Circuits, Systems, and Signal Processing (2022) 41:725–742
https://doi.org/10.1007/s00034-021-01818-2

Fully Distributed Event-Triggered Consensus for a Class of Second-Order Nonlinear Multi-agent Systems

Hua-Jun Sun[1] · **Rong Xia**[2] · **A-Long Yu**[1]

Received: 23 May 2020 / Revised: 5 August 2021 / Accepted: 7 August 2021 / Published online: 23 August 2021

Abstract

The fully distributed control of networked nonlinear systems under the sampled-data control mechanism is a challenging task. The lack of global information, the unavailability of continuous communication, and the influence of nonlinear factors make most of the existing control schemes invalid. This article proposes an event-triggered consensus protocol for a class of second-order nonlinear multi-agent systems, where all global information is unavailable in the protocol design. In order to deal with the unknown nonlinear term in the differential equation which describes each subsystem, the neural network approximation method is used. It is proved that the states of all subsystems can reach bounded consensus with only intermittent and local information exchange among subsystems. In addition, the proposed event-triggered mechanism can run without continuous monitoring the triggered condition. The Zeno behavior is then excluded to guarantee the implementability of the given protocol. Finally, a demonstrative example is provided to illustrate the effectiveness of the developed control scheme.

Keywords Multi-agent systems · Event-triggered control · Nonlinear systems · Zeno behavior · Neural network approximation

✉ Hua-Jun Sun
huajunsun427@163.com

Rong Xia
xiarong825@163.com

A-Long Yu
yal@hytc.edu.cn

[1] School of Physics and Electronic Electrical Engineering, Huaiyin Normal University, Huai'an 223300, China

[2] School of Foreign Languages, Huaiyin Normal University, Huai'an 223300, China

Birkhäuser

1 Introduction

Consensus of multi-agent systems has been a hot topic in control theory community for the past few years because of the wide applications in formation control, distributed monitoring and so forth [7,19,24,27,34]. The objective of consensus control is to solve a protocol via which the states of all subsystems can achieve synchronization in some sense [2,23,38,44,48].

As a class of typical multi-agent systems, second-order nonlinear multi-agent systems have been widely investigated in recent years. In [43], a finite-time protocol was proposed for a class of second-order nonlinear multi-agent systems, where the estimations of the convergence time and final state were given. The periodically intermittent pinning control strategy was applied to a second-order nonlinear multi-agent systems with time delays in [31]. Consensus for second-order nonlinear multi-agent systems with fractional-order dynamics was investigated in [10]. Wang et al. [29] presented a protocol with only aperiodically intermittent position measurements for second-order nonlinear multi-agent systems with time delays. By using the stochastic theory, the means square consensus problem was addressed for a class of second-order nonlinear multi-agent systems subject to white noises in [13]. It is noted that in these literature, the eigenvalues of Laplacian matrix are necessary in the protocol design, so the protocols are not fully distributed. To overcome this drawback, many schemes based on the adaptive control strategy have been developed. In [12,50], fully distributed consensus for second-order nonlinear multi-agent systems with unmodeled dynamics was investigated. Iterative learning-based fully distributed consensus protocols were proposed in [14,35]. By the protocol in [42], consensus for the second-order nonlinear multi-agent systems can be reached in finite time without global information. Two edge-based distributed adaptive protocols were, respectively, designed for leaderless and leader-following multi-agent systems in [39]. For a second-order nonlinear multi-agent system with position constraints and unknown control directions, the fully distributed protocol was presented in [1]. In [32], a fully distributed protocol was derived without using velocity measurements. However, continuous data transmissions among subsystems are required in the implementation of these fully distributed protocols.

Event-triggered control is one of the important techniques to avoid the continuous data transmission [5,25,26,33]. In recent years, the event-triggered control strategy has been widely used in cooperative control of multi-agent systems. For instance, the event-triggered formation control for multiple robots was considered in [37]; the containment control for multi-agent systems with event-triggered protocols was investigated in [15,45]; for the cooperative output regulation problem, event-triggered protocols were given in [21,41]; in [9,49], the event-triggered consensus tracking control problem was addressed. As a matter of fact, some efforts have been made on fully distributed event-triggered consensus of multi-agent systems. In [46], fully distributed event-triggered consensus for double-integrator multi-agent systems was studied. The higher-order linear multi-agent systems were considered in [17,20,30,36]. In [3,18], the fully distributed consensus for nonlinear multi-agent systems was addressed; however, the nonlinear terms are required to be bounded and satisfy the Lipschitz condition, respectively. There are also many results focusing on the event-triggered consensus for nonlinear multi-agent systems without these strict conditions, such as in [8,16,28]. However, the protocol design or the convergence performance relies on some global information, so the protocols in these results are not fully distributed. To the best of our knowledge, the fully distributed event-triggered consensus for more general nonlinear multi-agent systems has not been considered in existing literature. Designing fully distributed consensus protocol for totally unknown second-order nonlinear multi-agent systems by event-triggered strategies is thus the task of this article. The main work and contributions of the article are given as follows. a) A new consensus protocol is developed for a class of nonlinear multi-agent systems, where the implementation of protocol does not depend on any global information or continuous communications. b) Compared with [1,12,14,32,35,39,42,50], the continuous data transmissions among subsystems are avoided in this article. Compared with [8,16,28], the protocol is designed in the fully distributed manner. c) Compared with [17,20,30,36,46], the nonlinear multi-agent system is considered here. In addition, the nonlinear terms are more general than those in [3,18]. The main difficulty of the work lies in that the proposed protocol needs to deal with both the unknown topology information and the more general nonlinearities, and ensure that no Zeno behavior occurs.

We organize the rest of the article as follows. The model, objective and some mathematical tools are introduced in Sect. 2. In Sect. 3, the main results are given. Simulation results are shown in Sect. 4. Conclusions are summarized in Sect. 5.

Notations: $\mathbf{R}$ denotes the set of all real numbers. $\mathbf{R}^{m\times n}$ represents the set of all $m \times n$ real matrices. $\mathbf{1}_n$ is an n-dimension vector with all elements being 1 and I_n is the n-dimension identity matrix. The symbol $\otimes$ is used to denote the Kronecker product operator. Denote by $\text{sig}(\cdot)$ the sign function.

2 Problem Formulation and Preliminaries

This article considers a multi-agent system consisting of n subsystems described by:

$$\begin{aligned} \dot{\xi}_i &= \psi_i \\ \dot{\psi}_i &= c_i \upsilon_i + \phi_i(\xi_i, \psi_i) + \rho_i, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where ξ_i and $\psi_i \in \mathbf{R}$ are position and velocity of the ith subsystem, respectively; $\upsilon_i \in \mathbf{R}$ is the input to be designed, and $c_i \geq \underline{c}_i > 0$ is an unknown gain coefficient ($\underline{c}_i$ is a known positive constant); $\phi_i(\cdot) \in \mathbf{R}$ is an unknown continuous nonlinear function, and ρ_i is the external disturbance satisfying that $|\rho_i| \leq \bar{\rho}_i$ ($\bar{\rho}_i > 0$ is a constant).

