

Mixing Bismarck and child pension systems: an optimum taxation approach

Robert Fenge · Jakob von Weizsäcker

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Abstract Pensions with a strong tax–benefit link (Bismarck pensions) minimise the labour–leisure distortion of the public pension system. By contrast, pensions with a strong link of benefits to the number of children (child pensions) minimise the fertility distortion. When both types of distortion are present, we obtain a Corlett–Hague result regarding the optimal mix of the two pension formulae: the Bismarck pension should be given a positive weight if and only if children are more complementary to leisure than consumption. Alternative fertility instruments such as child benefits turn out to be perfect substitutes to a child pension.

Keywords Pay-as-you-go pension · Fertility · Optimal taxation

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1 Introduction

Pay-as-you-go pensions (PAYG) have the potential to distort both the labour–leisure decision and the fertility decision of its members. The labour–leisure

Responsible editor: Alessandro Cigno

R. Fenge (✉)
University of Munich, Schackstr. 4, 80539 Munich, Germany
e-mail: fenge@ifo.de

J. von Weizsäcker
Bruegel, Rue de la Charité 33, 1210 Brussels, Belgium

distortion is caused by the fact that returns to PAYG contributions are typically lower than the interest rate in a dynamically efficient economy. In addition, the returns to marginal pension contributions may be further depressed as the result of a weak tax–benefit link.

The fertility distortion of PAYG pensions arises because parents only obtain a negligible fraction of the pension contributions of their own children. Thereby part of the benefits of having children is socialised whilst the cost of raising children remains private. As a result, fertility could be expected to be distorted downwards.

It is conceivable that this effect may have contributed somewhat to the extremely low fertility rates in countries such as Germany, Italy and Spain (see Cigno and Rosati 1996; Cigno et al. 2003). The resulting demographic imbalance further reduces the implicit rate of return of PAYG pension in their respective pension systems. In that sense, the distortion towards reduced fertility can exacerbate the labour–leisure distortion.

To date, the economic literature has only treated these two distortions separately. In models with endogenous labour supply but exogenous fertility, Fenge (1995), Brunner (1996) and Sinn (2000) have shown that a move towards pension with a maximum tax–benefit link (Bismarck pensions) minimises the labour–leisure distortion. By contrast, in models with endogenous fertility and an exogenous labour supply, Kolmar (1997) and von Auer and Büttner (2004) have shown that the fertility distortion of public pensions should be fully eliminated by linking pension to fertility.

This raises the question what efficient PAYG pension systems look like when both labour supply and fertility are endogenous.¹ The present paper analyses this question using the modelling framework developed by Fenge and Meier (2005, 2008): conceptually, the pension system is split up into a contribution-related Bismarckian pension and a fertility-related child pension component. By increasing the weight of the Bismarck pension, the labour–leisure distortion can be reduced, and by increasing the share of the child pension, the fertility distortion can be reduced. However, at the possibility frontier, a stronger contribution–benefit link implies a weaker child–benefit link and vice versa.

Interestingly, our model immediately yields the insight that the policy optimisation problem at any time t is equivalent to the classic Corlett and Hague (1953) problem from optimal tax theory where consumption and children are two consumption goods that can be taxed whilst leisure cannot be taxed. Hence, it is found that a pure child pension system is efficient when consumption and children are equally complementary to leisure. When children are more complementary to leisure than consumption, a partial Bismarckian

¹In our model, fertility is deterministic. For moral hazard problems that arise in pension schemes with stochastic fertility, see Cremer et al. (2006).

pension system is efficient. And when consumption is more complementary to leisure than children, then a hyper-child pension system is efficient where children are subsidised at the expense of consumption. If, as seems plausible, children are indeed more complementary to leisure than consumption, this may help to rationalise the historical emphasis on the tax–benefit link.

Since the representative agent framework does not capture distributional objectives, a model with two productivity types is also discussed, relying on the papers by Edwards et al. (1994) and Nava et al. (1996) for the equivalent optimal taxation problem. Again, albeit for different reasons, the Bismarck pension component should be positive if children are more complementary to leisure than consumption.

The significance of these results is further reinforced as we show that the child pension discussed above is equivalent to child benefits. Hence, these instruments are interchangeable within the present modelling framework, and the often controversial claims that one of these instruments should be favoured over another would need to rely on additional assumptions.

The next section presents the model and discusses the welfare criterion. In Section 3, the second-best mix of Bismarck and child pension elements is analysed in an optimal taxation framework. Section 4 relates our findings to the existing literature by exploring the extreme cases of inelastic fertility or labour–leisure decisions. Section 5 compares child pensions to child benefits and Section 6 concludes.

2 The model

A standard two-period overlapping generations model forms the basis of our considerations. For simplicity, we consider a one-sex population. In period t , the size of the population is N_t . The growth factor of the young generation in period $t + 1$ is defined as $N_{t+1}/N_t \equiv \bar{n}_t$. Each generation t works and reproduces during period t and lives in retirement during period $t + 1$. The lifetime utility of a representative parent of generation t depends on own consumption in period t , c_t , own consumption in period $t + 1$, c_{t+1} , the number of children, n_t ,² and an index of the children’s quality, q_t :

$$U(c_t, c_{t+1}, n_t, q_t). \tag{1}$$

We assume the utility function to be continuous, strictly concave and strictly increasing in all arguments. The quality per child q_t can be understood as a good produced domestically by the parent who uses as inputs own time, h_t , spent with the child and a child-specific consumption good, z_t , bought on the

²Note that this individual number of children has to be distinguished from the population growth factor \bar{n}_t which is equal to the average number of children.

