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ORIGINAL PAPER

Differences in fertility behavior and uncertainty: an economic theory of the minority status hypothesis

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Abstract We revisit the question of why fertility behaviors and educational decisions appear to vary systematically across ethnic groups. We assess the possibility that differences in fertility across groups remain even though their socio-economic characteristics are similar. More specifically, we consider that parents' fertility decisions are affected by the uncertainty concerning the future economic status of their offspring. We assume that this uncertainty varies across groups and is linked to the size of the group one belongs to. We find theoretical support for the minority status hypothesis according to which members of large minorities usually have a higher fertility than those in the majority facing low potential for social mobility while small minorities have lower fertility.

Keywords Fertility · Minority status · Uncertainty

JEL Classification D81 · J13 · J15

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1 Introduction

Differences in fertility behavior and educational attainment between minority and majority populations around the world are persistent and have existed during most of history. Ethnic minorities are often found to exhibit higher fertility levels and lower educational investments than the majority counterpart; it is, however, the contrary in some cases. For instance, in the USA in 2006, the Black and Hispanic minorities had a completed fertility rate[1] of, respectively, 2.003 and 2.300 children, while the White had 1.765, and the Asian, 1.689. As already ascertained in the quality/quantity trade-off literature, fertility is inversely related to educational investment in children. Data in 2006[2] gives that 81.7 % of the Black and 59.3 % of the Hispanic were high school graduates or more, while 86.1 % of the White and 87.4 % of the Asian.[3]

Socio-economic characteristics of parents, by which we intend level of income and education, are often thought to drive differences in fertility. More educated and/or wealthier parents may have a higher opportunity cost of raising children and thus prefer smaller families. It could also be that educated parents have a higher taste for educated children. They, thus, make less children but invest more in their education. In both cases, if characteristics of parents converge, then so do the fertility rates. In de la Croix and Doepke (2003, 2004) for instance, differences in fertility collapse as inequality across households disappears. In this view, persistent differentials in fertility and educational investment therefore arise only if socio-economic characteristics do not converge across groups.

More recently, an emerging strand of the literature has underlined the important role of culture in fertility choices. Fernàndez and Fogli (2006, 2009) find that cultural proxies are significant determinants of fertility controlling for socio-economic characteristics. For instance, fertility and labor force participation rates in the origin country of migrants or number of siblings of parents have been found to be significantly correlated to fertility decisions of migrant women in the USA. In this view, differences in fertility may persist as beliefs and norms of different groups evolve differently than socio-economic characteristics. In this paper, we do not deny the potential effects of culture; nevertheless, we depart from this kind of explanation in order to check whether an economic mechanism, other than parental characteristics, may be useful to explain the remaining cross-race differences in fertility.

In our opinion, uncertainty about future economic conditions of children may be a potential determinant of differences in fertility behaviors. The literature on endogenous fertility already introduced uncertainty but in a different fashion. Kalemli-Ozcan (2003) builds a stochastic model where the number

[1] Number of children ever born to 1,000 women aged 40–44. Source: U.S. Census Bureau.

[2] Source: U.S. Census Bureau.

[3] These differentials become even bigger when we look at the proportion of college graduates or more.

of surviving children is a random variable. She finds that the risk stemming from the loss of a child generates a precautionary demand for children that leads parents to overshoot their desired fertility. A key result is that an exogenous decrease in the infant mortality rate reduces the uncertainty faced by parents and thus decreases the precautionary demand for children in favor of educational investment. Doepke (2005) shows that precautionary demand is likely to be quantitatively small and cannot be responsible alone for large variations of fertility as experienced during demographic transitions. Nevertheless, we think that it may play a role in a post-demographic transition contexts, like the one we study. On the other hand, Hondroyiannis (2010) documents that uncertainty about macro-economic variables may be detrimental to childbearing, as responsible parents would not decide to have one more child if they were to face a high risk of unemployment or low income in the future. He uses a panel data of European countries to show that measures of economic uncertainty such as output volatility and the unemployment rate are negatively related to total fertility rates. In such frameworks, the fact that groups face different levels of uncertainty would explain persistent fertility gaps.

The sociological literature has viewed the behavior of minorities in terms of fertility and educational decisions from two complementary perspectives. The first, known as *assimilation* or *characteristics hypothesis* asserts that differences in fertility arise from differences in socio-economic achievement. According to this view, as a minority group reaches the socio-economic characteristics of the majority group, fertility differences disappear.[4] A second approach, namely the *minority status hypothesis*, argues that minority group membership can have an independent effect on fertility. This theory, as formulated in the seminal work of Goldscheider and Uhlenberg (1969), admits the relevance of socio-economic factors in explaining differences in fertility behaviors between majority and minority groups but highlights the importance of several other factors: desire for social and economic mobility, psychological insecurity, segregation, and intragroup and intergroup socio-economic relationships that may induce fertility differences to persist even though socio-economic characteristics of the minority converge towards those of the majority.

In this view, belonging to a minority has an independent effect on fertility. One possible reason might be the presence of barriers to minorities for upward economic mobility. These obstacles may lead minority members to modify their fertility and educational investment in order to compensate for potential limitations to economic achievement even when social, economic, and demographic attributes for majority and minority populations are very similar. On the one hand, according to Goldscheider and Uhlenberg (1969), "the residual lower fertility of minority groups may result from the insecurity associated with minority group status". This mechanism of fertility limitation among minority

[4]See, for instance, Gordon (1964, 1978), Ryder (1973), Bean and Marcum (1978), Bean and Swicegood (1985), and Trovato (1987) for an exhaustive discussion on the social characteristics hypothesis in sociology.

