



Hermite Expansion Technique for Model Reduction of Circuit Systems with Delay Components

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Abstract

Model order reduction technique provides an effective way to reduce computational complexity in large-scale circuit simulations. This paper proposes a new model order reduction method for delay circuit systems based on Hermite expansion technique. The presented method consists of three steps i.e., first the delay elements are approximated using the recursive relation of Hermite polynomials, then in the second step, the reduced order is estimated for the delay circuit system using a delay truncation in the Hermite domain and in the third step, a multi-order Arnoldi process is computed for obtaining the projection matrix. In the following, the reduced order delay circuit model is obtained by the projection matrix. Moment matching and passivity properties of the reduced circuit system are also analyzed. Two circuit examples with delay components are performed to verify the effectiveness of the proposed MOR approach.

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1 Introduction

The development of Electronic Design Automation (EDA) techniques has had a significant impact on the design of integrated circuits (ICs) [10], enabling their progress and advancement. However, modern IC simulations which become increasingly complex bring significant challenges to EDA tools. As is known to all, mathematical models of ICs need to consider the dynamical processes and interactions of circuit devices, especially with the reduction in IC structure sizes and increase in packing density and driving frequency. This requires refined mathematical models that account for secondary and parasitic effects [2]. The complexity of modern IC mathematical models is enormous, with small portions of a chip alone requiring millions of linear, non-linear, and time-delay differential equations for accurate modeling. Verification of IC behavior through solving these large-scale model equations in the time or frequency domain is crucial, but the high-dimensional mathematical problems involved may not be solvable within a reasonable amount of time using available computational resources. Naturally, this surge in demand for IC simulation places a substantial burden on computational science and engineering researchers.

Model order reduction (MOR) is a powerful technique that can significantly reduce the computational cost of simulating complex circuits. It involves creating a simplified model of the circuit that accurately captures its behavior while using fewer computational resources and the important properties of the original circuit must be preserved in the model reduction process. By this means, redundancies are resolved, less relevant quantities are replaced by the most significant ones. Then, the high dimension mathematical problem's complexity is reduced, keeping the main characteristics. Solving lower dimensional problems one can get statements on the circuit's behaviours more quickly. Based on these theoretical results, Wil H. A. discussed the future needs of the electronics industry with regard to model order reduction [39].

Early in 1990, model reduction method which based on explicit moment matching was applied to circuit simulation in [30]. In [7], the technique based on coefficient comparison is proposed to obtain reduced order system from a linear time invariant SISO system. Since then, MOR methods for linear circuit systems were extensively researched over the past several decades, mainly consist of moments matching and balanced truncation MOR methods. However, explicit moment matching technique suffers from numerical instability as elaborated in [16]. To eliminate this drawback, implicit moment matching based MOR methods were presented, of which PVL [16] and algorithm proposed in [19] were examples. Later, structure and passivity preservation model reduction processes were developed. For example, PRIMA [28] and SPRIM [18] methods could meet passivity preservation requirement for RLC circuits. Balanced truncation methods were also developed, of which PABTEC [32] and PMTBR [29] were examples. These methods could preserve the stability property of the original circuit systems and provided a global computable error bound. Besides, Laguerre type algorithms [11, 25], general orthogonal polynomial based MOR [22]

and other moment matching based MOR methods [20, 23, 24] were also presented. The paper [9] discussed the reduction of large-scale circuit equations with many terminals and applied it to the design process of integrated circuit power supply networks. This work emphasized the necessity of model order reduction for such systems within the context of integrated circuit design and simulation.

It is essential to consider the delay phenomena caused by propagation delays in high-speed interconnect circuits and this is particularly important when modeling and designing interconnects in circuit packaging and printed circuit boards [1, 35]. In recent years, there have been significant efforts to accelerate the simulation of delay circuit systems. In [10], the authors formulated a compact delayed-differential macromodel through approximating each response partition with a low order sum of exponentials. In [13], the delay macromodel was obtained by iterative weighted least squares process. Both of the method proposed in [10, 13] were tabulated data driven. That is to say, it is necessary to pre-calculate the response data of the circuit system at certain points in the frequency domain or time domain. Projection based MOR method can directly address the mathematical model of the circuit system without pre-calculating the system response. Considering the special structure of the time-delay mathematical model, model reduction algorithms for linear systems can not be directly used. So, researchers proposed various approximation strategies to deal with the delay elements. Such as *Padé* approximation [26], Taylor expansion [14] and Laguerre expansion techniques [33]. However, Taylor expansion based MOR method proposed in [14] may lose the delay structure of the original system and the physical informations can not be preserved. In [33], the authors using the Laguerre expansion technique to approximate the delay elements and then Krylov subspace technique is used to formulated the reduced system. Although this method can preserve the structure of the original system, the passivity property of the reduced system is not considered. Lombardi, etc. [27] adapted *Padé* approximation technique to model reduction of parameterized time-delay circuits while the stability of reduced circuit system was not analyzed. In [17], the authors construct the projection matrices by iterative interpolation method and the interpolation points are selected by a new greedy algorithm. This interpolation based MOR method achieved an excellent results both in reduction accuracy and stability. Additionally, model reduction methods for the delay system which based on balanced truncation [15] and Hermite expansion in time domain [38] techniques are also researched.

Some novel model reduction approaches based on optimization techniques have also been proposed. For example, in [6], an optimal reduced order H-infinity controller, based on Hankel singular values (HSV), has been proposed using genetic algorithm. Sikander and Prasad [34] propose a new method for order reduction of higher-order linear time invariant systems based on stability equation and particle swarm optimization algorithm, this MOR method preserves the stability property of the original system. In [21], the authors combine the modified Big bang big crunch optimization algorithm and Pade approximation technique to reduced the order of the power systems. In [3], the authors propose a hybrid order reduction technique based on time moment matching method and salp swarm optimization technique. Other model reduction methods based on Cuckoo search [4] and Routh Criterion [8] are also pro-

posed. It is worth noting that [5] contributes a order reduction technique based on Salp Swarm Optimization and the proposed method is also applied to time-delay systems.

Although the existing delay circuit model reduction methods provide a powerful computational simplification tool, they leave some problems as follows:

- (1) Are there any other high-precision approximation methods for handling delay elements?
- (2) Does the stability of the reduced-order system remain consistent with the original system?
- (3) Can the reduced order system maintain the structural properties of the original system?

Motivated by these issues, this paper proposes a Hermite expansion based MOR method for delay circuit systems in frequency domain. The primary contributions of this article are outlined as follows. Firstly, we innovatively adapt the Hermite expansion technique to approximate the delay elements. Secondly, we theoretically prove that the passive property of the reduced order system is consistent with that of the original system. Finally, the structure of the original circuit system is maintained and the physical informations can also be preserved. The MOR process in this paper is summarized as follows. The delay elements are approximated by Hermite polynomials after a reasonable transformation. Then, the relationship between the moments of the original delay circuit is formulated. By applying multi-order Arnoldi orthogonalization technique to the moments, the projection matrix V is formulated. Based on the projection matrix V , the compact circuit model is obtained.

