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CORRECTION

Correction to: A Short Proof of Commutator Estimates

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The Paley–Littlewood square function g_λ^* is *not* bounded on L^p spaces if $p \le 1$, as wrongly stated in the paper (see Theorem 2.5). A weaker estimate holds though, namely, g_λ^* is bounded from the H^p real Hardy space to the usual L^p space if $p \le 1$. This is proved in references [17, 24]. Thus it is necessary to make two changes in the paper:

(1) In Theorem 1.1, after formula (1.5) add the sentence: where the L^{p_j} norms at the right must be replaced by Hardy space norms H^{p_j} if $p_j \le 1$.
(2) In Theorem 2.5, at the end of statement (i), add the sentence: where the L^p norm at the right must be replaced by a Hardy space H^p norm if $p \le 1$.

For completeness, here are the correct statements of Theorems 1.1 and 2.5.

Theorem 1.1 *Let $n \ge 1$. Assume s, s_1, s_2 and r, p_1, p_2 satisfy*

$$s = s_1 + s_2 \in (0, 2), \quad s_j \in (0, 1), \qquad \frac{1}{r} = \frac{1}{p_1} + \frac{1}{p_2}, \quad \frac{2n}{n + 2s_j} < p_j < \infty.$$

Then for all $u, v \in \mathscr{S}(\mathbb{R}^n)$ we have

$$\|D^s(uv) - uD^s v - vD^s u\|_{L^r} \lesssim \|D^{s_1} u\|_{L^{p_1}} \|D^{s_2} v\|_{L^{p_2}} \tag{1.1}$$

where the L^{p_j} norms at the right must be replaced by Hardy space norms H^{p_j} if $p_j \le 1$.

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Moreover, if we define

$$q_j = p_j\left(\tfrac{1}{2} + \tfrac{s_j}{n}\right) \quad \textit{if } n \geq 2, \qquad q_j = \min\left\{p_j, p_j\left(\tfrac{1}{2} + s_j\right)\right\} \quad \textit{if } n = 1,$$

and we assume in addition $p_1, p_2 > 1$ *when* $n = 1$*, then for any weights* $w_j \in A_{q_j}$ *we have*

$$\|D^s(uv) - uD^s v - vD^s u\|_{L^r(w_1^{r/p_1} w_2^{r/p_2} dx)} \lesssim \|D^{s_1} u\|_{L^{p_1}(w_1 dx)} \|D^{s_2} v\|_{L^{p_2}(w_2 dx)}. \tag{1.2}$$

Theorem 2.5 *Let* $n \geq 1$*,* $\lambda > 1$*. For any* $u \in \mathscr{S}(\mathbb{R}^n)$*,* $g_\lambda^*(u)$ *satisfies the following estimates, with constants independent of* u*:*

(i) $\|g_\lambda^*(u)\|_{L^p} \lesssim \|u\|_{L^p}$ *for* $\lambda > \max\{1, \frac{2}{p}\}$ *and* $0 < p < \infty$*, where the* L^p *norm at the right must be replaced by a Hardy space* H^p *norm if* $p \leq 1$*.*

(ii) $\|g_\lambda^*\|_{L^p(wdx)} \lesssim \|u\|_{L^p(wdx)}$ *for* $\lambda > \max\{1, \frac{2}{p}\}$*,* $1 < p < \infty$ *and* $w \in A_{\min\{p, \frac{p\lambda}{2}\}}$*.*