Table 1 Children ever born per 1,000 women aged 35–44 by race and Hispanic origin and educational attainment of mothers in the USA 2006

Education	White	Asian	Hispanic	Black
Total	1.764	1.765	2.286	1.978
Not high school graduated	2.197	2.689	2.758	2.735
High school (4 years)	1.860	1.905	2.244	2.105
Some college (no degree)	1.829	1.848	1.970	1.827
Bachelor degree	1.626	1.596	1.834	1.648

Source: US Census Bureau, Current Population Survey, 2006

members operates only when it is thought as a means to improve socio-economic perspectives.[5] On the other hand, when the obstacles to upward mobility are substantial enough, the marginality and the insecurity associated with the minority group status may induce minority members to choose a higher fertility than majority members with similar characteristics as a way to self-insure against potential discriminations.

We develop a model of endogenous fertility that gives theoretical support to the minority status hypothesis. More specifically, we model an uncertainty mechanism that has an heterogeneous effect on fertility across groups: we find that small minorities display a lower fertility than comparable majority members, while large minorities exhibit a higher fertility rate.[6] This theory has been tested in several empirical studies, but, to our knowledge, it has never been investigated in a theoretical framework.[7] We revisit the question of why fertility behaviors and educational decisions appear to vary systematically across ethnic groups. We assess the possibility that differences in fertility across groups remain even though their socio-economic characteristics are similar. In this work, we claim that a way to obtain these persistent fertility differences is to take into account uncertainty about future earnings or expected socio-economic conditions of children.

The data on completed fertility by race and Hispanic origin and educational attainment shown in Table 1 seem to support this hypothesis. Indeed, on the one hand, for low levels of education, all the minority groups exhibit a higher fertility than the White. On the other hand, for higher levels of education, some groups still exhibit a higher fertility (the Hispanic), while others adopted a fertility rate that is similar to that of the majority or even lower (the Black and the Asian, respectively). Notice that the data also clearly

[5]In a study on Chinese-American women, Espenshade and Ye (1994) explain the lower fertility of this ethnic group with the sacrificial effort in their pursuit of social and economic equality.

[6]This result is not in contradiction with Hondroyiannis (2010) as he documents a negative average effect of aggregate uncertainty on total fertility rates, while we are looking at the heterogeneous effect of uncertainty on completed fertility for different groups. It could perfectly be the case that these heterogeneous effects net out and give a negative overall effect, but this is not the scope of our paper.

[7]See Sly (1970), Roberts and Lee (1974), Jiobu and Marshall (1977), Johnson and Nishida (1980), and Boyd (1994).

show the interdependency between fertility and education: the more educated women are, the lower the fertility rate. Georgiadis and Manning (2011) find similar results for the UK. Indeed, their study shows that only the Chinese community exhibits a lower fertility than the UK-born, while all other communities, Bangladeshi, Pakistani, Indian, and Black Caribbean, display a higher fertility.

In our economy, parents decide on their number of children and on the education level of each child. We take a standard model of endogenous fertility, to which we add that parents care about the future earnings of their children. Unlike the existing literature, this paper stresses the role of uncertainty and, in particular, of the insecurity about the expected economic status of children.[8] Assuming that this uncertainty depends on the size of one's group allows to infer that minority status has an independent effect on parental behavior. To this purpose, we model labor market outcomes of children as a restricted pool matching process.[9] It provides, first, a convenient link between the level of uncertainty and the size of the group and furthermore fairly general conclusions as random matching is nothing but a particular case of the restricted pool matching. We use a logarithmic utility function, which allows us to cancel out the effect of parental human capital on fertility decisions in order to focus on the risk mechanism. Furthermore, this formulation implies that our agents are prudent in the sense of Kimball (1990), meaning that facing risk, they have a precautionary demand, here, for children. We show that the risk term associated to the matching process displays an inverted U-shaped relationship with respect to the size of one's group, meaning that the risk is perceived to be the highest for groups of intermediate size. Consequently, as fertility serves as an insurance mechanism, it also displays the same inverted U-shaped relationship. We compare this result to an example of the USA: large enough minorities (Black and Hispanic) have higher fertility than White, while Asian have a lower fertility. As a result, this paper formulates a contribution to the theoretical literature on endogenous fertility: we argue that non-convergence in the socio-economic characteristics might not be the only determinant of persistent fertility gaps. We claim that minority membership may influence fertility and educational investment behaviors in various directions, which constitutes a theoretical support of the minority status hypothesis. We link group size and uncertainty about future economic status of the children and introduce the fact that this uncertainty may influence fertility decisions. Section 2 introduces the static model, while Section 3 solves the dynamics. Section 4 gives a parametrization of the model for the US case. Section 5 concludes.

[8] Some papers already introduced uncertainty in endogenous fertility models, but, to our knowledge, the uncertainty focuses on the survival rate, never on the expected economic status of children.

[9] We discuss in Section 5 another possible interpretation of our model in which uncertainty arises from the matching process in the marriage market.