Remark 1 Second-order nonlinear multi-agent systems have drawn much attention because of the typicality and wide applicability. In the existing literature, the similar multi-agent systems have been widely investigated.

Definition 1 *(bounded consensus* [38,48]) Multi-agent systems with subsystems in the form of (1) are said to reach bounded consensus if there are two positive constants ε_1 and ε_2 such that

$$\lim_{t\to\infty} \left|\xi_i - \xi_j\right| \leq \varepsilon_1 \quad \text{and} \quad \lim_{t\to\infty} \left|\psi_i - \psi_j\right| \leq \varepsilon_2. \tag{2}$$

This article aims to design an appropriate event-triggered protocol without using the global information such that the bounded consensus can be reached. Moreover, the convergence errors are supposed to be as small as possible by adjusting the parameters in the protocol appropriately.

Remark 2 The task of this article is inspired by [1,3,12,14,17,18,20,30,32,35,36,39,42,46,50]. Compared with [1,12,14,32,35,39,42,50], the continuous data transmissions among subsystems are avoided in this article. Compared with [17,20,30,36,46], the nonlinear multi-agent system is considered here. In addition, the nonlinear terms are more general than those in [3,18], and the neural network method was used to deal with the nonlinearities. In [8,16,28], the neural network-based event-triggered consensus protocols were given for nonlinear systems; however, these protocols are not fully distributed.

Lemma 1 [11] *For any vectors* $\chi_1, \chi_2 \in \mathbf{R^n}$, *the following inequality holds:*

$$\chi_1{}^T \chi_2 \leq \gamma \chi_1{}^T \chi_1 + \frac{1}{\gamma} \chi_2^T \chi_2, \tag{3}$$

where $\gamma > 0$ *is a positive real number.*

2.1 Graph Theory

The subsystems are communicated under an undirected graph topology in this article. $\mathcal{G} = \{ \mathcal{V}, \mathcal{E} \}$ is used to denote a graph, where $\mathcal{V} = \{ \mathrm{v}_1, \ldots \mathrm{v}_n \}$ denotes the nodes set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. For each edge, there is a corresponding weight $a_{ij} > 0$ if $(\mathrm{v}_j,\mathrm{v}_i) \in \mathcal{E}$, otherwise, $a_{ij} = 0$. A path is a series of linked edges with positive weights. An undirected graph is connected if there is at lease a path between any two nodes. An important matrix, Laplacian matrix is defined as $L = [l_{ij}]_{n\times n} \in \mathbf{R}^{n\times n}$, where $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$.

Lemma 2 [6] *If graph* $\mathcal{G}$ *is undirected and connected, then* 0 *is a simple eigenvalue of the Laplacian matrix. Denote by* λ_q *the* q*th smallest eigenvalue of* L, *then we have* $0 = \lambda_1 < \lambda_2 \leq \ldots \leq \lambda_n$ *and* $\lambda_2 = \min_{\mathbf{1}_n^T x=0, x\neq 0} \frac{x^T L x}{x^T x}$.

2.2 Radial Basis Function Neural Networks

The Radial basis function neural networks will be used to approximate the unknown continuous functions in this article. A necessary lemma is given as follows.

Lemma 3 [40] *For any continuous function* $\phi(y)$ *defined on a compact set* $\mathcal{C} \in \mathbf{R}^h$, *the neural network* $\vartheta^T \Upsilon(y)$ *can approximate* $\phi(y)$ *with a bounded error* $\sigma(y)$, *that is,*

$$\phi(y) = \vartheta^T \Upsilon(y) + \sigma(y), \quad |\sigma(y)| \leq \sigma, \tag{4}$$

where $\vartheta \in \mathbf{R}^h$ is the ideal weight vector, $\Upsilon(y) = [\Upsilon_1(y), \ldots, \Upsilon_h(y)]^T \in \mathbf{R}^h$ is the basis function vector, and $\sigma > 0$ is a constant. The basis function is chosen as $\Upsilon_j(y) = exp\left(-\frac{(y-c_j)^T(y-c_j)}{\mu_j^2}\right)$, where c_j and μ_j are the center and the width, respectively.

3 Main Results

3.1 Protocol Design

To show the design protocol clearly, we introduce the following parameters and variables: $M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $N = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $\Omega = \begin{bmatrix} \omega_1 & \omega_2 \\ \omega_2 & \omega_3 \end{bmatrix}$ is a positive definite matrix satisfying

$$M^T\Omega + \Omega M - \Omega N N^T \Omega \leq -\beta I_n, \tag{5}$$

where $\beta > 0$ is a constant.

A fully distributed consensus protocol is proposed in the following form:

$$\begin{aligned} \dot{\varsigma}_i &= \Phi_i \\ \dot{\Phi}_i &= \upsilon_i{}^o \triangleq l\hat{\theta}_i e_i(t_k^i) \\ \upsilon_i &= f_i(\xi_i, \varsigma_i, \psi_i, \Phi_i, \hat{\vartheta}_i), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{6}$$

where $t \in [t_k^i, t_{k+1}^i)$, t_k^i is the triggered point for subsystem i, $k = 0, 1, \ldots$, $t_0^i = 0$; ς_i, Φ_i, $\hat{\theta}_i$ and $\hat{\vartheta}_i$ are adaptive variables to be designed; $e_i = \omega_2 \sum_{j=1}^{n} a_{ij}(\varsigma_j - \varsigma_i) + \omega_3 \sum_{j=1}^{n} a_{ij}(\Phi_j - \Phi_i)$ is a constructed error variable; $l > 0$ is a gain constant and $f_i(\cdot)$ is a operator to be designed.

Remark 3 In this article, the protocol form is well motivated by [48], in which the finite-time consensus for switched nonlinear multi-agent systems. Compared with [48], the input in (1) is considered with the unknown gain. Moreover, the protocol in this article is fully distributed and designed based on the event-triggered control strategy.

In the implementation of protocol (6), the operator $f_i(\cdot)$, variables $\hat{\theta}_i$, $\hat{\vartheta}_i$ and the triggered condition should be determined. Then, we will derive the components to be determined in the protocol by the Lyapunov functional method.

Define $\Lambda = I_n - \frac{1}{n}\mathbf{1}_n^T\mathbf{1}_n$ and $z = (\Lambda \otimes I_2)[\varsigma_1, \Phi_1, \ldots, \varsigma_n, \Phi_n]^T$. Consider the following function

$$V_0 = z^T(\mathcal{L} \otimes \Omega)z + \frac{\gamma}{2}\sum_{i=1}^{n}(\hat{\theta}_i - \theta)^2, \tag{7}$$

where γ and θ are two positive constants to be given later.

Define $\eta_i = \begin{bmatrix} \eta_{i,1} \\ \eta_{i,2} \end{bmatrix} = \left[\sum_{j=1}^{n} a_{ij}(\varsigma_j - \varsigma_i), \ \sum_{j=1}^{n} a_{ij}(\Phi_j - \Phi_i) \right]^T$, $\eta = [\eta_1^T, \ldots, \eta_n^T]^T$, $\tilde{\eta} = \eta - \eta(t_k^i) = [\tilde{\eta}_1^T, \ldots, \tilde{\eta}_n^T]^T$, $\Xi_i = [\varsigma_i, \Phi_i]^T$ and $\Xi = [\Xi_1^T, \ldots, \Xi_n^T]^T$.