market (Becker 1991; Cigno 1991; Balestrino et al. 2002). The price on the market for the child-specific consumption good is B_t . The domestic production function is given by:

$$q_t = q(h_t, z_t) \tag{2}$$

and increases monotonically in both arguments. The parent disposes of a time endowment which is normalised to unity. In the first period, she allocates this time to working which yields wage at the rate w_t and to leisure. We assume that the parent spends her leisure completely with the children. If she has n_t children, her parental time is $h_t n_t$. The rest of the total time is working time and given by $1 - h_t n_t$. From wage income $w_t (1 - h_t n_t)$, the parent has to pay a contribution to a public pension scheme. The contribution rate is τ_t . Next to costs of optional child consumption and parental time, the upbringing of children incurs also a fixed cost per child D_t which covers essential child expenditure without which the child would and could not exist. In the second period $t + 1$, the consumption of the parent is financed by savings which yield an interest at the factor R_{t+1} and by a public pension p_{t+1} . The budget constraints in both periods are:

$$c_t + s_t + B_t z_t n_t + D_t n_t = w_t (1 - \tau_t) (1 - h_t n_t), \tag{3}$$

$$c_{t+1} = R_{t+1} s_t + p_{t+1}. \tag{4}$$

The government runs a PAYG financed pension scheme. The contribution revenues in period $t + 1$ are used to finance the pension p_{t+1} of the N_t retirees of generation t . The average pension of the representative parent is given by $\bar{n}_t \tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})$. The upper bar of \bar{h}_{t+1} denotes the average parental time that a generation $t + 1$ spends with its children. Hence, the average pension of the representative individual of generation t is given by the average number of children of the members of generation t , \bar{n}_t , multiplied by the average pension contribution of the next generation $t + 1$, $\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})$. A share $(1 - \alpha_t)$ of this pension is paid out as a Bismarck pension where benefits are determined by individual contributions compared to the average contribution of the generation, and a share α_t represents the child pension where benefits are determined by the number of children raised compared to the generational average number. The total PAYG pension of the representative parent of generation t can thus be written as:

$$\begin{aligned}
 p_{t+1} &= \bar{n}_t \tau_{t+1} w_{t+1} \left(1 - \bar{h}_{t+1} \bar{n}_{t+1}\right) \left[(1 - \alpha_t) \frac{w_t \tau_t (1 - h_t n_t)}{w_t \tau_t (1 - \bar{h}_t \bar{n}_t)} + \alpha_t \frac{n_t}{\bar{n}_t} \right] \\
 &= \tau_{t+1} w_{t+1} \left(1 - \bar{h}_{t+1} \bar{n}_{t+1}\right) \left[(1 - \alpha_t) (1 - h_t n_t) \frac{\bar{n}_t}{(1 - \bar{h}_t \bar{n}_t)} + \alpha_t n_t \right]. \tag{5}
 \end{aligned}$$

Combining Eqs. 3 and 4 by substituting for s_t and using Eq. 5, the inter-temporal budget constraint of the representative parent of generation t can be written as:

$$\begin{aligned}
 c_t + \frac{c_{t+1}}{R_{t+1}} + B_t z_t n_t + \left[D_t - \alpha \frac{\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})}{R_{t+1}} \right] \cdot n_t \\
 = w_t (1 - h_t n_t) \left[(1 - \tau_t) + (1 - \alpha) \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot \frac{\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})}{R_{t+1}} \right].
 \end{aligned}
 \tag{6}$$

The parent chooses own consumption in both periods, c_t and c_{t+1} , the number of children, n_t , child-specific consumption, z_t , and parental time, h_t , so as to maximise lifetime utility $U(c_t, c_{t+1}, n_t, q(z_t, h_t))$ by taking account of child quality production (Eq. 2) and the inter-temporal budget constraint (Eq. 6). The first-order conditions yield the following necessary and sufficient conditions of the concave maximisation problem:

$$\frac{U_{c_{t+1}}}{U_{c_t}} = \frac{1}{R_{t+1}},
 \tag{7}$$

$$\begin{aligned}
 \frac{U_{n_t}}{U_{c_t}} = B_t z_t + D_t - \alpha \frac{\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})}{R_{t+1}} \\
 + w_t h_t \left[(1 - \tau_t) + (1 - \alpha) \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot \frac{\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})}{R_{t+1}} \right],
 \end{aligned}
 \tag{8}$$

$$\frac{U_q}{U_{c_t}} q_{z_t} = B_t n_t,
 \tag{9}$$

$$\frac{U_q}{U_{c_t}} q_{h_t} = w_t n_t \left[(1 - \tau_t) + (1 - \alpha) \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot \frac{\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})}{R_{t+1}} \right].
 \tag{10}$$

All conditions (7), (8), (9) and (10) have the well-known meaning that the marginal rate of substitution between the respective decision variables have to be equal to the marginal rates of transformation at the utility maximum. With respect to condition (8), it is assumed that the marginal cost per child in terms of parental consumption in period t is higher than the average pension payment per child $\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1}) / R_{t+1}$. If we confine our attention to inner solutions, this means that the price for child-specific consumption

B_t , the direct cost D_t and the opportunity cost of forgone wage income and Bismarckian pension claims (the last term on the RHS of Eq. 8) is sufficiently large to over-compensate the average pension payment per child. This assures that the optimal number of children is finite since children are consumption goods with a positive net cost.