2 The model

2.1 The certainty case

Let us take a simple OLG model of endogenous fertility with uncertainty, where we consider the future wage rate of children as stochastic.[10] Parents maximize their lifetime utility, choosing their own consumption c_t, the number of offsprings n_t, and the schooling time per child r_t, given an ad hoc altruism parameter γ. Let us assume a production function that is linear in human capital, as it is often assumed in this literature. The labor market is competitive, so that agents act as price takers with respect to the wage rate per unit of human capital. Future earnings of the children, denoted W_{t+1}, are therefore well captured by the expression $h_{t+1}w^e_{t+1}$, where h_{t+1} is the human capital of children and w^e_{t+1} is their expected wage rate per unit of human capital. Parents decide how much time they devote to the labor market and to raising children. Bearing a child implies a fixed time cost $\phi \in [0, 1]$; while the time spent on education per child, r, is chosen by parents. Human capital of children is produced according to the technology described in Eq. 3, where ψ and τ are positive parameters whose sum is less or equal than 1, so that returns to parental human capital and education are non-increasing and η is a positive multiplier.

$$\max_{c_t, n_t, r_t} (1-\gamma)\ln(c_t) + \gamma \ln(W_{t+1}) \tag{1}$$

s.t.

$$c_t = (1 - n_t(\phi + r_t))\, h_t w_t \tag{2}$$

$$W_{t+1} = \sum_{j=0}^{n_t} h_{t+1} w^e_{j,t+1} \tag{3}$$

$$h_{t+1} = \eta r_t^{\tau} h_t^{\psi} \tag{4}$$

As the expected wage rate per unit of human capital is a priori equal for all children of the same family, we obtain that children of the same cohort are identical. We can therefore write $W_{t+1} = n_t h_{t+1} w^e_{t+1}$. Furthermore, we treat n_t as a continuous variable. We do so for convenience and without loss of generality. Indeed a continuous fertility may be interpreted as the average of the (discrete) optimal fertilities of all members of an heterogeneous group (in terms of wealth for instance). Our results remain valid as long as as the mechanism we propose has a constant effect on fertility along the heterogeneous dimension.

[10]The modeling strategy is similar to that of Kalemli-Ozcan (2003), but instead of the survival rate, we take the future wage rate of children as stochastic.

Taking the first order conditions and checking that the second order conditions are verified, we obtain the following optimal decisions:

$$r_t^* = \frac{\tau\phi}{1-\tau} \tag{5}$$

$$n_t^* = \frac{\gamma(1-\tau)}{\phi}. \tag{6}$$

Notice that optimal decisions do not depend on parental human capital nor on expected wage rate. The intuition is straightforward (and was already given by Kalemli-Ozcan (2003) about the survival probability) in the sense that any exogenous change in w_{t+1}^e represents a change in the price of having well-off children, and this price change has two contradicting effects: income and substitution, which exactly cancel out with logarithmic utility functions. This means that we do not observe a quality/quantity trade-off in the sense that more educated parents will not prefer having less children in order to educate them more, at least as long as w_{t+1}^e is certain.[11] Studying the logarithmic utility case cancels out the effect of parental human capital on optimal decisions and allows us to focus essentially on the role of uncertainty on differential fertility. Another feature of the logarithmic utility function is that agents in our model are prudent in the sense of Kimball (1990), which implies that agents have a precautionary behavior.[12]

2.2 Uncertainty and matching in the labor market

Let us now introduce uncertainty, assuming w_{t+1}^e is a random variable. We consider that the economy is populated by several groups (be they ethnic, cultural, religious, etc.). Everyone needs to match someone in the society in order to produce output and get paid, which represents a reasonable assumption about the job market. The production function is assumed to be linear in own human capital and to depend on the quality of the match. Everyone matches somebody with probability one, but we assume individuals may be biased towards matching someone belonging to their own group. Indeed, it is now established that networks play an important role in the job seeking process.[13]

[11] The logarithmic formulation implies that a fixed fraction of parental time (and thus income) is devoted to children, so that consumption is never affected. Furthermore, if the price of well-off children falls (through an increase in the expected wage rate), the relative price of education with respect to childbearing has not changed, which is why the trade-off between quality and quantity of children is not affected by the expected outcome on the labor market and that optimal decisions remain the same.

[12] This feature certainly constrains our results in the sense that we rule out the possibility that, facing a greater risk, parents would decide to decrease their demand for children, but generalizing our model to a less specific utility function would require further research.

[13] See Simon and Warner (1992).

These networks usually appear within a given community. This is why, as in intermarriage models,[14] agents will first try to match someone from their group and, only in a second attempt, they will match anybody in the society. More specifically, there is an exogenous probability π that an individual matches in the restricted pool of coworkers from the same group, in which case the outcome is μ_s. With probability $1-\pi$ instead, an agent of group i matches anybody in the society, that is someone from the same group with probability $(1-x_i)$, where x_i represents the proportion of non-i agents in the society, and someone from another group with probability x_i, in which case the wage rate is μ_d.

$$w^e = \mu_d + (\mu_s - \mu_d)X \quad \text{where} \quad X \sim \text{Bern}\,(\pi + (1-\pi)(1-x_i))$$

where π proxies the bias towards own group and x_i is the proportion of non-i agents in the society. The wage rate per unit of human capital is therefore a linear transformation of a Bernoulli distribution with probability $\pi + (1-\pi)(1-x_i)$.[15] We interpret a high value of π as a situation in which there are less opportunities and/or more costs for two agents from different groups to match, due to physical or cultural distance. For instance, it is often considered in labor economics that language creates an extra cost to match outside of one's own group. Lang (1986) shows that a competitive labor market tends to minimize communication costs through segregation. As for μ_s and μ_d, we are agnostic with respect to their ranking. Indeed, it could be the case that there is a gain to diversity in the workplace ($\mu_d > \mu_s$), or on the contrary, a gain to homogeneity or even purely discriminative behavior across groups ($\mu_s > \mu_d$). We only require that there exist some systematic cross-group variability in the wage rate per unit of human capital. In the case in the USA, Neal and Johnson (1996) and Johnson and Neal (1998) show that Black–White wage gaps remain substantial even after controlling for some measures of human capital, which gives credit to our assumption.

