour supply from households denotes as $l_{j,t}$. On the expenditure side of the pension budget equation are pension payments. Pensions are defined by an earnings point system. The paid-out pension is calculated by multiplying the number of acquired earnings points, $p_{j,t}$, with the pension value, $b_{j,t}$. The pension value consists of the replacement rate, γ_t, times the wage (after pension contributions) at time t divided by $J_k^R - 1$, the number of years in a standardized working life. In each period of its working life, a fully working household would collect one earnings point.

$$p_{j+1,t+1} = \begin{cases} l_{j,t} + p_{j,t} & \text{if } j < J_k^R \\ p_{j,t} & \text{if } j \geq J_k^R \end{cases} \tag{13}$$

Rewriting the budget constraint of the pension system gives

$$\phi_t w_t L_t = \gamma_t (1-\phi_t) w_t P_t \tag{14}$$

with $L_t = \sum_{j=1}^{J_k^R-1} l_{j,t} N_{j,t}$ defined as the number of contributors and $P_t = \frac{1}{J_k^R-1} \sum_{j=J_k^R}^{J^T} p_{j,t} N_{j,t}$ defined as the number of retirees.

In the following quantitative exercise, three pension systems are analysed. They differ in how the pension level and the contribution rate are determined.

Fixed benefit system In a fixed benefit system, the pension level is set to a constant level

$$\gamma_t = \overline{\gamma}. \tag{15}$$

The contribution rate is determined endogenously to balance the pension system's budget.

$$\phi_t = \left[1 + \frac{L_t}{\overline{\gamma} P_t}\right]^{-1}. \tag{16}$$

Fixed contribution system In the fixed contribution system, the contribution rate is fixed.

$$\phi_t = \overline{\phi}. \tag{17}$$

To balance the budget, the pension level has to adjust accordingly

$$\gamma_t = \frac{L_t}{P_t} \frac{\overline{\phi}}{1-\overline{\phi}}. \tag{18}$$

German pension system In the German pension system (DRV), the pension (value) annual adjustment is determined according to the following formula

$$b_t = b_{t-1} \frac{w_t}{w_{t-1}} \frac{1-\phi_{t-1}}{1-\phi_{t-2}} \left[\left(1 - \frac{Q_{t-1}}{Q_{t-2}} \right) \times 0.25 + 1 \right] \tag{19}$$

with retiree ratio, Q_t, defined as the ratio of retirees to contributors

$$Q_t = \frac{P_t}{L_t}. \tag{20}$$

It can be seen as a summary statistic of the demographic and labour market developments.[15] The adjustment formula for the replacement rate is obtained by inserting (12) into (19)

$$\gamma_t = \gamma_{t-1} \frac{1-\phi_{t-1}}{1-\phi_{t-2}} \left[\left(1 - \frac{Q_{t-1}}{Q_{t-2}} \right) \times 0.25 + 1 \right] \frac{1-\phi_{t-1}}{1-\phi_t}. \tag{21}$$

The contribution rate is determined endogenously so that the pension system's budget constraint is balanced in each period[16]

$$\phi_t = \left[1 + \frac{L_t}{\gamma_t P_t} \right]^{-1}. \tag{22}$$

4.3 The firm sector

Firms produce with a Cobb-Douglas production function employing capital and labour

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{23}$$

where K_t denotes the aggregate capital stock, L_t the aggregate labour input at time t. The output elasticity of capital is α. The total factor productivity (TFP) level is A_t and its growth rate is $\mu = \frac{A_{t+1}}{A_t} - 1$.

Aggregate labour input is the labour supply of households, $l_{j,t}$, times the population structure at time t, $N_{j,t}$.

$$L_t = \sum_{j=1}^{J^R-1} l_{j,t} N_{j,t} \tag{24}$$

A static firm maximises profits subject to capital accumulation condition

$$K_{t+1} = (1-\delta) K_t + I_t \tag{25}$$

where I_t is net investment, δ is the capital depreciation rate.[17]

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[15] The retiree ratio is closely related to the old-age dependency ratio, which relates the population share above and below the statutory retirement age. An increase in the OADR due to the demographic change also realizes in Q.

[16] This is a deviation from the German pension system which has a fluctuation reserve. The actual adjustment rule is that the contribution rate must be raised if the fluctuation reserves would otherwise fall below their minimum permissible size. In light of the demographic situation, the reserves are likely to dwindle from their current high level to their minimum over the next few years.

[17] Capital adjustment costs in the firm sector are not considered.

