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SHORT PAPER

Distributed Dimensionality Reduction Fusion Kalman Filtering With Quantized Innovations

Xinhao Yan[1] · **Bo Chen**[2] · **Xiang Qiu**[1]

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Abstract

This paper is concerned with the distributed fusion Kalman filtering problem for networked systems with communication constraints. A dimensionality reduction strategy and a uniform quantization strategy are introduced to reduce communication traffic. To overcome the unboundedness of estimates/measurements in unstable systems, it is proposed to quantize the innovations that are sent to the fusion center through limited bandwidth channels. Then, a recursively distributed dimensionality reduction fusion Kalman filtering algorithm is developed by using a model uncertainty method to process quantization noises. Finally, a target tracking system is employed to demonstrate the effectiveness of the proposed methods.

Keywords Fusion Kalman filtering · Limited bandwidth · Dimensionality reduction · Quantized innovations · Networked multi-sensor fusion systems

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✉ Xiang Qiu
qiuxiang@zjut.edu.cn

Xinhao Yan
yanxinhao1998@aliyun.com

Bo Chen
bchen@aliyun.com

1 Department of Automation, Zhejiang University of Technology, Hangzhou 310023, People's Republic of China

2 Institute of Cyberspace Security, Zhejiang University of Technology, Hangzhou 310023, People's Republic of China

Birkhäuser

1 Introduction

Multi-sensor fusion estimation is how to make the best use of the information contained in local sensors to obtain an optimal estimation of an event or a parameter [17]. Some local and fusion estimation methods have been studied (see [1,9,11,13–15,18,21,23, 24], and the references therein). Due to the rapid development of communication technology, communication network is widely used to connect individual sensors, because it can effectively improve the scalability and reduce wiring complexity. In this case, some networked multi-sensor fusion systems (NMFSs) are put forward for solving fusion problems, and they have found applications in a wide range, such as cyber-physical systems [5], Internet of Things [3] and SINR estimation [25]. Then, a great number of fusion estimation algorithms have been developed based on NMFSs (see [4,6,28,29], and the references therein). However, along with the introduction of networks, some inevitable problems which will directly degrade the fusion estimation performance arise [2,7,22], such as bandwidth constraints, packet losses and time delays. Under this context, this paper will focus on the distributed fusion estimation for a class of NMFSs with limited communication capacity.

In general, there are two strategies to solve the problem of limited bandwidth: quantization [12,16] and dimensionality reduction [10,19]. The core idea of the dimensionality reduction is to convert a high-dimensional signal into a low-dimensional signal, while the idea of the quantization is to reduce the number of coding bits, but the dimension of the quantized signal is the same as that of the original signal. In this regard, to more greatly reduce communication traffic, the dual data compression strategy (DDCS) which includes both quantization and dimensionality reduction was proposed [8]. However, most of the above methods are based on stable systems, where the state estimates can be effectively quantized, while that of unstable systems cannot. Therefore, instead of directly quantizing the local estimates/measurements, the innovation quantization strategy was utilized to design the distributed fusion estimator in [26], but the dimensionality reduction scheme which can largely reduce the traffic was not considered.

Motivated by the aforementioned analysis, we shall study the distributed dimensionality reduction fusion estimation problem under quantized innovations, where the addressed dynamical systems are unstable. The main contributions of this paper can be summarized as follows: (1) The DDCS based on innovation is adopted to reduce the communication traffic for unstable NMFSs. (2) An uncertain approach is given to process the uncertainty of quantization, and a robust distributed dimensionality reduction fusion Kalman filter is derived under quantized innovations.

Notations The superscript "T" stands for the transpose, and "I" stands for the identity matrix. The notation diag$\{\cdot\}$ represents a block diagonal matrix, and col$\{a_1, \ldots, a_n\}$ represents a column vector whose elements are $a_1, \ldots, a_n$. E$\{\cdot\}$ denotes the mathematical expectation, while Tr$\{\cdot\}$ represents the trace of matrix. The notation rank$\{\cdot\}$ denotes the rank of matrix. $X > (<)0$ means a positive-definite (negative-definite) matrix, while $X \geq (\leq)0$ means a non-negative definite (non-positive definite) matrix.

