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Rotten spouses, family transfers, and public goods

Helmuth Cremer[1] · Kerstin Roeder[2]

Abstract We show that once interfamily exchanges are considered, Becker's rotten kids mechanism has some remarkable, hitherto unnoticed, implications. Specifically, Cornes and Silva's (J Polit Econ 107(5):1034–1040, 1999) result of efficiency in the contribution game amongst siblings extends to a setting where the contributors (spouses) belong to different families. More strikingly still, the mechanism may also have dramatic redistributive implications. In particular, we show that the rotten kids mechanism combined with a contribution game to a household public good may lead to an astonishing equalization of consumptions between and within families, even when their parents' wealth levels differ. The most striking results obtain when wages are equal and when parents' initial wealth levels are not too different. For very large wealth differences, the mechanism must be supplemented by a (mandatory) transfer that brings them back into the relevant range. When wages differ but are similar, the outcome will be near efficient (and near egalitarian).

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✉ Helmuth Cremer
helmuth.cremer@tse-fr.eu

Kerstin Roeder
kerstin.roeder@wiwi.uni-augsburg.de

[1] Toulouse School of Economics, 21 allée de Brienne, 31015 Toulouse, France

[2] University of Augsburg, Universitätsstr. 16, 86159 Augsburg, Germany

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1 Introduction

Becker's (1974, 1991) "rotten kid theorem" has by now become one of the corner-stones in family economics. In his seminal paper, Becker presents the challenging idea that intergenerational exchanges within a family may be efficient even when the children are purely selfish and the altruistic parents lack the power to commit to a reward scheme that might provide the children with the proper incentives to behave according to the "common good". The extensive subsequent literature has both qualified and extended this result.[1]

Probably, the most prominent qualification is due to Bergstrom (1989) who shows that the result rests on a certain number of restrictive assumptions (single good, interior solution, etc.).[2] While the outcome may not be efficient under realistic assumptions, the fundamental mechanism continues to be at work and spontaneously yields some "cooperative" behavior in a world which is otherwise biased towards totally selfish conduct.[3]

Amongst the various extensions, one of the most remarkable ones is by Cornes and Silva (1999) who show that the rotten kid theorem holds in a world with a private *and* a public good. The siblings non-cooperatively contribute to the family public good. By transferring the private good after the children have chosen their contributions to the public good, the benevolent parent achieves fulfillment of the Samuelson condition. In other words, the rotten kid mechanism may even be an effective way to achieve efficient contributions to (household) public goods in a non-cooperative world (where Nash equilibria are otherwise typically not efficient).[4]

So far, this literature has essentially concentrated on the exchanges within a single family.[5] We show that once interfamily exchanges are considered, the rotten kid mechanism has some remarkable implications that have gone hitherto unnoticed. Specifically, we establish that Cornes and Silva's result of efficiency in the contribution game amongst siblings extends to a setting where the contributors (spouses)

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[1] An excellent overview of this literature is given by Laferrère and Wolff (2006).

[2] For example, Hirshleifer (1977) was the first to point out that the efficient solution is implemented only when the parents have the last word, that is, when they move last. In a similar vein, Bruce and Waldman (1990) show that in a model of intertemporal consumption decisions, the rotten kid mechanism leads to the so-called Samaritan's dilemma in that the child does not save enough.

[3] For instance, in a recent paper, Cremer and Roeder (2013) show that when there are several goods, including family aid (and long-term care services in general), the outcome is likely to be inefficient. Still, the rotten kid mechanism is at work and ensures that a positive level of aid is provided as long as the bequest motive is operative.

[4] Efficiency is, however, only guaranteed if the solution to the kids problem is interior, that is, if all children make contributions to the family public good. Chiappori and Werning (2002) provide examples when this is or is not the case.

[5] A notable exception is Cornes et al. (2012) who consider two families and different scenarios of contributors to a (general) public good. They focus on the neutrality result by Warr (1983) and show that it continues to hold in their setting. This result says that lump-sum redistributions between participants in a Nash game of private provision of a public good are allocatively neutral when all participants make positive contributions and have the same productivity in producing the public good.

belong to different families. More strikingly still, the mechanism does not just have consequences for *efficiency* but it may have dramatic *redistributive* implications. In particular, we show that the rotten kid mechanism combined with a contribution game to a household public good may lead to an astonishing equalization of consumption levels between the spouses and their parents, even when their parents' original wealth levels are quite different.

We consider a setting with two families each consisting of a retired parent and an adult child who are "linked" by the young spouses. Children contribute part of their time to a household (couple) public good like child care or other domestic duties. Additionally, they provide attention (or caregiving services) to their respective parents "in exchange" for a bequest. Spouses behave towards their respective parents like Becker's rotten kids; they are purely selfish and anticipate that their altruistic parents will leave them a bequest. Parents cannot commit to a rule linking this bequest to the amount of attention provided by the child. In other words, a threat to, say, disinherit (or otherwise punish) the child who does not provide some specified level of attention is not credible because children anticipate that the estate and its allocation will be determined by the altruistic parent.

We start by determining the set of Pareto-efficient allocations which are used as a benchmark. Not surprisingly, the levels of aid are set to equalize marginal costs (the child's wage) to the marginal benefits incurred by the parent. The optimal provision of the family public good satisfies the Samuelson rule. When children differ in wages, Pareto-efficiency requires that only the lower-wage spouse contributes to the household public good. When children have equal wages, only the total provision of the household public good is uniquely defined and any allocation of this total level between the individual spouses is equally efficient.

We then study the (subgame perfect) Nash equilibrium that occurs when parents and children play a two-stage game, the timing of which reflects the rotten kids approach. First, the children (spouses) choose simultaneously and non-cooperatively the time spend with their parents, and their contribution to the family public good. Second, the parents set (simultaneously and non-cooperatively) the bequest left to their respective child.

