Circuits, Systems, and Signal Processing (2022) 41:661–683
https://doi.org/10.1007/s00034-021-01803-9

Finite-Time Stabilization of Multi-rate Networked Control System Based on Predictive Control

Hairui Zhao[1,2] · Dongyan Chen[1,2] · Jun Hu[1,2] · Junhua Du[3]

Received: 4 December 2020 / Revised: 16 July 2021 / Accepted: 19 July 2021 /
Published online: 5 August 2021

Abstract

This paper deals with the problem of finite-time stabilization for the multi-rate networked control system with network-induced time delays. In order to guarantee the finite-time stability and overcome the adverse impact of network-induced time delays in both sensor-to-controller channel and controller-to-actuator channel, a networked predictive control strategy is proposed based on multi-rate sampling mechanism. By utilizing the techniques of lifting and augmenting, the augmented closed-loop system is derived with a uniform sampling rate. By applying the Lyapunov stability theorem, sufficient conditions are established such that the multi-rate networked control system can be finite-time stabilized. Furthermore, the gain matrices are determined for the observer and the controller by solving bilinear matrix inequality or linear

This work was supported in part by the National Natural Science Foundation of China under Grants 12071102 and 61673141, the Natural Science Foundation of Heilongjiang Province of China under Grant A2018007, the Fundamental Research Funds in Heilongjiang Provincial Universities of China under Grant 135509121 and the Educational Research Project of the Qiqihar University of China under Grant YB201904.

✉ Dongyan Chen
dychen_2004@hotmail.com

✉ Jun Hu
hujun2013@gmail.com

Hairui Zhao
zhaohairui@stu.hrbust.edu.cn

Junhua Du
dujunhua1975@126.com

1 Department of Mathematics, Harbin University of Science and Technology, Harbin 150080, China

2 Heilongjiang Provincial Key Laboratory of Optimization Control and Intelligent Analysis for Complex Systems, Harbin University of Science and Technology, Harbin 150080, China

3 College of Science, Qiqihar University, Qiqihar 161006, China

matrix inequality. Finally, a numerical example is presented to show the validity of the obtained results.

Keywords Finite-time stabilization · Multi-rate networked control system · Predictive control · Network-induced time delays

1 Introduction

At present, most of the existing studies have assumed that the networked control system has a single sampling rate for the convenience of theoretical research [7,18–21]. However, due to the different physical characteristics of each system component (such as plant, sensor, controller and actuator), it is quite complicated to unify the sampling rate. The network environment and location of each system component are different, so it is generally difficult to meet the performance requirement of each system component at the same sampling rate. From a practical point of view, it is sometimes unnecessary to sample all the different kinds of signals at the same rate. In addition, the multi-rate networked control system has the advantage on balancing resource consumption and system performance [33,35]. Consequently, the multi-rate sampling mechanism (MRSM) has been proposed instead of using the synchronous sampling mechanism by some researchers. During the past few decades, the great concern among scholars has been aroused by the analysis problems of the multi-rate sampling on account of their successful applications in practice [6,25,32,34,39,52]. For example, the problems of H_∞ state estimation and distributed set-membership filtering have been discussed in [24] and [42] for the multi-rate systems with different sampling rate of the plant, the sensor and the filter, respectively. It's worth mentioning that the lifting technique proposed in [26] has become a pivotal instrument in developing the MRSM by converting the multi-rate sampling system into an equivalent system with single-rate sampling.

As is well known, the stability of the system is one of the primary concerns in the design and the analysis of the control system [30,44]. Up to now, with the help of the Lyapunov stability theorem, a large amount of efforts have been dedicated to the stabilization problem for multi-rate networked control system (MRNCS) with some results of significance presented in [10,27,54]. For instance, in order to guarantee the stability of the MRNCS, the state feedback controller has been designed in [54], where the MRNCS with time delays and packet dropout has been modelled as a switched system. Based on previous study, the stabilization problem has been dealt with in [2] for a discrete-time linear MRNCS whose sets of sampling rates are the integer multiples of those operating on all the preceding substates. However, it is necessary to consider the finite-time behaviors of state variables in some networked control systems. For example, for those systems that work in a short time (such as missile system, robot operating system) in practical engineering, if their overshoot is extremely large, it maybe bring about that the control system cannot work normally. Over the last few decades, the finite-time stability problem has triggered an intense discussion among a great of scholars because of its fast convergence [3,9,14,28,29,46,51]. Among them, the finite-time stability and stabilization problems have been studied for a class of linear discrete time-varying stochastic systems in [51] by employing the state transition matrix approach and the Lyapunov function method, respectively. In [28], by using the Lyapunov stability theorem, the finite-time stability criteria have been presented in the forms of linear matrix inequalities for a class of linear systems with time delays.

In the backdrop of the prevailing network environment, the network-induced phenomena (such as network-induced time delays (NITDs), data packet dropout, data packet disordering and so on) are occurred frequently during the signal transmission via the network [22,23,40,41]. It will weaken the control performance of the system, even lead to instability [16,47,50]. Accordingly, various approaches have been proposed by scholars to deal with these disadvantages, such as switched system method [4], time-delay system method [5], jump system method [37] and so on. Another effective method is the networked predictive control which has been widely employed to address the negative impact of network-induced phenomena [11–13,38,45,48,49]. Among them, according to the state space model, some networked predictive control strategies have been designed to focus on the analysis problems of stability or stabilization. Recently, the problems of networked predictive control and finite-time stabilization have been studied in [17] by using networked predictive control method, where the effects of network-induced phenomena have been overcome and sufficient conditions have been given such that the networked control system is finite-time stabilized. It is worth mentioning that most of the existing results on finite-time stabilization problem have been concerned with single-rate system [3,9,14,28,29,46,51]. However, the finite-time stabilization problem of MRNCS has not been received adequate attention. This is one of motivations of this paper. In addition, most of the designed networked predictive control strategies are applicable to single-rate system [11–13,38,45,48,49]. To the best of our knowledge, the MRNCS has seldom been taken into account in the design of NPCS, and a few proposed NPCS can be used directly to deal with the NITDs in MRNCS. Consequently, another motivations of this paper is to shorten such a gap by developing a new NPCS based on MRSM.

Influenced by the above discussion, we aim to address the finite-time stabilization problem for MRNCS with NITDs by employing networked predictive control method. The main challenges and difficulties from the following three aspects. (1) How to design the NPCS to overcome the negative effects for system performance caused by NITDs? (2) Due to different sampling rates of the plant and the sensor, how to transform the MRNCS into a equivalent single rate system? (3) How to guarantee the finite-time stability of MRNCS? To answer the mentioned three questions, the major contributions of this paper can be outlined as follows. The major contributions of this paper can be outlined as follows. (1) A novel NPCS based on MRSM is designed to overcome the NITDs. (2) With the help of the Lyapunov stability theorem, sufficient conditions are derived to ensure that the MRNCS is finite-time stable. (3) The gain matrices of observer and controller are not only obtained by solving the bilinear matrix inequality, but also explicitly expressed in terms of the solution to linear matrix inequality.

The rest of this article is introduced as follows. In Sect. 2, the discrete-time system model and the output model are mentioned, respectively. The NPCS is designed based on MRSM as well. In Sect. 3, the crucial theorems are proposed to cope with the finite-time stabilization problem for the MRNCS with NITDs. Next, the simulation example of this paper is expressed to demonstrate the validity of the main results in Sect. 4. In Sect. 5, the conclusion is given.

Notations The notations used in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ stand for the n-dimensional Euclidean space and the set of all $n \times m$ matrices, respectively. R^T and R^{-1} represent the transpose and inverse of the matrix R, respectively. $R > 0$ denotes that R is a real positive-definite symmetric matrix. I and 0 stand for the identity matrix and the zero matrix with appropriate dimensions, respectively. In a symmetric matrix, “*" is used to describe symmetric term. $cond(R)$ represents the ratio of $\lambda_{\max}(R)$ to $\lambda_{\min}(R)$, where $\lambda_{\max}(R)$ and $\lambda_{\min}(R)$ stand for the maximum eigenvalue and minimum eigenvalue of the matrix R, respectively.

2 Problem Formulation and Preliminaries

The structure of the MRNCS with NITDs in this paper is shown in Fig.1.

In Fig. 1, the scalars d_1 and d_2 are used to characterize the NITDs in the sensor-to-controller channel (feedback channel) and controller-to-actuator channel (forward channel), respectively. For the convenience of future research, we assume that the NITDs are integer multiples of the sampling period. Specifically, the scalars τ_1 and τ_2 are introduced to account for the upper bounds of delays form the feedback channel and the forward channel, respectively.

The plant is described in the form of

$$x(k_{i+1}) = Ax(k_i) + Bu(k_i) \tag{1}$$

where $x(k_i) \in \mathbb{R}^n$ accounts for the state vector, and $u(k_i) \in \mathbb{R}^m$ denotes the control input. Without loss of generally, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are two matrices given previously. Besides, for further analyzing, define $h \triangleq k_{i+1} - k_i$ $(\forall i = 0, 1, \cdots)$ as the sampling period of system (1) with $k_0 = 0$.