The paper is organized as follows. In Sect. 2, some basic backgrounds on Hermite polynomials are reviewed. In Sect. 3, a practical transformation is found to approximate the delay element using Hermite polynomials. Based on the new approximate strategy, a new MOR process is proposed for delay circuit systems and some important properties of the reduced circuit system are also analyzed. Two numerical time-delay circuit examples are computed in Sect. 4 to verify the effectiveness of the proposed MOR method. Some conclusions are drawn in Sect. 5.

2 Preliminary

2.1 Hermite Polynomials

Hermite polynomials ensure that the interpolating polynomial function has the same value as the original function at the interpolation node, as well as the same derivative value at that point. Based on this property, the delay element can be approximated effectively by fewer Hermite interpolation term. However, the generation function should be modified to approximate the delay element and this will be illustrated in Sect. 3. In this section, we briefly introduce the Hermite polynomials used in this paper. Hermite polynomials $h_i(x)$ are defined by the following formulas explicitly over $(-\infty, +\infty)$

$$h_i(x) = (-1)^i e^{x^2/2} \frac{d^i}{dx^i} e^{-x^2/2}, \quad i = 0, 1, \dots$$

Alternatively, they can also be formulated by the recursion

$$h_{i+1}(x) = xh_i(x) - \frac{d}{dx}h_i(x)$$

and the first two Hermite polynomials are $h_0(x) = 1$, $h_1(x) = x$. Besides, Hermite polynomials satisfy the orthogonal condition

$$\langle h_i(x), h_j(x) \rangle = \int_{-\infty}^{+\infty} h_i(x)h_j(x)\omega(x)dx = 0 \quad i \neq j,$$

where $\omega(x) = e^{-x^2/2}$ is the weight function. Further, an orthogonal basis functions of the Hilbert space formed by Hermite polynomials satisfying

$$\int_{-\infty}^{+\infty} |f(x)|^2\omega(x)dx < \infty,$$

where $f(x)$ can be expanded as $f(x) = \sum_{i=0}^{\infty} f_i h_i(x)$ and the coefficients f_i satisfy

$$f_i = \frac{\langle h_i(x), f(x) \rangle}{\langle h_i(x), h_i(x) \rangle} = \langle h_i(x), f(x) \rangle / \sqrt{2\pi i!}.$$

The generation function of Hermite polynomials is given by

$$e^{-u^2+2ux} = \sum_{n=0}^{\infty} h_n(x) \frac{u^n}{n!}. \quad (1)$$

In this paper, the generate function of Hermite polynomials is used to approximate the delay elements in frequency domain and it plays an important role in the proposed MOR method.

2.2 Mathematical Formulation

SPICE uses the modified nodal analysis (MNA) technique for formulating circuit equations. This subsection focuses on SPICE's circuit modeling principle for transient simulation [12]. In such simulations, the MNA-generated equation adopts a standard format, represented by a differential algebraic equation (DAE).

$$f(x(t)) + \frac{d}{dt}q(x(t)) = u(t),$$

where t denotes time, with $x(t)$ as the unknown variable array including node voltages and branch currents. The term $f(x(t))$ represents a nonlinear vector function that reflects the impact of static devices in the circuit, while $q(x(t))$ signifies another nonlinear vector function, indicating the influence of dynamic devices. Additionally,

$u(t)$ describes the circuit's input. The equation can be modified according to the practical circuit. Let the incidence matrix $A = [A_R, A_C, A_L, A_V, A_I]$ be partitioned accordingly, where the subscripts R, C, L, V and I stand for resistors, capacitors, inductors, voltage sources and current sources, respectively. Then Kirchhoff's current and voltage laws can be expressed in the compact form as [36]

$$Ai = 0, \quad A^T \eta = v,$$

respectively, where η denotes the vector of potentials of all nodes excepting the reference node. The branch constitutive relations for the capacitors, inductors and resistors are given by

$$C \frac{d}{dt} v_c(t) = i_c(t), \quad L \frac{d}{dt} i_l(t) = v_l(t), \quad v_R(t) = Ri_R(t),$$

The behaviour of a linear RLC circuit can be described as following based on MNA technique,

$$E \frac{dx(t)}{dt} = Ax(t) + Bu(t), \\ y(t) = Cx(t),$$

where

$$E = \begin{bmatrix} A_c C A_c^T & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -A_R G A_R^T & -A_L & -A_v \\ A_L^T & 0 & 0 \\ A_v^T & 0 & 0 \end{bmatrix}, \\ C = \begin{bmatrix} -A_I^T & 0 & 0 \\ 0 & 0 & -I \end{bmatrix} = B^T.$$

3 MOR Method for Time-Delay Differential Algebra Systems

In this section, an efficient frequency domain model reduction method is proposed for delay circuit systems based on Hermite expansion technique. Firstly, we present a new approximation sketch to approximate the time-delay elements. Secondly, based on the transfer function in u domain, the MOR process for delay circuit systems is elaborately described. In the following, moment matching and passivity properties of the reduced system are analyzed.

3.1 Moments of the Parametric System

Based on the circuit modeling procedure presented in Sect. 2.2, mathematical modeling of delay circuit system can be summarized as following. The lumped element part of the delay circuit system can be formulated as

$$C\dot{x}(t) + Gx(t) = bu(t),$$

where C is formulated by inductance and capacitance. In general, the elements of matrix C are non-negative and it is a symmetric matrix, so matrix C is a non-negative definite matrix [37]. G is the incidence matrix, $x(t)$ is the node voltage or the branch current. $u(t)$ represents the input and b is the input matrix. The mathematical model of the time-delay part of the circuit system is

$$A\dot{\xi}(t) + \sum_{k=1}^m A_k \xi(t - \alpha_k) + Bi(t) = 0,$$

where A is formulated by characteristic impedance of the transmission lines, A_k represents the relationship between the transmission lines and $i(t)$ represents the port current. The entire delay circuit system can be constructed through the following relationship

$$P^T x(t) = B^T \xi(t),$$

where matrices P and B map the relationship between the lumped element circuit part and the delay circuit part. Further, the delay circuit system can be represented as

$$\begin{bmatrix} C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} G & 0 & P \\ 0 & -A & -B \\ P^T & -B^T & 0 \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ \xi(t) \\ i(t) \end{bmatrix} + \sum_{k=1}^m \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_k & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x(t - \alpha_k) \\ \xi(t - \alpha_k) \\ i(t - \alpha_k) \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} u(t).$$