The dynamical equation of Ξ can be obtained as

$$\dot{\Xi} = (I_n \otimes M)\Xi + (\hat{\theta} \otimes l N N^T \Omega)(\eta - \tilde{\eta}), \tag{8}$$

where $\hat{\theta} = diag\{\hat{\theta}_1, \ldots, \hat{\theta}_n\}$.

Then, we can get

$$\dot{z} = (I_n \otimes M)z + (\Lambda\hat{\theta} \otimes l N N^T \Omega)(\eta - \tilde{\eta}), \tag{9}$$

where the fact that $L\Lambda = \Lambda L = L$ was used.

Combining (7) and (9), we have

$$\begin{aligned} \dot{V}_0 = & z^T(L \otimes (\Omega M + M^T \Omega))z - 2\eta^T(\hat{\theta} \otimes l\Omega N N^T \Omega)(\eta - \tilde{\eta}) \\ & + \gamma \sum_{i=1}^{n} (\hat{\theta}_i - \theta)\dot{\hat{\theta}}_i. \end{aligned} \tag{10}$$

By Lemma 1, we have that

$$\begin{aligned} & 2\eta^T(\hat{\theta} \otimes l\Omega N N^T \Omega)\tilde{\eta} \\ & \le \eta^T(\hat{\theta} \otimes l\Omega N N^T \Omega)\eta + \tilde{\eta}^T(\hat{\theta} \otimes l\Omega N N^T \Omega)\tilde{\eta}. \end{aligned} \tag{11}$$

If the variable $\hat{\theta}_i$ evolves as

$$\dot{\hat{\theta}}_i = m e_i^2(t_k^i), \quad \hat{\theta}_i(0) > 0, \ \forall t \in [t_k^i, t_{k+1}^i), \tag{12}$$

we can obtain that

$$\begin{aligned} \dot{V}_0 \le & z^T(L \otimes (\Omega M + M^T \Omega))z - \eta^T(\hat{\theta} \otimes l\Omega N N^T \Omega)\eta \\ & + \tilde{\eta}^T(\hat{\theta} \otimes l\Omega N N^T \Omega)\tilde{\eta} + m\gamma \sum_{i=1}^{n} (\hat{\theta}_i - \theta) e_i^2\left(t^i_{k_t^i}\right), \end{aligned} \tag{13}$$

where $t \in [t_k^i, t_{k+1}^i)$, $m > 0$ is a constant, $k_t^i = \arg_q \min_{q \in N, t \ge t_k^i}\{t - t_q^i\}$.

In addition,

$$\begin{aligned} & m\gamma \sum_{i=1}^{n} \hat{\theta}_i e_i^2\left(t^i_{k_t^i}\right) \\ & = m\gamma \sum_{i=1}^{n} \hat{\theta}_i \left(N^T \Omega \eta_i\left(t^i_{k_t^i}\right)\right)^2 \end{aligned}$$

$$
\begin{aligned}
&= m\gamma \sum_{i=1}^{n} \hat{\theta}_i \Big(N^T \Omega (\eta_i - \tilde{\eta}_i) \Big)^2 \\
&\le 2m\gamma \eta^T (\hat{\theta} \otimes \Omega N N^T \Omega) \eta + 2m\gamma \tilde{\eta}^T (\hat{\theta} \otimes \Omega N N^T \Omega) \tilde{\eta}.
\end{aligned}
\tag{14}
$$

The triggered rule is designed as:

$$
t_{k+1}^i = \inf_{t > t_k^i} \left\{ t \,\middle|\, g_i(e_i, e_i(t_k^i), t) = 0 \right\}, \tag{15}
$$

where $g_i(e_i, e_i(t_k^i), t) = (e_i - e_i(t_k^i))^2 - \delta_1 e_i^2 - \frac{\delta_2}{\hat{\theta}_i} \mathrm{e}^{-\delta_3 t}$, $0 < \delta_1 < 1$, $\delta_2 > 0$ and $\delta_3 > 0$ are constants.

It can be concluded from (13) to (15) that

$$
\begin{aligned}
\dot{V}_0 \le\; & z^T (L \otimes (\Omega M + M^T \Omega)) z - (l - 2m\gamma) \eta^T (\hat{\theta} \otimes \Omega N N^T \Omega) \eta \\
& + (l + 2m\gamma) \tilde{\eta}^T (\hat{\theta} \otimes \Omega N N^T \Omega) \tilde{\eta} - m\gamma\theta \sum_{i=1}^{n} e_i^2 \left(t_{k_t^i}^i \right) \\
\le\; & z^T (L \otimes (\Omega M + M^T \Omega)) z - m\gamma\theta \sum_{i=1}^{n} e_i^2 \left(t_{k_t^i}^i \right) + n\delta_2 (l + 2m\gamma) \mathrm{e}^{-\delta_3 t} \\
& - ((1 - \delta_1) l - 2(1 + \delta_1) m\gamma) \eta^T (\hat{\theta} \otimes \Omega N N^T \Omega) \eta.
\end{aligned}
\tag{16}
$$

According to Lemma 1 and triggered rule (15), it can be obtained that

$$
\begin{aligned}
-e_i^2(t_k^i) &\le -\frac{p}{1+p} e_i^2 + p \Big(e_i - e_i \left(t_k^i \right) \Big)^2 \\
&\le p \left(\Big(e_i - e_i \left(t_k^i \right) \Big)^2 - \delta_1 e_i^2 \right) - \left(\frac{p}{1+p} - \delta_1 p \right) e_i^2 \\
&\le \frac{p\delta_2}{\hat{\theta}_i} \mathrm{e}^{-\delta_3 t} - \left(\frac{p}{1+p} - \delta_1 p \right) e_i^2,
\end{aligned}
\tag{17}
$$

where $0 < p < \frac{1}{\delta_1} - 1$ is a constant.

It is noted that

$$
\begin{aligned}
e^T e &= \sum_{i=1}^{n} (N^T \Omega \eta_i)^2 \\
&= \eta^T (I_n \otimes \Omega N N^T \Omega) \eta \\
&= \Xi^T (L^2 \otimes \Omega N N^T \Omega) \Xi \\
&= z^T (L^2 \otimes \Omega N N^T \Omega) z.
\end{aligned}
\tag{18}
$$

If we choose γ such that $(1-\delta_1)l > 2(1+\delta_1)m\gamma$, then we can get from (16) to (18) that

$$\dot{V}_0 \leq z^T\left(L\otimes\left(\Omega M+M^T\Omega\right)\right)z-\theta\kappa_1 z^T\left(L^2\otimes\Omega NN^T\Omega\right)z+\kappa_2 \mathrm{e}^{-\delta_3 t}, \quad (19)$$

where $\kappa_1 = m\gamma\left(\frac{p}{1+p}-\delta_1 p\right)$, $\kappa_2 = n\delta_2(l+2m\gamma)+m\gamma\theta\sum_{i=1}^{n}\frac{p\delta_2}{\hat{\theta}_i(0)}$. It is noted that $\kappa_1 > 0$ and $\hat{\theta}_i(0) \leq \hat{\theta}_i$, because of $0 < p < \frac{1}{\delta_1}-1$ and $\dot{\hat{\theta}}_i \geq 0$, respectively. Here, we choose $\theta \geq \frac{1}{\kappa_1\lambda_2}$. From Lemma 2, we know that $\lambda_2 > 0$.