This equilibrium turns out to have a number of interesting properties some of which are rather surprising. Levels of family aid are always efficient; this is perfectly in line with the rotten kid specification and not surprising. The most stunning results arise when wages are equal. Unless parents' wealth levels are very different, we obtain a (unique) interior equilibrium where both spouses contribute to the public good. This equilibrium is efficient (the Samuelson condition is satisfied), which is otherwise typically not the case in non-cooperative contribution games (see Bergstrom et al. 1986). More surprisingly still, it always corresponds to the utilitarian (equal individual weights) Pareto-efficient allocation. Consequently, consumption levels are equalized within and across families, in spite of the fact that the spouses have parents with different wealth levels. The equalization of the contributing spouses' consumption levels is in line with a result stated by Bergstrom et al. (1986). They show that when preferences are identical, all contributors to a public good will consume the same amount of private good. The striking new feature of our

result is that this equalization "spills over" to the spouses' parents: their equilibrium consumption levels will be equal even when they differ in initial wealth.[6]

Both properties arise because a rotten kid-like mechanism is at work under which spouses' contributions are effectively subsidized through adjustments in the bequests. This is reminiscent of the results obtained by Cornes and Silva (1999) within a single family setting. The striking feature of our results is that this property extends to a setting where the contributors have different parents (they are spouses rather than siblings). In addition, the rotten kid mechanism proofs not only to promote efficiency but also to spontaneously achieve a "perfect" redistribution not only between spouses but also between their respective parents. In other words, the initial wealth differences are spontaneously washed out by the interplay of contributions, aid, and bequests.[7]

These interfamily results crucially depend on the existence of the household public good; it is the contribution game which creates the strategic link between the two parents, via their children. These results occur when the contribution equilibrium is interior, which in turn is the case when the difference in parents' wealth does not exceed a certain threshold. The level of this threshold increases with the significance of the expenditure on the household public good; when these expenditures are sufficiently large, the contribution to the household public good can neutralize initial wealth differences.

The idea that dynasties may be interconnected and that this gives rise to various neutrality results was also presented by Bernheim and Bagwell (1988). Formally, the effects that give rise to these neutrality results are also present in our setting. Our focus, however, is not on neutrality; instead, we concentrate on the impact of the linkage between families on efficiency and inequality. In any event, the "everything neutral" result is based on the property that all individuals belong to several dynasties so that at the end, everyone is "connected" to everyone. In our setting, individuals belong to a single couple and thus indirectly to two dynasties. However, unless we extend our model to a polygamic society, we do not obtain the chain effects described by Bernheim and Bagwell (1988). The interdynasty links in our setting are of much more limited scope which, at the same time, makes them more relevant for reality.[8]

When wealth differences are large, there will be a (unique) corner equilibrium where only the spouse with the richest parents contributes. This equilibrium is no longer efficient, and consumption levels are not equalized between parents. We also show that in this case, some *ex ante* redistribution (at stage 0) between families can restore efficiency. Interestingly, to accomplish this, it is not necessary to fully equalize wealth levels. The redistribution must just bring them within the range that yields

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[6]And recall that this comes on top of the efficiency property which is at odds with conventional results and in particular with Bergstrom et al. (1986).

[7]Our result is also related to Caplan et al. (2000). These authors consider a federation where regional governments contribute to the public good. This setting is somewhat similar to a rotten kids model. The federation plays a role similar to the altruistic parents, while the regions replace the rotten kids. But it also introduces new specific features like labor mobility. The major difference to our approach lies once again in the interfamily aspect which has no direct counterpart in the (single) federation setting.

[8]Recall that Bernheim and Bagwell's point was to take dynastic models *ad absurdum* by showing that the interdynasty links would have policy implications that are obviously unrealistic.

an interior equilibrium. The contribution game then takes care of the rest, namely achieving efficiency and perfect equalization of consumption levels; we return to the case where the equilibrium corresponds to the utilitarian allocation.

The results are more complex in the case where spouses differ in wage. While Pareto-efficiency requires that only the lower-wage spouse contributes to the public good, the equilibrium can yield any pattern of contributions. Depending on the wealth and wage heterogeneity, we can have an interior or a corner solution, with either of the spouses (even the high-wage one) as sole contributor. This equilibrium is (almost) never efficient, even when the solution is of the right type. However, when spouses' wages are not exactly equal but sufficiently similar, the solution will be interior and close to the utilitarian allocation.[9] In any event, whatever the wage, differential efficiency can once again be reestablished with a transfer in stage 0, but unlike in the previous case, there is no longer a whole range of possible transfers but only a single level which does the job.

While our paper concentrates on theoretical and methodological issues, the underlying problem is also of significant practical importance. The number of dependent elderly is predicted to increase dramatically in the coming decades. Currently, long-term care (LTC) is mainly provided by family members and many couples will face the situation that they have to assist the dependent parents of *both* spouses, while contributing at the same time to the household chores. Because of a variety of societal changes, informal aid may not be sustainable at its current level and alternative arrangements have to be found, including public and private insurance. To assess the need for public LTC policy and study its design, it is important to understand the underlying nature of exchanges between generations (see Cremer et al. 2012 for a detailed discussion and references).

Our model is concerned with informal care. Empirically, the exact extent of family care is extremely difficult to quantify, precisely, because it is by definition informal and not traded in a market. Still, it is widely acknowledged that it represents an important part of total care services. For instance, according to Norton (2000) "A general rule of thumb is that about two-thirds of care for the elderly is informal care." There is also ample evidence that intergenerational transfers are used as implicit payment for aid. For instance, Norton et al. (2014) use data from the National Longitudinal Survey of Mature Women to show that informal caregivers are 20 % points more likely to receive any transfer and 9 % points more likely to receive a financial transfer.[10]

Cigno (2014) discusses similar issues within the broader context of "old-age security." He presents a comprehensive survey of the relevant literature and forcefully shows that intra-family exchanges and the various patterns along which they may be organized play a crucial role that cannot be ignored when policies intended to support the elderly are designed. We add a building block to the study of this problem by considering both exchanges between and within families.

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[9]Provided that parent's wealth differences are not too large.

[10]See also Bernheim et al. (1985), who present econometric and other evidence that strongly suggests that bequests are often used as compensation for services rendered by beneficiaries.

Our paper proceeds as follows. Section 2 sets up the model. Section 3 determines the Pareto-efficient allocations, while Section 4 analyzes the *laissez-faire* solution. Section 5 shows how the Pareto-efficient solution can be implemented whenever the *laissez-faire* is not Pareto-efficient. Section 6 concludes and an Appendix contains most of the proofs.