Then, it can be rewritten as

$$\begin{cases} \mathcal{E} \dot{\mathcal{X}}(t) = \mathcal{A} \mathcal{X}(t) + \sum_{k=1}^m \mathcal{A}_k \mathcal{X}(t - \alpha_k) + \mathcal{B}u(t), \\ y = \mathcal{B}^T \mathcal{X}(t). \end{cases} \quad (2)$$

where $\mathcal{X}(t)$ is the state variable, represents the node voltage or the branch current, α_k is the delay element. $\mathcal{E} \in \mathbb{R}^{n \times n}$, $\mathcal{A} \in \mathbb{R}^{n \times n}$ and $\mathcal{A}_k \in \mathbb{R}^{n \times n}$. $\mathcal{B} \in \mathbb{R}^{n \times p}$, $\mathcal{B}^T \in \mathbb{R}^{p \times n}$ are input and output matrices respectively, $u(t)$ is the input. Besides, we also set $\mathcal{X}(0) = \mathcal{X}_0$ and $\mathcal{X}(t) = g(t)$, $t \in [-h, 0)$ to determine the state solution. Directly solving the aforementioned differential-algebraic equations is quite challenging [40]. Researchers have proposed model order reduction methods to address this issue.

Next, we propose a new approximation strategy for the delay elements arising in system (2). Applying Laplace transformation to system (2), we can obtain its transfer function

$$H(s) = \mathcal{B}^T \left(s\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-s\alpha_k} \right)^{-1} \mathcal{B}. \quad (3)$$

Traditional MOR methods, such as Krylov subspace methods and Balance Truncation methods, can not be directly used to this kind of systems. Researchers proposed various strategies to deal with the delay elements $e^{-s\alpha_k}$, such as Taylor expansion [37] and Laguerre expansion [33] techniques. Here, we proposed a new strategy to approximate the delay element $e^{-s\alpha_k}$ based on the recursive relation of Hermite polynomials. Apparently, generate function (1) can not be directly used to approximate $e^{-s\alpha_k}$. Considering the mathematical expressions of Eq. (1), we present a transformation $s = u^2 - 2u$. Substitute it into $e^{-s\alpha_k}$, we get

$$\begin{aligned} e^{-s\alpha_k} &= e^{-(u^2-2u)\alpha_k} \\ &= e^{2\alpha_k u - \alpha_k u^2} \\ &= e^{2\sqrt{\alpha_k}\sqrt{\alpha_k}u - (\sqrt{\alpha_k}u)^2}. \end{aligned}$$

According to Eq. (1), one obtains

$$e^{2\sqrt{\alpha_k}\sqrt{\alpha_k}u - (\sqrt{\alpha_k}u)^2} = \sum_{n=0}^{\infty} h_n(\sqrt{\alpha_k}) \frac{(\sqrt{\alpha_k}u)^n}{n!} \tag{4}$$

where $h_n(x)$ are the Hermite polynomials, i.e.,

$$\begin{aligned} h_0(x) &= 1, \quad h_1(x) = 2x, \quad h_2(x) = 4x^2 - 2, \\ h_3(x) &= 8x^3 - 12x, \quad h_4(x) = 16x^4 - 48x^2 + 12 \\ &\dots \end{aligned}$$

Thus, the delay element $e^{-s\alpha_k}$ can be approximated by Eq. (4). We give an example to illustrate the effectiveness of this approximation. Figure 1 shows the approximation curves considering $n = 3$ and $n = 7$ with $\alpha = 0.3$ ns.

Naturally, delay elements of the transfer function (3) can be approximated by the generation function of Hermite polynomials. Using the transformation $s = u^2 - 2u$, we can obtain a new transfer function in u domain which is equivalent to the original transfer function.

3.2 MOR Through Moment Matching

In this subsection, we present the MOR process for delay circuit systems in Hermite domain. Consider the transfer function of the delay circuit system

$$H(s) = \mathcal{B}^T \left(s\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-s\alpha_k} \right)^{-1} \mathcal{B}.$$

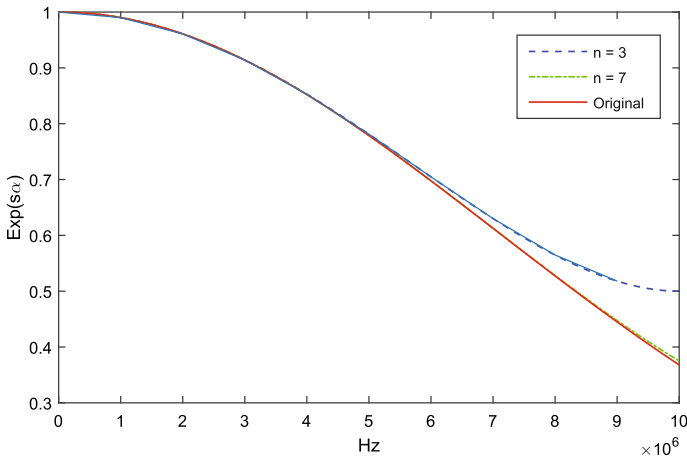


Fig. 1 Delay elements approximated by Hermite polynomial

Using the transformation $s = u^2 - 2u$, the original transfer function can be rewritten as

$$H(u) = \mathcal{B}^T \left((u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-(u^2-2u)\alpha_k} \right)^{-1} \mathcal{B}. \tag{5}$$

Expanding the delay elements using Eq. (4), it has

$$e^{-(u^2-2u)\alpha_k} = \sum_{n=0}^{\infty} h_n(\sqrt{\alpha_k}) \frac{(\sqrt{\alpha_k}u)^n}{n!}.$$

Further, $H(u)$ can be represented as

$$H(u) = \mathcal{B}^T \left((u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k \sum_{n=0}^{\infty} h_n(\sqrt{\alpha_k}) \frac{(\sqrt{\alpha_k}u)^n}{n!} \right)^{-1} \mathcal{B}.$$

We choose the first l items to approximate the delay element

$$e^{-(u^2-2u)\alpha_k} \approx \sum_{n=0}^l h_n(\sqrt{\alpha_k}) \frac{(\sqrt{\alpha_k}u)^n}{n!}.$$

Then, the transfer function of the delay system can be approximated as

$$H(u) \approx \mathcal{B}^T \left((u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k \sum_{n=0}^l h_n(\sqrt{\alpha_k}) \frac{(\sqrt{\alpha_k}u)^n}{n!} \right)^{-1} \mathcal{B}.$$

It should be noted that one can select different items to approximate the delay element according to the required accuracy. In this paper, we set $l = 3$. The moments M_i of

the transfer function are given by

$$\begin{aligned} & \mathcal{B}^T \left((u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k \sum_{n=0}^3 h_n(\sqrt{\alpha_k}) \frac{(\sqrt{\alpha_k}u)^n}{n!} \right)^{-1} \mathcal{B} \\ &= \mathcal{B}^T \sum_{i=0}^{\infty} M_i u^i. \end{aligned}$$