In what follows, the operator $f_i(\cdot)$ and the dynamical system for variable $\hat{\vartheta}_i$ will be designed. Considering the following function

$$V_{i,1} = \frac{1}{2}(\xi_i - \varsigma_i)^2, \quad i = 1, \ldots, n. \quad (20)$$

The derivation of $V_{i,1}$ can be obtained as $\dot{V}_{i,1} = (\xi_i-\varsigma_i)(\psi_i-\Phi_i)$.

Define $\varpi_{i,1} = \xi_i - \varsigma_i$, $\varpi_{i,2} = \psi_i - \Phi_i$, $\varpi_{i,2}^* = -r(\xi_i-\varsigma_i)$ and $\tilde{\varpi}_{i,2} = \varpi_{i,2}-\varpi_{i,2}^*$, where $r > 1$ is a constant. Considering the function

$$V_{i,2} = V_{i,1} + \frac{1}{2c_i}\tilde{\varpi}_{i,2}^2, \quad (21)$$

we can obtain that

$$\begin{aligned}\dot{V}_{i,2} = {} & \tilde{\varpi}_{i,2}\left(\upsilon_i + \frac{\phi_i(\xi_i,\psi_i)}{c_i} + \frac{\rho_i}{c_i} - \frac{l}{c_i}\hat{\theta}_i e_i\left(t_k^i\right) + \frac{r}{c_i}\varpi_{i,2}\right)\\ & -r\varpi_{i,1}^2 + \varpi_{i,1}\tilde{\varpi}_{i,2}. \end{aligned} \quad (22)$$

By Lemma 1, it is easy to get

$$\varpi_{i,1}\tilde{\varpi}_{i,2} \leq \frac{1}{2}\varpi_{i,1}^2 + \frac{1}{2}\tilde{\varpi}_{i,2}^2, \quad (23)$$

$$\frac{r}{c_i}\tilde{\varpi}_{i,2}\varpi_{i,2} = \frac{r}{c_i}\tilde{\varpi}_{i,2}(\tilde{\varpi}_{i,2}-r\varpi_{i,1}) \leq \frac{r}{c_i}\tilde{\varpi}_{i,2}^2 + \frac{r^4}{2c_i^2}\tilde{\varpi}_{i,2}^2 + \frac{1}{2}\varpi_{i,1}^2. \quad (24)$$

It follows from (22)-(24) that

$$\begin{aligned}\dot{V}_{i,2} \leq {} & \tilde{\varpi}_{i,2}\left(\upsilon_i + \frac{\phi_i(\xi_i,\psi_i)}{c_i} + \frac{\rho_i}{c_i} - \frac{l}{c_i}\hat{\theta}_i e_i\left(t_k^i\right)\right)\\ & -(r-1)\varpi_{i,1}^2 + \left(\frac{r^4}{2c_i^2}+\frac{r}{c_i}+\frac{1}{2}\right)\tilde{\varpi}_{i,2}^2. \end{aligned} \quad (25)$$

Since $\phi_i(\xi_i, \psi_i)$ and c_i are unknown, the neural network approximation strategy will be adopted. Here, we use the neural network $\vartheta_i^T \Upsilon_i(\xi_i, \psi_i)$ to approximate $\frac{\phi_i(\xi_i, \psi_i)}{c_i}$, that is,

$$\frac{\phi_i(\xi_i, \psi_i)}{c_i} = \vartheta_i^T \Upsilon_i(\xi_i, \psi_i) + \sigma_i(\xi_i, \psi_i), \tag{26}$$

where Lemma 3 was used, $\Upsilon_i(\xi_i, \psi_i) = [\Upsilon_{i,1}(\xi_i, \psi_i), \ldots, \Upsilon_{i,h}(\xi_i, \psi_i)]^T$ is the basis function vector, ϑ_i is the ideal weight vector, $|\sigma_i(\xi_i, \psi_i)| \leq \sigma_i$ is the approximation error, $\sigma_i > 0$ is a constant.

By Lemma 1, we can get that

$$\varpi_{i,2}\sigma_i(\xi_i, \psi_i) \leq \frac{1}{2}\varpi_{i,2}^2 + \frac{1}{2}\sigma_i^2. \tag{27}$$

Construct the following Lyapunov function candidate

$$V_i = V_{i,2} + \frac{1}{2}\tilde{\vartheta}_i^T \tilde{\vartheta}_i, \tag{28}$$

where $\tilde{\vartheta}_i = \vartheta_i - \hat{\vartheta}_i$, $\hat{\vartheta}_i$ is the adaptive law to be designed.

If the adaptive law $\hat{\vartheta}_i$ and input υ_i are designed as

$$\dot{\hat{\vartheta}}_i = \tilde{\varpi}_{i,2}\Upsilon_i(\xi_i, \psi_i) - \iota_2\hat{\vartheta}_i, \tag{29}$$

$$\begin{aligned} \upsilon_i &= f_i(\xi_i, \varsigma_i, \psi_i, \Phi_i, \hat{\vartheta}_i) \\ &= -\left(\frac{r^4}{2\underline{c}_i^2} + \frac{r}{\underline{c}_i} + \frac{1}{2} + \iota_1\right)\tilde{\varpi}_{i,2} - \hat{\vartheta}_i^T \Upsilon_i(\xi_i, \psi_i) + \upsilon_i^o, \end{aligned} \tag{30}$$

where $\iota_1 > 0$, $\iota_2 > 0$ are constants, $\upsilon_i^o = -\frac{\mathrm{sig}(\tilde{\varpi}_{i,2})}{\underline{c}_i}(\bar{\rho}_i + l\hat{\theta}_i \left|e_i(t_k^i)\right|)$ is the compensation component.

It can be concluded from (25) to (29) that

$$\dot{V}_i \leq -(r-1)\varpi_{i,1}^2 - \iota_1\varpi_{i,2}^2 + \iota_2\tilde{\vartheta}_i^T \hat{\vartheta}_i + \frac{1}{2}\sigma_i^2. \tag{31}$$

Remark 4 The proposed protocol is in fact composed by (6), (12), (15), (29) and (30). Based on the above analysis, the results on the bounded consensus can be easily derived, which will be shown in the next subsection.

3.2 Consensus Analysis

Theorem 1 *Protocol* (6) *ensures that the bounded consensus of multi-agent system* (1) *under a connected graph can be reached if variables* $\hat{\theta}_i$, $\hat{\vartheta}_i$ *and function* $f_i(\cdot)$ *are* *designed as in* (12)*,* (29) *and* (30)*, respectively, and the triggered rule is given as in* (15).