The first-order conditions from profit maximization give standard expressions for equilibrium factor prices. The return on capital is given by

$$r_t = r_t + \delta = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} = \alpha \frac{Y_t}{K_t}. \tag{26}$$

Wages are given by

$$w_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha} = (1-\alpha) \frac{Y_t}{L_t}. \tag{27}$$

4.4 The household sector

By choosing an optimal consumption path, each cohort c maximizes at any age j and point in time $t = c + j$ the sum of discounted future utility. The within-period utility function exhibits constant inter-temporal elasticity of substitution and preferences are additive and separable over time. Cohort c's maximization problem at $j = 1$ is given by

$$\max_{\{c_{j,t}\}_{j=1}^{J^T}} \sum_{j=1}^{J^T} \beta^j s_{j,t} U\left(c_{j,t}\right) \tag{28}$$

where β is the pure time discount factor. In addition to pure discounting households discount future utility with their unconditional survival probability, $s_{j,t+j} = \prod_{m=1}^{j} \pi_{m-1,t}$. $c_{j,t}$ denotes consumption.

All assets (including return on capital) of households that died at the end of one period are passed over to the next period's younger households. So in each period households up to a specific inheritance age, J^Q, receive bequests

$$q_{j,t} = \begin{cases} \frac{(1+r_t)\sum_{i=1}^{J^T}(1-\pi_{i,t-1})a_{i,t-1}N_{i,t-1}}{\sum_{i=1}^{J^Q}N_{i,t}} & \text{if } j \leq J^Q \\ 0 & \text{if } j > J^Q \end{cases}. \tag{29}$$

Denoting household assets by $a_{j,t}$, maximization of the household's inter-temporal utility is subject to a dynamic budget constraint given by

$$a_{j+1,t+1} = (1 + r_{t+1})\left(a_{j,t} + q_{j,t} + y_{j,t} - c_{j,t}\right). \tag{30}$$

Income, $y_{j,t}$, consists of labour income and pension income

$$y_{j,t} = (1 - \phi_t) w_t l_{j,t} + b_{j,t} p_{j,t}. \tag{31}$$

Households supply labour, $l_{j,t}$, exogenously. The labour supply varies over time and age.

4.5 Definition of equilibrium

Given the exogenous population distribution and survival rates in all periods $\{N_{j,t}, \pi_{j,t}\}$ and a world interest rate, $\overline{r}_t$, a competitive equilibrium of the economy is defined as a sequence of dis-aggregated variables, $\{c_{j,t}, a_{j,t}\}$, aggregate variables, C_t, L_t, K_t, a wage rate, w_t, and government/pension policies, $\{\phi_t, \tau_t^y\}$ such that

1. Given initial conditions household maximize utility with $c_{j,t}$ as the resulting optimal policies.
2. Pension policies satisfy Eq. (14) in every period.
3. Wages satisfy (26) and (27).
4. Markets clear and allocations are feasible in all periods

$$L_t = \sum_{j=1}^{J_k^R-1} l_{j,t} N_{j,t} \tag{32}$$

This small open economy setting requires that the rate of return on investment is equalized to the world interest rate,

$$r_t = \overline{r} \tag{33}$$

The net foreign assets, defined as the difference between assets and the capital stock is,

$$F_{t+1} = \sum_{j=1}^{J^T} a_{j+1,t+1} N_{j,t} - K_{t+1} \tag{34}$$

The goods market clears

$$G_t + C_t + I_t + \delta K_{t+1} = Y_t + (r - \delta) F_t \tag{35}$$

5 Calibration

The calibration aims to match the German economy and specifically its demographic structure. Calibration of the model requires (i) data for the exogenous demographic processes and (ii) the determination of values for several structural model parameters. The model period is 1 year and it is concentrated on the economic life of agents. Therefore, households enter the model at the biographical age of 20 which is model age of 1. The maximum biological age is 109, in model terminology, $J^T = 90$. By assumption, Germany was in a steady state in 1960. The population projections and their parameters are already discussed in Section 2.

5.1 Labour force participation and retirement age

For this analysis, it is necessary to see how labour force participation (LFP) for each cohort developed over time. The historical data used for the labour force participation are taken from the historical data set of the GFOS and are based on the German micro-census. Figure 3(a) shows the LFP for different cohorts over their life cycle. Solid lines represent actual data whereas dotted lines represent own projections (see below).