A remark to our interpretation of children as consumption goods is in order here. To the extent that the human race evolved by evolution, it is reasonable to assume that parents intrinsically enjoy having children (see , e.g. Dawkins 1976). In the absence of serious constraints, we would, therefore, expect the marginal child to have a positive cost for its parents (net of tax and government transfers) for the simple reason that, as long as children have a negative cost, the intrinsic joy of having them will induce people to have more of them. The literature confirms that the direct costs and opportunity costs of foregone wage income are substantial in developed economies. For several countries, including the United States, Calhoun and Espenshade (1988) and Calhoun (1994) find that forgone lifetime labour supply rises roughly proportionally in the number of children, at least in the range up to three children. Angrist and Evans (1998) estimate that the presence of a third child reduces the probability of work of married women by about 17 percentage points and family income by 13%. This feature is reflected in our model. Of course, the impact of children on (female) labour supply is less negative in countries with more generous provision of public childcare and in countries with a lower level of economic welfare—where the economic necessity to work is high (Uunk et al. 2005). Also, just because the cost of children is positive net of taxes and government transfers, this does not necessarily mean that children are costly for a society overall. In particular, as convincingly argued by Sinn (2001), the pension system heavily redistributes the incomes of children from their parents to childless pensioners which may distort fertility decisions.

From the government's perspective, a pension policy can be described as sequence of policy pairs $\{(\alpha_0, \tau_0), (\alpha_1, \tau_1), (\alpha_2, \tau_2), \dots\}$. The obvious question to ask for any given policy is whether it is efficient. The traditional Pareto criterion only allows comparisons between states with the same individuals and, therefore, a fortiori, also the same number of individuals. However, the policy parameters in our model generally impact fertility and thereby the size of the next generation.

Therefore, a modified efficiency criterion is used: a pension policy is said to be efficient if no policy reform exists that makes at least one representative agent of the presently living or future generations better off whilst not making any representative agent of any other generation worse off.³ This efficiency criterion is in fact equivalent to the Pareto criterion if it is assumed that for any generation those who remain unborn in a particular realisation of the

³Since all children are treated equally, this efficiency concept is equivalent to the one proposed by Baland and Robinson (2002) where the average utility of all living individuals in the future matters.

world have exactly the same level of utility as the representative agent of the members of the generation who are actually born.

From a political perspective, our efficiency criterion appears reasonable. In effect, the Pareto criterion is applied to everybody who is at present an actor in the political economy sense by virtue of being alive. Already deceased generations are protected through the irreversibility of the past. And whilst potential members of future generations are not completely protected since they have no guarantee of being born, they are at least to some extent protected through the restriction that their representative agent is not to be made worse off. Nevertheless, solutions that are efficient in the above sense but imply a rapid demographic implosion or explosion should be interpreted with some caution. As a next step, we define global and local efficiency of a pension policy.

Definition 1 A pension policy $\{(\tilde{\alpha}_0, \tilde{\tau}_0), (\tilde{\alpha}_1, \tilde{\tau}_1), (\tilde{\alpha}_2, \tilde{\tau}_2), \dots\}$ is called globally efficient if no globally reformed policy $\{(\alpha_0, \tau_0), (\alpha_1, \tau_1), (\alpha_2, \tau_2), \dots\}$ exists that makes at least one representative agent of the presently living or future generations better off whilst not making any representative agent of any other generation worse off.

Definition 2 A pension policy $\{(\tilde{\alpha}_0, \tilde{\tau}_0), (\tilde{\alpha}_1, \tilde{\tau}_1), (\tilde{\alpha}_2, \tilde{\tau}_2), \dots\}$ is called locally efficient if no local reform policy that only changes the pension parameters of a particular generation $\{(\tilde{\alpha}_0, \tilde{\tau}_0), (\tilde{\alpha}_1, \tilde{\tau}_1), (\tilde{\alpha}_2, \tilde{\tau}_2), \dots, (\tilde{\alpha}_{t-1}, \tilde{\tau}_{t-1}), (\alpha_t, \tau_t), (\tilde{\alpha}_{t+1}, \tilde{\tau}_{t+1}), \dots\}$ for any generation $t \geq 0$ exists that makes at least one representative agent of the presently living or future generations better off whilst not making any representative agent of any other generation worse off.

Clearly, a pension policy that is not locally efficient cannot be globally efficient. Hence, local efficiency is a necessary condition for global efficiency. However, local efficiency is not generally a sufficient condition for global efficiency. The essential reason for this is that local efficiency may be satisfied everywhere whilst the growth rate of the wage sum exceeds the interest rate for every generation—the so-called Aaron condition (Aaron 1966). In other words, the locally efficient policy might be dynamically inefficient and, therefore, not globally efficient since the expansion of the PAYG pension system would lead to an efficiency gain. However, the validity of the Aaron condition would be inconsistent with the small open economy assumption in the long run, provided that the rest of the world is to the left of the golden rule. It is true that the government of a small open economy could increase fertility beyond the golden rule by an appropriate pension policy without affecting factor prices. This generates an incentive to finance consumption by foreign debt which can be acquitted by the growing future generations. However, in the long-run equilibrium, this Ponzi game violates the small economy assumption. Since these issues are examined extensively by Kolmar (1997), we focus on local efficiency in this paper.

In order to test for local efficiency at time t , we examine the following maximisation problem for the state. The policy parameters (α_t, τ_t) are varied so as to maximise the utility of the representative agent of generation t whilst keeping the representative agents of all other generations indifferent compared to the status quo. Note that the policy parameters (α_t, τ_t) only affect the utility of the representative agents of generation t and that of generation $t - 1$. In fact, the utility of generation $t - 1$ is only influenced through the channel of the total pension contributions of generation t . Therefore, in order to keep the utility of all other generations constant, it suffices to introduce the additional constraint that the pension contributions of generation t must not vary as the policy parameters of time t are changed. Or more formally:

$$\left(w_t \tilde{\tau}_t (1 - \tilde{h}_t \tilde{n}_t) \right) \Big|_{(\tilde{\alpha}_t, \tilde{\tau}_t)} = (w_t \tau_t (1 - h_t n_t)) \Big|_{(\alpha_t, \tau_t)} \tag{11}$$

where $(1 - \tilde{h}_t \tilde{n}_t) \Big|_{(\tilde{\alpha}_t, \tilde{\tau}_t)}$ is the labour supply in the original state of the economy before the pension reform. Consequently, the local maximisation problem of the government can be written as:

$$\begin{aligned} & \max_{\alpha_t, \tau_t} \left[\begin{array}{l} \max_{c_t, c_{t+1}, n_t, z_t, h_t} U(c_t, c_{t+1}, n_t, q(z_t, h_t)) \\ \text{s.t. } c_t + \frac{c_{t+1}}{R_{t+1}} + B_t z_t n_t + (D_t - \alpha_t X_{t+1}) n_t = w_t (1 - h_t n_t) \left[(1 - \tau_t) + (1 - \alpha_t) \frac{\tilde{n}_t}{w_t (1 - \tilde{h}_t \tilde{n}_t)} X_{t+1} \right] \end{array} \right] \\ & \text{s.t. } \left(w_t \tilde{\tau}_t (1 - \tilde{h}_t \tilde{n}_t) \right) \Big|_{(\tilde{\alpha}_t, \tilde{\tau}_t)} = (w_t \tau_t (1 - h_t n_t)) \Big|_{(\alpha_t, \tau_t)} \end{aligned} \tag{12}$$

where X_{t+1} is the short notation for the present value of the pension contribution of the representative parent of generation $t + 1$: $X_{t+1} \equiv \frac{\tau_{t+1} w_{t+1} (1 - \tilde{h}_{t+1} \tilde{n}_{t+1})}{R_{t+1}}$.

3 The efficient mix of child pensions and Bismarck pensions

In this section, we analyse the optimal share of a child pension in a Bismarckian PAYG pension scheme when labour supply and the number of children are endogenous. It turns out that the local maximisation problem of the government (Eq. 12) can be equivalently rewritten in terms of the classic optimal tax problem of Corlett and Hague (1953) where the state optimises the relative tax on two consumption goods subject to the constraint that a total revenue of T_t needs to be raised and that leisure cannot be taxed.⁴

⁴Hence, parental time is the untaxed use of the primary factor. The reason for assuming that there is no tax on leisure (which is equivalent to parental time in our setting) has been debated controversially. The suggestion that the untaxed use of leisure is merely a matter of normalization seems to be a misinterpretation of statements by Auerbach (1985, p. 89) and Myles (1995, p. 123). Auerbach admits that there is an untaxable endowment of labour (and leisure). Myles simply states that one cannot infer from normalizing the tax of one good to zero that there is a real restriction to do so. Our interpretation of Corlett and Hague (1953) is that the real restriction on leisure which makes it untaxable comes from the unobservability of effective leisure (and effective labour time). Otherwise, there would be no optimal tax problem. See also Kaplov (2007).

We rewrite the government’s maximisation problem (12) by defining the tax rates on consumption and on children as follows:

$$t_{c_t} \equiv \frac{1}{1 - \tau_t + (1 - \alpha_t) \frac{\tilde{n}_t}{w_t(1 - \tilde{h}_t \tilde{n}_t)} X_{t+1}} - 1, \tag{13}$$

$$t_{n_t} \equiv \frac{1}{1 - \tau_t + (1 - \alpha_t) \frac{\tilde{n}_t}{w_t(1 - \tilde{h}_t \tilde{n}_t)} X_{t+1}} \frac{D_t - \alpha_t X_{t+1}}{D_t - X_{t+1}} - 1 \tag{14}$$

and the total tax revenue needed to hold the utility of the old generation $t - 1$ constant as:

$$T_t \equiv \left(w_t \tilde{\tau}_t \left(1 - \tilde{h}_t \tilde{n}_t \right) \right) \Big|_{(\tilde{\alpha}_t, \tilde{\tau}_t)}. \tag{15}$$

By using these definitions, the government’s maximisation problem (12) is shown to be formally equivalent to the Corlett–Hague optimisation problem in the [Appendix](#). In our pension framework, this translates into:

Proposition 1

- (a) *If children are as complementary to parental (leisure) time as consumption, then a pure child pension system $\alpha_t = 1$ is efficient.*
- (b) *If children are more complementary to parental (leisure) time than consumption, then a child pension with weight smaller than one, $\alpha_t < 1$, complemented by a positive Bismarck pension system, is efficient.*
- (c) *If children are less complementary to parental (leisure) time than consumption, then a child pension with greater weight than one, $\alpha_t > 1$, financed through a negative Bismarck pension system, is efficient.*

Proof See [Appendix](#). □

Since we assume that parental time is equal to leisure time, it is likely that children are more complementary to leisure at the margin. In view of the high time demands of proper parenting compared to most other forms of consumption, we would not be surprised if children were indeed more complementary to leisure than other forms of consumption. In that case, the optimal solution would indeed involve a positive Bismarck pension as is presently observed.

One possible approach to address this question empirically might be to assume that the utility function is separable in all three arguments. In this case, it can be shown that children are more (less) complementary to leisure than consumption if and only if the income elasticity of the number of children is smaller than the income elasticity of consumption. Hence, it would suffice to compare the more readily estimated income elasticities.

With only one type of representative agent, there is no endogenous justification for redistributive taxation. Therefore, it is reassuring that the reasoning

of Proposition 1 can be extended to a setting with two productivity types: $w_t^{\text{high}} > w_t^{\text{low}}$. If the utility function is separable in consumption goods, the number of children and leisure, then the Atkinson and Stiglitz (1976) argument applies that distortive taxation differentiating between children and the other consumption good would not be able to alleviate the incentive compatibility constraint. Therefore, a pure child pension system ($t_{k_t} = t_{c_t} \Leftrightarrow \alpha_t = 1$) would be optimal in this case. However, without separability of leisure, consumption good and the number of children, Proposition 1 is restored, as can be shown following the equivalent argument from optimum taxation theory by Edwards et al. (1994) and Nava et al. (1996).