The measurement model is described by

$$y(s_i) = Cx(s_i) \tag{2}$$

where $y(s_i) \in \mathbb{R}^l$ denotes the measurement output at sampling instant s_i, $C \in \mathbb{R}^{l\times n}$ is a constant matrix. Under the initial condition $s_0 = 0$, denote $\tilde{h} \triangleq s_{i+1} - s_i$ $(\forall\, i = 0, 1, \cdots)$ as the sampling period of the sensor.

Considering physical restrictions on different system components, the MRSM is discussed here. To be more specific, the relationship among h and $\tilde{h}$ is $\tilde{h} = ah$, in which $a \geq 2$ and $h \geq 2$ are known positive integers. Specially, Fig. 2 provides an illustration of the multi-rate sampling mechanism among different devices.

By using the lifting technique, we can obtain the following dynamics equations:

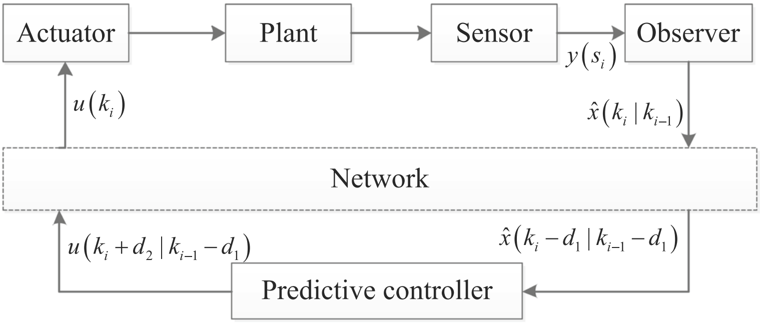

Fig. 1 The structure of MRNCS

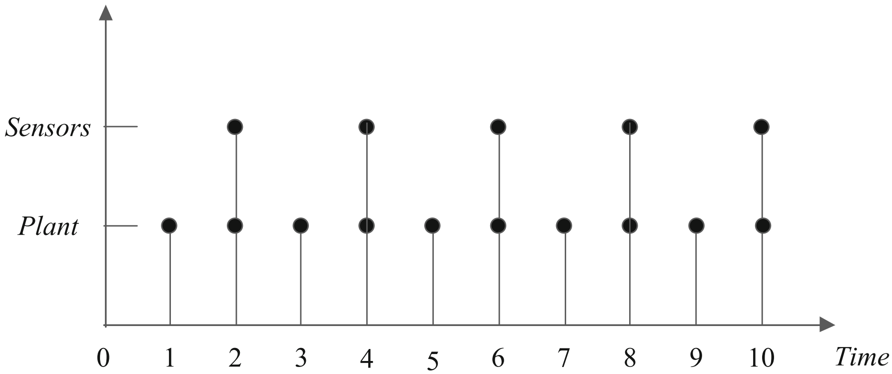

Fig. 2 MRSM for the plant and the sensor with $a = 2$

$$\begin{cases} x(s_{i+1}) = A^a x(s_i) + A^{a-1}Bu(s_i) + A^{a-2}Bu(s_i+h) \\ \qquad + \cdots + Bu(s_{i+1}-h) \\ x(s_{i+1}-h) = A^{a-1}x(s_i) + A^{a-2}Bu(s_i) + A^{a-3}Bu(s_i+h) \\ \qquad + \cdots + Bu(s_{i+1}-2h) \\ \quad \vdots \\ x(s_i+2h) = A^2 x(s_i) + ABu(s_i) + Bu(s_i+h) \\ x(s_i+h) = Ax(s_i) + Bu(s_i). \end{cases} \tag{3}$$

In this paper, the following observer is constructed:

$$\begin{cases} \hat{x}(s_{i+1}|s_i) = A^a\hat{x}(s_i|s_{i-1}) + A^{a-1}Bu(s_i) + A^{a-2}Bu(s_i+h) \\ \qquad + \cdots + Bu(s_{i+1}-h) + L_a(y(s_i) - C\hat{x}(s_i|s_{i-1})) \\ \hat{x}(s_{i+1}-h|s_i) = A^{a-1}\hat{x}(s_i|s_{i-1}) + A^{a-2}Bu(s_i) + A^{a-3}Bu(s_i+h) \\ \qquad + \cdots + Bu(s_{i+1}-2h) + L_{a-1}(y(s_i) - C\hat{x}(s_i|s_{i-1})) \\ \vdots \\ \hat{x}(s_i+2h|s_i) = A^2\hat{x}(s_i|s_{i-1}) + ABu(s_i) + Bu(s_i+h) \\ \qquad + L_2(y(s_i) - C\hat{x}(s_i|s_{i-1})) \\ \hat{x}(s_i+h|s_i) = A\hat{x}(s_i|s_{i-1}) + Bu(s_i) + L_1(y(s_i) - C\hat{x}(s_i|s_{i-1})) \end{cases} \tag{4}$$

where $\hat{x}(s_{i+1}|s_i)$ is the prediction of the state for time s_{i+1}, $L_a, L_{a-1}, \cdots, L_1$ are gain matrices of observer to be designed.

Recalling from the fact that NITDs are unavoidable in the feedback channel, the observer (4) is further rewritten as follows:

$$\begin{cases} \hat{x}(s_{i+1}-\tau_1|s_i-\tau_1) = A^a\hat{x}(s_i-\tau_1|s_{i-1}-\tau_1) + \cdots \\ \qquad + Bu(s_{i+1}-h-\tau_1) + L_a(y(s_i-\tau_1) \\ \qquad - C\hat{x}(s_i-\tau_1|s_{i-1}-\tau_1)) \\ \hat{x}(s_{i+1}-h-\tau_1|s_i-\tau_1) = A^{a-1}\hat{x}(s_i-\tau_1|s_{i-1}-\tau_1) + \cdots \\ \qquad + Bu(s_{i+1}-2h-\tau_1) + L_{a-1}(y(s_i-\tau_1) \\ \qquad - C\hat{x}(s_i-\tau_1|s_{i-1}-\tau_1)) \\ \vdots \\ \hat{x}(s_i+2h-\tau_1|s_i-\tau_1) = A^2\hat{x}(s_i-\tau_1|s_{i-1}-\tau_1) + ABu_h(s_i-\tau_1) \\ \qquad + Bu(s_i+h-\tau_1) + L_2(y(s_i-\tau_1) \\ \qquad - C\hat{x}(s_i-\tau_1|s_{i-1}-\tau_1)) \\ \hat{x}(s_i+h-\tau_1|s_i-\tau_1) = A\hat{x}(s_i-\tau_1|s_{i-1}-\tau_1) + Bu(s_i-\tau) \\ \qquad + L_1(y(s_i-\tau_1) - C\hat{x}(s_i-\tau_1|s_{i-1}-\tau_1)). \end{cases} \tag{5}$$

In order to handle the NITDs caused by the network, the state $\hat{x}(s_{i+1}-h|s_i-\tau), \cdots, \hat{x}(s_i|s_i-\tau)$ can be predicted by following equations

$$
\begin{cases}
\hat{x}(s_{i+1}+p-h-\tau_1|s_i-\tau_1) = A\hat{x}(s_{i+1}+p-2h-\tau_1|s_i-\tau_1) \\
\qquad + Bu(s_{i+1}+p-2h-\tau_1|s_i-\tau_1) \\
\hat{x}(s_{i+1}+p-2h-\tau_1|s_i-\tau_1) = A\hat{x}(s_{i+1}+p-3h-\tau_1|s_i-\tau_1) \\
\qquad + Bu(s_{i+1}+p-3h-\tau_1|s_i-\tau_1) \\
\qquad \vdots \\
\hat{x}(s_i+p+h-\tau_1|s_i-\tau_1) = A\hat{x}(s_i+p-\tau_1|s_i-\tau_1) \\
\qquad + Bu(s_i+n-\tau_1|s_i-\tau_1) \\
\hat{x}(s_i+p-\tau_1|s_i-\tau_1) = A\hat{x}(s_i+p-h-\tau_1|s_i-\tau_1) \\
\qquad + Bu(s_i+p-h-\tau_1|s_i-\tau_1)
\end{cases}
\tag{6}
$$

for $p = 2h, 3h, \cdots, \tau$, where $\tau = \tau_1 + \tau_2$, $u(f|g)$ $(f > g)$ denotes the predictive control input at time f.

On account of the predicted state from (6), the predictive controller is designed as

$$
\begin{cases}
u(s_{i+1}-h) = u(s_{i+1}-h|s_i-\tau) = K_a\hat{x}(s_{i+1}-h|s_i-\tau) \\
u(s_{i+1}-2h) = u(s_{i+1}-2h|s_i-\tau) = K_{a-1}\hat{x}(s_{i+1}-2h|s_i-\tau) \\
\qquad \vdots \\
u(s_i+h) = u(s_i+h|s_i-\tau) = K_2\hat{x}(s_i+h|s_i-\tau) \\
u(s_i) = u(s_i|s_i-\tau) = K_1\hat{x}(s_i|s_i-\tau)
\end{cases}
\tag{7}
$$

where $K_a, \cdots, K_1$ are gain matrices of controller to be designed.