Considering that the Bernoulli processes determining the wage of each child are uncorrelated, W_i thus becomes a linear transformation of a binomial distribution with n_t draws and probability $\pi + (1-\pi)(1-x_i)$.

$$\begin{aligned} W_i &= \sum_{j=0}^{n_t} h_{t+1} w^e_{j,t+1} \\ &= h_{t+1}[n_t \mu_d + (\mu_s - \mu_d)Y] \\ \text{where} \quad & Y \sim \text{B}\,(n_t, \pi + (1-\pi)(1-x_i)) \end{aligned}$$

[14] See Bisin and Verdier (2000).

[15] Recall that the Bernoulli distribution yields 1 with some probability p and 0 with probability $1-p$.

Y follows a binomial distribution and gives the number of children that will get an intragroup match. The idea is that all n_t offsprings of the same family get at least μ_d for matching someone and that Y of these n_t get the intragroup match "bonus" $(\mu_s - \mu_d)$ on top.[16]

We then derive the expected value and variance of W_i, recalling that the expected value of a binomial distribution with probability of success p and n draws is np and that its variance is $np(1-p)$:

$$E(W_i) = n_t h_{t+1}\left[(\pi + (1-\pi)(1-x_i))\mu_s + (1-\pi)x_i\mu_d\right]$$

$$V(W_i) = n_t\left[h_{t+1}(\mu_s - \mu_d)\right]^2(\pi + (1-\pi)(1-x_i))(1-\pi)x_i$$

The variance can also be retrieved using Bienayme's formula for the variance of the sum of uncorrelated random variables $V\left[\sum_{i=0}^{n} X_i\right] = \sum_{i=0}^{n} V[X_i]$ but the assumption of uncorrelated random variables is not crucial here. Indeed an issue would arise only in the case of perfectly correlated Bernoulli processes, in which case we would have n^2 instead of only n in the variance of the sum. This way, all the n would cancel out in the risk term and fertility would not allow to mitigate risk. It is enough to assume non-perfectly correlated Bernoulli processes to keep the qualitative results. To simplify the exposition, we stick to the uncorrelated case.

Notice that the variance is an inverted U-shaped function in x_i (as the variance of a binomial law is with respect to the probability of success). Intuitively, it means that, under some conditions that will become clear later on, the large majorities and small minorities will experience smaller variances.

We use the Delta method[17] to solve the maximization program defined by Eqs. 1, 2, 3, and 4, where $W_{i,t+1}$ is now a stochastic component. Specifically, we take a Taylor approximation of the expected utility around its mean. Dropping the time subscripts for convenience, we get

$$U(W_i) = U(E[W_i]) + (W_i - E[W_i])U_{W_i}(E[W_i])$$
$$+\frac{(W_i - E[W_i])^2}{2}U_{W_iW_i}(E[W_i]) + o\left(W_i^2\right).$$

[16] Again, thinking the other way around of a poor intragroup outcome and an extragroup match bonus would not change the result. The only crucial assumption is that there should be some variability in the outcome, that is $\mu_s \neq \mu_d$.

[17] Given the fact that wages are always positive regardless of the matching outcome so that utility is well-behaved, it is appropriate to use a local approximation like the Delta method.

Now, taking the expected value of the above expression, we obtain

$$E[U(W_i)] = U(E[W_i]) + \overbrace{E[W_i - E[W_i]]}^{=0} U_{W_i}(E[W_i])$$
$$- \frac{\overbrace{E[W_i - E[W_i]]^2}^{=V(W_i)}}{2!} \frac{\gamma}{(E[W_i])^2}$$
$$= U(E[W_i]) - \frac{\gamma(\mu_s - \mu_d)^2(\pi + (1-\pi)(1-x_i))(1-\pi)x_i}{2n_i\left[(\pi + (1-\pi)(1-x_i))\mu_s + (1-\pi)x_i\mu_d\right]^2}$$
$$= (1-\gamma)\ln(c_i) + \gamma\ln(E[W_i]) - \frac{f(x_i)}{n_i} \tag{7}$$

where $f(x_i) = \dfrac{\gamma(\mu_s - \mu_d)^2(\pi + (1-\pi)(1-x_i))(1-\pi)x_i}{2\left[(\pi + (1-\pi)(1-x_i))\mu_s + (1-\pi)x_i\mu_d\right]^2}$.

The expected value of the difference between a random variable and its mean is zero, that of the square of this difference is its variance. The second term consequently cancels out, while the third remains, representing the risk effect. Notice that, due to the logarithmic formulation, the two first terms of Eq. 7 are equivalent to the utility in the certainty case. The only difference is therefore the last term that represents the risk. It is composed of a function of the size of the rest of the population, $f(x_i)$, that is always positive, because it is a variance divided by the square of an expected value over n_i. One may already observe that agents will increase n_i in order to reduce the risk term and that this increase in n_i will be positively correlated to $f(x_i)$. This simple model, thus, proposes an economic mechanism that links fertility and the composition of the population through uncertainty.

Proposition 1 *The fertility obtained maximizing the expected utility Eq. 7 (under uncertainty) is always greater that or equal to the one that maximizes the utility under certainty. There exists a threshold* $\bar{x} = \dfrac{\mu_s}{(1-\pi)(\mu_d + \mu_s)} \in [0, 1]$ *in size of the remaining population below which fertility is increasing in* x_i *and above which it is decreasing if and only if* $\pi < \frac{\mu_d}{\mu_s + \mu_d}$.