Proof It can be easily obtained that there is a unitary matrix U such that $U^T L U = L_u = diag\{0, \lambda_2, \ldots, \lambda_n\}$. Define $z' = [{z_1'}^T, \ldots, {z_n'}^T]^T = (U^{-1} \otimes I_2)z$, we have that

$$\dot{V}_0 \leq {z'}^T (L_u \otimes (\Omega M + M^T \Omega))z' - \theta\kappa_1 {z'}^T ({L_u}^2 \otimes \Omega N N^T \Omega)z' + \kappa_2 \mathrm{e}^{-\delta_3 t}. \quad (32)$$

Since $\theta \geq \frac{1}{\kappa_1 \lambda_2}$, we can get that

$$\begin{aligned} \dot{V}_0 &\leq \sum_{i=2}^{n} \lambda_i {z'_i}^T (\Omega M + M^T \Omega - \Omega N N^T \Omega) z'_i + \kappa_2 \mathrm{e}^{-\delta_3 t} \\ &\leq -\beta \sum_{i=2}^{n} \lambda_i {z'_i}^T z'_i + \kappa_2 e^{-\delta_3 t}. \end{aligned} \quad (33)$$

From (33), we obtain that

$$V_0 \leq V_0(0) + \int_0^t \kappa_2 e^{-\delta_3 \tau} \mathrm{d}\tau \leq V_0(0) + \frac{\kappa_2}{\delta_3}. \quad (34)$$

The boundedness of V_0 means that monotonically nondecreasing $\hat{\theta}_i$ converges to a positive value and $e_i(t)$ is bounded. Then, it can be analyzed that $\lim_{t\to\infty} e_i(t_k^i) = 0$. From the triggered rule (15), we can get that

$$(1 - \delta_1)e_i^2(t) \leq 2e_i(t)e_i(t_k^i) - e_i^2(t_k^i) + \frac{\delta_2}{\hat{\theta}_i(0)} \mathrm{e}^{-\delta_3 t}, \quad (35)$$

which implies that $\lim_{t\to\infty} e_i(t) = 0$.

It can be obtained that

$$\lim_{t\to\infty} (\varsigma_i - \varsigma_j) = 0, \quad \lim_{t\to\infty} (\Phi_i - \Phi_j) = 0. \quad (36)$$

It is noted that

$$\iota_2 \tilde{\vartheta}_i^T \hat{\vartheta}_i = -\iota_2 \tilde{\vartheta}_i^T \tilde{\vartheta}_i + \iota_2 \tilde{\vartheta}_i^T \vartheta_i \leq -\frac{\iota_2}{2} \tilde{\vartheta}_i^T \tilde{\vartheta}_i + \frac{\iota_2}{2} \vartheta_i^T \vartheta_i. \quad (37)$$

It can be concluded from (31) and (37) that

$$\dot{V}_i \leq -\alpha_i V_i + d_i, \quad (38)$$

where $\alpha = \min\{2(r-1), 2c_i\iota_1, \iota_2\}$, $d_i = \frac{1}{2}\sigma_i^2 + \frac{\iota_2}{2}\vartheta_i^T \vartheta_i$.

We can obtain from (38) that $\lim_{t\to\infty} V_i \le \frac{d_i}{\alpha_i}$, which means that

$$
\begin{aligned}
\lim_{t\to\infty} |\xi_i - \varsigma_i| &\le \varepsilon_{i,o,1} = \sqrt{\frac{2d_i}{\alpha_i}},\\
\lim_{t\to\infty} |\psi_i - \Phi_i| &\le \varepsilon_{i,o,2} = \sqrt{\frac{\max\{4, 4c_i r^2\} d_i}{\alpha_i}}.
\end{aligned}
\tag{39}
$$

We can conclude from (36) and (39) that

$$
\lim_{t\to\infty} \left|\xi_i - \xi_j\right| \le \varepsilon_{i,o,1} + \varepsilon_{j,o,1}, \quad \lim_{t\to\infty} \left|\Phi_i - \Phi_j\right| \le \varepsilon_{i,o,2} + \varepsilon_{j,o,2}. \tag{40}
$$

The proof is complete. □

Remark 5 Since there are errors in the neural network approximation for the unknown functions, only bounded consensus can be reached. The consensus errors can be as small as possible by using as many neural network nodes as possible.

Then, we show that the Zeno behavior can be excluded.

Theorem 2 *If the protocol (6) is applied to multi-agent system (1) under a connected graph, where variables $\hat{\theta}_i$, $\hat{\vartheta}_i$ and function $f_i(\cdot)$ are designed as in (12), (29) and (30), respectively, and the triggered rule is given as in (15), then the Zeno behavior can be excluded.*

Proof Motivated by [20,22], we hope to exclude the Zeno behavior by showing that $\lim_{k\to\infty} t_k^i = \infty$. Since $e_i = \omega_2 \sum_{j=1}^{n} a_{ij}(\varsigma_j - \varsigma_i) + \omega_3 \sum_{j=1}^{n} a_{ij}(\Phi_j - \Phi_i)$, we have

$$
\begin{aligned}
\frac{d(e_i - e_i(t_k^i))^2}{dt} &= 2(e_i - e_i(t_k^i))\dot{e}_i\\
&\le 2(e_i - e_i(t_k^i))N^T \Omega \begin{bmatrix} \sum_{j=1}^{n} a_{ij}(\Phi_j - \Phi_i) \\ \sum_{j=1}^{n} a_{ij}(\dot{\Phi}_j - \dot{\Phi}_i). \end{bmatrix}
\end{aligned}
\tag{41}
$$

From Theorem 1, we can obtain that $\sum_{j=1}^{n} a_{ij}(\Phi_j - \Phi_i)$ is bounded. From (6), we can also obtain that $\dot{\Phi}_i$ is bounded since $\hat{\theta}_i$ converges to a constant. In combining with triggered rule (15), we can conclude that $\frac{d(e_i - e_i(t_k^i))^2}{dt}$ is bounded, that is, there is a constant π_i such that $\frac{d(e_i - e_i(t_k^i))^2}{dt} \le \pi_i$. Therefore, we have

$$
(e_i - e_i(t_k^i))^2 \le \pi_i (t - t_k^i), \quad t \in [t_k^i, t_{k+1}^i). \tag{42}
$$

According to triggered (15), it can be seen that

$$(e_i(t^i_{k+1}) - e_i(t^i_k))^2 = \delta_1 e_i^2(t^i_{k+1}) + \frac{\delta_2}{\hat{\theta}_i(t^i_{k+1})}\mathrm{e}^{-\delta_3 t^i_{k+1}}, \tag{43}$$

which means that

$$\frac{\delta_2}{\hat{\theta}_i(t^i_{k+1})}\mathrm{e}^{-\delta_3 t^i_{k+1}} \le (e_i(t^i_{k+1}) - e_i(t^i_k))^2 \le \pi_i(t^i_{k+1} - t^i_k). \tag{44}$$

If $\lim\limits_{k\to\infty} t^i_k = \bar{t}^i < \infty$, then from $\lim\limits_{k\to\infty}(t^i_{k+1} - t^i_k) = 0$, we have

$$0 < \lim_{k\to\infty} \frac{\delta_2}{\hat{\theta}_i(t^i_{k+1})}\mathrm{e}^{-\delta_3 \bar{t}^i} \le \lim_{k\to\infty} \pi_i(t^i_{k+1} - t^i_k) = 0, \tag{45}$$

which results in a contradiction. As such, the Zeno behavior can be excluded.