Remark 1 Generally speaking, the NITDs caused by network are time-varying. The time-varying delays can be transformed into the constant delays by using the dwell time method in [43]. That is, if the NITDs less than the upper bound of the delay, the signal with NITDs will be transmitted via network after the signals are forced to dwell such that the NITDs achieve the upper bound.

Set the error $e(s_i) = x(s_i) - \hat{x}(s_i|s_i-\tau)$. It is certain from (3) and (4) that the error dynamic equations can be expressed by

$$
\begin{cases}
e(s_{i+1}) = (A^a - L_aC)e(s_i) \\
e(s_{i+1}-h) = (A^{a-1} - L_{a-1}C)e(s_i) \\
\qquad \vdots \\
e(s_i+2h) = (A^2 - L_2C)e(s_i) \\
e(s_i+h) = (A - L_1C)e(s_i).
\end{cases}
\tag{8}
$$

Before proceeding, the following definition and lemma are introduced and they are useful for the later developments.

Definition 1 [1] For the discrete-time linear system

$$x(s+1) = Ax(s) \tag{9}$$

if $x^T(0)Rx(0) \leq \delta^2$ implies $x^T(s)Rx(s) \leq \varepsilon^2$ in the interval $0 \leq s \leq N$, then the system (9) is said to be finite-time stable with respect to $(\delta, \varepsilon, R, N)$, where matrix $R > 0$, integer N, and scalars $\delta > 0$, $\varepsilon > 0$ $(\delta < \varepsilon)$ and $\gamma > 1$ are given previously.

Remark 2 It should be noted that the definitions between finite-time stability and Lyapunov stability are different. On the one hand, the finite-time dynamic behaviors of system states are discussed in the discussion of finite-time stability. However, the infinite-time behaviors of the system states are addressed in the discussion of Lyapunov stability. On the other hand, the bound of state trajectory should be given in advance when discussing the finite-time stability (this bound is generally given based on practical situation), but the bound is not pre-determined in the discussion of Lyapunov stability.

Lemma 1 *[31] For a known matrix $H \in \mathbb{R}^{n\times m}$ with $\operatorname{rank}(H) = m$ and $H = U\begin{bmatrix} \Xi \\ 0 \end{bmatrix}V^T$, where $U \in \mathbb{R}^{n\times n}$ and $V \in \mathbb{R}^{m\times m}$ are orthogonal matrices, if the matrix Q can be written $Q = U\begin{bmatrix} Q_1 & Q_2 \\ 0 & Q_3 \end{bmatrix}U^T$, then there exists a matrix $M \in \mathbb{R}^{m\times m}$, such that $QH = HM$, and $M = V\Xi^{-1}Q_1\Xi V^T$, where $\Xi = \operatorname{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$, λ_i $(i = 1, 2, \cdots, m)$ with $\lambda_i \neq 0$ are singular values of H, $Q_1 \in \mathbb{R}^{m\times m}$, $Q_2 \in \mathbb{R}^{m\times(n-m)}$ and $Q_3 \in \mathbb{R}^{(n-m)\times(n-m)}$.*

The aim of this paper is to investigate the finite-time stabilization issue for MRNCS by using the networked predictive control method. Specially, the objectives of this paper can be given as follows.

1) For the MRNCS, a novel NPCS is designed to attenuate the negative effect from NITDs.
2) The sufficient conditions are given to guarantee the finite-time stability of the closed-loop system.

3 Main Results

In this section, sufficient conditions will be provided to ensure that the MRNCS is finite-time stable with respect to $(\delta, \varepsilon, R, N)$. Subsequently, the observer gain matrix and the controller gain matrix are determined simultaneously by solving bilinear matrix inequality or linear matrix inequality. In addition, an algorithm is provided for calculating the bilinear matrix inequality.

Based on (6) and replacing $s_i + \tau_2$ with s_i, the relationship is established between information from observer and the predictions of state $\hat{x}(s_{i+1}-h|s_i-\tau), \cdots, \hat{x}(s_i|s_i-\tau)$ by

$$\begin{cases} \hat{x}(s_{i+1}-h|s_i-\tau) = A^{\frac{\tau-h}{h}}\hat{x}(s_{i+1}-\tau|s_i-\tau)+\tilde{u}_a(s_i) \\ \hat{x}(s_{i+1}-2h|s_i-\tau) = A^{\frac{\tau-h}{h}}\hat{x}(s_{i+1}-h-\tau|s_i-\tau)+\tilde{u}_{a-1}(s_i) \\ \vdots \\ \hat{x}(s_i+h|s_i-\tau) = A^{\frac{\tau-h}{h}}\hat{x}(s_i+2h-\tau|s_i-\tau)+\tilde{u}_2(s_i) \\ \hat{x}(s_i|s_i-\tau) = A^{\frac{\tau-h}{h}}\hat{x}(s_i+h-\tau|s_i-\tau)+\tilde{u}_1(s_i) \end{cases} \tag{10}$$

where $\tilde{u}_j(s_i) = \sum_{p=2h}^{\tau+(j-1)h} A^{\frac{\tau+(j-1)h-p}{h}} Bu(s_i+p-2h-\tau+jh|s_{i-1}-\tau)$ $(j = 1, 2, \cdots, a)$.

On the basis of (10), substituting the observer state (5) into (10) yields

$$\begin{cases} \hat{x}(s_{i+1}-h|s_i-\tau) = A^{\frac{\tau-h}{h}}(A^a-L_aC)\hat{x}(s_{i+1}-\tau|s_i-\tau) \\ \qquad + A^{\frac{\tau-h}{h}}L_aCx(s_i-\tau)+\bar{u}_a(s_i) \\ \hat{x}(s_{i+1}-2h|s_i-\tau) = A^{\frac{\tau-h}{h}}(A^{a-1}-L_{a-1}C)\hat{x}(s_{i+1}-\tau|s_i-\tau) \\ \qquad + A^{\frac{\tau-h}{h}}L_{a-1}Cx(s_i-\tau)+\bar{u}_{a-1}(s_i) \\ \vdots \\ \hat{x}(s_i+h|s_i-\tau) = A^{\frac{\tau-h}{h}}(A^2-L_2C)\hat{x}(s_{i+1}-\tau|s_i-\tau) \\ \qquad + A^{\frac{\tau-h}{h}}L_2Cx(s_i-\tau)+\bar{u}_2(s_i) \\ \hat{x}(s_i|s_i-\tau) = A^{\frac{\tau-h}{h}}(A-L_1C)\hat{x}(s_{i+1}-\tau|s_i-\tau) \\ \qquad + A^{\frac{\tau-h}{h}}L_1Cx(s_i-\tau)+\bar{u}_1(s_i) \end{cases} \tag{11}$$

where $\bar{u}_j(s_i) = \sum_{p=h}^{\tau+(j-1)h} A^{\frac{\tau+(j-1)h-p}{h}} Bu(s_i+p-h-\tau|s_{i-1}-\tau)$ $(j = 1, 2, \cdots, a)$.

Bearing in mind (3), one has

$$\begin{cases} x(s_{i+1}-h) = A^{\frac{\tau+(a-1)h}{h}}x(s_i-\tau)+\bar{u}_a(s_i) \\ x(s_{i+1}-2h) = A^{\frac{\tau+(a-2)h}{h}}x(s_i-\tau)+\bar{u}_{a-1}(s_i)) \\ \vdots \\ x(s_i+h) = A^{\frac{\tau+h}{h}}x(s_i-\tau)+\bar{u}_2(s_i) \\ x(t_i) = A^{\frac{\tau}{h}}x(s_i-\tau)+\bar{u}_1(s_i). \end{cases} \tag{12}$$

Combining (11) with (12), the predictions of state $\hat{x}(s_{i+1}-h|s_i-\tau), \cdots, \hat{x}(s_i|s_i-\tau)$ can be described as

$$\begin{cases} \hat{x}(s_{i+1}-h|s_i-\tau) = A^{\frac{\tau-h}{h}}(A^a - L_aC)\hat{x}(s_i-\tau|s_{i-1}-\tau) \\ \qquad + A^{\frac{\tau-h}{h}}L_aCx(s_i-\tau) + x(s_{i+1}-h) \\ \qquad - A^{\frac{\tau+(a-1)h}{h}}x(s_i-\tau) \\ \hat{x}(s_{i+1}-2h|s_i-\tau) = A^{\frac{\tau-h}{h}}(A^{a-1} - L_{a-1}C)\hat{x}(s_i-\tau|s_{i-1}-\tau) \\ \qquad + A^{\frac{\tau-h}{h}}L_{a-1}Cx(s_i-\tau) + x(s_{i+1}-2h) \\ \qquad - A^{\frac{\tau+(a-2)h}{h}}x(s_i-\tau) \\ \qquad \vdots \\ \hat{x}(s_i+h|s_i-\tau) = A^{\frac{\tau-h}{h}}(A^2 - L_2C)\hat{x}(s_i-\tau|s_{i-1}-\tau) \\ \qquad + A^{\frac{\tau-h}{h}}L_2Cx(s_i-\tau) + x(s_i+h) \\ \qquad - A^{\frac{\tau+h}{h}}x(s_i-\tau) \\ \hat{x}(s_i|s_i-\tau) = A^{\frac{\tau-h}{h}}(A - L_1C)\hat{x}(s_i-\tau|s_{i-1}-\tau) \\ \qquad + A^{\frac{\tau-h}{h}}L_1Cx(s_i-\tau) + x(s_i) \\ \qquad - A^{\frac{\tau}{h}}x(s_i-\tau). \end{cases} \tag{13}$$