However, an important caveat needs to be applied to the latter two results. The assumption of linear commodity taxes is normally justified due to the possibility of private resale. This argument clearly cannot and should not be applied to the ‘consumption good’ children. The government, for good reason, tracks the identity and number of children in each household. This opens up the possibility of non-linear elements in the taxation of children. As explored in Balestrino et al. (2002), quite sophisticated tax schedules might arise in this way that go substantially beyond what is possible in the present paper. In particular, child taxes or subsidies can be made to depend on earnings, a possibility that is beyond the scope of this paper.

4 Hyper-child pension and Adenauer pension

It is instructive to now re-examine the polar cases of exogenous fertility and exogenous labour–leisure distortion within our model to link our findings to the previous literature.

From taxation literature, it is clear that a replacement of distortive by lump-sum taxes is efficient. Hence, the optimum taxation for an inelastic supply of labour with endogenous fertility is characterised by $t_{n_t}^* = 0$, $t_{c_t}^* > 0$ and, therefore, $\alpha_t^* > 1$. In other words, a hyper-child pension efficiently places all tax burden on labour, extending the result of von Auer and Büttner (2004).

By contrast, if labour is supplied, elastically and fertility is exogenous, the optimum taxation is characterised by $t_{n_t}^* > 0$, $t_{c_t}^* = 0$. In the following, we show that this implies $\alpha_t^* < 0$ in a dynamically efficient steady state.

If children are supplied inelastically at \bar{n}_t , then the representative parent’s maximisation problem can be written as:

$$\begin{aligned} & \max_{c_t, c_{t+1}, z_t, h_t} U(c_t, c_{t+1}, q(z_t, h_t)) \\ & \text{s.t. } c_t + \frac{c_{t+1}}{R_{t+1}} + B_t z_t \bar{n}_t + (D_t - \alpha_t X_{t+1}) \bar{n}_t \\ & = w_t (1 - h_t \bar{n}_t) - w_t (1 - h_t \bar{n}_t) \tau_t \left[1 - (1 - \alpha_t) \bar{n}_t \frac{\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})}{R_{t+1} \tau_t w_t (1 - \bar{h}_t \bar{n}_t)} \right]. \end{aligned} \tag{16}$$

At $\alpha_t = 0$, the implicit tax on labour due to the PAYG pension scheme remains positive if the payroll growth rate is smaller than the interest rate of the capital market, i.e. we are in a dynamic efficient equilibrium, and if contribution rates are not increased such that the loss in the return of the PAYG scheme is compensated (no-Ponzi game):

$$\left(\tau_t \left[1 - (1 - \alpha_t) \bar{n}_t \frac{\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})}{R_{t+1} \tau_t w_t (1 - \bar{h}_t \bar{n}_t)} \right] \right) \Big|_{\alpha_t=0} > 0 \text{ iff } R_{t+1} > \bar{n}_t \frac{\tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1})}{\tau_t w_t (1 - \bar{h}_t \bar{n}_t)}. \tag{17}$$

The last condition in Eq. 17 is clearly satisfied in countries with pension systems which define an upper bound for contribution rates, e.g. by constitutional terms against expropriation.

Using again the formulation of an optimal tax problem for two commodity goods, the following proposition can be stated:

Proposition 2 *If children are supplied inelastically, then a negative child pension component $\alpha^* < 0$ is efficient.*

Proof From optimal taxation theory, it is well-known that if lump-sum taxation is possible then all distortionary taxes should be optimally set to zero (see, e.g. Sandmo 1976; Auerbach 1985). In our framework, this means that the solution to the optimal tax problem:

$$\begin{aligned} & \max_{c_t, n_t} \left[\begin{array}{l} \max_{c_t, c_{t+1}, n_t, z_t, h_t} U(c_t, c_{t+1}, q(z_t, h_t)) \\ \text{s.t. } (1 + t_c) \left(c_t + \frac{c_{t+1}}{R_{t+1}} + B_t z_t \bar{n}_t \right) + (1 + t_n) (D_t - X_{t+1}) \bar{n}_t = w_t (1 - h_t \bar{n}_t) \end{array} \right] \\ & \text{s.t. } T_t = t_c \left(c_t + \frac{c_{t+1}}{R_{t+1}} + B_t z_t \bar{n}_t \right) + t_n (D_t - X_{t+1}) \bar{n}_t \end{aligned} \tag{18}$$

is a zero tax on consumption: $t_c^* = 0$ because the tax on children $t_n^* > 0$ is, by assumption, a lump-sum instrument. At the solution where the tax rate on consumption (Eq. 13) is zero, we obtain:

$$\alpha_t^* = 1 - \frac{\tau_t w_t (1 - \bar{h}_t \bar{n}_t)}{\bar{n}_t X_{t+1}} < 0 \text{ iff } \left(1 - (1 - \alpha_t) \bar{n}_t \frac{X_{t+1}}{\tau_t w_t (1 - \bar{h}_t \bar{n}_t)} \right) \Big|_{\alpha_t=0} > 0. \tag{19}$$

with $X_{t+1} \equiv \tau_{t+1} w_{t+1} (1 - \bar{h}_{t+1} \bar{n}_{t+1}) / R_{t+1}$. Since the implicit tax on labour on the RHS of Eq. 19 is positive at the point $\alpha_t = 0$ in a dynamically efficient equilibrium, the optimal child factor is negative: $\alpha_t^* < 0$. \square

In other words, a pension system that places all tax burden on children is efficient. If children are supplied inelastically, a child factor smaller than one is in fact a lump-sum tax on children. According to optimal tax theory, this lump-sum tax should be increased up to the point at which the distortionary implicit tax on labour is reduced to zero. In this case, the pension should be reduced by α^* for each child so that the reduction of pensions adds to the direct cost of having children.