Directly calculating M_i is complicated. Fortunately, we can find the relationship between the moments and the iterative formulas for computing M_i are given by

$$\begin{aligned} M_0 &= \phi_0^{-1} \mathcal{B}, \quad M_1 = -\phi_0^{-1} \phi_1 M_0, \\ M_2 &= -\phi_0^{-1} \phi_2 M_0 - \phi_0^{-1} \phi_1 M_1, \\ M_3 &= -\phi_0^{-1} \phi_1 M_2 - \phi_0^{-1} \phi_2 M_1 - \phi_0^{-1} \phi_3 M_0, \\ M_4 &= -\phi_0^{-1} \phi_1 M_3 - \phi_0^{-1} \phi_2 M_2 - \phi_0^{-1} \phi_3 M_1, \\ M_5 &= -\phi_0^{-1} \phi_1 M_4 - \phi_0^{-1} \phi_2 M_3 - \phi_0^{-1} \phi_3 M_2, \\ &\vdots \\ M_i &= -\phi_0^{-1} \phi_1 M_{i-1} - \phi_0^{-1} \phi_2 M_{i-2} - \phi_0^{-1} \phi_3 M_{i-3}. \end{aligned}$$

where

$$\begin{aligned} \phi_0 &= -\mathcal{A} - \sum_{k=1}^m \mathcal{A}_k h_0(\sqrt{\alpha}), \\ \phi_1 &= -2\mathcal{E} - \sum_{k=1}^m \mathcal{A}_k h_1(\sqrt{\alpha}) \sqrt{\alpha}, \\ \phi_2 &= \mathcal{E} - \sum_{k=1}^m \mathcal{A}_k h_2(\sqrt{\alpha}) \frac{(\sqrt{\alpha})^2}{2!}, \\ \phi_3 &= -\sum_{k=1}^m \mathcal{A}_k h_3(\sqrt{\alpha}) \frac{(\sqrt{\alpha})^3}{3!}. \end{aligned}$$

The relationship between the M_i satisfies a high-order Krylov subspace condition. The three order Krylov subspace can be constructed as

$$\begin{aligned} & \mathcal{K}_r \left(-\phi_0^{-1} \phi_1, -\phi_0^{-1} \phi_2, -\phi_0^{-1} \phi_3; \phi_0^{-1} \mathcal{B} \right) \\ &= \text{span}\{M_0, M_1, \dots, M_q\}. \end{aligned}$$

By orthogonalizing the Krylov subspace, the projection matrix V can be obtained. Here, we use a stable multi-order Arnoldi orthogonalization technique to formulated V . The process is summarized as following.

Algorithm 1 Multi-order Arnoldi based MOR method for delay circuit systems based on l-th Hermite approximation

INPUT: $\mathcal{E}, \mathcal{A}, \mathcal{A}_k, \mathcal{B}, \mathcal{B}^T$;
Step 1. Compute $U: \phi_0 U = \mathcal{B}$;
Step 2. QR factorization of $U, v_0 = QR(U)$;
Step 3. for $i = 1, 2, \dots, q$ do;
Step 4. $v_i^{(0)} = \sum_{\omega=1}^{\min\{l, q\}} (-\phi_0^{-1} \phi_\omega) v_{i-\omega}$;
Step 5. for $j = 1, 2, \dots, i$, do;
Step 6. $H = v_{i-j}^T v_i^{(j-1)}, v_i^{(j)} = v_i^{(j-1)} - v_{i-j} H$;
Step 7. end for;
Step 8. $v_i = QR(v_i^{(i)})$;
Step 9. end for;
Step 10. $V = [v_0, v_1, \dots, v_q]$;
OUTPUT: V

By projecting the original delay circuit system into the mathematics space spanned by $M_i (i = 0, \dots, q)$, i.e., V , it obtains a reduced order circuit system

$$\begin{cases} \tilde{\mathcal{E}} \dot{\tilde{\mathcal{X}}}(t) = \tilde{\mathcal{A}} \tilde{\mathcal{X}}(t) + \sum_{i=1}^m \tilde{\mathcal{A}}_k \tilde{\mathcal{X}}(t - \alpha_k) + \tilde{\mathcal{B}}u(t), \\ \tilde{y} = \tilde{\mathcal{B}}^T \tilde{\mathcal{X}}(t), \end{cases} \tag{6}$$

where $\tilde{\mathcal{X}} = V \mathcal{X}, \tilde{\mathcal{E}} = V^T \mathcal{E} V \in \mathbb{R}^{(q+1) \times (q+1)}, \tilde{\mathcal{A}} = V^T \mathcal{A} V \in \mathbb{R}^{(q+1) \times (q+1)}, \tilde{\mathcal{A}}_k = V^T \mathcal{A}_k V \in \mathbb{R}^{(q+1) \times (q+1)}, \tilde{\mathcal{B}} = V^T \mathcal{B} \in \mathbb{R}^{(q+1) \times p}, \tilde{\mathcal{B}}^T = \mathcal{B}^T V \in \mathbb{R}^{p \times (q+1)}$ and $(q + 1) \ll n$. Thus, we obtain a compact delay circuit model which can effectively approximate the original one. Benefiting from the sparsity of the coefficient matrices, the order of the original delay circuit system can be greatly reduced. In the following, we analyze some important properties of the reduced system.

3.3 Structure-Preserving MOR Method

In this subsection, we present a moment matching theorem between the original system and the reduced one. Considering the transfer function of the reduced circuit system (6)

$$\tilde{H}(u) = \tilde{\mathcal{B}}^T \left((u^2 - 2u)\tilde{\mathcal{E}} - \tilde{\mathcal{A}} - \sum_{k=1}^m \tilde{\mathcal{A}}_k e^{-(u^2 - 2u)\alpha_k} \right)^{-1} \tilde{\mathcal{B}}. \tag{7}$$

Further, the moments can be computed by

$$\begin{aligned} & \tilde{\mathcal{B}}^T \left((u^2 - 2u)\tilde{\mathcal{E}} - \tilde{\mathcal{A}} - \sum_{k=1}^m \tilde{\mathcal{A}}_k e^{-(u^2 - 2u)\alpha_k} \right)^{-1} \tilde{\mathcal{B}} \\ &= \tilde{\mathcal{B}}^T \sum_{i=0}^{\infty} \tilde{M}_i u^i. \end{aligned}$$

Since we use projection matrix V to generate the reduces circuit system as shown in system(6), the MOR method is a one-side projection method. In the following we present a theorem and prove that the reduced system and the original system match the first $q + 1$ moments.