All LFP profiles show a pronounced hump-shaped pattern. LFP increases between the beginning of working life between age 20 and age 30. The LFP profiles reach

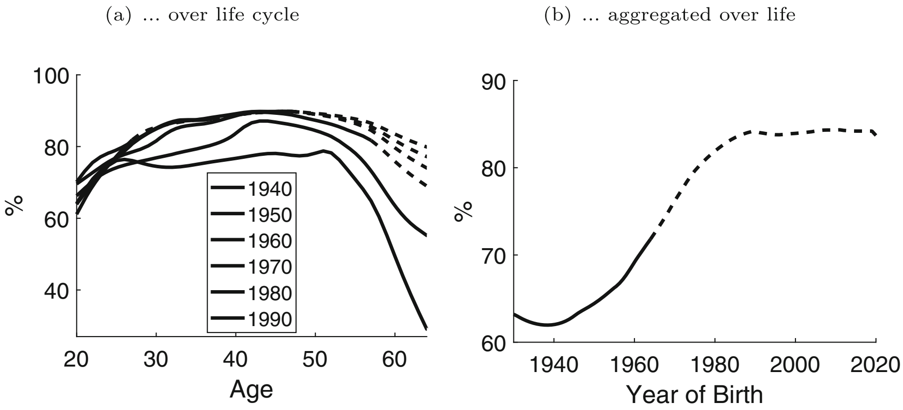

Fig. 3 Cohort labour force participation Panel (a) shows labour force participation for different cohorts over their life cycle. Solid lines represent actual data taken from GFOS; dotted lines represent projections. Panel (b) shows the sum of labour force participation over the life cycle. Solid lines are calculated for actual data only; the dotted line includes actual data and projections

their respective maximum between the age of 40 years and the age of 50 years. After the age of 50 years, the LFP falls until the end of the working life.

Over time, the hump-shaped profiles rotate and have an overall higher level of labour force participation. Employment among those below the age of 30 is lower and decreases over time.[18] The increase in the LFP between age 30 and age 50 was sustained almost exclusively by a higher level of female employment. While the labour force participation of males was stagnating at a high level, the female labour force participation rate rose especially after the age of 30.[19]

At the end of working life, especially between 55 and 65, disability and early retirement (for other reasons) explain the low level of employment. The employment rate between 55 and 65 reveals a marked upward trend over time. This could be a sign of improved health within the population. Another reason might be the reduced pressure to retire early owing to the considerably improved situation in the labour market. But it might also be the result of the more restrictive regulations on early retirement, cf. Bodnar and Nerlich (2020). While no more than roughly 40% of women aged between 55 and 60 were in employment in 1991, the figure was already just under 80% in 2015. The gradual convergence since 2000 of the statutory retirement age for women towards that for men is also likely to have played a part in this.[20]

[18]One explanation for this might be that many persons at this age are still enrolled in an institute of higher education. This means that the trend might be a sign of an increasingly large segment of the population attaining a high level of formal education.

[19]One reason for the higher level of female employment along with improved childcare facilities may have been social reforms which made it less attractive to leave the labour force at a younger age.

[20]The fraction of people older than 65 years also rises over time. In 2010, only 3.4% of those over 65 were working. In 2016, this figure was as high as 6.4%. The fact that the statutory retirement age was raised from 65 years to 65 years and 5 months during this period is likely to have had a significant impact in this context.

The basis of the extrapolation assumed in the model is the observed labour force participation of the cohorts (Fig. 3(a) solid lines). There are three main assumptions on future LFP development based on the observed past trends. First, the labour force participation rate decreases in younger years. Secondly, the labour force participation rate in the middle period of life increased in the past but now stabilises around 90%. And, thirdly, there is a strong and continuing increase in LFP between the age of 50 and 65.

On aggregate, these assumptions lead to an increase in the overall LFP over time. The black solid line in Fig. 3(b) shows the sum of labour force participation over the life cycle (age 20 – 65) for different birth cohorts. Solid lines are calculated with actual data only and dotted line signals that actual data and projections are used. The labour force participation increases from 64% for the 1940 cohort to almost 84% for the cohort born in 1990. The assumptions made result in more or less constant life cycle labour force participation for all cohorts after 1990.

The statutory retirement age, J_k^R, of each household is set to $J_k^R = 45$ (age 65 years).

5.2 Technology & preferences

The TFP parameter A determines the level of output and is calibrated to match the gross value added in 1960 of Euro 185bn.[21] The TFP growth rate and the world interest rate are set to zero $r_t = \mu_t = 0$. The production elasticity of capital is calibrated such that the model matches the labour income share in Germany's national accounts in 1960, $1 - \alpha = 1 - 60\% = 0.4$.[22] The model assumes a capital to output ratio of 2.5. This capital to output ratio will be attained in the model by appropriate calibration of the preference parameters. Using data on output, capital, and national income, VE_t, the implied yearly depreciation rate is $\delta = \frac{Y_t - \text{VE}_t}{K_t} = 4.1\%$.

The within period utility function is given by

$$U\left(c_{j,t}\right) = ln\left(c_{j,t}\right) \tag{36}$$

Bequests are distributed to households within the first 10 periods of their life, $J^Q = 20$.

6 Rate of return within PAYG systems

The quantitative model allows us to calculate the return generated by cohort c within the PAYG system. It compares the contributions that a cohort makes over its lifetime

[21] The basis of this calibration is the GDP of West Germany. It is then extrapolated to also account for the GDP of East Germany.