It should be noted that this would be a more extreme version of the Bismarck pension advocated by Fenge (1995) where child taxes subsidise the pension systems to such an extent that the implicit tax rate is reduced to zero. In memory of the first chancellor of the Federal Republic of Germany, Konrad Adenauer, who famously quipped ‘Kinder bekommen die Leute immer’ (People will always have children), this hyper-Bismarck pension could be called Adenauer pension.

5 Equivalence of various fertility instruments

Furthermore, it is worth noting that child pensions are not the only instrument available to set incentives for fertility. Alternative fertility incentives could be provided via direct child benefits or by making child-rearing costs deductible from the pension contributions. Whilst the introduction of the latter instrument is a trivial inter-temporal substitution in a framework of perfect credit markets, the substitutability of child benefits is not so obvious (see Fenge and Meier 2008). This raises the question which of these instruments would be best.

Introducing child benefits in the model modifies the inter-temporal budget constraint (6) to:

$$\begin{aligned}
 c_t + \frac{c_{t+1}}{R_{t+1}} + B_t z_t n_t + D_t n_t \\
 = w_t (1 - h_t n_t) (1 - \tau_t) + \beta_t n_t + \left[(1 - \alpha_t) \bar{n}_t \frac{(1 - h_t n_t)}{(1 - \bar{h}_t \bar{n}_t)} + \alpha_t n_t \right] X_{t+1}
 \end{aligned}
 \tag{20}$$

where β_t denotes the child benefit per child that is granted to the parent. Maximising utility with respect to the budget constraint (20) yields the following proposition:

Proposition 3 *Child benefits and child pensions are equivalent instruments of fertility policy within the present model.*

Proof See [Appendix](#). □

Of course, there may be reasons outside the scope of this paper that would favour one instrument over others. For example, credit-constrained families might be better off with child benefits early on in life instead of child pensions later in life. In the framework in this study, however, all instruments turn out to be perfect substitutes so that there is a priori no reason to favour one of the instruments over the other.

6 Conclusion

In this paper, we discuss the second-best implications of a pension type which is contingent on the individual numbers of children of a pensioner. In an overlapping generations model with endogenous fertility and labour supply, we transform the government's decision problem of choosing the relevant pension parameters in a standard optimal taxation problem with taxes on consumption and children. Our main results are the following.

A child pension should never completely replace a Bismarckian pension scheme in the most relevant case of children being more complementary to leisure than consumption goods. The reason is that the elimination of a Bismarck pension in favour of a child pension increases labour-leisure distortions by too much. Indeed, a mix of both pension types would balance the distortions of labour supply and fertility and produce a second-best optimum.

Under the extreme hypothesis that fertility behaviour is inelastic, the taxation of children means to implement a lump-sum tax on children. Optimally, such a tax should be used to subsidise the Bismarckian PAYG pension scheme to an extent that the distortive implicit tax on wage income vanishes.

Furthermore, alternative measures of family policy—such as child benefits—are equivalent instruments to set fertility incentives.

It should be noted that some features of child pensions have not been captured by our model and deserve further examination. Child pensions set financial incentives for fertility relatively late in an individual's life cycle. Thus, they are equivalent to other instruments of family policy which come to the aid of families earlier in life only if capital markets are perfect. If liquidity is especially tight for young families and future pensions cannot be perfectly advanced by capital markets, policy instruments like child benefits (or a rebate of pension contributions per child) may be preferable to child pensions.

Moreover, we did not analyse the political credibility of pension reforms. The introduction of child pensions is a promise to help parents in the distant future when they retire. This promise is obviously subject to a substantial political risk. Other financial aids with immediate execution are more credible and, hence, might allow one to influence fertility more effectively.

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Appendix

Proof to Proposition 1 First, we need to introduce some notation. The tax payment per unit of consumption and per child is denoted as: $\theta_{c_t} \equiv t_{c_t}$ and $\theta_{n_t} \equiv t_{n_t} (D_t - X_{t+1})$. The gross prices of consumption and children are abbreviated as: $P_{c_t} \equiv 1 + \theta_{c_t}$ and $P_{n_t} \equiv D_t - X_{t+1} + \theta_{n_t}$. Furthermore, we define aggregate child consumption as $Z_t \equiv z_t n_t$ and aggregate parental (leisure) time as $H_t \equiv h_t n_t$. Now, we can transform the utility function of the representative parent (Eq. 1) to a utility function V by taking account of the child quality (Eq. 2):

$$U(c_t, c_{t+1}, n_t, q(z_t, h_t)) \equiv V(c_t, c_{t+1}, n_t, Z_t, H_t).$$

Note that V is monotone, increasing in all arguments and strictly concave. Using the definitions (13), (14) and (15), the government’s maximisation problem (12) can be written as:

$$\begin{aligned} & \max_{c_t, n_t} \left[\begin{array}{l} \max_{c_t, c_{t+1}, n_t, Z_t, H_t} V(c_t, c_{t+1}, n_t, Z_t, H_t) \\ \text{s.t. } P_{c_t} \left(c_t + \frac{c_{t+1}}{R_{t+1}} + B_t Z_t \right) + P_{n_t} n_t = w_t (1 - H_t) \end{array} \right] \\ & \text{s.t. } T_t = \theta_{c_t} \left(c_t + \frac{c_{t+1}}{R_{t+1}} + B_t Z_t \right) + \theta_{n_t} n_t. \end{aligned} \tag{21}$$