2 The model

We consider two families $i = 1, 2$ each consisting of one parent (superscript "p") and one child (superscript "c"). Parents are altruistic, while children are purely selfish. The young constitute a couple who non-cooperatively produces a household public good, G, like housework.[11] The production of this household public good is linear and costs $g_i \in [0, \tau]$ units of time. The total amount of time available is $\tau \in \mathbf{R}^+$. Children may also spend some time $a_i \in [0, \tau]$ with their own parents providing them simply with attention or with aid in case of illness or dependency. The (monetary) value of this time for their parents is given by $h(a_i)$ with $h' > 0$, $h'' < 0$. The residual time $\tau - g_i - a_i$ is spend on the labor market for which the child receives the wage rate w_i. Parents own a wealth of x_i and may leave a transfer $b_i \geq 0$ to their own child. We refer to this as a bequest, but it can also represent an inter vivos gift.[12] Wages of the children as well as wealth of the parents may differ between families, implying $w_1 \lesseqgtr w_2$ and $x_1 \leq x_2$. Both generations derive utility from consumption of a numeraire commodity, while the young couple additionally enjoys consumption of the household public good. The altruistic parent maximizes the welfare function $W_i^p = U_i^p + U_i^c$. The parent's "own" utility (not including the altruistic element) is given by

$$U_i^p = u(x_i + h(a_i) - b_i) \quad \forall i, \tag{1}$$

while the utility of the child is represented by

$$U_i^c = u(w_i(\tau - g_i - a_i) + b_i) + \varphi(G) \quad \forall i. \tag{2}$$

The utility functions satisfy $u', \varphi' > 0$ and $u'', \varphi'' < 0$, and we have $G = g_1 + g_2$. Both families are perfectly informed about each other's characteristics, which allows us to focus on the efficiency and distributional issues.

The timing of the game is as follows. In stage 1, the children (spouses) choose simultaneously and non-cooperatively the time spend with their parents, a_i, and their contribution to the family public good, g_i. In stage 2, the parents set simultaneously and non-cooperatively the bequest, b_i, left to their respective child (see Fig. 1 for an illustration).

The player's payoff functions are defined by W_i^p for the parents and by U_i^c for the children. To determine the subgame perfect Nash equilibrium, we solve this game by backward induction. Before we turn our attention to the *laissez-faire* solution, we

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[11] In other words, we assume descending altruism and truncate the analysis after the parent's generation.
[12] As long as it occurs sufficiently late to be consistent with the timing of our model. Put differently, its timing is such that it can represent a "payment" for informal care.
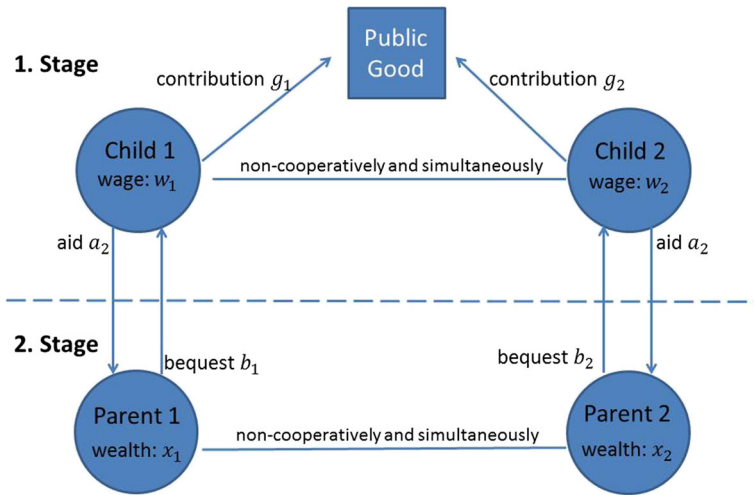
Fig. 1 The setup

will study the Pareto-efficient allocations which provide a benchmark against which we can compare the Nash equilibrium outcome.

3 Pareto-efficient allocations

Denoting consumption levels of the parents by m_i and of the children by d_i, Pareto-efficient allocations solve the following maximization problem[13]

$$\max_{m_1,m_2,d_1,d_2,a_1,a_2,g_1,g_2} \quad \mathcal{W} = \sum_{i=1}^{2} \left\{ \pi_i^p u(m_i) + \pi_i^c [u(d_i) + \varphi(G)] \right\}$$

$$\text{s.t.} \quad \sum_{i=1}^{2} \{ w_i(\tau - g_i - a_i) + x_i + h(a_i) \} \geq \sum_{i=1}^{2} \{ m_i + d_i \},$$

$$G = \sum_{i=1}^{2} g_i, \quad \text{and} \quad a_i + g_i \leq \tau \quad \forall\, i.$$

$$(3)$$

where $\pi_i^c, \pi_i^p \in (0, 1)$ denote the weights attached to the child's and parent's utility of family $i = 1, 2$. They are normalized to sum up to one:

$$\sum_{i=1}^{2} \left\{ \pi_i^c + \pi_i^p \right\} = 1.$$

Solving this problem for a given vector of weights yields a specific Pareto-efficient allocation and the full set of efficient allocations can be described by varying the

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[13]Throughout the paper, we assume that the time constraint $a_i + g_i \leq \tau$ will be never binding.

weights. Denoting $\mathcal{L}$ the Lagrangian expression associated with problem (3), the first-order conditions (FOCs) are given by

$$\frac{\partial \mathcal{L}}{\partial m_i} = \pi_i^p u'(m_i) - \mu = 0 \quad \forall\, i, \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial d_i} = \pi_i^c u'(d_i) - \mu = 0 \quad \forall\, i, \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial a_i} = \mu \pi_i^c (h'(a_i) - w_i) = 0 \quad \Rightarrow \quad h'(a_i) = w_i \quad \forall\, i, \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial g_i} = -\mu w_i + (\pi_1^c + \pi_2^c)\varphi'(G) \leq 0 \quad \forall\, i, \tag{7}$$

where μ is the Lagrangian multiplier with respect to the resource constraint. Equations 4 and 5 state that the (weighted) marginal utilities between and across families should be equalized. Equation 6 shows that attention should be chosen such that its marginal benefit to the parent is equal to the marginal costs of its provision. It shows that the level of a_i is the same in all Pareto-efficient allocations (it does not depend on the weights). Equation 7 determines the Pareto-efficient public good contributions for both spouses; it can be easily verified that $g_1 > 0$ and $g_2 = 0$ if $w_1 < w_2$. In words, it is efficient that only the spouse with the lower wage rate (production costs) contributes to the family public good. Conditions (4), (5), and (7) can be simplified to

$$\min\{w_1, w_2\} = \frac{\varphi'(G)}{u'(d_1)} + \frac{\varphi'(G)}{u'(d_2)}, \tag{8}$$

which is the Samuelson rule, stating that the sum of the marginal rates of substitution between the public and the private good must be equal to the marginal costs of production. When children have equal wages ($w_1 = w_2$), G is uniquely defined (for a given set of weights) by Eq. 8 along with the FOCs (4)–(6), but individual contributions can take any values satisfying $g_1 + g_2 = G$.[14]

We denote the utilitarian solution that arises with equal weights ($\pi_1^c = \pi_2^c = \pi_1^p = \pi_2^p = 1/4$) with the superscript ^{e}. It is given by

$$u'(m_1^e) = u'(d_1^e) = u'(m_2^e) = u'(d_2^e), \tag{9}$$

$$h'(a_i^e) = w_i \quad \forall\, i, \tag{10}$$

$$\min\{w_1, w_2\} = 2\frac{\varphi'(G^e)}{u'(d_1^e)}. \tag{11}$$

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[14]The level of G will (in general) vary across Pareto-efficient allocations.