Proof Let us take the first order conditions of the maximization of the expected utility given in expression 7 subject to the constraints 2 and 3. We obtain

$$\frac{\gamma}{n_i} - \frac{(1-\gamma)(\phi + r_i)}{n_i(\phi + r_i) - 1} + \frac{f(x_i)}{n_i^2} = 0 \tag{8}$$
$$\frac{\gamma\tau}{r_t} - \frac{(1-\gamma)n_t}{n_t(r_t + \phi) - 1} = 0 \tag{9}$$

Solving Eq. 9 for r_i gives

$$r_i = \frac{\gamma\tau(1 - n_i\phi)}{n_i(1 - \gamma(1-\tau))}. \tag{10}$$

From which we observe that r_i and n_i are necessarily inversely correlated. Then substituting into Eq. 8:

$$\frac{2\phi n_i^2 + \gamma(\phi f(x_i) - 2(1-\tau))n_i - \gamma f(x_i)}{2n_i^2(n_i\phi - 1)} = 0.$$

Taking the discriminant $\Delta = [\gamma(\phi f(x_i) - 2(1-\tau))]^2 + 8\phi\gamma f(x_i)$, one may observe it is always positive and that there are two solutions of opposite signs. We keep the positive one and write the optimal fertility choice as follows:

$$n_i^{**} = \frac{-\gamma[\phi f(x_i) - 2(1-\tau)] + \sqrt{[\gamma(\phi f(x_i) - 2(1-\tau))]^2 + 8\phi\gamma f(x_i)}}{4\phi}. \tag{11}$$

First, notice that in the case $\mu_s = \mu_d$, the uncertainty disappears as the outcome becomes certain. Furthermore, observe that in this case, $f(x_i) = 0$ and the FOCs of the problem under uncertainty amount to those under certainty. It therefore follows that optimal decisions are the same in this case. As soon as $\mu_s \neq \mu_d$, then $f(x_i) > 0$ as it is a variance over the square of an expected value. And one may observe from Eq. 11 that n^{**} is necessarily greater than n^* because as $f(x_i)$ increases, the square root term becomes greater in absolute value than the first term of the numerator, which is always negative. The first part of Proposition 1 is therefore proven. Using the same argument, we can say that n^{**} varies in the same direction as $f(x_i)$.

Let us then study the variations of $f(x_i)$.

$$\lim_{x_i \to 0} f(x_i) = 0$$

$$\lim_{x_i \to 1} f(x_i) = \frac{\gamma(\mu_s - \mu_d)^2 \pi(1-\pi)}{2\,[\pi\mu_s + (1-\pi)\mu_d]^2} > 0$$

$$\frac{\partial f(x_i)}{\partial x_i} = \frac{(1-\pi)\gamma(\mu_d - \mu_s)^2[(1-\pi)x_i(\mu_d + \mu_s) - \mu_s]}{2((1-\pi)x_i(\mu_d - \mu_s) + \mu_s)^3}$$

The sign of the derivative depends only on the term $(1-\pi)x_i(\mu_d + \mu_s) - \mu_s$, as the rest is necessarily positive. We therefore have that the derivative is nil for $\bar{x} = \frac{\mu_s}{(1-\pi)(\mu_d + \mu_s)}$, positive (respectively negative) for $x < \bar{x}$ (respectively $x > \bar{x}$). f is consequently an inverted U-shaped function of x_i over $[0, 1]$ if and only if $\bar{x} \in (0, 1)$, i.e., $\pi < \frac{\mu_d}{\mu_s + \mu_d}$ (because $\bar{x} \geq 0$, the only restriction is on the upper bound). □

Under uncertainty, only a risk term is added to the utility function with respect to the certainty case. This risk term is an inverted U-shaped function of x_i and monotonically decreasing in n_t. So fertility has a similar inverted U-shape with respect to x_i. Intuitively, this result comes from the fact that the risk term is driven by the variance of the binomial process determining the labor market outcome. For a very small (respectively very large) group, the probability of an intergroup (respectively intragroup) match is very high and therefore there is little variability in the outcome. Instead, for average size groups, both types of matches are likely to happen, which increases unambiguously the variability of the outcome with respect to the previous situation. In other words, a member of a small minority has high chances to match with the other group, thus the perceived uncertainty is low. The same applies to a member of a big majority and a match within their own group. A member of a middle-sized group, instead, is about equally likely to match in either groups, which increases the perceived uncertainty. As the number of children represents also the number of draws in the matching process, parents react to a higher risk by increasing fertility because increasing the number of draws unambiguously decreases the risk term.[18] In other words, the uncertainty on the labor market is perceived differently by agents from different groups, as suggested by the minority status hypothesis, which induces different fertility behaviors.

Performing some comparative statics of our model allows to investigate what would be the effect of an anti-discrimination policy. Indeed, suppose the government imposes a ban on labor market discrimination, then one may imagine that the gap between μ_s and μ_d decreases. We show that such a policy would reduce the risk factor f and therefore the fertility of any group in the society. Indeed, let us assume $\mu_s > \mu_d$ and rewrite the function f in terms of the wage gap $d \equiv \mu_s - \mu_d > 0$ and any probability of intragroup match $p(x_i)$:

$$f(x_i) = \frac{\gamma p(x_i)(1 - p(x_i))d^2}{2[\mu_d + p(x_i)d]^2}$$

Taking the derivative with respect to d, we obtain

$$\frac{\partial f(x_i)}{\partial d} = \frac{\gamma d \mu_d p(x_i)(1 - p(x_i))}{[\mu_d + p(x_i)d]^3} > 0 \tag{12}$$

which is always positive. A symmetric analysis can be done for the case $\mu_s < \mu_d$ and would lead to the same conclusion. Finally, notice that a strict ban on discrimination that imposes equal mutual treatment of groups can be represented in the model by setting $\mu_s = \mu_d$, in which case there is no uncertainty, and fertility of all groups is equal to the certainty case.