The proof is complete. □

In the implementation of (6), the triggered condition should be checked continuously. If the information of ς_i and Φ_i is required to be transmitted continuously, the event-triggered scheme will be meaningless. In fact, the information can be accurately estimated by neighbored subsystems and the drawback can be overcome.

It is noted that

$$\begin{aligned}
\begin{bmatrix} \varsigma_j \\ \Phi_j \end{bmatrix} &= e^{M(t-t^j_k)} \begin{bmatrix} \varsigma_j \\ \Phi_j \end{bmatrix}(t^j_k) + \int_{t^j_k}^{t} N\dot{\Phi}_j(\tau)\mathrm{d}\tau \\
\hat{\theta}_j &= \hat{\theta}_j(t^j_k) + m e_j^2(t^j_k)(t - t^j_k).
\end{aligned} \tag{46}$$

It can be seen that the information of ς_i and Φ_i can be well estimated by neighbors under the discontinuous communication. Of course, when the ith subsystem is triggered, $e_i(t^i_k)$ is required to be transmitted to neighbors.

Remark 6 By the data transmissions on triggered points, the information of ς_i and Φ_i can be well estimated by neighbors. This method, which is an effective tool to avoid the continuous data transmissions, has been widely used, such as in [4,47]. In addition to saving communication resources, the protocol is in fully distributed form and of scalability. Since more general nonlinearities are considered in this paper, the proposed control scheme can be applied to more physical systems.

4 Simulation Study

In this section, an example is given to illustrate the effectiveness of the proposed protocol. Consider multi-agent system (1) with 6 subsystems, and the corresponding graph is shown in Fig. 1.

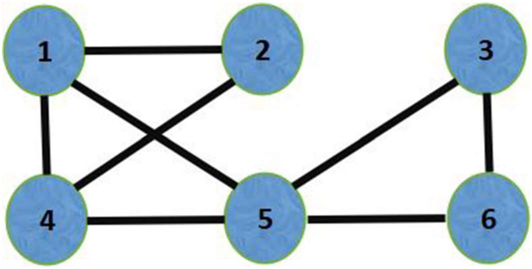

Fig. 1 The graph of the multi-agent system

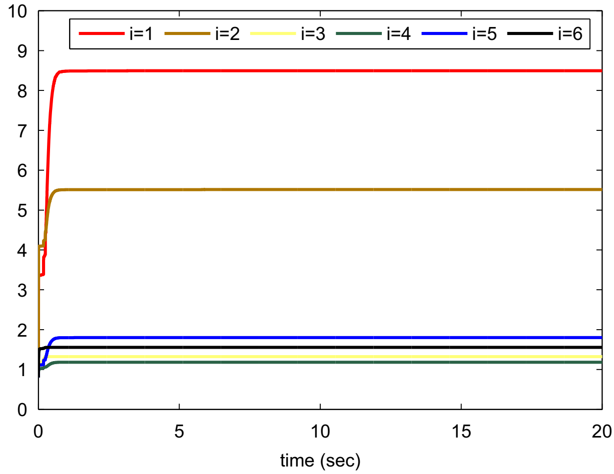

Fig. 2 State trajectories of $\hat{\theta}_i$

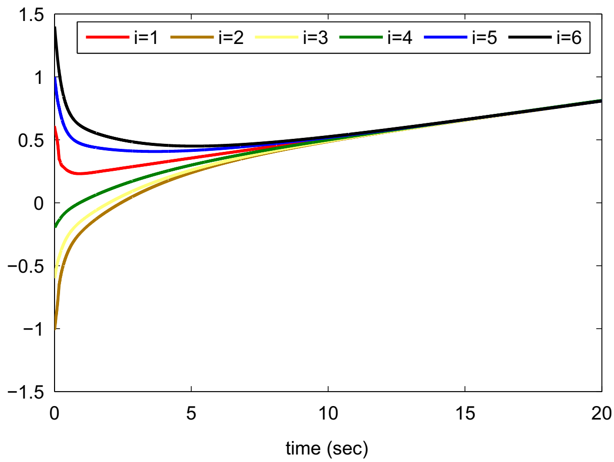

Fig. 3 State trajectories of ξ_i

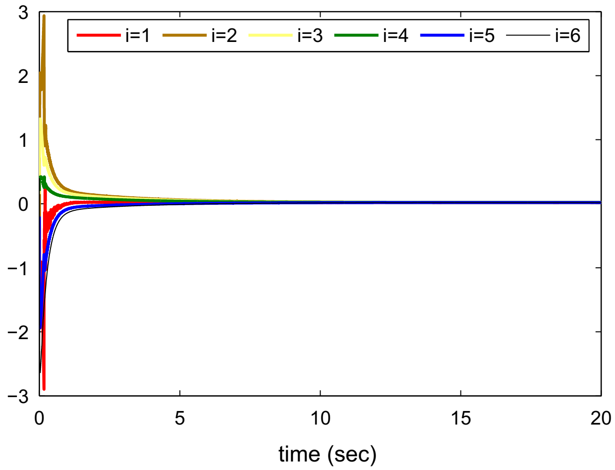

Fig. 4 State trajectories of ψ_i

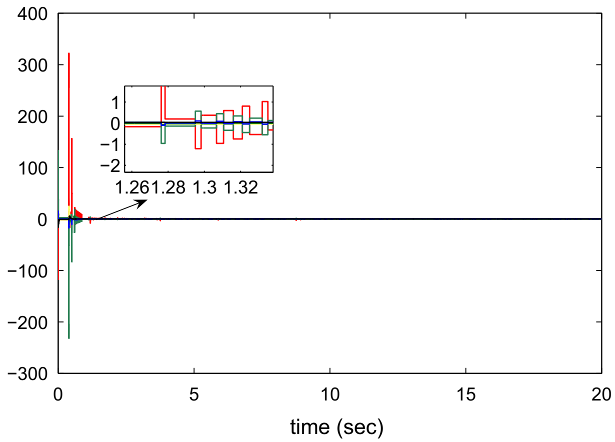

Fig. 5 Trajectories of ${\upsilon_i}^o$

The nonlinear functions are given as follows:

$$\begin{aligned}
&\phi_1(\xi_1, \psi_1) = \sin \xi_1 + \cos \psi_1, \ \ \phi_2(\xi_2, \psi_2) = \cos \xi_1 + \sin \psi_1, \\
&\phi_3(\xi_3, \psi_3) = \sin \xi_3 \cos \psi_3, \quad \phi_4(\xi_4, \psi_4) = 4\xi_4 + 2\psi_4, \\
&\phi_5(\xi_5, \psi_5) = \mathrm{e}^{-3|\xi_5|} + 2\psi_5, \quad \phi_6(\xi_6, \psi_6) = \mathrm{e}^{-3|\xi_6|}\cos^2\psi_6. \qquad (47)
\end{aligned}$$

We can see that the constraints on nonlinear terms in [3,18] are not satisfied here, so the proposed protocol is of wider applicability. The gain coefficients are given as $c_1 = \ldots = c_6 = 0.85$, and the external disturbances are given as $\rho_i(t) = 0.1 \sin t$, $i = 1, 2, 3, 4, 5, 6$.