2 Problem Statement

Consider the following discrete-time state-space model:

$$x(t+1) = Ax(t) + Bw(t) \tag{1}$$

$$y_i(t) = C_i x(t) + v_i(t)(i = 1, 2, \ldots, L). \tag{2}$$

$x(t) \in \mathbb{R}^n$ is the system state, and $y_i(t) \in \mathbb{R}^{m_i}$ is the measurement of the ith sensor at time t. The addressed system is not necessarily stable, i.e., $\lim_{t\to\infty} x(t) \to \infty$ and $\lim_{t\to\infty} y_i(t) \to \infty$. Moreover, let L denote the number of sensors. A, B and C_i are constant matrices with appropriate dimensions. $w(t)$ and $v_i(t)$ are uncorrelated white Gaussian noises satisfying

$$\mathrm{E}\{[w^{\mathrm{T}}(t)\ v_i^{\mathrm{T}}(t)]^{\mathrm{T}}[w^{\mathrm{T}}(t_1)\ v_j^{\mathrm{T}}(t_1)]\} = \delta_{tt_1}\mathrm{diag}\{Q, \delta_{ij}R_i\} \tag{3}$$

where $\delta_{ij} = 0$ and $\delta_{tt_1} = 0$ if $i \neq j$ and $t \neq t_1$; otherwise, $\delta_{ij} = 1$ and $\delta_{tt_1} = 1$. Assume that (A, B) is controllable and (A, C_i) is observable, that is

$$\begin{cases} \mathrm{rank}\{[B\ AB\ \cdots\ A^{n-1}B]\} = n \\ \mathrm{rank}\{\mathrm{col}\{C_i, C_iA, \ldots, C_iA^{n-1}\}\} = n \end{cases}. \tag{4}$$

In the sequel, the local estimate $\hat{x}_i(t)$ at the ith sensor is given by the Kalman structure:

$$\hat{x}_i(t+1) = A\hat{x}_i(t) + K_i(t+1)\varepsilon_i(t+1) \tag{5}$$

where $\varepsilon_i(t)$ represents the innovation sequence, and it can be described by the following form:

$$\varepsilon_i(t) = y_i(t) - C_iA\hat{x}_i(t-1). \tag{6}$$

2.1 DDCS Based on Innovation

Before each sensor message is transmitted to the FC via constrained communication channels, the following DDCS is proposed to satisfy the finite bandwidth.

According to the idea of dimensionality reduction in [10], it is similarly proposed in this paper that $r_i(1 \le r_i < m_i)$ components of the ith innovation $\varepsilon_i(t)$ are selected to be transmitted at a particular time. In this case, the allowed sending components have θ_i possible cases, where θ_i is taken as

$$\theta_i = \tfrac{m_i!}{r_i!(m_i-r_i)!}. \tag{7}$$

Hence, the reorganized innovation (RI) $\varepsilon_i^r(t)$ can only take one signal from the following finite set:

$$\chi_i(t) = \{\Theta_1^i(t)\varepsilon_i(t), \ldots, \Theta_{h_i}^i(t)\varepsilon_i(t), \ldots, \Theta_{\theta_i}^i(t)\varepsilon_i(t)\} \tag{8}$$

where $\Theta^i_{h_i}$ denotes a diagonal matrix which contains r_i diagonal elements "1" and $m_i - r_i$ diagonal elements "0."