Note that the level of G^e is unique for a *given* total level of wealth in society $(x_1 + x_2)$. When either x_i or w_i changes so does the optimal G^e.[15] Observe that while g_1^e and g_2^e are not uniquely defined when wages are equal, they are well defined when wages differ. Specifically, when $w_i < w_j$ $(i, j = 1, 2)$, we have $g_i^e = G^e$ and $g_j^e = 0$.

The following sections show that an equilibrium of the two-stage game will satisfy conditions (9)–(11) when children have the same wage rate, $w_1 = w_2$, while parents' wealth levels may differ but within a limited range. In other words, in these cases, the *laissez-faire* equilibrium corresponds to the utilitarian optimum. On the other hand, when children differ in wages, the contribution equilibrium is in general inefficient. However, efficiency of the *laissez-faire* solution and its coincidence with the utilitarian allocation can be reestablished through an appropriate lump-sum transfer between parents.

4 *Laissez-faire* solution

As usual in two-stage games, we begin by analyzing the second stage. The parent solves the following optimization problem

$$\max_{b_i} \quad U_i^p = u(x_i + h(a_i) - b_i) + u(w_i(\tau - g_i - a_i) + b_i) + \varphi(G) \quad \text{s.t.} \quad b_i \geq 0 \quad \forall i.$$

The FOC with respect to bequests is given by

$$\frac{\partial U_i^p}{\partial b_i} = -u'(m_i) + u'(d_i) \leq 0 \quad \forall i. \tag{12}$$

That is, bequests in both families are chosen so that consumption levels between the parent and the child are equalized. Denote $b_i^* \equiv b_i(g_i, a_i)$ the optimal bequest level. We assume throughout the paper that the bequest motive is operative so that b_i^* is given by an interior solution and Eq. 12 holds as equality.[16] Differentiating

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[15] For equal wages $(w_1 = w_2)$, G^e is determined by

$$-w_i u' \left(\frac{x_1 + x_2 + 2w_i(\tau - a_i^e) + 2h(a_i^e) - w_i G^e}{4} \right) + 2\varphi'(G^e) = 0.$$

Differentiating yields

$$\frac{dG^e}{dx_i} = \frac{\frac{w_i}{4} u''(d_i^e)}{\frac{w_i^2}{4} u''(d_i^e) + 2\varphi'(G^e)} = \frac{1}{w_i + \frac{8\varphi''(G^e)}{w_i u''(d_i^e)}} > 0.$$

[16] Recall that bequests are restricted to be nonnegative, and one obtains from Eq. 12

$$b_i > 0 \quad \Longleftrightarrow \quad x_i + h(a_i) > w_i(\tau - a_i - g_i) \quad \forall i.$$

In words, the net resources of the parents (including the monetary value of informal aid, if any) must be larger than that of the children otherwise the bequest motive is not operative.

this expression shows that the derivatives of bequests with respect to public good investments and attention are as follows

$$\frac{\partial b_i^*}{\partial g_i} = \frac{u''(d_i)w_i}{u''(m_i) + u''(d_i)} = \frac{w_i}{2} > 0 \quad \forall \, i, \tag{13}$$

$$\frac{\partial b_i^*}{\partial a_i} = \frac{u''(m_i)h'(a_i) + u''(d_i)w_i}{u''(m_i) + u''(d_i)} = \frac{h'(a_i) + w_i}{2} > 0 \quad \forall \, i. \tag{14}$$

When the child increases his contributions to the family public good, the parent compensates the child by half of his forgone wage income, w_i. Additionally, when the child increases his attention to the parent, the bequest increases by half of the parent's return, $h'(a_i)$, plus by half of the child's forgone wage income, w_i.

At stage 1, child i's problem is

$$\max_{a_i, g_i} \quad U_i^c = u(w_i(\tau - a_i - g_i) + b_i^*) + \varphi(G) \quad \text{s.t.} \quad g_i \geq 0 \quad \forall \, i.$$

A non-negativity constraint is imposed on g_i because a corner solution is possible. When choosing the attention to the parent and investments in the (own) family public good, the child takes into consideration the adjustments in bequests and takes the spouse's contributions g_{-i} as given

$$\frac{\partial U_i^c}{\partial a_i} = u'(d_i)\left(-w_i + \frac{\partial b_i^*}{\partial a_i}\right) = 0 \quad \forall \, i, \tag{15}$$

$$\frac{\partial U_i^c}{\partial g_i} = u'(d_i)\left(-w_i + \frac{\partial b_i^*}{\partial g_i}\right) + \varphi'(G) \leq 0 \quad \forall \, i. \tag{16}$$

With Eqs. 13 and 14, the above first-order conditions can be written as

$$-w_i + \frac{h'(a_i) + w_i}{2} = 0 \quad \Rightarrow \quad h'(a_i^*) = w_i \quad \forall \, i, \tag{17}$$

$$-u'(d_i)\frac{w_i}{2} + \varphi'(G) \leq 0 \quad \Rightarrow \quad 2\varphi'(G) \leq u'(d_i)w_i \quad \forall \, i. \tag{18}$$

Equation 17 directly determines a_i^*; the spouse's level of attention a_{-i}^* is of no relevance and there is effectively no strategic interaction on this variable. Substituting this level of attention into Eq. 18 and taking into account the constraint $g_i \geq 0$, we can solve for the spouses' best-response functions for the contributions to the family public good $\tilde{g}_1(g_2)$ and $\tilde{g}_2(g_1)$. The Nash equilibrium levels of contributions (g_1^*, g_2^*) are defined in the usual way by the mutual best reply conditions $g_1^* = \tilde{g}_1(g_2^*)$ and $g_2^* = \tilde{g}_2(g_1^*)$. Existence of this equilibrium is easily established.[17] The total equilibrium amount of the family public good produced by the couple is then given by $G^* = g_1^* + g_2^*$.