Moreover, from (8) and (13), we obtain

$$\begin{cases} \hat{x}(s_{i+1}-h|s_i-\tau) = x(s_{i+1}-h) - A^{\frac{\tau-h}{h}}e(s_{i+1}-\tau) \\ \hat{x}(s_{i+1}-2h|s_i-\tau) = x(s_{i+1}-2h) - A^{\frac{\tau-h}{h}}e(s_{i+1}-\tau-h) \\ \qquad \vdots \\ \hat{x}(s_i+h|s_i-\tau) = x(s_i+h) - A^{\frac{\tau-h}{h}}e(s_i-\tau+2h) \\ \hat{x}(s_i|s_i-\tau) = x(s_i) - A^{\frac{\tau-h}{h}}e(s_i-\tau+h). \end{cases} \tag{14}$$

Based on (14), the predictive control input can be derived as follows:

$$\begin{cases} u(s_{i+1}-h) = K_ax(s_{i+1}-h) - K_aA^{\frac{\tau-h}{h}}e(s_{i+1}-\tau) \\ u(s_{i+1}-2h) = K_{a-1}x(s_{i+1}-2h) - K_{a-1}A^{\frac{\tau-h}{h}}e(s_{i+1}-\tau-h) \\ \qquad \vdots \\ u(s_i+h) = K_2x(s_i+h) - K_2A^{\frac{\tau-h}{h}}e(s_i-\tau+2h) \\ u(s_i) = K_1x(s_i) - K_1A^{\frac{\tau-h}{h}}e(s_i-\tau+h). \end{cases} \tag{15}$$

According to (1), it is not difficult to obtain that

$$\begin{cases} x(s_{i+1}) = Ax(s_{i+1}-h) + Bu(s_{i+1}-h) \\ x(s_{i+1}-h) = Ax(s_{i+1}-2h) + Bu(s_{i+1}-2h) \\ \vdots \\ x(s_i+2h) = Ax(s_i+h) + Bu(s_i+h) \\ x(s_i+h) = Ax(s_i) + Bu(s_i). \end{cases} \tag{16}$$

Subsequently, based on (15) and (16), the closed-loop system is written as follows:

$$\begin{cases} x(s_{i+1}) = (A+BK_a)x(s_{i+1}-h) - BK_aA^{\frac{\tau-h}{h}}e(s_{i+1}-\tau) \\ x(s_{i+1}-h) = (A+BK_{a-1})x(s_{i+1}-2h) \\ \qquad - BK_{a-1}A^{\frac{\tau-h}{h}}e(s_{i+1}-\tau-h) \\ \vdots \\ x(s_i+2h) = (A+BK_2)x(s_i+h) - BK_2A^{\frac{\tau-h}{h}}e(s_i-\tau+2h) \\ x(s_i+h) = (A+BK_1)x(s_i) - BK_1A^{\frac{\tau-h}{h}}e(s_i-\tau+h). \end{cases} \tag{17}$$

That is to say

$$\begin{cases} x(k_{i+a}) = (A+BK_a)x(k_{i+a-1}) - BK_aA^{\frac{\tau-h}{h}}e(k_{i+a}-\tau) \\ x(k_{i+a-1}) = (A+BK_{a-1})x(k_{i+a-2}) - BK_{a-1}A^{\frac{\tau-h}{h}}e(k_{i+a-1}-\tau) \\ \vdots \\ x(k_{i+2}) = (A+BK_2)x(k_{i+1}) - BK_2A^{\frac{\tau-h}{h}}e(k_{i+2}-\tau) \\ x(k_{i+1}) = (A+BK_1)x(k_i) - BK_1A^{\frac{\tau-h}{h}}e(k_{i+1}-\tau). \end{cases} \tag{18}$$

Letting $E(s_i) = \text{col}\,\{e(s_i), e(s_i-h), \cdots, e(s_{i-1}+h)\}$, according to (8), the augmented error dynamics system is derived as follows:

$$E(s_{i+1}) = \Sigma E(s_i) \tag{19}$$

where $\Sigma = \text{col}\{\tilde{\Sigma}^T, 0, \cdots, 0\}^T$, $\tilde{\Sigma} = \text{col}\{A^a - L_aC, A^{a-1} - L_{a-1}C, \cdots, A - L_1C\}$.

According to (18), we have

$$\eta(k_{i+a}) = \Gamma\eta(k_{i+a-1}) + \Phi E(k_{i+a}-\tau) \tag{20}$$

where

$$\eta(k_{i+a}) = \text{col}\{x(k_{i+a}), x(k_{i+a-1}), \cdots, x(k_{i+1})\}$$

$$\begin{aligned}&\Gamma = \mathrm{diag}\{A + BK_a, A + BK_{a-1}, \cdots, A + BK_1\}\\&\Phi = \mathrm{diag}\{-BK_a A^{\frac{\tau-h}{h}}, -BK_{a-1}A^{\frac{\tau-h}{h}}, \cdots, -BK_1 A^{\frac{\tau-h}{h}}\}.\end{aligned}$$

Together with (19) and (20), the following compact form can be derived by means of augmenting technique

$$\xi(s_{i+1}) = \Pi\xi(s_i) \tag{21}$$

where $\xi(s_i) = [\eta^T(k_{i+a}) \quad E^T(s_{i+1} - \tau)]^T$, $\Pi = \begin{bmatrix} \Gamma & \Phi \\ 0 & \Sigma \end{bmatrix}$.

Remark 3 The lifting technique is used to deal with the multi-rate networked control system described by (1) and (2), ultimately, the equivalent single-rate system (19) is obtained. According to (19), (20) and letting $\xi(s_i) = [\eta^T(k_{i+a}) \quad E^T(s_{i+1} - \tau)]^T$, we can get the system (21) with the help of augmenting technique. As a matter of fact, the system (21) can be considered as an autonomous system. Therefore, all the eigenvalues of matrices Γ and Σ are within the unit circle that constitute the necessary and sufficient condition of asymptotic stability for the system (21).

Remark 4 According to (21), it is very evident that $x(k_i)$ is the element of $\xi(s_i)$. In other word, the finite-time stabilization problem for the MRNCS described by (1) and (2) has been transformed into the finite-time stability issue of the system (21).

The following two theorems provide sufficient conditions that can guarantee the finite-time stability of the MRNCS described by (1) and (2), respectively.

Theorem 1 *For given positive definite matrix $R \in \mathbb{R}^{2na\times 2na}$, integer $N > 0$, scalars $\varepsilon > \delta > 0$, $\gamma > 1$, the system (21) is finite-time stable with respect to $(\delta, \varepsilon, R, N)$, if there exist positive definite matrix $P \in \mathbb{R}^{2na\times 2na}$, real matrices $K_j \in \mathbb{R}^{m\times n}$ and $L_j \in \mathbb{R}^{n\times l}$ ($j = 1, 2, \cdots, a$), such that*

$$\Theta = \begin{bmatrix} -\gamma P & * \\ P\Pi & -P \end{bmatrix} < 0 \tag{22}$$

$$cond(\tilde{P}) < \frac{\varepsilon^2}{\gamma^N \delta^2} \tag{23}$$

where $cond(\tilde{P}) = \frac{\lambda_{\max}(\tilde{P})}{\lambda_{\min}(\tilde{P})}$ and $\tilde{P} = R^{-\frac{1}{2}}PR^{-\frac{1}{2}}$.

Proof The Lyapunov function is chosen in the following form

$$V(s_i) = \xi^T(s_i)P\xi(s_i). \tag{24}$$

Letting $\Delta V(s_i) = V(s_{i+1}) - \gamma V(s_i)$, along the trajectory of system (21), we can obtain

$$\Delta V(s_i) = \xi^T(s_i)(\Pi^T P\Pi - \gamma P)\xi(s_i). \tag{25}$$

If $\Pi^T P\Pi - \gamma P < 0$, we have $\Delta V(s_i) < 0$. Using the Schur complement lemma [53], $\Pi^T P\Pi - \gamma P < 0$ is equivalent to $\Theta < 0$. Assume that $\xi^T(0)R\xi(0) \leq \delta^2$. Further, we can obtain

$$V(s_{i+1}) < \gamma V(s_i) < \gamma^{s_i} V(0) < \gamma^N \xi^T(0)P\xi(0) \leq \gamma^N \lambda_{\max}(\tilde{P})\delta^2. \qquad (26)$$

On the other hand,

$$V(s_i) = \xi^T(s_i)P\xi(s_i) \geq \lambda_{\min}(\tilde{P})\xi^T(s_i)R\xi(s_i). \qquad (27)$$

According to (26) and (27), if $cond(\tilde{P}) < \frac{\varepsilon^2}{\gamma^N \delta^2}$, we have $\xi^T(s_i)R\xi(s_i) < \varepsilon^2$. Based on the Definition 1, the system (21) is finite-time stable with respect to $(\delta, \varepsilon, R, N)$. The proof is now complete. □

Note that the formula (22) is a bilinear matrix inequality, which includes the bilinear item $P\Pi$. As far as we know, the parameters $K_a, \cdots, K_1, L_a, \cdots, L_1$ cannot be acquired directly from the formula (22) by using Matlab LMI Toolbox. For the sake of convenience, letting $K = \mathrm{diag}\{K_a, \cdots, K_1\}$ and $L = \mathrm{col}\{L_a, \cdots, L_1\}$, the following iterative algorithm is needed to obtain the gain matrices via Theorem 1 for the observer and the controller.