3 Dynamics

Given initial conditions on human capital, $h_{i,0}$, and on population share $x_{i,0}$, an intertemporal equilibrium is a sequence of temporary equilibria of the static

[18]This mechanism is actually similar to the precautionary demand for children that arises when the survival rate is stochastic as in Kalemli-Ozcan (2003).

problem defined by Eqs. 10 and 11 that satisfies for all $t \geq 0$ and all i the following laws of motion:

$$h_{i,t+1} \equiv \xi(x_{i,t}, h_{i,t}) = \eta r_{i,t}^{\tau} h_{i,t}^{\psi} \tag{13}$$

$$x_{i,t+1} \equiv \quad \chi(x_{i,t}) \quad = \frac{x_{i,t}\bar{n}_{i,t}}{x_{i,t}\bar{n}_{i,t} + (1 - x_{i,t})n_{i,t}} \tag{14}$$

where $\overline{n}_{i,t}$ represents the average fertility of the other groups in the society. Notice that $x_{i,t+1}$ is a function of $x_{i,t}$, $\bar{n}_{i,t}$ and the parameters, while $h_{i,t+1}$ depends on $x_{i,t}$, $h_{i,t}$ and the parameters.

Let us now study the two-group case, say i and j, in order for us to derive analytically the steady states and their stability properties. $x_{i,t+1}$ boils down to a function of $x_{i,t}$ only. Indeed, in the two-group case, $\overline{n}_{i,t} = n_{j,t}$ depends only on $x_{i,t}$ because $x_{i,t} = 1 - x_{j,t}$ while, in a N group generalization, it would depend also on the relative sizes of the other groups.[19] In Eq. 14, one may see that for x_i to be at a steady state, one needs $x_i = 0$ or $x_i = 1$ or $n_i = \bar{n}_i \equiv n_{j,t}$. For this last equality to occur in the two-group case, it is necessary and sufficient that the risk term be the same for the two groups. Indeed, both types of agents then solve the same problem and necessarily have the same fertility. We therefore have that x_i^* is a steady state if and only if $x_i^* = 0$, $x_i^* = 1$ or x_i^* is such that $f(x_i^*) = f(x_j^*) \equiv f(1 - x_i^*)$. We are looking for the intersections over [0, 1] between two inverted U-shaped functions that are symmetric with respect to 1/2. Recall that $f(x_i)$ starts at 0 (for $x_i = 0$), increases and reaches a maximum at $\bar{x}$, and then decreases to a positive limit at $x_i = 1$. We therefore have only two possible cases: either $\bar{x} > 1/2$, in which case the two functions intersect only once for $x = 1/2$, or $\bar{x} < 1/2$ and then $f(x_i)$ and $f(1 - x_i)$ would intersect three times: $x_1 < 1/2$, 1/2 and $x_2 > 1/2$. Notice that $\bar{x} = 1/2$ if and only if $\mu_s = \mu_d$, which we rule out because it amounts to the certainty case.

(1) $\bar{x} > 1/2$ (illustrated in the left panel of Fig. 1)
$f(x_i)$ starts at 0 and increases monotonically until reaching a maximum at $\bar{x}$ then decreases to $f(1) > 0$, while $f(1 - x_i)$ is the symmetric graph with respect to $x = 1/2$. We therefore have that $f(x_i) < f(1 - x_i)$ for $x_i < \bar{x}$, while the opposite holds for the other side, and the only intersection is for $x_i = 1 - x_i = 1/2$. Consequently, $n_i < \bar{n}_i$ (respectively $>$) to the left (respectively right) of $\bar{x}$. We infer from Eq. 14 that $\chi(x_i) > x_i$ (respectively $<$) for $x_i < \bar{x}$ (respectively $>$). From which we conclude that 1/2 is a globally stable steady state.

[19] Whatever the number of groups, it is noticeable from Eq. 13 that the only way to have a society divided into steady shares is that either one group tends to 1 and all the others to 0, or fertility of the various groups should be equal. In this latter case, the risk term should be the same for all groups and thus they should all be of equal size. We are, thus, able to say that a one-group society or a society divided among groups of equal size are potential steady states. It is, however, very difficult to derive more analytical results with more than two groups.

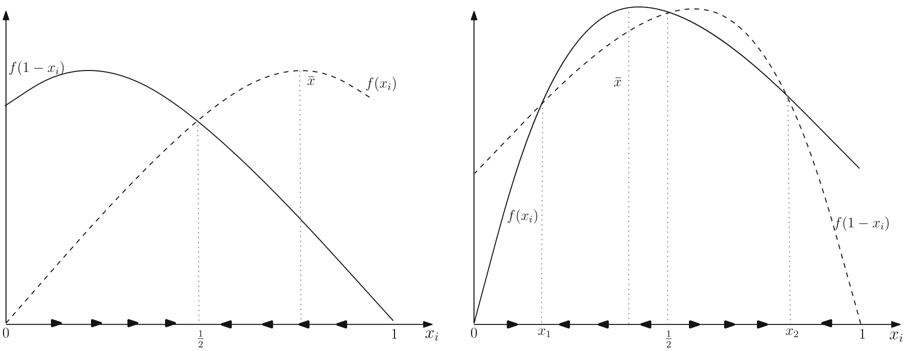


Fig. 1 Graphical illustration of the possible steady states

(2) $\bar{x} < 1/2$ (illustrated in the right panel of Fig. 1)
Over $[0, 1/2]$, $f(x_i)$ increases from 0 to its maximum and then decreases towards $f(1/2)$, while $f(1-x_i)$ increases monotonically from $f(1) > 0$ to $f(1/2)$. By continuity, $f(x_i)$ and $f(1-x_i)$ need to have crossed at $x_1 \in (0, 1/2)$. The same reasoning may be applied symmetrically over the interval $[1/2, 1]$ to obtain $x_2 \in (1/2, 1)$ such that $f(x_2) = f(1-x_2)$. We can therefore infer that $n_i > \bar{n}_i$ and consequently $\chi(x_i) < x_i$ over $(x_1, 1/2) \cup (x_2, 1)$, while the opposite holds over $(0, x_1) \cup (1/2, x_2)$. From which we conclude that x_1 (respectively x_2) is a stable steady state over the interval $(0, 1/2)$ (respectively $(1/2, 1)$), while 0, 1/2 and 1 are unstable.