Two distinct types of equilibria are possible; an interior solution, that is, one in which both spouses contribute to the household public good and a corner solution in

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which only one of the spouses contributes. For future reference note that with Eq. 12 an interior Nash equilibrium satisfies

$$u'(d_1^*)w_1 = 2\varphi'(G^*) \quad \Leftrightarrow \quad u'\left(\frac{(\tau - a_1^* - g_1^*)w_1 + h(a_1^*) + x_1}{2}\right)w_1 = 2\varphi'(G^*), \quad (19)$$

$$u'(d_2^*)w_2 = 2\varphi'(G^*) \quad \Leftrightarrow \quad u'\left(\frac{(\tau - a_2^* - g_2^*)w_2 + h(a_2^*) + x_2}{2}\right)w_2 = 2\varphi'(G^*). \quad (20)$$

We shall now examine the properties of the Nash equilibrium and analyze the efficiency of the induced allocation. We start with the case where children have identical wages and then consider the case where wages differ.

4.1 Identical children

Assume children are equally productive in the labor market, $w_1 = w_2 \equiv w$. Recall that subscript 2 is used for families with higher wealth ($x_1 \leq x_2$). To simplify notation, we fix x_2 at some arbitrary level and then study the Nash equilibrium and its properties as a function of x_1. Observe that as long as $\widetilde{g}_1$ is an interior solution for which Eq. 18 holds as equality, we have

$$\frac{\partial \widetilde{g}_1}{\partial x_1} = \frac{u''(d_1)w}{u''(d_1)\frac{w_1^2}{2} + 2\varphi''(G)} > 0. \quad (21)$$

Thus, for a given level of x_2, the best response of spouse 1 to any level of g_2 decreases as x_1 becomes smaller. Consequently, we expect that the equilibrium moves from the interior one to the corner solution when spouse 1's wealth falls. This conjecture is confirmed in the following proposition which is established in the Appendix. It shows that the equilibrium is interior when wealth levels are not too different, while a corner solution may arise when x_1 is sufficiently small.

Proposition 1 *The Nash equilibrium is unique and an interior solution ($g_1^* > 0; g_2^* > 0$) if $x_1 > \widehat{x}_1$, while a corner solution ($g_1^* = 0; g_2^* > 0$) arises if $x_1 \leq \widehat{x}_1$, where $\widehat{x}_1 \equiv x_2 - \widetilde{g}_2(0)w_2$.*

To get an intuitive understanding of this proposition, consider Eq. 18 defining the best responses for $w_1 = w_2$. Assume that spouse 2 contributes $\widetilde{g}_2(0)$, i.e., her best response to $g_1 = 0$. Equations 19 and 20 then show that $(0, \widetilde{g}_2(0))$ is an interior equilibrium if x_1 is at exactly the level which yields equal consumption levels (including the respective bequests) across spouses, $d_1 = d_2$ for these respective contributions. With equal wages $a_1^* = a_2^*$ so that $d_1^* = d_2^*$ occurs when $x_1 = x_2 - \widetilde{g}_2(0)w_2$. In words, the wealth difference $x_2 - x_1$ corresponds to the costs of the spouses contributions: $\widetilde{g}_2(0)w_2 - 0w_1$. Taking into account (21), it is plain that for a level of wealth smaller than $\widehat{x}_1$ the best (interior) response of spouse 1 to $\widetilde{g}_2(0)$ is negative, which along with the non-negativity constraint brings us to a corner solution. Conversely, when $x_1 > \widehat{x}_1$, the poorer spouse wants to contribute a positive amount as response to $\widetilde{g}_2(0)$ and we get an interior equilibrium.

We now turn to the study of the properties of the equilibrium. It will turn out that they crucially depend on the type of equilibrium, interior, or corner, and thus ultimately on the wealth difference between the spouses' parents (see Proposition 1).

Let us start with the special case where parents have equal wealth $x_1 = x_2 \equiv x$ (in which case we necessarily have an interior solution). It can be easily verified that the subgame-perfect equilibrium of the two-stage game coincides with the Pareto-efficiency conditions (9)–(11) for equal weights; marginal utilities are equalized within and across families, and time is optimally allocated to the parent and to the production of the family public good. In other words, the *laissez-faire* solution corresponds to the utilitarian optimum. Via an adjustment in bequests, the old not only induce the efficient amount of attention from their children but they also achieve that the young couple produces the efficient amount of their family public good.

The intuition behind this outcome is as follows. The positive bequest equalizes consumption levels (between parents and children and between spouses) within each family. Since due to the adjustment in bequests, the child bears only half of the costs of higher attention but also receives half of its return, he opts for the efficient amount of $a_i^* \equiv a^e$. This resembles the famous rotten kid theorem by Becker (1974, 1991). However, in our setting, also public good investments within the young generation are efficient. Again, via the adjustment in bequests the child effectively bears only half of the costs of higher public good investments. Since each child equalizes his own marginal costs of investments with his own marginal benefits, the tradeoff by Eq. 18 becomes effectively the efficient one. Recall that from Eqs. 19–20, we have $u'(d_1^*) = u'(d_2^*)$. Consequently, both spouses have the same marginal benefit of the public good.[18] In other words, public good investments by each spouse are chosen such that the Samuelson rule, Eq. 8, is satisfied implying $G^* = G^e$.

To see this, note that for equal wages, Eqs. 19 and 20 imply

$$d_1^* = d_2^* \quad \Leftrightarrow \quad (\tau - g_1^*)w + x_1 = (\tau - g_2^*)w + x_2. \tag{22}$$

With $x_1 = x_2$ and $G^* = g_1^* + g_2^*$, we have $g_1^* = g_2^* = G^*/2$. That is, both spouses equally contribute to the family public good.

Interestingly, this result also holds when parents differ in their wealth levels, $x_1 < x_2$, as long as the difference is not too large so that the solution continues to be interior for both g_1 and g_2. This in turn is more likely, if the benefits $\varphi(G)$ of the public good are large implying a high G^e.[19] In this case, the spouse who expects the higher bequest (spouse 2) contributes more to the family public good than the one with the lower bequest. More precisely, the contributions to the family public good by spouse 2 are chosen so that consumption levels between the couple and between their parents are equalized and the *laissez-faire* allocation again coincides with the (utilitarian) Pareto-efficient solution. The equalization of spouses' consumption levels is in line with the result stated by Bergstrom et al. (1986). They show that when preferences are identical, all contributors to a public good will consume the same amount of

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[18] So that the social benefit is exactly twice the individual benefit.

[19] To see this, observe that the benchmark level of x_1, $\widehat{x}_1 \equiv x_2 - \widetilde{g}_2(0)w_2$ is decreasing in $\widetilde{g}_2(0)$, which will be, the larger is $\varphi(G)$.

private good (Theorem 5, point *(ii)*). The striking new feature of our result is that this equalization "spills over" to the spouses' parents (even though they differ in initial wealth). Another striking difference between our result and those obtained by Bergstrom et al. (1986) is that we obtain an efficient Nash equilibrium public good provision which is highly unusual in the voluntary contribution literature. This is exactly where the rotten kid mechanism discussed above comes in.