Iterative algorithm:

Step 1. Set $i = 0$. Choose the initial value of the matrix P and let $P = P^0$.
Step 2. Set $i = i + 1$. In virtue of P^{i-1}, calculate the gain matrices K and L by solving optimization problem: min t_1, subject to $\Theta - t_1 I < 0$. Let $K^i = K$, $L^i = L$.
Step 3. If $t_1 < 0$, exit algorithm. Otherwise, go to next step.
Step 4. In virtue of K^i and L^i, calculate the positive definite matrix P by solving optimization problem: min t_2, subject to $\Theta - t_2 I < 0$. Let $P^i = P$.
Step 5. If $t_2 < 0$, exit algorithm, Otherwise, return to Step 2.

Remark 5 The iterative algorithm proposed in this paper has provided a strategy to solve bilinear matrix inequality. By adding unknown parameters, the trouble of solving bilinear matrix inequality is transformed into an optimization issue based on linear matrix inequality. The iteration number of the algorithm is affected by the initial value selected. It should be noted that the feasible solution is gained by this way rather than the optimal solution of bilinear matrix inequality.

Theorem 2 *For given positive definite matrix $R \in \mathbb{R}^{2na \times 2na}$, integer $N > 0$, scalars $\varepsilon > \delta > 0$, $\gamma > 1$ and $\mathrm{rank}(B) = m$ $(m < n)$, the system (21) is finite-time stable with respect to $(\delta, \varepsilon, R, N)$, if there exist positive definite matrix $P_j \in \mathbb{R}^{n \times n}$, $Q \in \mathbb{R}^{na \times na}$, real matrices $W_j \in \mathbb{R}^{m \times n}$ and $Y \in \mathbb{R}^{na \times al}$, such that the following linear matrix inequality*

$$\Phi = \begin{bmatrix} -\gamma P & * \\ \Phi_1 & -P \end{bmatrix} < 0 \qquad (28)$$

and

$$cond(\tilde{P}) < \frac{\varepsilon^2}{\gamma^N \delta^2} \tag{29}$$

hold. Furthermore, the corresponding gain matrices of controller and observer are presented by $K_j = V_j^T \varXi P_{j1}^{-1} \varXi^{-1} V_j W_j$, $L_j = E_j L$ $(j = 1, 2, ..., a)$, *respectively, where*

$$P = \mathrm{diag}\{P_a, \cdots, P_1, Q\},\ \Phi_1 = P\varPi_1 + \varPsi_1 + \varPsi_2 + \varPsi_3,\ \varPsi_1 = \mathrm{diag}\{\bar{B}W, 0\}$$
$$\varPsi_2 = \begin{bmatrix} 0 & -\bar{B}W\bar{A} \\ 0 & 0 \end{bmatrix},\ \varPsi_3 = \mathrm{diag}\{0, Y\tilde{C}\},\ \varPi_1 = \mathrm{diag}\{A_1, A_2\},\ \tilde{C} = \{C, \underbrace{0, \cdots, 0}_{a-1}\}$$
$$W = \mathrm{diag}\{W_1, \cdots, W_a\},\ A_1 = \mathrm{diag}\{\underbrace{A, \cdots, A}_{a}\},\ \bar{A} = \mathrm{diag}\{\underbrace{A^{\frac{\tau-h}{h}}, \cdots, A^{\frac{\tau-h}{h}}}_{a}\}$$
$$A_2 = [\tilde{A}_2\, \underbrace{0, \cdots, 0}_{a-1}],\ \tilde{A}_2 = \mathrm{col}\{A^a, A^{a-1}, \cdots, A\},\ \bar{B} = \mathrm{diag}\{\underbrace{B, \cdots, B}_{a}\}$$
$$P_j = U_j \begin{bmatrix} P_{j1} & P_{j2} \\ 0 & P_{j3} \end{bmatrix} U_j^T,\ \tilde{P} = R^{-\frac{1}{2}} P R^{-\frac{1}{2}},\ E_j = [0, \cdots, 0, I_{l\times n}^j, 0, \cdots, 0]$$

and U_j, $\varXi$, V_j *are defined by* $B = U_j \begin{bmatrix} \varXi \\ 0 \end{bmatrix} V_j^T$.

Proof Assume that $\xi^T(0)R\xi(0) \leq \delta^2$. If $\begin{bmatrix} -\gamma P & * \\ P\varPi & -P \end{bmatrix} < 0$ and $cond(\tilde{P}) < \frac{\varepsilon^2}{\gamma^N\delta^2}$, according to Theorem 1, the system (21) is finite-time stable with respect to $(\delta, \varepsilon, R, N)$. Subsequently, the condition $\begin{bmatrix} -\gamma P & * \\ P\varPi & -P \end{bmatrix} < 0$ will be processed.

For simplicity, the following notations are introduced

$$\begin{aligned} &\varPi = \varPi_1 + \varPi_2 + \varPi_3 + \varPi_4,\ \bar{P} = \mathrm{diag}\{P_a, \cdots, P_1\},\ \varPi_2 = \tilde{B}\tilde{K}_1 \\ &\varPi_3 = -\tilde{B}\tilde{K}_2\tilde{A},\ \varPi_4 = \mathrm{diag}\{0, -L\tilde{C}\},\ \tilde{B} = \mathrm{col}\{\bar{B}, 0\} \\ &\tilde{K}_1 = [K\ 0],\ \tilde{K}_2 = [0\ K],\ \tilde{A} = \mathrm{diag}\{0, \bar{A}\}. \end{aligned}$$

It is not intricate to derive that

$$\begin{aligned} P\varPi_2 &= P\tilde{B}\tilde{K}_1 = \mathrm{diag}\{\bar{P}\bar{B}K, 0\} \\ P\varPi_3 &= -P\tilde{B}\tilde{K}_2\tilde{A} = \begin{bmatrix} 0 & -\bar{P}\bar{B}K\bar{A} \\ 0 & 0 \end{bmatrix} \\ P\varPi_4 &= \mathrm{diag}\{0, QL\tilde{C}\}. \end{aligned}$$

Since $\mathrm{rank}(B) = m$, it is obvious that $\mathrm{rank}(\bar{B}) = am$. For all $j = 1, \cdots a$, from $P_j = U_j \begin{bmatrix} P_{j1} & P_{j2} \\ 0 & P_{j3} \end{bmatrix} U_j^T$, $B = U_j \begin{bmatrix} \varXi \\ 0 \end{bmatrix} V_j^T$ and Lemma 1, there exists Z_j, such that

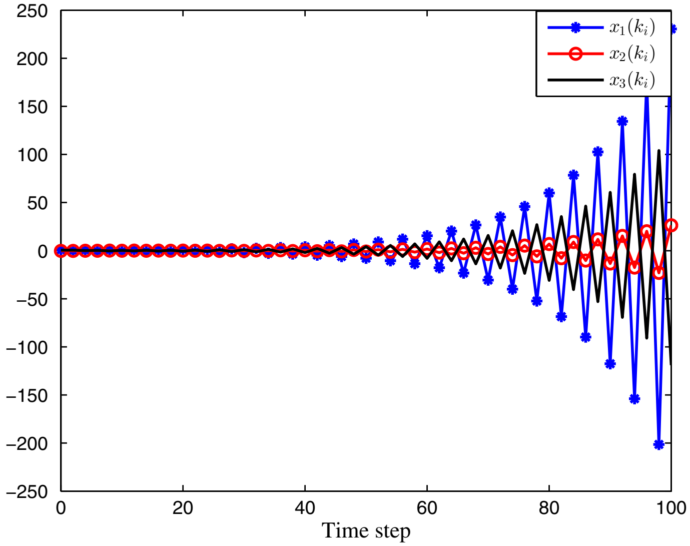

Fig. 3 The state trajectories of the open-loop system

$P_j B = B Z_j$. Subsequently, setting $Z = \mathrm{diag}\{Z_a, \cdots, Z_1\}$, we have $\bar{P}\bar{B} = \bar{B}Z$. Letting $W_j = Z_j K_j$ and $Y = QL$, the linear matrix inequality (28) can be obtained. From Lemma 1, one can obtain $Z_j = V_j \Xi^{-1} P_{j1} \Xi V_j^T$ and $L = Q^{-1}Y$. Therefore, $K_j = V_j^T \Xi P_{j1}^{-1} \Xi^{-1} V_j W_j$, $L_j = E_j L$. The proof is now complete. □

Remark 6 Up till now, Theorems 1 and 2 have provided sufficient conditions to ensure the finite-time stability of the system (21), respectively. In order to obtain the gain matrices from Theorem 1 for the observer and the controller, we need to employ the proposed iterative algorithm to solve the bilinear matrix inequality (22). It is worthwhile noting that the initial value of matrix P must be determined in advance and it will affect the solution of (22). Compared with the Theorem 1, due to the form of the positive definite matrix P and the rank of B are constrained, the sufficient condition given by Theorem 2 is more conservative. But the sufficient condition of Theorem 2 is made up of the linear matrix inequality (28), which is simpler to solve than formula (22). In addition, the design methods of observer and controller parameters have been given in Theorem 2.