To describe the RI in a simple way, suppose that θ_i elements of set χ_i are indexed from 1 to θ_i. Then, the following indicator function is introduced:

$$\sigma^i_{h_i}(t) = \begin{cases} 1 & \text{if } \varepsilon^r_i(t) = \Theta^i_{h_i}(t)\varepsilon_i(t) \\ 0 & \text{if } \varepsilon^r_i(t) \neq \Theta^i_{h_i}(t)\varepsilon_i(t) \end{cases} \tag{9}$$

It can be realized from (9) that if the h_ith element of set $\chi_i(t)$ is chosen as $\varepsilon^r_i(t)$, $\sigma^i_{h_i}(t) = 1$; otherwise, $\sigma^i_{h_i}(t) = 0$. Meanwhile, the binary variables $\sigma^i_{h_i}(t)(h_i = 1, 2, \ldots, \theta_i)$ satisfy

$$\sigma^i_{h_i}(t)\sigma^i_{h_{i0}}(t) = 0(h_i \neq h_{i0}), \sum_{h_i=1}^{\theta_i} \sigma^i_{h_i}(t) = 1\,. \tag{10}$$

It can be guaranteed from (10) that the RI only takes one signal from the set $\chi_i(t)$ at each time. Then, the RI $\varepsilon^r_i(t)$ can be calculated by the following form:

$$\varepsilon^r_i(t) = \Theta_i(t)\varepsilon_i(t) \tag{11}$$

where $\Theta_i(t)$ is described by:

$$\Theta_i(t) = \sum_{h_i=1}^{\theta_i} \sigma^i_{h_i}(t)\Theta^i_{h_i}(t)\,. \tag{12}$$

Since $\Theta_i(t)$ is diagonal, it can be simplified as follows:

$$\Theta_i(t) = \mathrm{diag}\{\xi^{h_i}_{i1}(t), \xi^{h_i}_{i2}(t), \ldots, \xi^{h_i}_{im_i}(t)\} \tag{13}$$

where $\xi^{h_i}_{ij}(t)(j = 1, 2, \ldots, m_i)$ are the binary variables satisfying

$$\xi^{h_i}_{ij}(t) \in \{0, 1\}, \sum_{j=1}^{m_i} \xi^{h_i}_{ij}(t) = r_i\,. \tag{14}$$

Subsequently, the RI will be quantized after dimensionality reduction, and Fig. 1 shows the relationship between the input and the output of uniform quantizer. Here, the uniform quantizer can be described by [20]:

$$Q(x) = \bar{x} + \mathrm{sgn}(x - \bar{x}) \cdot \delta \cdot \lfloor \tfrac{\|x-\bar{x}\|}{\delta} + \tfrac{1}{2} \rfloor \tag{15}$$

where sgn$\{\cdot\}$ is the sign function. x denotes the input, while $\bar{x}$ denotes the mid-value of certain interval. The parameter $\delta = \frac{l}{2}$ is the maximum error, where l is the length of the quantization interval. Thus, the quantization error Δ is bounded, that is

$$\|\Delta\| = \|x - Q(x)\| < \delta = \tfrac{l}{2}\,. \tag{16}$$

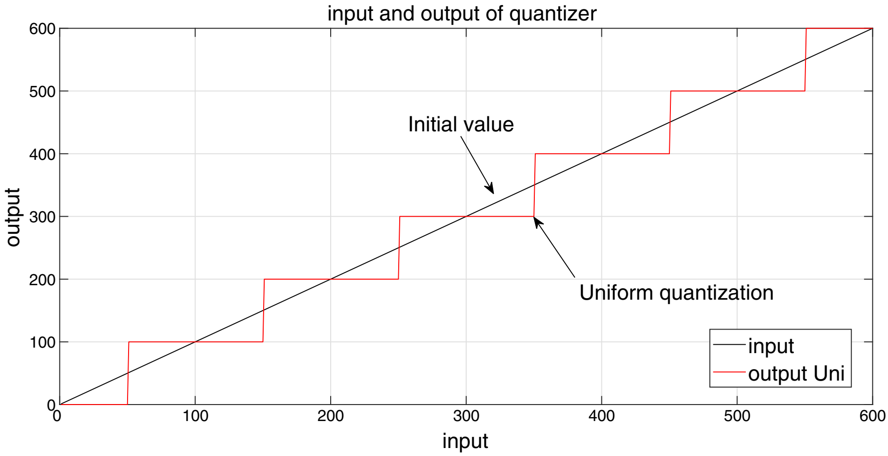

Fig. 1 The relationship between the input and output of the uniform quantizer

Accordingly, $Q(x)$ can be expressed as $(I + \Delta)x$ for certain Δ which satisfies (16).