[22] See Grömling (2006).

with the benefits that a cohort receives. The return in a PAYG system is calculated as the internal rate of return produced by the following equation:

$$\sum_{j=1}^{90} \pi_{j,k} \frac{\mathcal{C}_{j,k+j} - \mathcal{B}_{j,k+j}}{(1+i_k)^{j-1}} = 0 \tag{37}$$

where $\pi_{j,k}$ stands for the unconditional survival probability of cohort c becoming j years old, $\mathcal{C}$ are contributions to the pension system, and $\mathcal{B}$ are benefits from the pension system.

6.1 Full demographic change

Figure 4 first maps the key variables of the pension system in the context of complete demographic change (longevity effect + cohort effect + changing labour force participation). The general pattern in all described PAYG systems is that between 1960 and 2004 pension systems benefit from the demographic change. Afterwards, PAYG pension systems experience enormous financing pressure. This pressure is especially strong in the period between 2020 and 2035. In an FB system, this pressure is borne by the contribution rate. Initially, demographics and increased labour force participation result in a slightly lower contribution rate. In the long run, however, the contribution rate has to more than double to balance the pension budget. In an FC system, the endogenous pension level initially increases. As time progresses the financing pressure due to the demographic change cuts the pension level in half in the long run. In the DRV, the pension level eventually decreases by a third and the contribution rate almost doubles.

Figure 4(c) shows the implicit rates of return cohorts realize within the PAYG pension systems. The sign and the magnitude of the rate of return vary over time and depend on the design of the particular pension system. In the long term, in all PAYG systems rates of return converge towards wage growth (here 0). It is however noteworthy that for the majority of cohorts, the demographic change has a positive overall impact on their returns. In an FB system, all cohorts benefit from the ageing population and the increase in labour force participation. The cohort that profits the

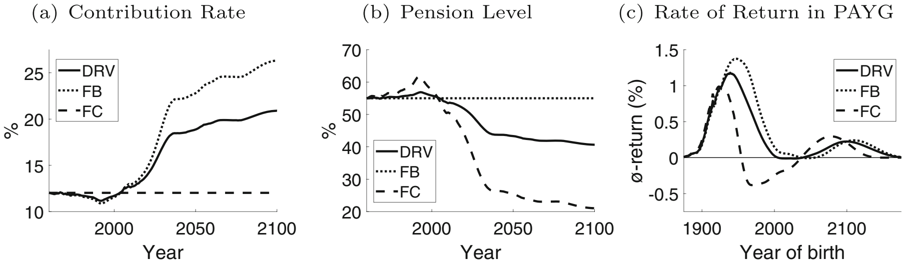

Fig. 4 Full demographic change Panel (a) shows the contribution rate to the pension system. Panel (b) shows the pension level. Panel (c) shows the implicit rate of return in PAYG systems. Black solid line: DRV; red dotted line: FB system, green dashed line: FC system

most is born in 1948 with a return of 1.4%. The returns decrease for cohorts that are born later and will be almost neutral for the cohort born in 2035.

In an FC system, it is especially profitable to be born early. The maximum return in an FC system is 1.0% for cohort 1924. Afterwards, the rates of return decrease as in the FB system. Contrary to the FB system, in an FC system, the returns of cohorts born between 1955 and 2037 are even negatively affected. The return of the cohort of 1969 is −0.4%. The course of the rates of return of the German DRV system confirms its hybrid nature. The cohort born in 1939 benefits the most from demographic change. It is 1.2 percentage points higher than without the demographic change.

6.2 Longevity effect

For the most part, the overall effect is determined by the longevity effect. To isolate the longevity effect, now all cohorts have the same size (the size of the 1960 cohort). The demographic structure only changes owing to the rising life expectancy as projected by GFOS. The old-age dependency ratio thus increases monotonically over time and, from 2100, remains at a consistently high level. Additionally, the life cycle profile of labour force participation is kept constant over time (LFP of the 1960 cohort).

Figure 5 shows the pension variables as they would develop only based on the longevity effect. The rising and then constantly high life expectancy result in a higher number of retirees. This leads to a constantly increasing contribution rate in an FB system and a constantly declining pension level in an FC system. The German pension system occupies the middle ground here, with a rising contribution rate and a declining pension level. The shown trend would continue as long as life expectancy increases.