4 An Illustrative Example

In this part, an example is utilized to illustrate the validity of the results proposed in this paper. For the MRNCS described by (1) and (2), the related parameters are chosen by

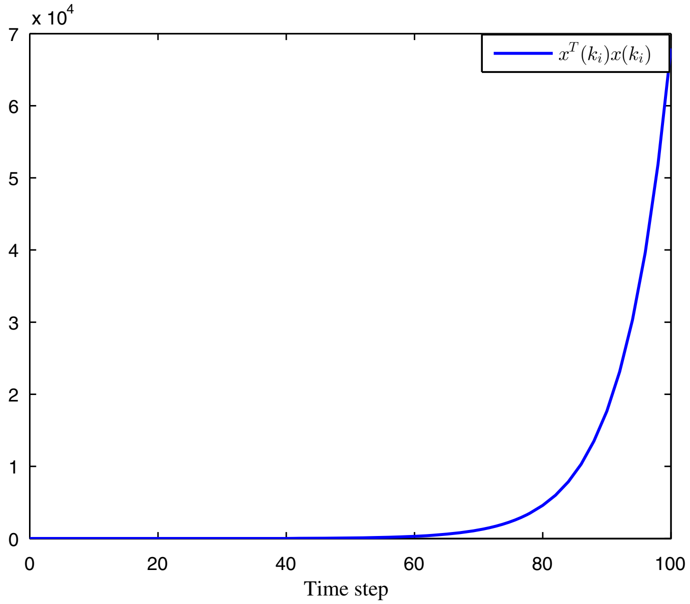

Fig. 4 The energy curve of the open-loop system

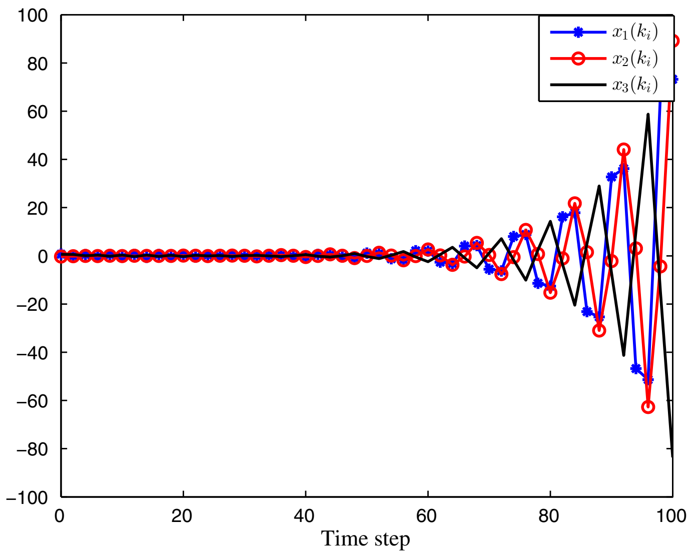

Fig. 5 The state trajectories of the closed-loop system in case 1

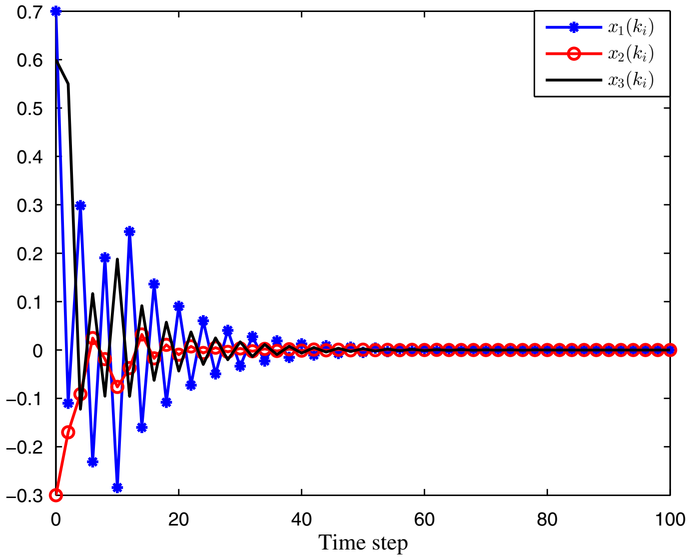

Fig. 6 The state trajectories of the closed-loop system determined by Theorem 1

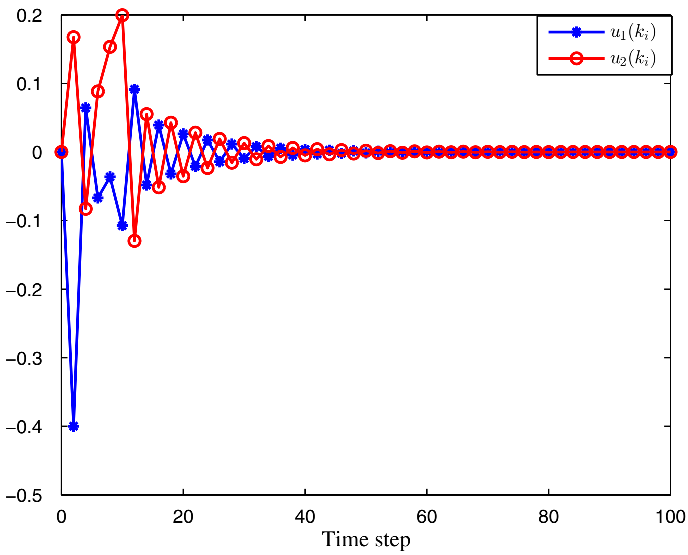

Fig. 7 The trajectories of the predictive control input determined by Theorem 1

$$A = \begin{bmatrix} -0.8 & -0.3 & 0.6 \\ -0.2 & -0.3 & -0.2 \\ 0.7 & 0.4 & 0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 0.23 & -0.47 \\ -0.04 & -0.42 \\ 0.67 & -0.75 \end{bmatrix}, \quad C = \begin{bmatrix} -1.84 & -0.55 & 0.87 \end{bmatrix}.$$

Assuming $a = 2$ and the sampling period of the plant is 2, i.e., $h = 2$, The NITDs are $\tau_1 = \tau_2 = 4$. The initial values of the MRNCS are selected as $x(0) = \begin{bmatrix} 0.7 & -0.3 & 0.6 \end{bmatrix}^T$,

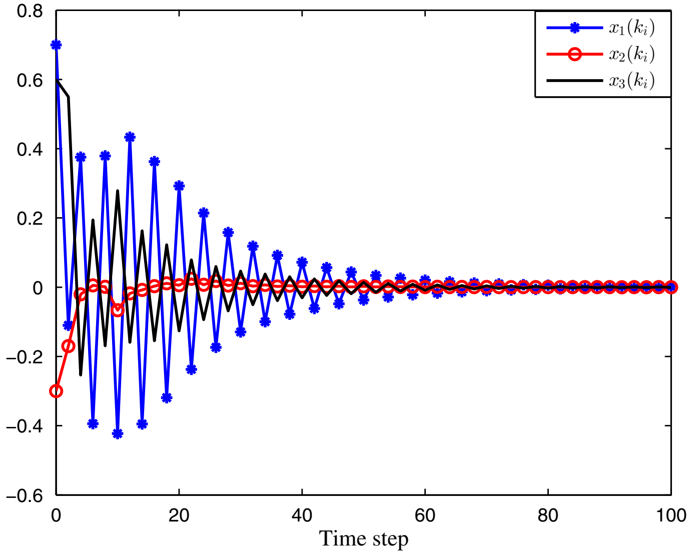

Fig. 8 The state trajectories of the closed-loop system determined by Theorem 2

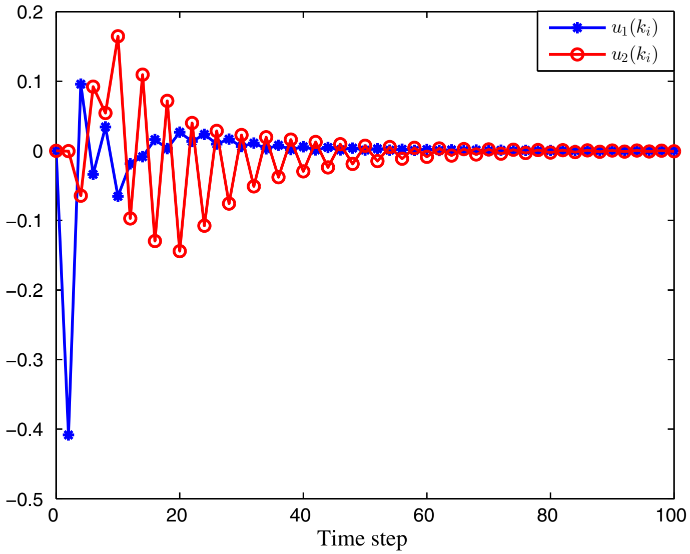

Fig. 9 The trajectories of the predictive control input determined by Theorem 2

$u(0) = 0$ and $\hat{x}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$. The other parameters are selected as $R = I$, $\delta = 1$, $N = 100$, $\gamma = 1.01$, $\varepsilon = 2.32$.