Theorem 1 *Given the reduced system (6), if the projection matrix V is obtained by Algorithm 1 and the delay elements are approximated by the first l elements in Eq. (4), the transfer functions of the reduced system (6) and the original system (2) match the first $q + 1$ moments, i.e., $\mathcal{B}^T M_i = \tilde{\mathcal{B}}^T \tilde{M}_i, i = 0, 1, \dots, q$.*

Proof If $q \leq l$, we have

$$\begin{bmatrix} \phi_0 & & & & \\ \phi_1 & \phi_0 & & & \\ \vdots & \vdots & \ddots & & \\ \phi_q & \phi_{q-1} & \cdots & \phi_0 & \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ \vdots \\ M_q \end{bmatrix} = \begin{bmatrix} \mathcal{B} \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Since the projection matrix V is formulated by

$$\text{colspan}\{V\} = \text{colspan}\{M_0, M_1, M_2, \dots, M_q\},$$

there exists $\xi_i \in \mathbb{R}^{n \times p}$ that satisfies $M_i = V\xi_i$ for all M_i , where $i = 0, 1, \dots, q$. Further, we have

$$\begin{bmatrix} \tilde{\phi}_0 & & & & \\ \tilde{\phi}_1 & \tilde{\phi}_0 & & & \\ \vdots & \vdots & \ddots & & \\ \tilde{\phi}_q & \tilde{\phi}_{q-1} & \cdots & \tilde{\phi}_0 & \end{bmatrix} \begin{bmatrix} \xi_0 \\ \xi_1 \\ \vdots \\ \xi_q \end{bmatrix} = \begin{bmatrix} \tilde{\mathcal{B}} \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

This system of linear equations has a unique solution, then it obtains $\tilde{M}_i = \xi_i$ and $\mathcal{B}^T M_i = \mathcal{B}^T V\xi_i = \tilde{\mathcal{B}}^T \tilde{M}_i$.

If $q > l$, it has

$$\begin{bmatrix} \phi_0 & & & & & & & & & & \\ \phi_1 & \phi_0 & & & & & & & & & \\ \vdots & \vdots & \ddots & & & & & & & & \\ \phi_l & \phi_{l-1} & \cdots & \phi_0 & & & & & & & \\ 0 & \phi_l & \cdots & \phi_1 & \phi_0 & & & & & & \\ \vdots & \vdots & \ddots & \dots & \ddots & \ddots & & & & & \\ 0 & 0 & \cdots & \phi_l & \dots & \dots & \phi_0 & & & & \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ \vdots \\ M_l \\ M_{l+1} \\ \vdots \\ M_q \end{bmatrix} = \begin{bmatrix} \mathcal{B} \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Similarly, we can prove $\tilde{M}_i = \xi_i$ and $\mathcal{B}^T M_i = \mathcal{B}^T V\xi_i = \tilde{\mathcal{B}}^T \tilde{M}_i$. So the first $q + 1$ moments of the original circuit system and the reduced circuit system matched. This concludes the proof. □

4 Property Analysis

Passivity is an important property of the circuit systems. The preservation of the passivity of the original system is essential for MOR method. Next we prove that the proposed MOR method can preserve the passivity property.

Firstly, we give a definition that define the passivity of the delay system.

Definition 1 [37] If the transfer function $H(s)$ of the delay system (3) satisfy the following conditions,

- (1) $H(s^*) = H^*(s)$, where $*$ represents complex conjugate operator;
- (2) $H(s)$ is a positive-real matrix in the right-half plane of the s domain, which satisfies $[H(s) + H^T(s^*)] \geq 0$, i.e., $[H(s) + H^T(s^*)]$ is nonnegative definite;
- (3) $H(s)$ is analytic for all values of s with $Re(s) > 0$. Then, the delay system (3) is passive.

According to Definition 1, we know that if the transfer function $H(u)$ in u domain satisfies the following three conditions

- (1) $H(u^*) = H^*(u)$ where $(*)$ is the complex conjugate operator,
- (2) $[H(u) + H^T(u^*)] \geq 0$, which means $[H(u) + H^T(u^*)]$ is nonnegative definite,
- (3) $H(u)$ is analytic for all values of u with $R(u) > 0$, the corresponding delay system is passive.

In the following, we present a theorem about the passivity of the reduced circuit system.

Theorem 2 Given the time-delay circuit system (2), if the matrix $[(-A - \sum_{i=1}^m A_k e^{-(u^2-2u)\alpha_k}) + (-A^T - \sum_{i=1}^m A_k^T e^{-(u^2-2u)\alpha_k})]$ in the transfer function (5) is non-negative definite, the corresponding reduced system obtained(6) is passivity.

Proof For the original transfer function in u domain

$$H(u) = \mathcal{B}^T \left((u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-(u^2-2u)\alpha_k} \right)^{-1} \mathcal{B},$$

the corresponding reduced transfer function is

$$\tilde{H}(u) = \tilde{\mathcal{B}}^T \left((u^2 - 2u)\tilde{\mathcal{E}} - \tilde{\mathcal{A}} - \sum_{k=1}^m \tilde{\mathcal{A}}_k e^{-(u^2-2u)\alpha_k} \right)^{-1} \tilde{\mathcal{B}}.$$

It is easy to know that if the coefficient matrices of the original circuit system is a real matrix, the coefficient matrices of the reduced circuit system is also a real matrix. Further, we know that $\tilde{H}(u^*) = \tilde{H}^*(u)$. The first condition of passivity is satisfied.

In the following, we prove that for $z \in \mathbb{R}^n$ and $Re(u) > 0$, the transfer function of the reduced system $\tilde{H}(u)$ satisfies $z^*T[\tilde{H}(u) + \tilde{H}^T(u^*)]z \geq 0$.

Based on the transfer function of the reduced system and $z^{*T}[\tilde{H}(u) + \tilde{H}^T(u^*)]z$, we can obtain

$$z^{*T} \left[\tilde{\mathcal{B}}^T \left((u^2 - 2u)\tilde{\mathcal{E}} - \tilde{\mathcal{A}} - \sum_{k=1}^m \tilde{\mathcal{A}}_k e^{-(u^2-2u)\alpha_k} \right)^{-1} \tilde{\mathcal{B}} + \left(\tilde{\mathcal{B}}^T \left((u^2 - 2u)^*\tilde{\mathcal{E}} - \tilde{\mathcal{A}} - \sum_{k=1}^m \tilde{\mathcal{A}}_k e^{-(u^2-2u)^*\alpha_k} \right)^{-1} \tilde{\mathcal{B}} \right)^T \right] z.$$

Then, it has

$$z^{*T} \left[\tilde{\mathcal{B}}^T \left((u^2 - 2u)\tilde{\mathcal{E}} - \tilde{\mathcal{A}} - \sum_{k=1}^m \tilde{\mathcal{A}}_k e^{-(u^2-2u)\alpha_k} \right)^{-1} \tilde{\mathcal{B}} + \tilde{\mathcal{B}}^T \left((u^2 - 2u)^*\tilde{\mathcal{E}}^T - \tilde{\mathcal{A}}^T - \sum_{k=1}^m \tilde{\mathcal{A}}_k^T e^{-(u^2-2u)^*\alpha_k} \right)^{-1} \tilde{\mathcal{B}} \right] z$$