Under the above analysis, the RI after quantization becomes the quantized innovation (QI) $\varepsilon_i^q(t)$, i.e., the innovation based on the DDCS, and the QI can be expressed by the following form:

$$\varepsilon_i^q(t) = Q(\varepsilon_i^r(t)) = (I + \Delta_i(t))\Theta_i(t)\varepsilon_i(t) = (I + H_i(t)F_i(t)E_i(t))\Theta_i(t)\varepsilon_i(t) \tag{17}$$

where $H_i(t)$, $F_i(t)$, $E_i(t)$ are known matrices with appropriate dimensions.

Remark 1 Generally, local estimates were quantized for satisfying the limited bandwidth [10]. But in this paper, the innovations were proposed to be quantized through finite communication channels. In fact, the inputs of quantizer should be bounded at each time. This means that the quantization density and the upper bound should be determined in advance, and then the quantizer can correctly judge which interval the output is in. However, for unstable systems, the estimates/measurements will diverge as time goes to ∞. As an alternative, innovation sequences can be bounded due to the stability of Kalman filtering when the system is controllable and observable. In this case, the innovation quantization strategy was adopted in [26] such that the quantization method can be applicable for unstable NMFSs. Although the fusion estimation problem was also discussed in [26], the dimensionality reduction strategy which greatly reduces the traffic more than the quantization strategy was not considered. Therefore, the DDCS based on innovation is proposed in this paper.

Remark 2 Obviously, with the dimension reduced by dimensionality reduction strategy, the amount of useful information will be simultaneously lost and the estimation performance in FC will be compromised. To overcome this shortage, the one-step prediction compensation strategy is commonly proposed to reduce the loss [24]. However, different from the above-mentioned view, there is no need to compensate the reorganized information caused by dimensionality reduction in this paper, because it is the innovations instead of the estimates that are going to be transmitted. In this case, whether the FC receives the innovations or not, the one-step prediction will be computed when recursing Kalman filter, i.e., the local estimates, will be potentially replaced by the one-step predictions. Moreover, the stability and the accuracy are influenced by the dimension r_i. Hence, the dimension r_i needs to be carefully determined to balance the communication traffic and the accuracy.

2.2 Problem of Interests

Consider the following local filter with quantized innovation [26]:

$$\hat{x}_i(t+1) = A\hat{x}_i(t) + K_i(t+1)\varepsilon_i^q(t+1) \tag{18}$$

where $\hat{x}_i(t+1)$ is the local state estimate and $K_i(t+1)$ is the optimal Kalman filter gain. Notice that the Kalman gain in the linear minimum variance sense can be solved in two steps. First, a filter with quantized innovation needs to be designed to eliminate the multiplicative uncertainties such that for all admissible uncertainties, there exists a positive-definite matrix $\tilde{P}_{ii}(t)$ satisfying

$$\bar{P}_{ii}(t) = \mathrm{E}\{(x_i(t) - \hat{x}_i(t))(x_i(t) - \hat{x}_i(t))^{\mathrm{T}}\} \le \tilde{P}_{ii}(t) \tag{19}$$

because

$$\mathrm{Tr}\{\bar{P}_{ii}(t)\} = \mathrm{E}\{(x_i(t) - \hat{x}_i(t))^{\mathrm{T}}(x_i(t) - \hat{x}_i(t)\} \le \mathrm{Tr}\{\tilde{P}_{ii}(t)\} \tag{20}$$

where $\bar{P}_{ii}(t)$ represents the state estimation error covariance matrix of ith sensor, and $\tilde{P}_{ii}(t)$ is the corresponding finite upper bound. Second, we shall minimize $\mathrm{Tr}(\tilde{P}_{ii}(t))$ to calculate the optimal gain $K_i(t)$ and $P_{ii}(t)$, the minimum of upper bound, and then construct the optimal recursive Kalman filter.