Fig. 3 depicts the state trajectories of the open-loop system, where x_l $(l = 1, 2, 3)$ is the l-th element of the state. It is very obvious that the state trajectories are divergent for the open-loop system. We are convinced that the energy curve $x^T(k_i)x(k_i)$ is unbounded from Fig. 4.

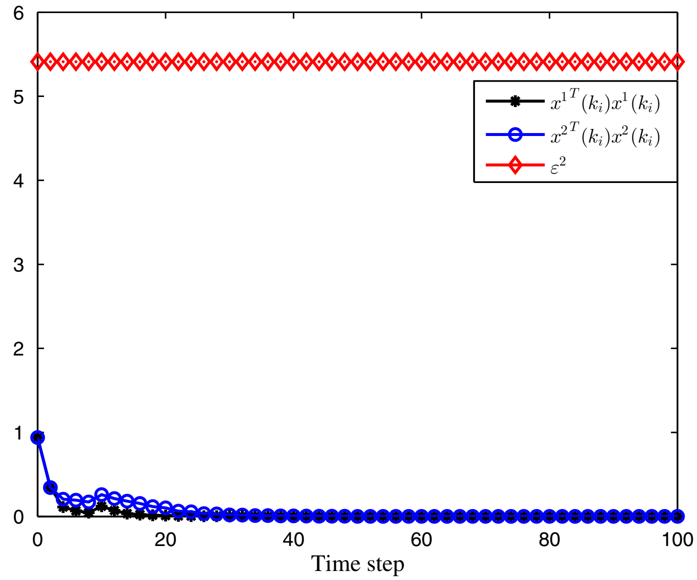

Fig. 10 The energy curves of the closed-loop system in different cases

Two cases are considered for the purpose of comparison. Case 1: there exist NITDs in the feedback channel and forward channel, and the controller without handling the NITDs is utilized as follows:

$$\begin{cases} u(s_{i+1} - h) = u(s_{i+1} - h - \tau) \\ \vdots \\ u(s_i + h) = u(s_i + h - \tau) \\ u(s_i) = u(s_i - \tau). \end{cases}$$

Case 2: considering the NITDs in the MRNCS, the following predictive controller is chosen by employing the networked predictive control method

$$\begin{cases} u(s_{i+1} - h) = u(s_{i+1} - h | s_i - \tau) \\ \vdots \\ u(s_i + h) = u(s_i + h | s_i - \tau) \\ u(s_i) = u(s_i | s_i - \tau). \end{cases}$$

Selecting the initial value of positive definite matrix $P = I$, by solving the bilinear matrix inequality (22), the following gain matrices are obtained

$$L_1^1 = \begin{bmatrix} -0.6164 \\ -0.0654 \\ 0.2844 \end{bmatrix}, \quad K_1^1 = \begin{bmatrix} -0.1136 & -0.0060 & -0.7981 \\ -0.3919 & -0.4829 & 0.0826 \end{bmatrix}$$

$$L_2^1 = \begin{bmatrix} 0.4803 \\ 0.1163 \\ -0.3286 \end{bmatrix}, \quad K_2^1 = \begin{bmatrix} -0.0918 & -0.0208 & -0.7522 \\ -0.3946 & -0.4810 & 0.0769 \end{bmatrix}.$$

By solving the linear matrix inequality (28), the following gain matrices are acquired

$$L_1^2 = \begin{bmatrix} 0.5811 \\ 0.0653 \\ -0.2881 \end{bmatrix}, \quad K_1^2 = \begin{bmatrix} -0.3489 & -0.2083 & -0.8756 \\ -0.3518 & -0.5000 & -0.2256 \end{bmatrix}$$

$$L_2^2 = \begin{bmatrix} -0.4857 \\ -0.0808 \\ 0.2805 \end{bmatrix}, \quad K_2^2 = \begin{bmatrix} -0.3489 & -0.2083 & -0.8756 \\ -0.3518 & -0.5000 & -0.2256 \end{bmatrix}.$$

For case 1, the state trajectories are shown in Fig. 5. Apparently, the closed-loop MRNCS is unstable. This result is principally because that the effect of NITDs are neglected. For case 2, the simulation results are described in Figs. 6, 7, 8, 9 and 10. Among them, Fig. 6 and Fig. 7 describe the trajectories of state and predictive control input determined by Theorem 1, respectively, where u_l $(l = 1, 2)$ is the l-th element of the predictive control input. As such, by applying the gain matrices of observer and controller from Theorem 2, the trajectories of state and predictive control input are shown in Fig. 8 and Fig. 9, respectively. From Figs. 6 and 8, the closed-loop system is finite-time stable under the action of the predictive controller. Besides, the convergence rate of state trajectories in Fig. 6 is faster than in Fig. 8, this also means that the condition obtained from Theorem 2 is more conservative than Theorem 1. Figure 10 displays that the energy curve $x^T(k_i)x(k_i)$ is bounded, where the energy curves $x^{1T}(k_i)x^1(k_i)$ and $x^{2T}(k_i)x^2(k_i)$ are calculated from Theorems 1 and 2, respectively. In summary, this example shows that the NPCS proposed in this paper compensates the negative effect of NITDs actively.

5 Conclusions

The finite-time stabilization problem has been investigated for discrete-time MRNCS with NITDs in this paper. A new NPCS has been designed based on MRSM to handle the effects of NITDs. By applying the lifting technique and augmenting method, the MRNCS has been changed into a single-rate system. With the help of the Lyapunov theorem, two sufficient criteria have been derived to guarantee finite-time stability for the given MRNCS. Moreover, the observer gain matrices and the controller gain matrices have been designed based on the solution to some matrix inequalities. Finally, the effectiveness and feasibility has been shown for the results proposed in this paper by a numerical example. It should be pointed out that the depth of the results needs to be further expanded. Further research directions include the extension of the current results to MRNCS with communication constraints [15], and to the nonlinear multi-agent systems [3], the nonlinear complex networks [8] and the nonlinear systems over sensor networks [36]. Besides, it is also interesting to consider the influences of model uncertainty and external disturbances on the design of NPCS.

Declarations

Conflicts of interest The authors claim that there are no potential conflicts of interest. In addition, this submission has been approved by all co-authors.

Data Availability Statement All data generated or analyzed during this study are included in this published article.

References

1. F. Amato, M. Ariola, Finite-time control of discrete-time linear systems. IEEE Trans. Autom. Control **50**(5), 724–729 (2005)
2. I.S.M. De, On some sufficiency-type stability and linear state-feedback stabilization conditions for a class of multirate discrete-time systems. Mathematics **6**(5), 1–18 (2018)
3. L. Dong, D. Yu, H. Hao, Stability analysis of nonlinear multi-agent relay tracking systems over a finite time interval. Int. J. Control **93**(3), 519–527 (2020)
4. M.C.F. Donkers, W.P.M.H. Heemels, D.W.N. Van, L. Hetel, Stability analysis of networked control systems using a switched linear systems approach. IEEE Trans. Autom. Control **56**(9), 2101–2115 (2011)
5. D. Freirich, E. Fridman, Decentralized networked control of discrete-time systems with local networks. Int. J. Robust Nonlinear Control **28**(1), 365–380 (2018)
6. H. Geng, Y. Liang, Y. Liu, F.E. Alsaadi, Bias estimation for asynchronous multi-rate multi-sensor fusion with unknown inputs. Inf. Fusion **39**, 139–153 (2018)
7. J. Hu, Z. Wang, G. Liu, Delay compensation-based state estimation for time-varying complex networks with incomplete observations and dynamical bias. IEEE Trans. Cybern. (2020). https://doi.org/10.1109/TCYB.2020.3043283
8. J. Hu, Z. Wang, G. Liu, C. Jia, J. Williams, Event-triggered recursive state estimation for dynamical networks under randomly switching topologies and multiple missing measurements. Automatica (2020). https://doi.org/10.1016/j.automatica.2020.108908
9. M. Hu, J. Cao, A. Hu, Y. Yang, Y. Jin, A novel finite-time stability criterion for linear discrete-time stochastic system with applications to consensus of multi-agent system. Circuits Syst. Signal Process. **34**(1), 41–59 (2015)
10. J. Jia, W. Chen, H. Dai, J. Li, Global stabilization of high-order nonlinear systems under multi-rate sampled-data control. Nonlinear Dyn. **94**(4), 2241–2453 (2018)
11. G. Liu, Consensus and stability analysis of networked multiagent predictive control systems. IEEE Trans. Cybern. **47**(4), 1114–1119 (2017)
12. G. Liu, Predictive control of networked multiagent systems via cloud computing. IEEE TransD. Cybern. **47**(8), 1852–1859 (2017)
13. G. Liu, Predictive control of networked nonlinear multiagent systems with communication constraints. IEEE Trans. Syst. Man Cybern. Syst. **50**(11), 4447–4457 (2020)
14. H. Li, L. Bai, Q. Zhou, R. Lu, L. Wang, Adaptive fuzzy control of stochastic nonstrict-feedback nonlinear systems with input saturation. IEEE Trans. Syst. Man Cybern. Syst. **47**(8), 2185–2197 (2017)