We therefore derive the following proposition:

Proposition 2 *The dynamics of x_i converges to* $1/2$ *if and only if* $\bar{x} > 1/2$. *If* $\bar{x} < 1/2$ *then x_i converges to x_1 (respectively x_2) for an initial value* $x_{i,0} < 1/2$ *(respectively >).*

Proof Proposition 2 stems directly from the previous discussion. □

Once we have solved for the population share, the remaining problem concerning human capital is straightforward. Indeed, the long-run value for human capital depends only on the population share and the parameters. From Proposition 1, we know that fertility and education vary in opposite direction following a shock on the parameters. We, thus, observe dynamics of opposite direction for human capital as the group shares evolve towards the steady state.

Steady states 0 and 1 in population shares are always unstable. In turn, a bifurcation occurs as the point at which $f(x_i)$ reaches its maximum, namely

$\bar{x}$, which depends only on parameters (π, μ_s and μ_d), is shifted from the right to the left of 1/2. There, the stable steady state 1/2 becomes unstable and splits into two new stable steady states x_1 and x_2, which is a case of supercritical pitchfork bifurcation.[20] It means that if π, μ_s and μ_d are such that the maximum of $f(x_i)$ occurs for sizes of the remaining population greater than 1/2 (that is for groups smaller than half the population), then the f of the small group is always higher than that of the large group. The small group, thus, has a higher fertility and catches up with the majority. If the maximum of $f(x_i)$ occurs instead for sizes of the remaining population smaller than 1/2 (that is for a majority group), then we observe distinct steady states for each group. The initially large one converges towards $x_1 < 1/2$, while the initially small to $x_2 = 1 - x_1 > 1/2$. This is intuitive because in this case, small majorities have a higher f than large minorities, inducing the group sizes to diverge until f for majorities eventually decreases and crosses that of the minority.

We can now assess the effect of an anti-discrimination policy in a dynamic context. Indeed, the bifurcation occurs as the parameter $\bar{x} \equiv \dfrac{\mu_s}{(1-\pi)(\mu_d + \mu_s)}$ goes below 1/2, which happens if $\pi < \dfrac{\mu_d - \mu_s}{\mu_d + \mu_s}$. Notice that case (2), with two stable steady states at x_1 and x_2, may occur only if $\mu_d > \mu_s$, that is if there exists a productivity gain in an heterogeneous match. This comes from the fact that the risk factor f is composed of the variance at the numerator, which is always larger for minorities ($x_i < 1/2$), and of the square of the expected value at the denominator, which is necessarily smaller for minorities if there is discrimination or a productivity loss in an heterogeneous match ($\mu_s > \mu_d$). A necessary condition for case (2) to arise is therefore that an intergroup match should be more productive than an intragroup one, a sufficient condition is that π should be small enough and the productivity gap large enough. In terms of comparative statics, we can therefore conclude that an anti-discrimination policy reducing the wage gap when $\mu_s > \mu_d$ does not affect the long-run distribution of shares in the society (1/2 is globally stable) unless a strong enough positive discrimination policy or a policy that subsidizes cross-group cooperation is led, in which case the group that is initially a minority may remain a minority forever.

Looking more closely at the case $\mu_s > \mu_d$ because it is by definition more suitable for an anti-discrimination policy to apply, we can show that a reduction in the wage gap reduces the fertility gap and therefore the speed of convergence to the steady state. Indeed, let us take the case of a minority of size $x_i > 1/2$. We can see in Fig. 1 that $f(x_i) > f(1 - x_i)$, so that $n_i > n_j$. Using the expression for $\dfrac{\partial f(x_i)}{\partial d}$ given by Eq. 12, it is possible to compare it

[20] See Wiggins (1990).

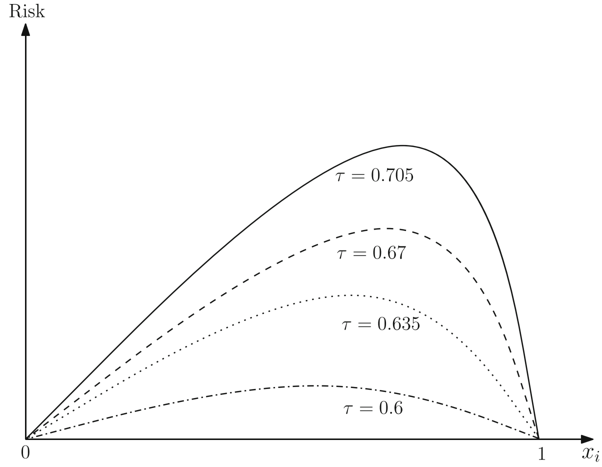
to $\frac{\partial f(1-x_i)}{\partial d}$. First notice that $p(x_i) < p(1-x_i)$, that is an intragroup match in more likely for a majority member, so that the denominator of $\frac{\partial f(x_i)}{\partial d}$ is smaller than that of $\frac{\partial f(1-x_i)}{\partial d}$. Comparing the numerator boils down to signing $p(x_i)(1-p(x_i)) - p(1-x_i)(1-p(1-x_i))$, which can be written $\pi(1-\pi)(2x_i-1)$. We, therefore, have that the numerator of $\frac{\partial f(x_i)}{\partial d}$ is larger than that of $\frac{\partial f(1-x_i)}{\partial d}$ because $x_i > 1/2$. Consequently, following a drop in d due to an anti-discrimination policy, $f(x_i)$ decreases more than $f(1-x_i)$, so that n_i decreases more than n_j. As n_i was initially bigger than n_j, we have a decrease in the fertility gap.