By solving (5), we can get that $\Omega = \begin{bmatrix} \omega_1 & \omega_2 \\ \omega_2 & \omega_3 \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ 2 & 6 \end{bmatrix}$ is a solution for $\beta = 4$. The other parameters in the protocol are chosen as follows: $l = 3$, $m = 3$, $\iota_1 = 2$, $\iota_1 = 4$, $r = 4$, $\bar{\rho}_i = 0.1$, $\delta_1 = 0.02$, $\delta_2 = 0.8$ and $\delta_3 = 10$. The neural networks are constructed by the basis functions

$$\Upsilon_{i,j}(\xi_i, \psi_i) = \mathrm{e}^{-\frac{(\xi_i-3+j)^2+(\psi_i-3+j)^2}{5^2}}, \quad j = 1, 2, 3, 4, 5.$$

The initial values of designed dynamical variables and states for subsystems are set as:

$$\begin{aligned}
&\hat{\theta}_i(0) = 0.8, \quad i = 1, 2, 3, 4, 5, 6, \\
&\hat{\vartheta}_i(0) = [0, 0, 0, 0, 0]^T, \quad i = 1, 2, 3, 4, 5, 6, \\
&[\varsigma_1(0), \varsigma_2(0), \varsigma_3(0), \varsigma_4(0), \varsigma_5(0), \varsigma_6(0)] = [0.3, -0.5, -0.3, -0.1, 0.5, 0.7], \\
&[\Phi_1(0), \Phi_2(0), \Phi_3(0), \Phi_4(0), \Phi_5(0), \Phi_6(0)] = [0.4, -0.75, 0.075, 0.1, -0.1, 0.2], \\
&[\xi_1(0), \xi_2(0), \xi_3(0), \xi_4(0), \xi_5(0), \xi_6(0)] = [0.6, -1, -0.6, -0.2, 1, 1.4], \\
&[\psi_1(0), \psi_2(0), \psi_3(0), \psi_4(0), \psi_5(0), \psi_6(0)] = [0.8, -1.5, 0.15, 0.2, -0.2, 0.4].
\end{aligned}$$

The simulation is carried out by the M-file tool from Matlab. In the simulation, the step length is chosen as 0.0001s. The trajectories of $\hat{\theta}_i$, $i = 1, 2, 3, 4, 5, 6$, are shown in Fig. 2. Figures 3 and 4 display the positions and velocities of 6 subsystems, respectively. The simulation results show that the proposed protocol can make the multi-agent system achieve bounded consensus successfully. Since the event instants can be reflected by signals υ_i^o, $i = 1, 2, 3, 4, 5, 6$, we depict the trajectories in Fig. 5, where the jump instants are event instants. From the simulation data, we can get that the minimum time interval is 0.0011 s, which is much lager than the step length. Therefore, there is no Zeno behavior in this numerical example.

5 Conclusion

In this article, an event-triggered protocol was proposed, under which the practical consensus of the considered multi-agent system can be reached. Due to that the global information is unavailable, the protocol is of fully distributed feature. The consumption of communication resources among subsystems can be well reduced by the developed control scheme. Via a numerical example, the validity of the protocol was verified. In four future work, fully distributed event-triggered consensus for nonlinear multi-agent systems with switching networks will be investigated.

Funding This work was financially supported by the Scientific Research Significant Projects of Universities and Colleges in Jiangsu Province (16KJA460003).

Availability of Data and Materials The data sets generated during and analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. X. Cai, C. Wang, G. Wang, D. Liang, Distributed consensus control for second-order nonlinear multi-agent systems with unknown control directions and position constraints. Neurocomputig **306**, 61–67 (2018)
2. S. Chen, B. Chen, F. Shi, Distributed fault-tolerant consensus protocol for fuzzy multi-agent systems. Circuits Syst. Signal Process. **38**(2), 611–624 (2019)
3. B. Cheng, Z. Li, Designing fully distributed adaptive event-triggered controllers for networked linear systems with matched uncertainties. IEEE Trans. Neural Netw. Learn. Syst. **30**(12), 3645–3655 (2019)
4. Y. Cheng, V. Ugrinovskii, Event-triggered leader-following tracking control for multivariable multi-agent systems. Automatica **70**, 204–210 (2016)
5. D.V. Dimarogonas, E. Frazzoli, K.H. Johansson, Distributed event-triggered control for multi-agent systems. IEEE Trans. Autom. Control **57**(5), 1291–1297 (2012)
6. M. Fiedler, A property of eigenvectors of non negative symmetric matrices and its application to graph theory. Czechoslov. Math. J. **25**, 619–633 (1975)
7. G. Freudenthaler, T. Meurer, PDE-based multi-agent formation control using flatness and backstepping: analysis, design and robot experiments. Automatica (2020). https://doi.org/10.1016/j.automatica.2020.108897
8. F. Gao, W. Chen, Z. Li, J. Li, B. Xu, Neural network-based distributed cooperative learning control for multiagent systems via event-triggered communication. IEEE Trans. Neural Netw. Learn. Syst. **31**(2), 407–419 (2020)
9. H. Gao, A. Hu, Event-triggered pinning bipartite tracking consensus of the multi-agent system subject to input saturation. Int. J. Control Autom. Syst. **18**(9), 2195–2205 (2020)
10. P. Gong, K. Wang, Output feedback consensus control for fractional-order nonlinear multi-agent systems with directed topologies. J. Frankl. Inst. **357**(3), 1473–1493 (2020)
11. R. Horn, C. Johnson, *Matrix Analysis* (Cambridge University Press, New York, 1985)
12. C. Hua, X. You, X. Guan, Adaptive leader-following consensus for second-order time-varying nonlinear multiagent systems. IEEE Trans. Cybern. **47**(6), 1532–1539 (2017)
13. H. Ji, H. Zhang, Z. Ye, H. Zhang, B. Xu, G. Chen, Stochastic consensus control of second-order nonlinear multiagent systems with external disturbances. IEEE Trans. Control Netw. Syst. **5**(4), 1585–1596 (2018)
14. J. Li, J. Li, Coordination control of multi-agent systems with second-order nonlinear dynamics using fully distributed adaptive iterative learning. J. Frankl. Inst. **352**(6), 2441–2463 (2015)
15. T. Li, Z. Li, S. Fei, Z. Ding, Second-order event-triggered adaptive containment control for a class of multi-agent systems. ISA Trans. **96**, 132–142 (2020)
16. S. Li, Y. Pan, H. Liang, Y. Tian, Event-triggered adaptive consensus tracking control for non-affine multi-agent systems. Neurocomputing **393**, 46–53 (2020)
17. W. Li, H. Zhang, S. Sun, J. Zhang, Fully distributed event-triggered consensus protocols for multi-agent systems with physically interconnected network. Neurocomputing **418**, 191–199 (2020)
18. T. Li, H. Zhao, A novel event-triggered communication strategy for second-order multiagent systems. ISA Trans. **97**, 93–101 (2020)
19. D. Liu, H. Liu, F.L. Lewis, Y. Wan, Robust fault-tolerant formation control for tail-sitters in aggressive flight mode transitions. IEEE Trans. Ind. Inf. **16**(1), 299–308 (2020)
20. X. Li, Y. Tang, H. Karimi, Consensus of multi-agent systems via fully distributed event-triggered control. Automatica (2020). https://doi.org/10.1016/j.automatica.2020.108898
21. Y. Ma, J. Zhao, Distributed adaptive integral-type event-triggered cooperative output regulation of switched multiagent systems by agent-dependent switching with dwell time. Int. J. Robust Nonlinear Control **30**(6), 2550–2569 (2020)
22. C. Nowzari, E. Garcia, J. Cortés, Event-triggered communication and control of networked systems for multi-agent consensus. Automatica **105**, 1–27 (2019)