In all PAYG systems, the return is positive for all cohorts, with a maximum return of 1.0% for the 1939 cohort in the fixed benefit system. The reason for this positive return is that longer life expectancy increases the amount paid out under the PAYG system. Similar to the introduction of a PAYG system, a permanent expansion creates gains. These gains are paid out to households over time, thus raising the implicit returns in the PAYG system. In an FC system, the expansion of the system is much

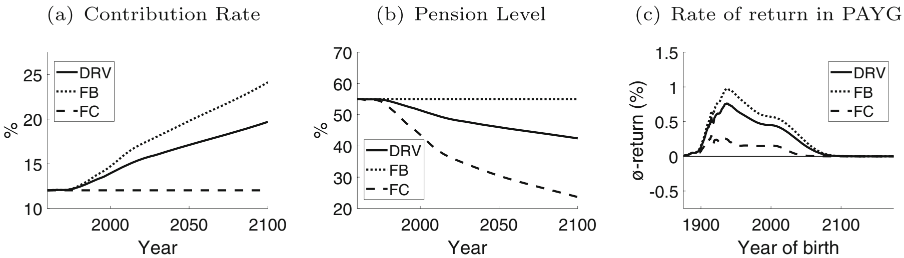

Fig. 5 **Longevity effect** Panel (a) shows the contribution rate. Panel (b) shows the pension level. Panel (c) shows the implicit rate of return in PAYG systems. Black solid line: DRV; red dotted line: FB system, green dashed line: FC system

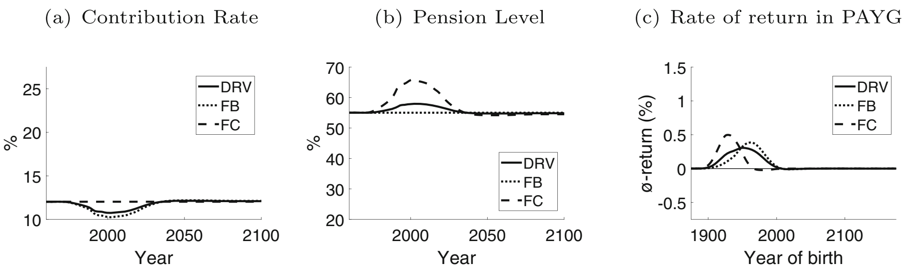

Fig. 6 Labour force participation effect Panel (a) shows the contribution rate. Panel (b) shows the pension level. Panel (c) shows the implicit rate of return in PAYG systems. Black solid line: DRV; red dotted line: FB system, green dashed line: FC system

smaller and therefore the additional return is lower. The maximum additional return would be 0.3% for the 1934 cohort.

6.3 Labour force participation effect

In this section, the labour force participation effect is isolated. This is achieved by assuming that the mortality rates and therefore the life expectancies are identical for all households to that of the cohort born in 1960. It is also assumed that all cohorts have the same size (the size of the 1960 cohort). Only the labour force participation varies over time (Fig. 6).

An increase in LFP is for pension systems equivalent to an increase in the cohort size. The main difference to a fluctuation in cohort size is that the labour force participation is assumed to stay high and not bounce back.[23] In this sense, the increase in LFP expands the PAYG system permanently and generates gains like the longevity effect. Contrary to the longevity effect, the pension level and the contribution rate are not disturbed in the long run. The reason for this is that higher LFP increases eventually the claims against the pension system.

The LFP effect on the implicit rates of return is also similar to the longevity effect. No cohort loses due to the LFP effect and some cohorts benefit from it. Depending on the pension system, the magnitude of the benefits differs. In an FB system, the benefits are distributed broader over cohorts. The maximum return is 0.4% for the cohort born in 1962. In an FC system, cohorts born early benefit more than in the FB system. Here, the maximum implicit rate of return is 0.5% for the cohort born in 1929.

6.4 Cohort effect

In the following section, the cohort effect will now also be analysed separately. It is assumed that the mortality rates and labour force participation of all households are

[23] A different result would occur if the observed increase in employment is not permanent. In this case, the LFP effect resembles in terms of the rate of return the cohort effect.

identical to that of the cohort born in 1960. The size of the relevant cohorts is taken from the data and the projection of the GFOS.

The contribution rate and the pension level are not monotonic. For the fixed benefit system, the endogenous contribution rate is initially decreasing. Beginning in the late 1990s, it starts to increase until 2037. Then, it converges back to its initial value. A mirrored but reversed pattern shows the endogenous pension level in a fixed contribution system. It initially increases and then drops to eventually return to its initial value. Again, the German pension system holds the middle ground.

As demonstrated by the simple model, the cohort effect (Fig. 7) has winners and losers. The quantitative model shows that the size of a cohort can only ever be seen in the context of the overall population. The baby boomers (cohorts from 1955–1969, grey shaded area) are at an advantage in FB systems. Here, there is a certain delay before the setup of the system takes effect. The ordering of the pension systems in terms of rate of return changes several times. The variance of the rate of return in the FC system is higher with a maximum excess return of 0.5% for the cohort born in 1924. For some cohorts, the FC system is best, for other cohorts, it is the FB system. The greatest return differential between FB and FC system of 0.8% would occur for the 1967 cohort, for example. While a positive return of 0.2% would be generated in an FB system owing to the cohort effect, it would be −0.6% in an FC system. Here, too, it is clear that the German pension system plays a kind of intermediary role between an FB and an FC system. Even though the contribution rate and the pension level vary less than in the longevity effect case, the size of the variation in rates of return is comparable.