To sum up, we get efficiency and equalization of consumption levels within and across families. The intra-family equalization is already an implication of by Cornes and Silva's (1999) results, even though they do not highlight it. With identical utilities, consumption levels are equalized across siblings. The most significant novelty of our result lies in the interfamily equalization of consumption levels.

If, however, the difference in parent's wealth is strong, such that $x_1 \leq \widehat{x}_1 = x_2 - w\widetilde{g}_2(0)$, the spouse who expects the lower bequests (spouse 1) contributes nothing to the household public good; we have a corner solution, and condition (8) is no longer satisfied because the two spouses no longer have the same willingness to pay for the public good. The *laissez-faire* allocation then not only implies an inefficient level of the family public good but also unequal consumption levels within the couple and thus across families. However, even in that case, the rotten kids mechanism continues to be at work and enhances the provision of the household public good.[20] Similarly, since only the spouse with the richest parents contributes to the family public good (of which half is effectively paid by his parents), it continues to mitigate wealth differences.[21] The following proposition summarizes our results.

Proposition 2 *The laissez-faire solution (subgame perfect equilibrium) of the two-stage game with two families consisting of altruistic parents and selfish children (the latter constituting a couple who non-cooperatively produces a household public good) is Pareto-efficient if the children have the same wage rates, $w_1 = w_2 \equiv w$, and the parents' wealth is such that $x_1 \geq \widehat{x}_1 \equiv x_2 - w\widetilde{g}_2(0)$ where $\widetilde{g}_2(0)$ is the best-response of spouse 2 to $g_1 = 0$. Specifically, for an operative bequest motive in both families $i = 1, 2$*

- *(i)* *attention provided by the child satisfies $h'(a_i) = w_i \ \forall i$.*
- *(ii)* *consumption levels between and across families are equalized, i.e., we have $d_1 = d_2 = m_1 = m_2$.*
- *(iii)* *public good investments by the children satisfy the Samuelson rule, and*
- *(iv)* *the spouse with the richer parents provides more of the family public good. For $x_1 < \widehat{x}_1$, the subgame perfect equilibrium is not Pareto-efficient, the time allocation within families continues to be efficient, but the time devoted to the*

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[20] This follows because the term $\partial b_i^*/\partial g_i$ appears in Eq. 16. In words, the adjustment in bequests is formally equivalent to a subsidy on contributions which is well known to enhance provision (recall that individual contributions are strategic substitutes).

[21] Wealth is of course not the only conceivable source of heterogeneity across parents. They could also differ in their degree of altruism. This would be a departure from the rotten kid hypothesis, which relies on perfect altruism, and the full efficiency and equalization results would no longer obtain. Still, as long as the parent is altruistic (even though not perfectly), the rotten kid mechanism is at work and inter- and intra-generational transfers are higher (closer to the first-best) than without an operative bequest motive.

household public good is no longer interior but at a corner and the Samuelson rule is not satisfied.

4.2 Heterogenous children

When children differ in wages $w_1 < w_2$, the pattern of equilibria that can arise is more complex. We can have *(i)* a corner equilibrium with only the *lower*-wage spouse contributing, *(ii)* an interior solution with both spouses contributing, and even *(iii)* a corner equilibrium with only the *higher*-wage spouse contributing. Roughly speaking, one can expect the interior solution to arise when wage and parents' wealth are not too different. Equilibrium *(iii)* can be expected if wages are not too different and the high-wage spouse has much richer parents. In all other cases, equilibrium *(i)* can be anticipated. A precise characterization of the parameter values yielding the different type of equilibria is tedious and not necessary for the issues we are dealing with. We shall thus restrict ourselves to presenting an example illustrating that the different cases can indeed arise.

Example 1 Assume the following functional forms for utility $u(d) = 4 \ln d$, $\varphi(G) = \frac{1}{2} \ln G$ and $h(a) = 4\sqrt{a} - 2$. Additionally, assume $w_1 = 1 < w_2 = 2$, $x_2 = 20$ and the total amount of time available is $\tau = 8$. With Eq. 17, we have for the optimal attention

$$h'(a_i^*) = 2(a_i^*)^{-1/2} = w_i \quad \Rightarrow \quad a_1^* = 4, \quad a_2^* = 1$$

implying $h(a_1^*) = 6$ and $h(a_2^*) = 2$. With our functional forms for utility Eq. 18 amounts to

$$(\tau - a_i^* - g_i^*)w_i + h(a_i^*) + x_i \leq 8w_i G.$$

With the above parameters, we can write the optimal response function for spouses 1 and 2 as

$$(8 - 4 - g_1) + 6 + x_1 \leq 8G, \tag{23}$$

$$(8 - 1 - g_2)2 + 2 + 20 \leq 16G. \tag{24}$$

For $x_1 = 15$, we have a corner equilibrium with only the lower-wage spouse contributing: $g_1^* = 3$ and $g_2^* = 0$; case (i). For $x_1 = 6\frac{5}{6}$, both spouses contribute: $g_1^* = \frac{3}{2}$ and $g_2^* = \frac{2}{3}$; case (ii). For $x_1 = 3$, we have a corner equilibrium with only the higher-wage spouse contributing: $g_1^* = 0$ and $g_2^* = 2$; case (iii).

From our perspective, the interesting feature is that equilibria of types *(ii)* and *(iii)* are *never* efficient: the spouse with the higher time cost contributes at least partly to the public good production. As to type *(i)* equilibria, they are in general inefficient. The equilibrium is efficient (and corresponds to the utilitarian optimum) only when

$$d_1^e = \frac{(\tau - a_1^e - g_1^e)w_1 + h(a_1^e) + x_1}{2} = d_2^e = \frac{(\tau - a_2^* - g_2^e)w_2 + h(a_2^e) + x_2}{2}.$$

Since g_1^e and g_2^e are uniquely defined in the unequal wage case, this can occur only "by coincidence" (see Section 5.2 for further details). Inefficiency arises for exactly the same reasons as in the corner solution case with identical wages considered in the previous subsection. Marginal utilities between spouses are no longer

equalized (see Eqs. 19 and 20). Thus, the Samuelson condition is not satisfied in the Nash equilibrium and the allocation in the *laissez-faire* is not Pareto-efficient. Notice, however, that the levels of attention continue to be at their efficient levels (we have $a_i^* = a_i^e$).