23. R. Olfati-Saber, R. Murray, Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. Autom. Control **49**(9), 1520–1533 (2004)
24. D. Shin, S. Bagchi, C. Wang, Toward optimal distributed monitoring of multi-channel wireless networks. IEEE Trans. Mob. Comput. **15**(7), 1826–1838 (2016)
25. S. Song, J. Hu, D. Chen, W. Chen, Z. Wu, An event-triggered approach to robust fault detection for nonlinear uncertain Markovian jump systems with time-varying delays. Circuits Syst. Signal Process **39**, 3445–3469 (2020)
26. P. Tabuada, Event-triggered real-time scheduling of stabilizing control tasks. IEEE Trans. Autom. Control **52**(9), 1680–1685 (2007)
27. Y. Tan, V. Venkatesh, X. Gu, Resilient self-compressive monitoring for large-scale hosting infrastructures. IEEE Trans. Parallel Distrib. Syst. **24**(3), 576–586 (2013)
28. W. Wang, Y. Li, S. Tong, Neural-network-based adaptive event-triggered consensus control of nonstrict-feedback nonlinear systems. IEEE Trans. Neural Netw. Learn. Syst. (2020). https://doi.org/10.1109/TNNLS.2020.2991015
29. F. Wang, Z. Liu, Z. Chen, Leader-following consensus of second-order nonlinear multi-agent systems with intermittent position measurements. Sci. China Inf. Sci. **62**(10), 1–16 (2019)
30. X. Wang, H. Su, X. Wang, G. Chen, Fully distributed event-triggered semiglobal consensus of multi-agent systems with input saturation. IEEE Trans. Ind. Electron. **64**(6), 5055–5064 (2017)
31. X. Wang, H. Su, X. Wang, B. Liu, Second-order consensus of multi-agent systems via periodically intermittent pinning control. Circuits Syst. Signal Process. **35**(7), 2413–2431 (2016)
32. X. Xu, S. Chen, L. Gao, Observer-based consensus tracking for second-order leader-following nonlinear multi-agent systems with adaptive coupling parameter design. Neurocomputig **156**, 297–305 (2015)
33. S. Yan, G. Zhang, T. Li, M. Shen, L. Li, Static output control of discrete-time networked control systems with an event-triggered scheme. Circuits Syst. Signal Process **37**(2), 553–568 (2018)
34. Y. Yang, B. Ding, Tracking and formation of multi-agent systems with collision and obstacle avoidance based on distributed RHC. Circuits Syst. Signal Process. **38**(7), 2951–2970 (2019)
35. N. Yang, J. Li, Fully distributed coordination learning control of second-order nonlinear multi-agent systems with input saturation. Asian J. Control (2020). https://doi.org/10.1002/asjc.2330
36. R. Yang, L. Liu, G. Feng, Leader-following output consensus of heterogeneous uncertain linear multiagent systems with dynamic event-triggered strategy. IEEE Trans. Syst. Man Cybern.: Syst. (2020). https://doi.org/10.1109/TSMC.2020.3034352
37. J. Yang, F. Xiao, T. Chen, Event-triggered formation tracking control of nonholonomic mobile robots without velocity measurements. Automatica (2019). https://doi.org/10.1016/j.automatica.2019.108671
38. P. Ye, A. Sheng, Y. Li, Bounded consensus tracking of second-order multi-agent systems using rectangular impulsive control. Nonlinear Dyn. **95**, 1189–1202 (2019)
39. Y. Yu, D. Huang, H. Jiang, C. Hu, Consensus of second-order multi-agent systems with nonlinear dynamics via edge-based distributed adaptive protocols. J. Frankl. Inst. **353**(18), 4821–4844 (2016)
40. T. Zhang, S.S. Ge, C.C. Huang, Adaptive neural network control for strict-feedback nonlinear systems using backstepping design. Automatica **36**(12), 1835–1846 (2000)
41. J. Zhang, H. Zhang, Y. Cai, Y. Lu, Distributed cooperative output regulation of heterogeneous linear multi-agent systems based on event- and self-triggered control with undirected topology. ISA Trans. **99**, 191–198 (2020)
42. Y. Zhao, Y. Liu, G. Wen, T. Huang, Finite-time distributed average tracking for second-order nonlinear systems. IEEE Trans. Neural Netw. Learn. Syst. **30**(6), 1780–1789 (2019)
43. Y. Zhao, Y. Liu, G. Wen, W. Ren, G. Chen, Edge-based finite-time protocol analysis with final consensus value and settling time estimations. IEEE Trans. Cybern. **50**(4), 1450–1459 (2020)
44. Z. Zhong, L. Sun, J. Wang, P. Lv, H. Zheng, Consensus for first- and second-order discrete-time multi-agent systems with delays based on model predictive control schemes. Circuits Syst. Signal Process. **34**(1), 127–152 (2015)
45. Y. Zhou, Y. Pan, S. Li, H. Liang, Event-triggered cooperative containment control for a class of uncertain non-identical networks. Appl. Math. Comput. (2020). https://doi.org/10.1016/j.amc.2020.125065
46. W. Zhu, Q. Zhou, D. Wang, G. Feng, Fully distributed consensus of second-order multi-agent systems using adaptive event-based control. Sci. China Inf. Sci. **61**(12), 1–3 (2018)
47. W. Zou, Y. Huang, C. Ahn, Z. Xiang, Containment control of linear multiagent systems with stochastic disturbances via event-triggered strategies. IEEE Syst. J. **14**(4), 4810–4819 (2020)

48. W. Zou, P. Shi, Z. Xiang, Y. Shi, Finite-time consensus of second-order switched nonlinear multi-agent systems. IEEE Trans. Neural Netw. Learn. Syst. **31**(5), 1757–1762 (2020)
49. W. Zou, P. Shi, Z. Xiang, Y. Shi, Consensus tracking control of switched stochastic nonlinear multiagent systems via event-triggered strategy. IEEE Trans. Neural Netw. Learn. Syst. **31**(3), 1036–1045 (2020)
50. W. Zou, Z. Xiang, C. Ahn, Mean square leader-following consensus of second-order nonlinear multi-agent systems with noises and unmodeled dynamics. IEEE Trans. Syst. Man Cybern.: Syst. **49**(12), 2478–2486 (2019)