The previous results illustrate that both a fixed benefit system as well as a fixed contribution system exhibits cohort-dependent rates of return. To provide equal rates of return for all cohorts, one has to adjust the pension system by allowing for a pension fund. In this pension system, the contribution and the replacement rate are fixed to the values in the initial steady state. In this example, the contribution rate is set to $\phi_t = \overline{\phi} = 12\%$, and the replacement rate is set to $\gamma_t = \overline{\gamma} = 55\%$.

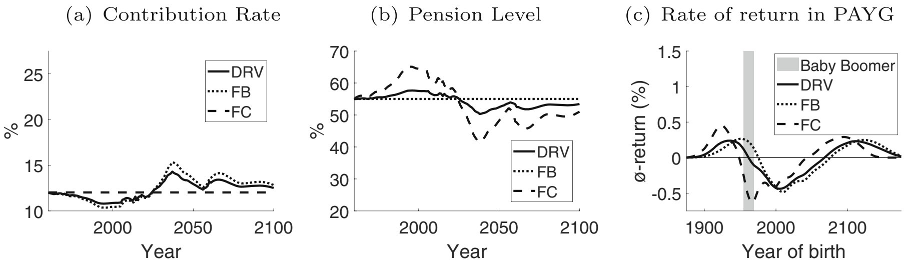

Fig. 7 **Cohort effect** Panel (a) shows the contribution rate. Panel (b) shows the pension level. Panel (c) shows the implicit rate of return in PAYG systems. Black solid line: DRV; red dotted line: FB system, green dashed line: FC system

In contrast to the previous pension systems, the budget of the pension system is not balanced in every period. The only condition is that the present value of contribution and expenditures must be equal.

$$\overline{\phi}\sum_{t=1}^{T}\frac{w_t L_t}{\Pi_{i=1}^{t}(1+r_i)}=\overline{\gamma}\sum_{t=1}^{T}\frac{w_t P_t}{\Pi_{i=1}^{t}(1+r_i)} \tag{38}$$

The pension fund, D_t, develops according to following equation

$$D_{t+1}=(1+r_t)D_t+\overline{\phi}w_t L_t-\overline{\gamma}(1-\overline{\phi})w_t P_t \tag{39}$$

with $D_0=0$.

The pension budget is not balanced with a constant replacement rate and contribution rate due to the cohort effect. The differences in rates of return within the pension system due to the cohort effect cannot be sufficiently addressed by simple parameter adjustments. This makes it necessary to have a pension fund to buffer variation in cohort size. We assume that the pension system would have introduced the pension fund in 1960. The development of the pension fund is shown in Fig. 8(c). It starts at zero in 1960 and then quickly accumulates assets until 2026. There, the pension fund would amount to 38% of GDP. With the entry of the baby boomer cohort in the following years, the pension fund would meltdown. After 2060, the assets would have been exhausted completely and the pension fund has to issue debt to pay the pension. In 2114, the debt of the pension fund would reach its maximum and amounts to 43% of GDP. In the following, the pension would pay back the debt and after 2200 the pension fund is again zero.

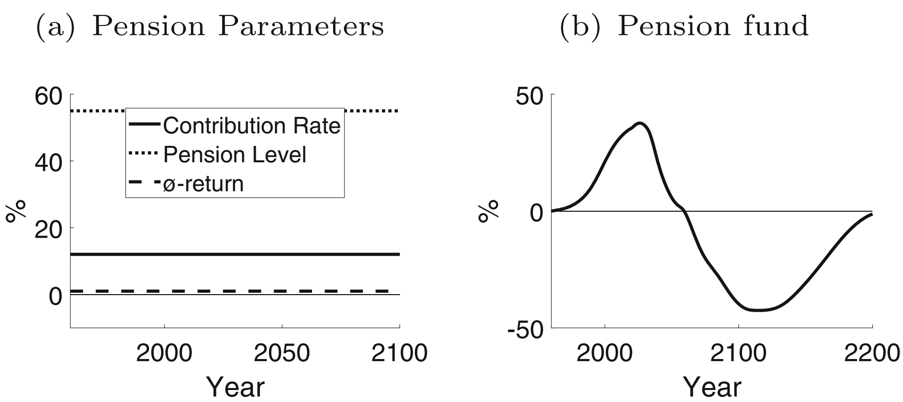

Fig. 8 **Reform in 1960** Panel (a) shows the contribution rate, the pension level and the implicit rate of return in PAYG systems. Panel (b) shows the pension fund in relation to GDP