After QI has been sent to the FC through constrained communication channel, the local estimate recursively calculated in FC, denoted as $\hat{x}_{fi}(t+1)$, will be described by the following form:

$$\hat{x}_{fi}(t+1) = A\hat{x}_{fi}(t) + K_{fi}(t+1)\varepsilon_i^q(t+1)\,. \tag{21}$$

When $\hat{x}_{fi}(0) = \hat{x}_i(0)$, there must be $\hat{x}_{fi}(t+1) = \hat{x}_i(t+1)$ at each time. In the sequel, the weighting fusion estimate in FC can be calculated by:

$$\hat{x}_m(t) = \sum_{i=1}^{L} W_i(t)\hat{x}_{fi}(t) \tag{22}$$

where $W_i(t)(i = 1, \ldots, L)$ are weight matrices meet $\sum_{i=1}^{L} W_i(t) = I$, and they need to be designed by the linear unbiased minimum variance criterion. Then, the distributed dimensionality reduction fusion structure with quantized innovations is shown in Fig. 2.

Consequently, the problems to be solved in this paper are summarized as follows: (1) Design a dimensionality reduction Kalman filter with quantized innovation based

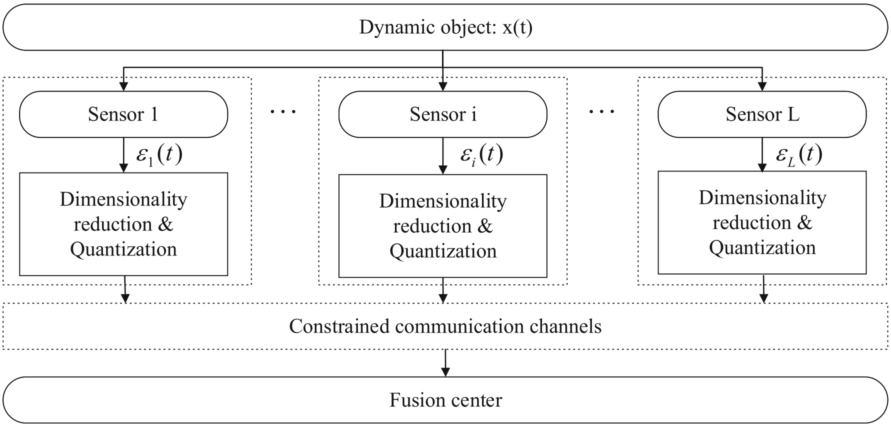

Fig. 2 The distributed dimensionality reduction fusion structure with quantized innovations

on the model uncertainty processing method in each sensor. (2) Design a distributed dimensionality reduction fusion Kalman filtering with quantized innovations based on the local estimates and the fusion criterion.

Remark 3 In most cases, the stable local estimator with standard form of Kalman filter is designed by $\hat{x}_i^s(t+1) = A\hat{x}_i(t) + K_i^s(t+1)\varepsilon_i(t+1)$. However, the stability of the local estimator $\hat{x}_{fi}(t+1)$ as (21) in FC cannot be guaranteed due to the introduction of DDCS. Suppose that the Kalman gain in FC is also computed as $K_i^s(t)$. Then, to prove the instability between the standard Kalman filter and filter (21), their relative error $\delta_i(t+1)$ is described as follows:

$$\begin{aligned} \delta_i(t+1) &= \hat{x}_i^s(t+1) - \hat{x}_{fi}(t+1) \\ &= A\delta_i(t) + K_i^s(t+1)\varepsilon_i(t+1)(I - (I + \Delta_i(t+1))\Theta_i(t+1)) \\ &= A^{t+1}\delta_i(0) + \sum_{n=0}^{t} A^{t-n}K_i^s(n+1)\varepsilon_i(n+1)\Delta_i(n+1)\Theta_i(n+1). \end{aligned} \tag{23}$$