4 Parametrization

The theoretical results in the previous sections show that persistent differences in fertility may be explained through other channels than differences in socio-economic characteristics. Indeed, in our model, fertility relies on how parents perceive the uncertainty about the job market opportunities their children will face. In this section, we construct a numerical example that aims at replicating qualitatively the fertility behaviors and group shares of the US society. We first calibrate the "traditional" parameters and then find values for the remaining parameters that allow to replicate the ranking of fertility differences across ethnic groups. As already stated, we ignore the effect of human capital and income on fertility. For this reason, this exercise rather constitutes a parametrization of our model that allows to assess what might be the extent of the channel we illustrate here than a proper calibration.

We extend the model to four groups (White, Black, Hispanic, and Asian). The only heterogeneity comes from the size of each group within the society. According to the U.S. Census (2006), we observe the following group shares: White, 0.687, Black, 0.128, Hispanic, 0.143, Asian, 0.041. Our model is parameterized so that a period has a length of 30 years. The time cost parameter ϕ for bearing a child is set to 0.075 based on the evidence in Haveman and Wolfe (1995) according to which the time cost of having a child is approximatively 15 % of a parent's time endowment. If we assume that a child lives 15 years with the parents, this opportunity cost amounts to 15 % of 15 years, which represent 7.5 % of a parent's total time endowment. The return to parental time τ is calibrated in de la Croix and Doepke (2003). While they set $\tau = 0.635$, we let τ vary from the value of 0.600 to 0.705 in order to observe the effect of the return to parental time on the fertility behaviors of different groups. In our model, we know that agents will spend optimally γ percent of their income on children and the rest on consumption. We therefore choose γ to match the ratio of expenditures on education over total expenditures (expenditures on private consumption plus on education). We obtain roughly $\gamma = 0.18$.

Fig. 2 Risk term



For every value of τ, we then choose the μ in order to fit the observed fertility for each group. We therefore solve the following minimization program:

$$\min_{\mu_s, \mu_d} \sum_i \left[n_i(\mu_s, \mu_d) - \tilde{n}_i\right]^2$$

where $\tilde{n}_i$ are the observed fertilities. The minimization gives us, respectively, for $\tau = 0.6$, $\mu_s = 1.691$, and $\mu_d = 1.062$; for $\tau = 0.635$, $\mu_s = 1.991$, and $\mu_d = 0.882$; for $\tau = 0.67$, $\mu_s = 2.284$, and $\mu_d = 0.837$; and for $\tau = 0.705$, $\mu_s = 2.465$, and $\mu_d = 0.783$. As we can observe, the gap needed between the expected outcome of an intergroup and an intragroup match is increasing in the return to parental time. Figure 2 plots the risk term in function of x_i for these scenarios.

As already discussed in the previous sections, the relation between the risk term and the size of the other group has an inverted U-shape. Observe that the risk term shifts up as we switch to a higher τ scenario. The reason is that the gap between intragroup and intergroup match given by the minimization program is positively related to the return on parental time. This enlarged gap allows the risk term to fluctuate more, which makes our mechanism quantitatively more plausible.

Given the parameter values, we are able to compute the fertilities predicted by the model. As shown in Table 2, we obtain the correct ranking of fertilities if τ is high enough. Obviously, we are not able to match precisely the observed fertilities, but we can say that if returns to parental time are important enough,

Table 2 Predicted fertility

τ	White	Black	Hispanic	Asian
0.600	2.004	1.996	2.000	1.950
0.635	1.972	2.036	2.052	1.882
0.670	1.900	2.064	2.084	1.830
0.705	1.804	2.088	2.110	1.792
Data	1.765	2.003	2.300	1.684

our mechanism may explain non-negligible differences in fertility across groups without taking into account any effect of socio-economic characteristics.

5 Conclusion

The aim of this paper was to study the fertility and educational investment differentials between racial and ethnic minorities within an endogenous fertility framework that explicitly accounts for uncertainty about future earnings or expected socio-economic conditions. The analysis led to three main conclusions. First, our model may be interpreted as a theoretical support of the minority status hypothesis according to which minority members usually have a higher fertility facing low potential for social mobility but may, in some instances, strategically decrease their fertility. This mechanism arises from the fact that the insecurity associated with the minority status may influence individual expectations about potential social mobility. A feature of our model is that it is not specific to the labor market. Actually, one may think of any other matching process that generates uncertainty differentials across groups. For instance, if μ_s and μ_d were interpreted as the surplus stemming from intragroup and intergroup marriages, the overall implications of the model would remain unchanged. Second, unlike previous results in the literature, we observed the possibility of persistent differences in fertility and educational investment even though socio-economic characteristics of parents are equivalent. Indeed, we link uncertainty about future earnings to the size of one's group and find that the risk associated has an inverted U-shape with respect to the size of one's group. As we model fertility as an insurance device, we obtain that large majorities and small minorities may both have a lower fertility and higher educational investment than intermediate size groups. Third, a parametrization exercise for the model shows that our model may qualitatively reproduce observed differences in fertility, although the effect of socio-economic characteristics on fertility is not modeled. Including an interaction between fertility and educational investment decisions may help to reproduce more closely the data. Finally, a natural extension of this article would be to incorporate the effect of parental human capital on fertility in order to quantitatively assess the respective importance of socio-economic characteristics versus minority status as determinants of differences in fertility.

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