In the following, we omit the time index in the price and tax variables. Since $\partial P_c / \partial \theta_c = \partial P_n / \partial \theta_n = 1$, the first-order condition of the Lagrange function of Eq. 21 with respect to θ_c is given by:

$$\begin{aligned} \frac{\partial L}{\partial \theta_c} &= \frac{\partial V}{\partial c_t} \frac{\partial c_t}{\partial P_c} + \frac{\partial V}{\partial c_{t+1}} \frac{\partial c_{t+1}}{\partial P_c} + \frac{\partial V}{\partial n_t} \frac{\partial n_t}{\partial P_c} + \frac{\partial V}{\partial Z_t} \frac{\partial Z_t}{\partial P_c} + \frac{\partial V}{\partial H_t} \frac{\partial H_t}{\partial P_c} \\ &+ \mu \left[\theta_c \left(\frac{\partial c_t}{\partial P_c} + \frac{\partial c_{t+1}}{\partial P_c} \frac{1}{R_{t+1}} + B_t \frac{\partial Z_t}{\partial P_c} \right) + \theta_n \frac{\partial n_t}{\partial P_c} + c_t + \frac{c_{t+1}}{R_{t+1}} + B_t Z_t \right] = 0 \end{aligned} \tag{22}$$

where μ is the Lagrangian parameter of the government’s problem (21). Solving the parent’s maximisation problem in the squared brackets of Eq. 21 yields the first-order conditions:

$$\begin{aligned} \frac{\partial V}{\partial c_t} &= \lambda P_c; & \frac{\partial V}{\partial c_{t+1}} &= \lambda \frac{P_c}{R_{t+1}}; & \frac{\partial V}{\partial n_t} &= \lambda P_n; \\ \frac{\partial V}{\partial Z_t} &= \lambda P_c B_t; & \frac{\partial V}{\partial H_t} &= \lambda w_t \end{aligned} \tag{23}$$

where λ is the Lagrangian parameter of the parent’s optimisation problem. By substitution, Eq. 22 can be written as:

$$\lambda \left[P_c \frac{\partial c_t}{\partial P_c} + \frac{P_c}{R_{t+1}} \frac{\partial c_{t+1}}{\partial P_c} + P_n \frac{\partial n_t}{\partial P_c} + P_c B_t \frac{\partial Z_t}{\partial P_c} + w_t \frac{\partial H_t}{\partial P_c} \right] + \mu \left[\theta_c \left(\frac{\partial c_t}{\partial P_c} + \frac{\partial c_{t+1}}{\partial P_c} \frac{1}{R_{t+1}} + B_t \frac{\partial Z_t}{\partial P_c} \right) + \theta_n \frac{\partial n_t}{\partial P_c} + c_t + \frac{c_{t+1}}{R_{t+1}} + B_t Z_t \right] = 0. \tag{24}$$

The partial derivative of the parent’s budget constraint in the squared brackets of Eq. 21 with respect to P_c yields:

$$c_t + \frac{c_{t+1}}{R_{t+1}} + B_t Z_t + P_c \left(\frac{\partial c_t}{\partial P_c} + \frac{1}{R_{t+1}} \frac{\partial c_{t+1}}{\partial P_c} + B_t \frac{\partial Z_t}{\partial P_c} \right) + P_n \frac{\partial n_t}{\partial P_c} + w_t \frac{\partial H_t}{\partial P_c} = 0. \tag{25}$$

The family lifetime consumption of the parent in both periods and the children in the first period can be aggregated to a composite commodity good where the relative prices of the consumption components, $1/R_{t+1}$ and B_t , are constant. Family consumption is denoted as $C_t \equiv c_t + \frac{c_{t+1}}{R_{t+1}} + B_t Z_t$. Solving Eq. 25 for C_t and inserting in Eq. 24, the first-order condition of the government’s optimisation problem can be rewritten as:

$$-\lambda C_t + \mu \left(\theta_c \frac{\partial C_t}{\partial P_c} + \theta_n \frac{\partial n_t}{\partial P_c} + C_t \right) = 0$$

$$\text{or } \theta_c \frac{\partial C_t}{\partial P_c} + \theta_n \frac{\partial n_t}{\partial P_c} = \nu C_t \tag{26}$$

where $\nu \equiv \frac{\lambda - \mu}{\mu}$. Now, we use the Slutsky equation to note that:

$$\frac{\partial C_t}{\partial P_c} = S_{CC} - C_t \frac{\partial C_t}{\partial I}; \quad \frac{\partial n_t}{\partial P_c} = S_{nC} - C_t \frac{\partial n_t}{\partial I} \tag{27}$$

and I is the full income equal to w_t . Substituting the Slutsky terms gives:

$$\theta_c S_{CC} + \theta_n S_{nC} = C_t \cdot \kappa \tag{28}$$

where $\kappa \equiv \nu + \theta_c \frac{\partial C_t}{\partial I} + \theta_n \frac{\partial n_t}{\partial I}$. Note that κ is negative because the Slutsky matrix is negative semi-definite. Following the same procedure with θ_n , we get:

$$\theta_c S_{Cn} + \theta_n S_{nn} = n_t \cdot \kappa.$$

Note that $S_{Cn} = S_{nC}$ due to the symmetry of the Slutsky substitution matrix. Solving both equations, the tax rates are implicitly characterised by:

$$\theta_c = \frac{\kappa}{S} (C_t S_{nn} - n_t S_{nC}),$$

$$\theta_n = \frac{\kappa}{S} (n_t S_{CC} - C_t S_{Cn}) \tag{29}$$

where $S \equiv S_{CC}S_{nn} - S_{Cn}S_{nC}$ is positive by the negative semi-definiteness of the Slutsky matrix. Now, we define the elasticity of the compensated demand:

$$\varepsilon_{nn}^c \equiv S_{nn} \frac{P_n}{n}; \quad \varepsilon_{CC}^c \equiv S_{CC} \frac{P_c}{C}; \quad \varepsilon_{nC}^c \equiv S_{nC} \frac{P_c}{n}; \quad \varepsilon_{Cn}^c \equiv S_{Cn} \frac{P_n}{C}. \tag{30}$$

Substituting in Eq. 29 and rearranging yields:

$$\begin{aligned} \frac{\theta_c}{P_c} &= \gamma [\varepsilon_{nn}^c - \varepsilon_{Cn}^c], \\ \frac{\theta_n}{P_n} &= \gamma [\varepsilon_{CC}^c - \varepsilon_{nC}^c] \end{aligned} \tag{31}$$

where $\gamma = \frac{\kappa}{S} \frac{C \cdot k}{p_k P_c}$.