By extracting V and V^T , we get

$$z^{*T} \mathcal{B}^T \left((u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-(u^2-2u)\alpha_k} \right)^{-1} \left[(u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-(u^2-2u)\alpha_k} + (u^2 - 2u)^*\mathcal{E}^T - \mathcal{A}^T - \sum_{k=1}^m \mathcal{A}_k^T e^{-(u^2-2u)^*\alpha_k} \right] ((u^2 - 2u)^*\mathcal{E}^T - \mathcal{A}^T - \sum_{k=1}^m \mathcal{A}_k^T e^{-(u^2-2u)^*\alpha_k})^{-1} \mathcal{B}z.$$

Finally, it obtains

$$Q^{*T} \left[(u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-(u^2-2u)\alpha_k} + (u^2 - 2u)^*\mathcal{E}^T - \mathcal{A}^T - \sum_{k=1}^m \mathcal{A}_k^T e^{-(u^2-2u)^*\alpha_k} \right] Q \tag{8}$$

where $Q = V[(u^2 - 2u)^*\tilde{\mathcal{E}}^T - \tilde{\mathcal{A}}^T - \sum_{k=1}^m \tilde{\mathcal{A}}_k^T e^{-(u^2-2u)^*\alpha_k}]^{-1} \tilde{\mathcal{B}}z$. It should prove Eq. (8) is non negative definite.

We only should prove equation

$$\left[\begin{aligned} &(u^2 - 2u)\mathcal{E} - \mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-(u^2-2u)\alpha_k} + (u^2 - 2u)^* \mathcal{E}^T - \\ &\mathcal{A}^T - \sum_{k=1}^m \mathcal{A}_k^T e^{-(u^2-2u)^*\alpha_k} \end{aligned} \right]$$

is non-negative definite.

Given the matrix

$$\mathcal{E} = \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and circuit coefficient matrix C is a non-negative definite matrix, for $Re(u) > 0$, it is easy to know that $(u^2 - 2u)\mathcal{E} + (u^2 - 2u)^* \mathcal{E}^T$ is non-negative definite. Considering the structures of \mathcal{A} and \mathcal{A}_k , and let $\mathcal{H} = -A - \sum_{k=1}^m A_k e^{-(u^2-2u)\alpha_k} - A^T - \sum_{k=1}^m A_k^T e^{-(u^2-2u)^*\alpha_k}$, we have

$$\begin{aligned} &-\mathcal{A} - \sum_{k=1}^m \mathcal{A}_k e^{-(u^2-2u)\alpha_k} - \mathcal{A}^T - \sum_{k=1}^m \mathcal{A}_k^T e^{-(u^2-2u)^*\alpha_k} \\ &= \begin{bmatrix} G + G^T & 0 & 0 \\ 0 & \mathcal{H} & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

In the circuit model, matrix $G + G^T$ is non-negative definite and the expression $\mathcal{H} = -A - \sum_{k=1}^m A_k e^{-(u^2-2u)\alpha_k} - A^T - \sum_{k=1}^m A_k^T e^{-(u^2-2u)^*\alpha_k}$ is also non-negative definite. So $[\tilde{H}(u) + \tilde{H}^T(u^*)]$ is non-negative definite.

If $H(u)$ is nonsingular in a certain region in u domain, then $H(u)$ is also analytic in the certain region. Since $H(u)$ is nonsingular in the right half plane, $H(u)$ is analytic in $Re(u) > 0$.

So far, we prove that the transfer function of the reduced circuit system satisfies the three passivity conditions. This concludes the proof. □

5 Numerical Examples

In this section, two numerical circuit examples are performed to demonstrate the effectiveness of this proposed algorithm. We compare the proposed method with the widely used time-delay MOR method presented in [33]. Relative errors which defined as $\|y_r - y\|_2 / \|y\|_2$, where y_r represents the output of the reduced circuit system and y represents the output of the original circuit system, are also analyzed to illustrate the accuracy of the new MOR method.

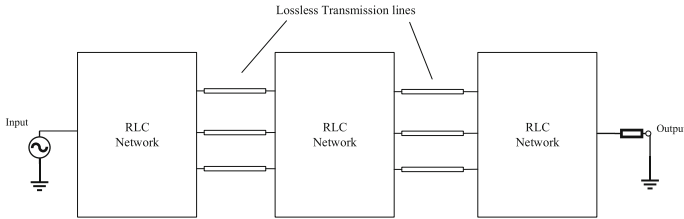


Fig. 2 Time-delay circuit systems

Example 4.1 We first consider a delay circuit model shown in Fig. 2 [37]. The circuit model is composed of three RLC circuit networks and two lossless multiconductor interconnect lines. For the circuit network, the mathematical model are obtained by lumped circuit modeling method, and for the interconnect circuit parts, we use the method introduced in Sect. 3.1 to model it. Then, we merge the two circuit models through the port connection relationship of the circuit system. The order of this time-delay circuit system is 2625 with the delay element $\alpha_k = 0.1$ ns and the parameters for interconnect lines are

$$L = \begin{bmatrix} 494.6 & 63.3 & 7.8 \\ 63.3 & 494.6 & 63.3 \\ 7.8 & 63.3 & 494.6 \end{bmatrix} \text{ nH/m,}$$

$$C = \begin{bmatrix} 63.8 & -4.9 & -0.3 \\ -4.9 & 63.8 & -4.9 \\ -0.3 & -4.9 & 63.8 \end{bmatrix} \text{ pF/m.}$$

We reduce the order of the original circuit system from 2625 to 52 through the method proposed in this paper and the reference MOR algorithms proposed in [33]. Runge–kutta algorithm is used to solve the large circuit model and the two compact circuit models. Figure 3 plots the outputs of the two reduced circuit systems and the original circuit system. Figure 4 presents the corresponding relative errors. The solution time for the original circuit system is 50.082 s, while the solution time for the reduced system is 13.684 s.

With the same input $u(t) = \frac{\pi}{4}(\sin(2\pi t) + \frac{1}{3}\sin(6\pi t))$ applied to the three circuit models, we observe the output waveforms of the circuit systems. As shown in Fig. 3, both the outputs of the reduced order circuit models can approximate the output of the original system well, indicating the effectiveness of the proposed algorithm. Then we analyze the relative error between the outputs of reduced circuit models and the original circuit model. From Fig. 4, we can see that the relative error corresponding to the proposed method is around 10^{-4} while the relative error corresponding to the reference method is around 10^{-3} . Relative errors demonstrate that the new MOR method for time-delay system can achieve a better model reduction accuracy.