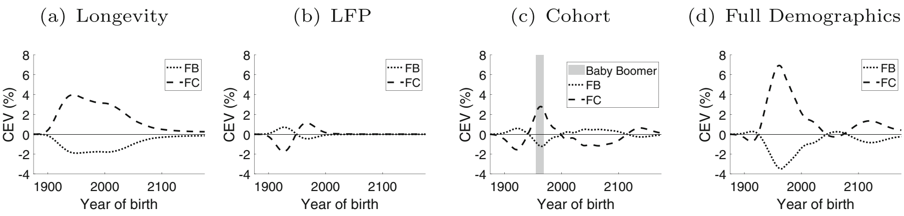

Fig. 9 **Consumption equivalent variation** Panel (a) shows CEV for the longevity effect. Panel (b) shows CEV for labour force participation effect. Panel (c) shows CEV for the cohort effect. Panel (d) shows CEV for the complete demographic change effect. Red dotted line: FB system, green dashed line: FC system

7 Welfare and inequality analysis

Besides the positive analysis, the quantitative model can also be applied to evaluate the welfare consequences of different pension systems and the interplay with the population scenario. This is done in an ex-ante and an ex-post view, following (Pestieau and Ponthiere 2016).

First, an perspective is adopted that is based on expectations (ex-ante view). Figure 9 shows consumption equivalent variations (CEV). The CEVs are computed as the percentage change in consumption over the life cycle required as compensation for individuals to be indifferent between the benchmark DRV pension system and an alternative pension system, i.e. FC and FB. Positive numbers indicate welfare losses from being born into the respective pension system compared to the DRV system. For every cohort, the CEVs are calculated based on discounted lifetime utility.

In the demographic scenario where only the life expectancy is increasing, cohorts that were born early clearly favour the fixed benefits system. A cohort that was born in 1950 would forego 1.9% of their lifetime consumption in case they could choose to live in a FB system rather than the DRV system. The opposite is the case for the FC system where the cohort that was born in 1944 would demand 4% more of lifetime consumption. For the labour force participation scenario and the cohort scenario, the pictures are ambiguous. For some cohorts, the FC system is favourable in terms of CEV for other cohorts it is the FB. Panel (d) of Fig. 9 shows the CEV for the complete

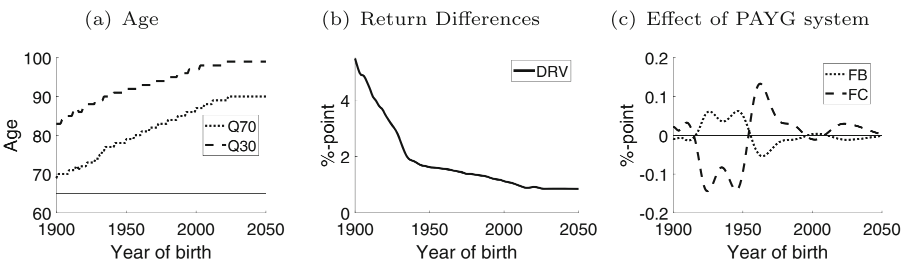

Fig. 10 **Return difference between quantiles** Panel (a) shows the age at least 70% (30%) of a cohort reaches over time. Panel (b) shows the rate of return differences between a cohort member that reaches age Q_{70} and age Q_{30}. Panel (c) shows differences in this return differential that is due to the pension system. Red dotted line: FB system, green dashed line: FC system

demographic change effect. For most of the cohorts, the fixed benefits system would be better compared to the DRV system. On the other hand, only a few cohorts would favour the fixed contribution system compared to the DRV system.

Second, the model shows distributional consequences from a perspective based on realizations (ex-post view). The increasing survival rates over time result in more people reaching retirement age and the time spent in the retirement phase is prolonged. Panel (a) in Fig. 10 shows the age at least 70% (30%) of a cohort reach. The ratio of the time spent in retirement between the two groups shrinks. Panel (b) in Fig. 10 shows that the rate of return differential decreases significantly. The return differential was over 5 percentage points for the cohort that was born in 1900 and is less than one percentage point for those born after 2050.[24] The differences between the PAYG system play only a minor role and the effect changes in sign over time.

8 Limitations of the analysis

The scope of this paper is to give a quantitative assessment of the direct effect of demographic change on the payoffs in PAYG pension systems. However, demographic change affects variables that are important for the pension system also on many other indirect ways.