With three goods of family consumption, children and parental time, we have:

$$\begin{aligned} \varepsilon_{CH}^c + \varepsilon_{CC}^c + \varepsilon_{Cn}^c &= 0, \\ \varepsilon_{nH}^c + \varepsilon_{nn}^c + \varepsilon_{nC}^c &= 0 \end{aligned} \tag{32}$$

because by the Euler theorem and homogeneity of degree 1 of the expenditure function:

$$\begin{aligned} S_{CH} \frac{w}{C} + S_{CC} \frac{P_c}{C} + S_{Cn} \frac{P_n}{C} &= \frac{1}{C} (S_{HC}w + S_{CC}P_c + S_{nC}P_n) = 0, \\ S_{nH} \frac{w}{n} + S_{nn} \frac{P_n}{n} + S_{nC} \frac{P_c}{n} &= \frac{1}{n} (S_{Hn}w + S_{nn}P_n + S_{Cn}P_c) = 0. \end{aligned}$$

By substituting, we get:

$$\begin{aligned} \frac{\theta_C}{P_c} &= \gamma [\varepsilon_{nn}^c + \varepsilon_{CC}^c + \varepsilon_{CH}^c], \\ \frac{\theta_n}{P_n} &= \gamma [\varepsilon_{CC}^c + \varepsilon_{nn}^c + \varepsilon_{nH}^c]. \end{aligned} \tag{33}$$

Using the definitions of θ and P and noting that $\gamma < 0$, we find:

$$\frac{t_c}{1 + t_c} > (\leq) \frac{t_n}{1 + t_n} \Leftrightarrow \gamma \varepsilon_{CH}^c > (\leq) \gamma \varepsilon_{nH}^c \Leftrightarrow \varepsilon_{CH}^c < (\geq) \varepsilon_{nH}^c. \tag{34}$$

If consumption is more (less) complementary to leisure than children, it should be taxed at a higher (lower) rate. From the definitions (13) and (14), it follows immediately that:

$$t_{n_i} \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} t_{c_i} \Leftrightarrow \alpha_t \left\{ \begin{array}{l} < \\ = \\ > \end{array} \right\} 1. \tag{35}$$

□

Proof to Proposition 3 The first-order conditions of maximising the utility function (1) with respect to budget constraint (20) modify the marginal rates of substitutions (8) and (10) to:

$$\frac{U_{n_t}}{U_{c_t}} = B_t z_t + D_t - \beta_t - \alpha X_{t+1} + w_t h_t \left[(1 - \tau_t) + (1 - \alpha) \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot X_{t+1} \right], \tag{36}$$

$$\frac{U_q}{U_{c_t}} q_{h_t} = w_t n_t \left[(1 - \tau_t) + (1 - \alpha) \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot X_{t+1} \right], \tag{37}$$

whilst conditions (7) and (9) remain unaffected. It suffices to show that any fertility instrument can be completely substituted by the other in an economically equivalent way if the contribution rate τ is varied appropriately at the same time so that the pension of generation $t - 1$ does not suffer. Therefore, we show that the child pension can be perfectly substituted by child benefits. Hence, we prove that a change of policy parameters $(\alpha_t, \beta_t, \tau_t) \rightarrow (0, \hat{\beta}_t, \hat{\tau}_t)$ will not change the first-order conditions (including the inter-temporal budget constraint):

$$\begin{aligned} & B_t z_t + D_t - \beta_t - \alpha X_{t+1} + w_t h_t \left[(1 - \tau_t) + (1 - \alpha) \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot X_{t+1} \right] \\ &= B_t z_t + D_t - \hat{\beta}_t + w_t h_t \left[(1 - \hat{\tau}_t) + \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot X_{t+1} \right], \end{aligned} \tag{38}$$

$$\begin{aligned} & w_t n_t \left[(1 - \tau_t) + (1 - \alpha) \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot X_{t+1} \right] \\ &= w_t n_t \left[(1 - \hat{\tau}_t) + \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} \cdot X_{t+1} \right], \end{aligned} \tag{39}$$

$$\begin{aligned} & w_t (1 - h_t n_t) (1 - \tau_t) + \beta_t B_t z_t n_t \tau_t + \left[(1 - \alpha_t) \bar{n}_t \frac{(1 - h_t n_t)}{(1 - \bar{h}_t \bar{n}_t)} + \alpha_t n_t \right] X_{t+1} \\ & - c_t - \frac{c_{t+1}}{R_{t+1}} - (B_t z_t + D_t) n_t \\ &= w_t (1 - h_t n_t) (1 - \hat{\tau}_t) + \hat{\beta}_t B_t z_t n_t \hat{\tau}_t + \bar{n}_t \frac{(1 - h_t n_t)}{(1 - \bar{h}_t \bar{n}_t)} X_{t+1} \\ & - c_t - \frac{c_{t+1}}{R_{t+1}} - (B_t z_t + D_t) n_t. \end{aligned} \tag{40}$$

Equation 39 implies:

$$\hat{\tau}_t = \tau_t + \alpha_t \frac{\bar{n}_t}{w_t (1 - \bar{h}_t \bar{n}_t)} X_{t+1}. \quad (41)$$

Substituting for $\hat{\tau}_t$ in Eq. 38 yields:

$$\hat{\beta}_t = \beta_t + \alpha_t X_{t+1}. \quad (42)$$

Substituting for $\hat{\tau}_t$ and $\hat{\beta}_t$ shows that Eq. 40 is identically satisfied. \square

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