Finally, the case where wages differ but are sufficiently close deserves some attention. Since the best-response functions are continues in wages, the equilibrium allocation will also be a continuous function of both wages.[22] Consequently, when w_1 is sufficiently close to w_2 and when wealth differences are not "too large" the equilibrium will be interior and it will be "almost" or "near" efficient and utilitarian. To be more precise as w_1 tends to w_2, the outcome will tend to the one described in Section 4.1. While this result is rather trivial from a theoretical perspective, it is quite important for the practical implications of our analysis. In reality, the case where wages are *exactly* equal may be very rare, but under suitable mating patterns, wages may often be close enough so that the notion of near efficiency applies and has relevant implications.

The next section studies those cases where the *laissez-faire* solution is inefficient and shows how the efficient solution can be implemented through lump-sum transfers across families.

5 Implementation of the efficient solution

Assume now that some public authority can put in place policies *before* the game between children and parents takes place.

5.1 Corner solution with identical children

We have shown in Section 4.1 that with identical children, the equilibrium is inefficient when it corresponds to a corner solution and $x_1 < \widehat{x}_1$. This in turn occurs (for any given level of x_2) when the wealth difference between parents is sufficiently significant. This problem can be overcome if wealth is redistributed (at stage 0, before the game is played) to bring wealth differences within the range that yields an interior solution. We then know from Proposition 3 that this will induce an equilibrium which corresponds to the utilitarian solution.

The result is formally stated in the following proposition (which is established in the Appendix).

Proposition 3 *Assume that children have equal wages $w_1 = w_2 \equiv w$, but the parent's wealth difference is such that $x_1 < \widehat{x}_1$, then the utilitarian Pareto-efficient*

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[22] This requires some additional technical conditions, but since our best-reponse functions are "well-behaved," it is plain that the continuity applies in our setting.

solution can be implemented by a lump-sum transfer T from high- to low-wealth families, given by

$$T \in \left[\frac{x_2 - x_1 - wG^e}{2}, \frac{x_2 - x_1 + wG^e}{2} \right].$$

Observe that G^e while being the utilitarian public good level for the initial wealth levels x_1 and x_2, it is of course also the optimal level for the after transfer wealth levels (only total wealth matters for Pareto-efficiency). The fact that the transfer can take any value in the above interval resembles Warr's (1983) neutrality result. As long as the transfer induces an interior solution for the public good provision, income redistribution is irrelevant in the presence of a privately provided public good. The effect that a pure redistribution of wealth between families has on each child is absorbed entirely by adjustments to his public good investments. One can of course set $T = (x_2 - x_1)/2$ to make (after transfer) wealth levels equal, but this is not necessary.

5.2 Different wages

Now, we must design a transfer scheme that ensures that the equilibrium is such that (only) the low wage individual contributes *and* that spouses' (equilibrium) consumption levels are equal. Recall that this latter condition ensures that both spouses have the same willingness to pay for the public good, which in turn will ensure that the Samuelson condition, Eq. 8, holds. To understand why the sole contributor then provides the Pareto-efficient level, recall that his contribution is subsidized through the extra bequest so that he only bears half of its cost (see expression (13)). And with consumption levels equalized between spouses, his private benefits are precisely equal to half of the social benefits. The following proposition, established in the Appendix, states the required level of transfer which is equal to half the difference in "total income" between both families (evaluated at the optimal solution).

Proposition 4 *If parents differ in wealth, $x_1 < x_2$ and children in wages, $w_1 \lessgtr w_2$, the Pareto-efficient allocation with equal weights can be decentralized by a lump-sum transfer from high- to low-income families. This transfer is simply half the income difference between both families and given by*

$$T = \frac{x_2 - x_1 + (\tau - a_2^e - g_2^e)w_2 - (\tau - a_1^e - g_1^e)w_1 + h(a_2^e) - h(a_1^e)}{2}. \quad (25)$$

Observe that in this expression, one of the g_i^e's (the one associated with the higher-wages spouse) is always equal to zero. Intuitively, with this transfer, we achieve $d_1^* = d_2^*$ (which is necessary for the Nash equilibrium to satisfy the Samuelson condition) but for $w_j < w_i$ ($i, j = 1, 2$) also implies $w_j u'(d_j^*) < w_i u'(d_i^*)$ which from Eq. 18 ensures that only the low-wage spouse (type j) will contribute.

6 Concluding remarks

Our model and results can be considered from two perspectives. The first one is purely methodological. The rotten kid model is a prominent part of family economics. Consequently, a comprehensive study of its ramifications and limitations is interesting in itself; we provide a building block to this edifice. The main point we have made is that when applied to an interfamily setting (where families are "linked" by young spouses), the rotten kid mechanism may take care of both efficiency and redistribution (between the spouses' respective families). This effectively extends the results of Cornes and Silva (1999) to an interfamily context. When spouses have equal wages, it will yield an efficient outcome and wash out parent's wealth differences (as long as they are not too large). For larger wealth differences, the mechanism would have to be supplemented by a (mandatory) transfer scheme which brings the discrepancies back within the relevant range. Interestingly, the mechanism continues to be effective (though less "perfect") when spouses' wages are not exactly equal but sufficiently similar. The outcome will then be close to the utilitarian allocation. This remark is crucial when it comes to asses the practical implications of our results. In reality, it is of course unlikely that spouses have *exactly* the same wages. Still, assortative mating is the rule rather than the exception and cases where spouses have sufficiently similar wages is not uncommon (see, e.g., Schwartz and Mare 2005).

All these findings are admittedly achieved within a very stylized setting which explains in part the strong results. For instance, aid is efficient because both its costs and benefits are expressed in monetary terms (like in Becker's original setting). Under more general preferences, the underlying mechanism remains at work. Continuing with the case of aid, it may then no longer be efficient, but there will generally be a positive level in equilibrium even though children are selfish. From that perspective, we contribute to identifying both the strengths and weaknesses of the rotten kid mechanism.

The second perspective is to explore the real world implications. Surprisingly, our analysis shows that assortative mating can effectively be a factor that decreases inequalities at least within families and between families linked by marriage. Whether or not this is the case crucially depends on the specific mating pattern. In particular, when mating occurs mainly according to the spouses' wages, then this may have positive implications both for efficiency and redistribution. It may then contribute to eliminate wealth differences. However, when the dominant factor is the parent's wealth, mating behavior may be neither good for efficiency nor for redistribution.