For example, the structure of the population shapes the supply of input factors in the production function, capital and labour. In a closed economy framework, a reduction of labour and an increase in savings make PAYG pension systems relatively more attractive compared for example to fully funded systems. Ludwig and Vogel (2010) show that population variations have also important effects on the accumulation of physical and human capital. A good analysis of how longevity and fertility affect capital accumulations, labour supply for various pensions systems in a two-period OLG closed economy[25] model is presented by Dedry et al. (2017).

Another important dimension of how demographic change might affect the pension system is via endogenous labour supply. In a notional personal account system such as the German system, relatively higher rates of return in the pension system might change the incentives to work for households. Whether a higher rate of return within a Bismarckian type of pension system increases or decreases labour supply, however, depends on whether the wealth effect or the substitution effect dominates in the labour supply decision. Either way, the labour supply would be distorted by the longevity effect and the cohort effect. In turn, the rate of return would be additionally shifted.

[24] An important factor that is not considered in this analysis is that the mortality rates are negatively correlated with income, see Cutler et al. (2006). The return differentials are likely to be underestimated and should be seen as a lower bound. Recent studies, e.g. see Haan et al. (2020), also point out that the increase in life expectancy is mostly experienced by the top income groups. This means that the decreasing pattern would also be underestimated.

[25] The assumption of an open economy shuts down a feedback channel via the endogenous adjustment of factor prices, wage rate and interest rate. The assumption of an open economy is given the globally relatively small size of Germany and the good integration of its capital market a plausible approximation.

Changes in the retirement system might also not only affect the agent's labour supply. According to Bratti et al. (2018), changes in the retirement eligibility of grandparents affect the labour force participation of women with young children. Using a natural experiment, Atalay et al. (2019) show that pension reform can have important spillover effects on the labour force participation of spouses.

Similarly, changes in the payout structure of a pension system might affect the timing of the retirement of households. Whether households postpone their retirement decision depends, as the labour supply decision, on the unclear preferences of households and additionally on the exact regularity options of early/late retirement.[26] In a recent study, Seibold (2021) found even that the most important determinant of the retirement decision is not financial considerations but reference point dependence.

Another important channel would include endogenous migration. There is a huge literature discussing the complex relationship between migration and existing welfare systems, e.g. Storesletten (2000) and Preston (2014). With endogenous migration, the described effects of changes in the demographic structure of the population will affect pre-tax and after-tax income, thereby affecting the migration choice. Another potential question is whether the fertility rate could react to life expectancy and the rate of returns of the pension system.

These effects are important in describing the potential effects of demographic change. Even though these endogenous reactions are important in full economic analysis, they fundamentally rely on the specific model assumption, e.g. the degree of capital market integration. Some of those effects are not even clear in sign and could go in either way. The model used in this study does not include any indirect effect that stems from behavioural responses of households or general equilibrium effects. By ignoring general equilibrium and behavioural effects, this study determines the direct effects of the demographic change that would otherwise be disguised. This by no means indicates that indirect effects are negligible or not important.

9 Conclusions

The demographic change most developed countries face consists of two different coinciding developments: first, increasing life expectancy, and second, fluctuations in the size of birth year cohorts. The increase in life expectancy is slow and monotonic and also (presumably) a sustained trend. The number of births per year can both decrease and increase, and changes more rapidly over time. However, fluctuating birth rates of the past have less and less of an impact on the demographic structure as time progresses. This paper shows that the longevity effect has a positive impact on the returns households generate within the PAYG pension system. The cohort effect, by contrast, results in winners and losers in a PAYG system in terms of the return on that system.

[26] Retirement regulation itself can be subject to the economic pressure resulting from demographic change. However, this is out of the scope of this paper.

Overall, the demographic change in Germany has a positive impact on the rates of return participants realize in the German PAYG pension system. The distribution among the cohorts is unequal with a difference 1.3 percentage points per year over the whole life cycle. The cohort that benefited the most is the birth cohort of 1939 with an excess return of 1.2 percentage points compared to a world without demographic change. The birth cohort of 2026 will have a slightly negative excess return of −0.1 percentage points.

This paper illustrates the interconnectivity of demographic change, the requirement of PAYG pension systems to run a balanced budget and the contribution/benefits of individuals within the pension system. To focus solely on this direct effect, it assumes many processes that are affected by the demographic change and the pension system as exogenous. Although including these feedback channels would be a promising and interesting approach for future research it is beyond the scope of this paper.

Acknowledgements I would like to thank editor Gregory Ponthiere, three anonymous reviewers, Alexander Ludwig, Nikolai Stähler and conference participants at the International Conference on Macroeconomic Analysis and International Finance, and the Annual Congress of the International Institute of Public Finance.

Funding Open Access funding enabled and organized by Projekt DEAL.

Data availability The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests The author declares no competing interests.

Disclaimer This paper represents the views of the author and does not necessarily reflect the views of the Deutsche Bundesbank, or the Eurosystem.

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