Example 4.2 We investigate another circuit system containing a single transmission line presented in [31]. In the circuit system, for simplicity, we set $r = 10 \Omega$, $C = 1$ pF, $L = 1$ nH and $\alpha_k = 0.01$ ns. The new MOR method and the reference MOR method

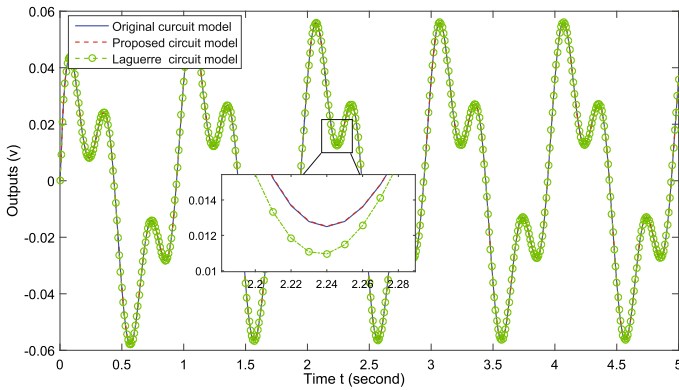


Fig. 3 Outputs of the original circuit system and the circuit reduced systems

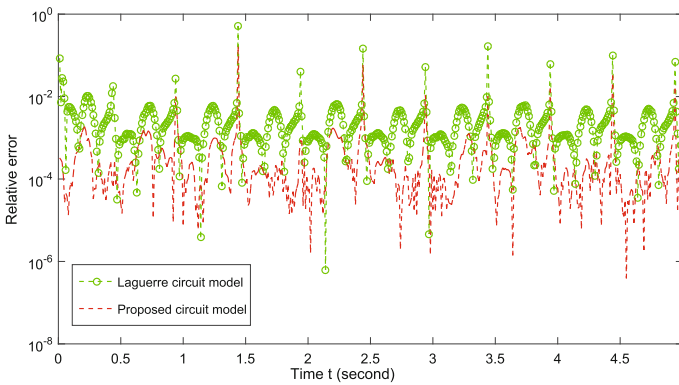


Fig. 4 Relative error

are used to reduce the order of the original delay circuit system from 1630 to 36. We investigate the outputs of the circuits models. Figure 5 compared the outputs of the original circuit system with the outputs of the compact circuit models obtained by the proposed method and the reference method. Figure 6 shows the corresponding relative errors. The original circuit model’s simulation time is 38.466 s, while the reduced circuit’s simulation time is 7.854 s.

Figure 5 illustrates that the outputs of the reduced order delay circuit system formulated by our new method can approximate the outputs of the original circuit system effectively. From Fig. 6, we observe that the relative error of the proposed method remains at a low value and it shows a higher accuracy than the reference algorithm.

6 Conclusion

This paper proposes a new MOR method for delay circuit systems. By transforming the transfer function of the original circuit system into u domain, the delay elements

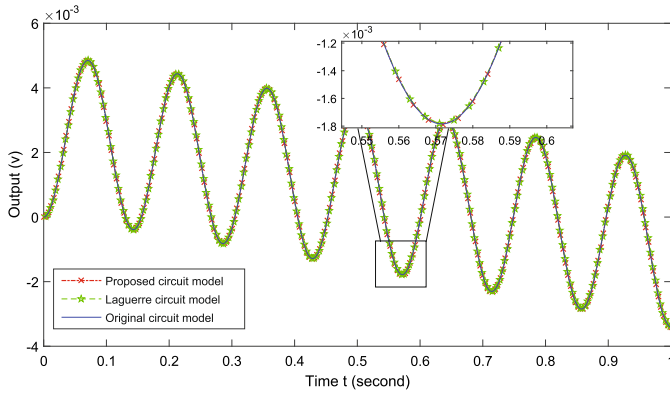


Fig. 5 Outputs of the original circuit system and the reduced circuit systems

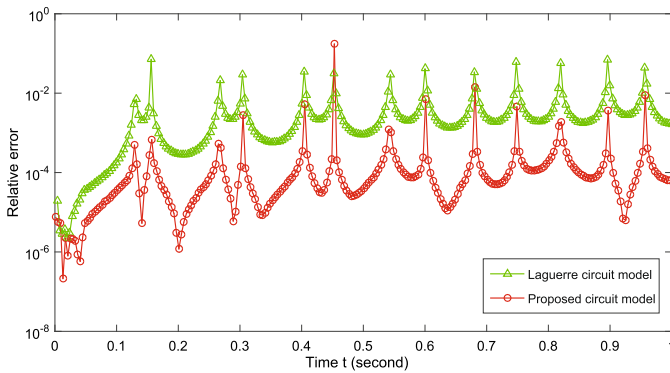


Fig. 6 Relative errors

are approximated by Hermite polynomials. Then, we found the iterative formulas of the moments of the delay circuit system. Projection matrix V is formulated through multi-order Arnoldi orthogonalization technique and then the order of the delay circuit system is reduced. The compact circuit model obtained by the proposed MOR method can approximate the original circuit more precisely than the compact model constructed by the reference method. Moment matching and passivity preserving property of this MOR method are also analyzed. Finally, the effectiveness of the proposed MOR algorithm is verified by time-delay circuit models.

Data availability Data sharing is applicable to this article and the authors will supply the relevant data in response to reasonable requests.

Declarations

Conflict of interest The authors declare that they have no Conflict of interest.