When $\delta_i(0) = 0$, $\delta_i(t)$ cannot be 0 due to the existence of quantization noise $\Delta_i(t)$ and dimensionality reduction matrix $\Theta_i(t)$. Nevertheless, if the standard local Kalman filter is replaced by the proposed filter with quantized innovation as (18), $\delta_i(t+1)$ will become

$$\begin{aligned} \delta_i(t+1) &= \hat{x}_i(t+1) - \hat{x}_{fi}(t+1) \\ &= A\delta_i(t) \\ &= A^{t+1}\delta_i(0). \end{aligned} \tag{24}$$

It is obvious that when $\delta_i(0) = 0$, $\delta_i(t) = 0$. In other words, the local filters recursively calculated in FC will be consistent with that in corresponding sensors. Thus, in this paper, recursive form (18) is considered for local state estimation instead of the standard Kalman filter.

3 Main Results

In this section, the distributed dimensionality reduction fusion Kalman filter with quantized innovations is designed for unstable NMFSs. Define the estimation error $e_i(t) = x(t) - \hat{x}_i(t)$. Then, the optimal weight matrices $W_i(t)$ in (22) can be obtained by resorting to the results in [21]:

$$W(t) = (I_a^{\mathrm{T}} P^{-1}(t) I_a)^{-1} I_a^{\mathrm{T}} P^{-1}(t) \tag{25}$$

where

$$I_a = [\underbrace{I_n^{\mathrm{T}} \cdots I_n^{\mathrm{T}}}_{L\ times}] \tag{26}$$

$$W(t) = \left[W_1(t) \ldots W_i(t) \cdots W_L(t)\right] \tag{27}$$

$$P(t) = \mathrm{E}\{[e_1^{\mathrm{T}}(t) \ldots e_L^{\mathrm{T}}(t)][e_1^{\mathrm{T}}(t) \ldots e_L^{\mathrm{T}}(t)]^{\mathrm{T}}\}. \tag{28}$$

Before deriving the main results, the following lemma is introduced.

Lemma 1 [27] *Given matrices A, H and E with compatible dimensions such that $FF^{\mathrm{T}} \leq I$. Let X be a symmetric positive-definite matrix and $\alpha > 0$ be an arbitrary positive constant such that $\alpha^{-1}I - EXE^{\mathrm{T}} > 0$, then the following inequality holds:*

$$\begin{aligned}&(A + HFE)X(A + HFE)^{\mathrm{T}} \\ &\quad \leq AXA^{\mathrm{T}} + AXE^{\mathrm{T}}(\alpha^{-1}I - EXE^{\mathrm{T}})^{-1}EXA^{\mathrm{T}} + \alpha^{-1}HH^{\mathrm{T}}.\end{aligned} \tag{29}$$

Theorem 1 *Given positive parameters α_{ii}, β_{ii} and γ_{ii} which satisfy*

$$\begin{cases} \alpha_{ii}^{-1}I - \tilde{E}_{i1}(t)P_{ii}(t)\tilde{E}_{i1}^{\mathrm{T}}(t) > 0 \\ \beta_{ii}^{-1}I - \tilde{E}_{i2}(t)Q\tilde{E}_{i2}^{\mathrm{T}}(t) > 0 \\ \gamma_{ii}^{-1}I - \tilde{E}_{i3}(t)R_i\tilde{E}_{i3}^{\mathrm{T}}(t) > 0 \end{cases}. \tag{30}$$

Then, the optimal estimator gain $K_i(t+1)$ in (18) is calculated by:

$$\begin{aligned} K_i(t+1) = {} & [AS_{ii1}(t)A^{\mathrm{T}}C_i^{\mathrm{T}}\Theta_i(t+1)^{\mathrm{T}} \\ & + BS_{ii2}(t)B^{T}C_i^{\mathrm{T}}\Theta_i(t+1)^{\mathrm{T}}][C_iAS_{ii1}(t)A^{\mathrm{T}}C_i^{\mathrm{T}} \\ & + C_iBS_{ii2}(t)B^{\mathrm{T}}C_i^{\mathrm{T}} + S_{ii3}(t) + (\alpha_{ii}^{-1} + \beta_{ii}^{-1} + \gamma_{ii}^{-1})H_i(t)H_i(t)^{\mathrm{T}}]^{-1} \end{aligned} \tag{31}$$