This being said, one has to keep in mind that the extent of redistribution achieved through this channel is limited (to families "linked" by marriage). Consequently, while it can eliminate some wealth differences, it cannot be considered as a substitute for a well-designed redistributive policy (which can be more or less egalitarian according to the society's preferences). Put differently, while family linkages (both between and within generations) have to be taken into account for policy design, rotten kid-like exchanges cannot realistically be expected to replace the need for public policy (like social insurance and more generally redistribution). While this limits the scope of our results' implications, it also strengthens their relevance. Specifically, our

results are not suffering from the "everything neutral" syndrome and do not lead to the unrealistic predictions of a dynastic approach à la Bernheim and Bagwell (1988).

These qualifications notwithstanding our results imply that the relationship between assortative mating and income inequality is more complex than it may appear at first. As Torche (2010) points out, there exist only few studies dealing with this issue and the studies there typically conclude that assortative mating increases inequality (see for instance Schwartz 2010 or Greenwood et al. 2014). Our results suggest that these studies miss part of the underlying effects.

First, we show that assortative mating will tend to reduce intra-family inequalities and, through marriage linkages, inequalities within an earnings class, but it does of course increase interfamily inequalities. Consequently, to examine the rather complex relationship between mating patterns and inequality, empirical studies would have to distinguish between the two types of inequality. To the best of our knowledge, no research paper has so far explored this avenue.

Second, we show that the impact on inequality does not only depend on wages but also on wealth. This is in line with Torche's observation: "This study highlights the limitations of using single aggregate measures of spousal educational resemblance (such as the correlation coefficient between spouses' schooling) to capture variation in assortative mating and its relationship with socioeconomic inequality."

To sum up, our results suggest that more studies are needed and they plead for looking at the data from a different perspective.

Appendix

Proof of Proposition 1

Assume $w_1 = w_2 = w$ and consider a given level of $x_2 > 0$ (and continue to assume without loss of generality that $x_1 \leq x_2$). From Eq. 18, we can see that a corner solution, $(g_1^* = 0, g_2^* > 0)$, prevails if

$$2\varphi'(G^*) = 2\varphi'(\widetilde{g}_2(0)) < u'(d_1^*)w, \tag{26}$$

where $G^* = g_2^* = \widetilde{g}_2(0)$. For $x_1 = x_2$, we have

$$u'(d_1^*)w = u'\left(\frac{(\tau - a_1^*)w + x_1}{2}\right)w < u'\left(\frac{(\tau - a_2^* - g_2^*)w + x_2}{2}\right)w = 2\varphi'(g_2^*),$$

so that condition (26) does *not* hold. Since $u'(d_1^*)$ increases as x_1 decreases, there exists at most one $\widehat{x}_1$ defined by $\widehat{x}_1 = x_2 - g_2^* w_2$ (yielding $d_1^* = d_2^*$) with $g_2^* = \widetilde{g}_2(0)$ and $g_1^* = 0$ for which Eq. 26 holds as equality. When $x_1 < \widehat{x}_1$, there exist then a corner solution (with only type 2 contributing). And since $\widetilde{g}_2(g_1)$ is decreasing, it is plain that there cannot also be an interior equilibrium (which would require $d_1^* = d_2^*$). When $x_1 > \widehat{x}_1$, condition (26) is violated and the equilibrium can only be interior.

Observe that for $x_1 = \hat{x}_1$, we have $g_1^* = 0$ and $g_2^* > 0$ but these levels also satisfy the conditions for an interior solution (the constraint that $g_1 \geq 0$ hold with equality but is not binding). This is where the "transition" between corner and interior solution occurs.

To complete the proof, it remains to show that an interior equilibrium is unique. Observe that the slopes of the reaction functions are (in absolute values) smaller than one. Substituting Eq. 17 into Eq. 18 and differentiating yields

$$\frac{dg_i}{dg_{-i}} = -\frac{2\varphi''(G)}{u''(d_i)\frac{w_i}{2} + 2\varphi''(G)} \in (-1, 0).$$

This means that the best reply map is a contraction which immediately implies uniqueness (see Vives 2001, pages 47–48).

Proof of Proposition 3

To determine the optimal transfers, (T_1, T_2) (the ones that implement the utilitarian Pareto efficient solution), we have to revisit the different stages of the game. In stage 2, parents leave a bequest to their children. This bequest is chosen so as to equalize consumption between the parent and the child,

$$m_i = d_i = \frac{(\tau - a_i - g_i)w + h(a_i) + x_i + T_i}{2} \quad \forall i.$$

Note that as long as bequests are interior, it is irrelevant whether the lump-sum transfer is paid by the children or by the parent.[23] With T_i set so that $T_1 = -T_2 \equiv T$, if follows from Eqs. 19 and 20 that the best-response functions of spouses 1 and 2 are implicitly defined by

$$u'\left(\frac{(\tau - a_1^* - g_1^*)w_1 + h(a_1^*) + x_1 + T}{2}\right) w_1 = 2\varphi'(G^*), \qquad (27)$$

$$u'\left(\frac{(\tau - a_2^* - g_2^*)w_2 + h(a_2^*) + x_2 - T}{2}\right) w_2 = 2\varphi'(G^*). \qquad (28)$$

The transfer must be chosen such that an interior solution for both g_1^* and g_2^* is guaranteed. At an interior solution, we have $d_1^* = d_2^*$, implying

$$\frac{(\tau - a_1^* - g_1^*)w_1 + h(a_1^*) + x_1 + T}{2} = \frac{(\tau - a_2^* - g_2^*)w_2 + h(a_2^*) + x_2 - T}{2}.$$

Since $w_1 = w_2 \equiv w$, we have $a_1^* = a_2^*$ and the above equation reduces to

$$x_1 + T - g_1^* w = x_2 - T - g_2^* w.$$

At an interior solution, $(g_1^*, g_2^*) \in (0, 1) \times (0, 1)$, the overall public good production, $g_1^* + g_2^*$, is uniquely determined by G^e. That is, we can write

$$T = \frac{x_2 - x_1 + (2g_1^* - G^e)w}{2}.$$

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[23] With operative bequests, Ricardian equivalence holds for the transfers.

Since $g_1^* \in (0, G^e)$, the optimal transfer is in the interval as stated in Proposition 2.

Proof of Proposition 4

The transfer across families must be chosen such that $d_1^* = d_2^*$, then from Eqs. 19 and 20, it can be seen that only the spouse with the lower-wage rate (spouse i) contributes to the family public good implying $g_i^* \equiv G^e$ and $g_j^* = 0$ ($i, j = 1, 2; i \neq j$). The transfer T must thus be chosen such that

$$\frac{(\tau - a_1^* - g_1^*)w_1 + h(a_1^*) + x_1 + T}{2} = \frac{(\tau - a_2^* - g_2^*)w_2 + h(a_2^*) + x_2 - T}{2}.$$

Solving for T yields expression (25) in Proposition 4.

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