References

1. R. Achar, Modeling of high-speed interconnects for signal integrity analysis: Part I. *IEEE Microw. Mag.* **12**(5), 61–74 (2011)
2. R. Achar, M.S. Nakhla, Simulation of high-speed interconnects. *Proc. IEEE* **89**(5), 693–728 (2001)
3. N. Ahamad, A. Sikander, A novel approach of order diminution using time moment concept with Routharray and salp swarm algorithm. *Turk. J. Electr. Eng. Comput. Sci.* **29**(2), 1077–1091 (2021)
4. N. Ahamad, A. Sikander, G. Singh, Substructure preservation based approach for discrete time system approximation. *Microsyst. Technol.* **25**, 641–649 (2019)
5. N. Ahamad, A. Sikander, G. Singh, Order diminution and its application in controller design using salp swarm optimization technique. *Int. J. Syst. Assur. Eng. Manag.* **13**(2), 933–943 (2022)
6. N. Ahamad, G. Singh, S. Khan, A. Sikander, Design and performance analysis of optimal reduced order H-infinity controller: L1 norm based genetic algorithm technique. In *International Conference on Power and Embedded Drive Control* (2017), pp. 8–13
7. N. Ahamad, G. Singh, A. Sikander, A new approach for order reduction of linear SISO system Coefficient Comparison method with one free variable. <https://www.researchgate.net/profile/Nafees-Ahamad-2/publication>
8. N. Ahmed, G. Singh, M. Samir, H. Ahmad, Performance analysis of reduced order aircraft bank angle control system. In *Conference on Advances in Communication and Control Systems (CAC2S 2013)* (2013), pp. 518–520
9. P. Benner, A. Schneider, Reduced representation of power grid models. *Syst. Reduct. Nanoscale IC Des.* 87–134 (2017)
10. A. Charest, D. Saraswat, M. Nakhla, R. Achar, N. Soveiko, Compact macromodeling of high-speed circuits via delayed rational functions. *IEEE Microw. Wirel. Compon. Lett.* **17**(12), 828–830 (2007)
11. Y. Chen, V. Balakrishnan, C.K. Koh, K. Roy, Model reduction in the time-domain using Laguerre polynomials and Krylov methods. In *Proceedings of the Conference on Design, Automation and Test, Europe* (2002), pp. 931–935
12. X. Chen, Y. Wang, H. Yang, *Parallel Sparse Direct Solver for Integrated Circuit Simulation* (Springer International Publishing, 2017), pp. 3–4
13. A. Chinae, P. Triverio, S. Grivet-Talocia, Delay-based macromodeling of long interconnects from frequency-domain terminal responses. *IEEE Trans. Adv. Packag.* **33**(1), 246–256 (2010)
14. J. Cullum, A. Ruehli, T. Zhang, A method for reduced-order modeling and simulation of large interconnect circuits and its application to PEEC models with retardation. *IEEE Trans. Circuits Syst. II Analog Digit. Signal Process.* **47**(4), 261–273 (2000)
15. J. Elias, T. Damm, W. Michiels, Model reduction of time-delay systems using position balancing and delay Lyapunov equations. *Math. Control Signals Syst.* **25**(2), 147–166 (2013)
16. P. Feldmann, R. Freund, Efficient linear circuit analysis by Padé approximation via the Lanczos process. *IEEE Trans. Comput Aided Des. Integr. Circuits Syst.* **14**, 639–649 (1995)
17. L. Feng, L. Lombardi, P. Benner, D. Romano, G. Antonini, Model order reduction for delayed PEEC models with guaranteed accuracy and observed stability. *IEEE Trans. Circuits Syst. I Regul. Pap.* **69**(10), 4177–4190 (2022)
18. R.W. Freund, SPRIM: structure-preserving reduced-order interconnect macromodeling. In *Proceedings of the IEEE/ACM International Conference on Computer Aided Design, CA, USA* (2004), pp. 80–87
19. P.K. Gunupudi, M. Nakhla, R. Achar, Simulation of high-speed distributed interconnects using Krylov-space techniques. *IEEE Trans. Comput. Aided Des. Integr. Circuits Syst.* **19**(7), 799–808 (2000)
20. Y. Huang, Y.L. Jiang, K.L. Xu, Model order reduction of RLC circuit system modeled by Port-Hamiltonian structure. *IEEE Trans. Circuits Syst. II Express Briefs* **69**(3), 1542–1546 (2022)
21. S. Jain, Y.V. Hote, Order diminution of LTI systems using modified big bang big crunch algorithm and Pade approximation with fractional order controller design. *Int. J. Control Autom. Syst.* **19**, 2105–2121 (2021)
22. Y.L. Jiang, H.B. Chen, Time domain model order reduction of general orthogonal polynomials for linear input–output systems. *IEEE Trans. Automat. Contr.* **57**(2), 330–343 (2012)
23. Y.L. Jiang, K.L. Xu, Frequency-limited reduced models for linear and bilinear systems on the Riemannian manifold. *IEEE Trans. Automat. Contr.* **66**(9), 3938–3951 (2021)
24. Y.L. Jiang, J.M. Yang, Asymptotic waveform evaluation with higher order poles. *IEEE Trans. Circuits Syst. I Regul. Pap.* **68**(4), 1681–1692 (2021)

25. L. Knockaert, D. De Zutter, Laguerre-SVD reduced-order modeling. *IEEE Trans. Microw. Theory Tech.* **48**(9), 1469–1475 (2000)
26. J. Lam, Model reduction of delay systems using Pad approximants. *Int. J. Control* **57**(2), 377–391 (2012)
27. L. Lombardi, Y. Tao, B. Nouri, F. Ferranti, G. Antonini, M.S. Nakhla, Parameterized model order reduction of delayed PEEC circuits. *IEEE Trans. Electromagn. Compat.* **62**(3), 859–869 (2020)
28. A. Odabasioglu, M. Celik, L.T. Pileggi, PRIMA: Passive reduced-order interconnect macromodeling algorithm. *IEEE Trans. Comput. Aided Des. Integr. Circuits Syst.* **17**(8), 645–654 (1998)
29. J.R. Phillips, L.M. Silveira, Poor man's TBR: a simple model reduction scheme. *IEEE Trans. Comput. Aided Des. Integr. Circuits Syst.* **24**(1), 43–55 (2005)
30. L.T. Pillage, R.A. Rohrer, Asymptotic waveform evaluation for timing analysis. *IEEE Trans. Comput. Aided Des. Integr. Circuits Syst.* **9**, 352–366 (1990)
31. Z.Y. Qiu, Y.L. Jiang, ε —Embedding model reduction method for time-delay differential algebra systems. *Circuits Syst. Signal Process.* **39**(11), 5390–5405 (2020)
32. T. Reis, T. Stykel, PABTEC: passivity-preserving balanced truncation for electrical circuits. *IEEE Trans. Comput. Aided Des. Integr. Circuits Syst.* **29**(9), 1354–367 (2010)
33. E.R. Samuel, L. Knockaert, T. Dhaene, Model order reduction of time-delay systems using a Laguerre expansion technique. *IEEE Trans. Circuits Syst. I Regul. Pap.* **61**(6), 1815–1823 (2014)
34. A. Sikander, R. Prasad, Soft computing approach for model order reduction of linear time invariant systems. *Circuits Syst. Signal Process.* **34**, 3471–3487 (2015)
35. H. P. Singh, G.S. Virdi, RLC modeled interconnect delay analysis for high-speed on-chip VLSI interconnects. In *2017 International Conference on Energy, Communication, Data Analytics and Soft Computing* (2017), pp. 2199–2203
36. T. Stykel, Balancing-related model reduction of circuit equations using topological structure. In: *Model Reduction For Circuit Simulation* (2011), pp. 53–83
37. W. Tseng, C. Chen, E. Gad, M. Nakhla, R. Achar, Passive order reduction for RLC circuits with delay elements. *IEEE Trans. Adv. Packag.* **30**(4), 830–840 (2007)
38. X.L. Wang, Y.L. Jiang, An efficient hybrid reduction method for time-delay systems using Hermite expansions. *Int. J. Control* **92**(5), 1033–1043 (2019)
39. S. Wil, The need for novel model order reduction techniques in the electronics industry. In: *Model Reduction for Circuit Simulation* (Springer Netherlands, Dordrecht, 2011), pp. 3–23
40. H. Yang, Y. Zhang, X. Huang, S. Hong, Positivity and exponential stability of coupled homogeneous time-delay differential-difference equations of degree one. *Circuits Syst. Signal Process.* **41**(2), 762–788 (2022)

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