15. H. Li, L. Bai, Q. Zhou, R. Lu, L. Wang, Adaptive neural control of uncertain nonstrict-feedback stochastic nonlinear systems with output constraint and unknown dead zone. IEEE Trans. Syst. Man Cybern. Syst. **47**(8), 2048–2059 (2017)
16. H. Li, L. Wang, H. Du, A. Boulkroune, Adaptive fuzzy backstepping tracking control for strict-feedback systems with input delay. IEEE Trans. Fuzzy Syst. **25**(3), 642–652 (2017)
17. Y. Li, G. Liu, S. Sun, C. Tan, Prediction-based approach to finite-time stabilization of networked control systems with time delays and data packet dropouts. Neurocomputing **329**, 320–328 (2019)
18. H. Li, S. Zhao, W. He, R. Lu, Adaptive finite-time tracking control of full state constrained nonlinear systems with dead-zone. Automatica **100**, 99–107 (2019)
19. T. Li, X. Yao, L. Wu, J. Li, Improved delay-dependent stability results of recurrent neural networks. Appl. Math. Comput. **218**(19), 9983–9991 (2012)
20. T. Li, Y. Zhang, Fault detection and diagnosis for stochastic systems via output PDFs. J. Frankl. Inst.-Eng. Appl. Math. **348**(6), 1140–1152 (2011)
21. T. Li, W. Zheng, C. Lin, Delay-slope-dependent stability results of recurrent neural networks. IEEE Trans. Neural Netw. **22**(12), 2138–2143 (2011)
22. D. Liu, Z. Wang, Y. Liu, F.E. Alsaadi, Extended Kalman filtering subject to random transmission delays: dealing with packet disorders. Inf. Fusion **60**, 80–86 (2020)
23. H. Liu, L. Ma, Z. Wang, Y. Liu, F.E. Alsaadi, An overview of stability analysis and state estimation for memristive neural networks. Neurocomputing **39**, 1–12 (2020)
24. S. Liu, Z. Wang, G. Wei, M. Li, Distributed set-membership filtering for multirate systems under the round-robin scheduling over sensor networks. IEEE Trans. Cybern. **50**(5), 1910–1920 (2020)
25. S. Liu, Z. Wang, L. Wang, G. Wei, On quantized H_∞ filtering for multi-rate systems under stochastic communication protocols: the finite-horizon case. Inf. Sci. **459**, 211–223 (2018)
26. D.G. Meyer, A new class of shift-varying operators, their shift-invariant equivalents, and multirate digital systems. IEEE Trans. Autom. Control **35**(4), 429–433 (1990)
27. M.M.S. Pasand, M. Montazeri, Controllability and stabilizability of multi-rate sampled data systems. Syst. Control Lett. **113**, 27–30 (2018)
28. J. Puangmalai, J. Tongkum, T. Rojsiraphisal, Finite-time stability criteria of linear system with non-differentiable time-varying delay via new integral inequality. Math. Comput. Simul. **171**, 170–186 (2020)
29. N. Rong, Z. Wang, Finite-time stabilization of nonlinear systems using an event-triggered controller with exponential gains. Nonlinear Dyn. **98**(1), 15–26 (2019)
30. H. Sun, J. Li, J. Zhao, Stabilization for a class of discrete-time switched-systems. Circuits Syst. Signal Process. **36**(2), 834–844 (2017)
31. J. Sun, J. Chen, Networked predictive control for systems with unknown or partially known delay. IET Contr. Theory Appl. **8**(18), 2282–2288 (2014)
32. Y. Shen, Z. Wang, B. Shen, F.E. Alsaadi, H_∞ state estimation for multi-rate artificial neural networks with integral measurements: a switched system approach. Inf. Sci. **539**(4), 434–446 (2020)
33. Y. Shen, Z. Wang, B. Shen, F.E. Alsaadi, Nonfragile H_∞ filtering for discrete multirate time-delayed systems over sensor networks characterized by Gilbert-Elliott models. Int. J. Robust Nonlinear Control **30**(8), 3194–3214 (2020)
34. Y. Shen, Z. Wang, B. Shen, F.E. Alsaadi, F.E. Alsaadi, Fusion estimation for multi-rate linear repetitive processes under weighted try-once-discard protocol. Inf. Fusion **55**, 281–291 (2020)
35. Y. Shen, Z. Wang, B. Shen, F.E. Alsaadi, F.E. Alsaadi, Fusion estimation for multi-rate linear repetitive processes under weighted try-once-discard protocol. Inf. Fusion **55**, 281–291 (2020)
36. B. Shen, Z. Wang, D. Wang, H. Liu, Distributed state-saturated recursive filtering over sensor networks under round-robin protocol. IEEE Trans. Cybern. **50**(8), 3605–3615 (2020)
37. Y. Shi, B. Yu, Output feedback stabilization of networked control systems with random delays modeled by Markov chains. IEEE Trans. Autom. Control **54**(7), 1668–1674 (2009)
38. C. Tan, X. Yin, G. Liu, J. Huang, Y. Zhao, Prediction-based approach to output consensus of heterogeneous multi-agent systems with delays. IET Contr. Theory Appl. **12**(1), 20–28 (2018)
39. H. Tan, B. Shen, Y. Liu, A. Alsaedi, B. Ahmad, Event-triggered multi-rate fusion estimation for uncertain system with stochastic nonlinearities and colored measurement noises. Inf. Fusion **36**, 310–320 (2017)
40. D. Wang, Z. Wang, B. Shen, F.E. Alsaadi, Security-guaranteed filtering for discrete-time stochastic delayed systems with randomly occurring sensor saturations and deception attacks. Int. J. Robust Nonlinear Control **27**(7), 1194–1208 (2017)

41. L. Wang, Z. Wang, T. Huang, G. Wei, An event-triggered approach to state estimation for a class of complex networks with mixed time delays and nonlinearities. IEEE Trans. Cybern. **46**(11), 2497–2508 (2016)
42. L. Wang, Z. Wang, G. Wei, F.E. Alsaadi, Variance-constrained H_∞ state estimation for time-varying multi-rate systems with redundant channels: the finite-horizon case. Inf. Sci. **501**, 222–235 (2019)
43. R. Wang, B. Wang, G. Liu, W. Wang, D. Rees, H_∞ controller design for networked predictive control systems based on the average dwell-time approach. IEEE Trans. Circuits Syst. II-Express Briefs **57**(4), 310–314 (2010)
44. T. Wang, T. Li, B. Zhang, S. Fei, New Lyapunov-Krasovskii functional for mixed-delay-dependent stability of uncertain linear neutral systems. Circuits Syst. Signal Process. **37**(5), 1825–1845 (2018)
45. Y. Wei, G. Liu, Consensus tracking of heterogeneous discrete-time networked multiagent systems based on the networked predictive control scheme. IEEE Trans. Cybern. **47**(8), 2173–2184 (2017)
46. K. Wu, M. Na, L. Wang, X. Ding, B. Wu, Finite-time stability of impulsive reaction-diffusion systems with and without time delay. Appl. Math. Comput. **363**, 1–17 (2019)
47. Y. Xu, C. Chu, W. Li, Quantized feedback control scheme on coupled systems with time delay and distributed delay: a finite-time inner synchronization analysis. Appl. Math. Comput. **337**, 315–328 (2018)
48. H. Yang, X. Guo, L. Dai, Y. Xia, Event-triggered predictive control for networked control systems with network-induced delays and packet dropouts. Int. J. Robust Nonlinear Control **28**(4), 1350–1365 (2018)
49. H. Yang, S. Ju, Y. Xia, J. Zhang, Predictive cloud control for networked multiagent systems with quantized signals under DoS attacks. IEEE Trans. Syst. Man Cybern. -Syst. **51**(2), 1345–1353 (2021)
50. N. Yang, D. Chen, D. Ji, Z. Wu, Resilient state estimation for nonlinear complex networks with time-delay under stochastic communication protocol. Neurocomputing **346**, 38–47 (2019)
51. T. Zhang, F. Deng, W. Zhang, Finite-time stability and stabilization of linear discrete time-varying stochastic systems. J. Frankl. Inst. -Eng. Appl. Math. **356**(3), 1247–1267 (2019)
52. Y. Zhang, Z. Wang, L. Zou, H. Fang, Event-based finite-time filtering for multirate systems with fading measurements. IEEE Trans. Aerosp. Electron. Syst. **53**(3), 1431–1441 (2017)
53. Z. Zhao, Z. Wang, L. Zou, H. Liu, Finite-horizon H_∞ state estimation for artificial neural networks with component-based distributed delays and stochastic protocol. Neurocomputing **21**, 169–177 (2018)
54. Q. Zhu, K. Lu, Y. Zhu, G. Xie, Modeling and state feedback control of multi-rate networked control systems with both short time delay and packet dropout. Int. J. Innov. Comp. Inf. Control **12**(3), 779–793 (2016)