$$\begin{aligned} P_{ii}(t+1) = {} & \tilde{A}_{i1}(t)S_{ii1}(t)\tilde{A}_{i1}^{\mathrm{T}}(t) + \tilde{A}_{i2}(t)S_{ii2}(t)\tilde{A}_{i2}^{\mathrm{T}}(t) \\ & + \tilde{A}_{i3}(t)S_{ii3}(t)\tilde{A}_{i3}^{\mathrm{T}}(t) \\ & + \alpha_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i^{\mathrm{T}}(t) + \beta_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i^{\mathrm{T}}(t) \\ & + \gamma_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i^{\mathrm{T}}(t) \end{aligned} \tag{32}$$

where

$$\begin{cases} \tilde{A}_{i1}(t) = A - K_i(t+1)\Theta_i(t+1)C_i A \\ \tilde{A}_{i2}(t) = B - K_i(t+1)\Theta_i(t+1)C_i B \\ \tilde{A}_{i3}(t) = -K_i(t+1)\Theta_i(t+1) \\ \tilde{H}_i(t) = -K_i(t+1)H_i(t+1) \\ \tilde{E}_{i1}(t) = E_i(t+1)\Theta_i(t+1)C_i A \\ \tilde{E}_{i2}(t) = E_i(t+1)\Theta_i(t+1)C_i B \\ \tilde{E}_{i3}(t) = E_i(t+1)\Theta_i(t+1) \end{cases} \tag{33}$$

and

$$\begin{cases} S_{ii1}(t) = P_{ii}(t) + P_{ii}(t)\tilde{E}_{i1}^{\mathrm{T}}(t)[\alpha_{ii}^{-1}I - \tilde{E}_{i1}(t)P_{ii}(t)\tilde{E}_{i1}^{\mathrm{T}}(t)]^{-1}\tilde{E}_{i1}(t)P_{ii}(t) \\ S_{ii2}(t) = Q + Q\tilde{E}_{i2}^{\mathrm{T}}(t)[\beta_{ii}^{-1}I - \tilde{E}_{i2}(t)Q\tilde{E}_{i2}^{\mathrm{T}}(t)]^{-1}\tilde{E}_{i2}(t)Q \\ S_{ii3}(t) = R_i + R_i\tilde{E}_{i3}^{\mathrm{T}}(t)[\gamma_{ii}^{-1}I - \tilde{E}_{i3}(t)R_i\tilde{E}_{i3}^{\mathrm{T}}(t)]^{-1}\tilde{E}_{i3}(t)R_i \end{cases}. \tag{34}$$

Meanwhile, after similarly giving α_{ij}, β_{ij} and γ_{ij}, the cross-covariance matrix is computed by:

$$\begin{aligned} P_{ij}(t+1) = {} & \tilde{A}_{i1}(t)S_{ij1}(t)\tilde{A}_{j1}^{\mathrm{T}}(t) + \tilde{A}_{i2}(t)S_{ij2}(t)\tilde{A}_{j2}^{\mathrm{T}}(t) \\ & + \tilde{A}_{i3}(t)S_{ij3}(t)\tilde{A}_{j3}^{\mathrm{T}}(t) \\ & + \alpha_{ij}^{-1}\tilde{H}_i(t)\tilde{H}_j^{\mathrm{T}}(t) + \beta_{ij}^{-1}\tilde{H}_i(t)\tilde{H}_j^{\mathrm{T}}(t) \\ & + \gamma_{ij}^{-1}\tilde{H}_i(t)\tilde{H}_j^{\mathrm{T}}(t) \end{aligned} \tag{35}$$

where

$$\begin{cases} S_{ij1}(t) = P_{ij}(t) + P_{ij}(t)\tilde{E}_{j1}^{\mathrm{T}}(t)[\alpha_{ij}^{-1}I - \tilde{E}_{i1}(t)P_{ii}(t)\tilde{E}_{j1}^{\mathrm{T}}(t)]^{-1}\tilde{E}_{i1}(t)P_{ij}(t) \\ S_{ij2}(t) = Q + Q\tilde{E}_{j2}^{\mathrm{T}}(t)[\beta_{ij}^{-1}I - \tilde{E}_{i2}(t)Q\tilde{E}_{j2}^{\mathrm{T}}(t)]^{-1}\tilde{E}_{i2}(t)Q \\ S_{ij3}(t) = 0 \end{cases}. \tag{36}$$

Proof Under the distributed fusion structure, the fusion algorithm is based on the local estimates and weighting criterion. To start, let us define the original local Kalman filter before minimization:

$$\bar{x}_i(t+1) = A\hat{x}_i(t) + \bar{K}_i(t+1)\varepsilon_i^q(t+1) \tag{37}$$

where $\bar{x}_i(t+1)$ and $\bar{K}_i(t+1)$, respectively, represent the estimate and gain before minimization. The corresponding state estimation error is denoted by $\bar{e}_i(t+1) \triangleq x_i(t+1) - \bar{x}_i(t+1)$. After expanding uncertain term as (17), $\bar{e}_i(t+1)$ will become:

$$\begin{aligned} \bar{e}_i(t+1) = {} & (\tilde{A}_{i1}(t) + \tilde{H}_i(t)F_i(t+1)\tilde{E}_{i1}(t))e_i(t) \\ & + (\tilde{A}_{i2}(t) + \tilde{H}_i(t)F_i(t+1)\tilde{E}_{i2}(t))w(t) \\ & + (\tilde{A}_{i3}(t) + \tilde{H}_i(t)F_i(t+1)\tilde{E}_{i3}(t))v_i(t+1) \end{aligned} \tag{38}$$

where $\tilde{A}_{i1}(t)$, $\tilde{A}_{i2}(t)$, $\tilde{A}_{i3}(t)$, $\tilde{H}_i(t)$, $\tilde{E}_{i1}(t)$, $\tilde{E}_{i2}(t)$ and $\tilde{E}_{i3}(t)$ have been defined in (33). As analyzed in (17), $H_i(t+1)F_i(t+1)E_i(t+1) = \Delta_i(t+1)$ represents the quantization noise. More concretely, $F_i(t+1) = \frac{\Delta_i(t+1)}{\delta}$ meets the constraint $F_i(t+1)F_i^{\mathrm{T}}(t+1) \leq I$, and $H_i(t+1) = \delta$, $E_i(t+1) = I$. Then, after giving positive parameters α_{ii}, β_{ii} and γ_{ii} which satisfy (30), the upper bound $\tilde{P}_{ii}(t+1)$ of $\bar{P}_{ii}(t+1) \triangleq \mathrm{E}\{\bar{e}_i(t+1)\bar{e}_i^{\mathrm{T}}(t+1)\}$, the original estimation error covariance matrix, can be derived by utilizing Lemma 1:

$$
\begin{aligned}
\bar{P}_{ii}(t+1) &\leq \tilde{A}_{i1}(t)S_{ii1}(t)\tilde{A}_{i1}^{\mathrm{T}}(t) + \tilde{A}_{i2}(t)S_{ii2}(t)\tilde{A}_{i2}^{\mathrm{T}}(t) + \tilde{A}_{i3}(t)S_{ii3}(t)\tilde{A}_{i3}^{\mathrm{T}}(t) \\
&\quad + \alpha_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i^{\mathrm{T}}(t) + \beta_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i^{\mathrm{T}}(t) + \gamma_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i^{\mathrm{T}}(t) \\
&= \tilde{P}_{ii}(t+1)
\end{aligned} \tag{39}
$$

where $S_{ii1}(t)$, $S_{ii2}(t)$ and $S_{ii3}(t)$ have been defined in (34). To obtain the minimum covariance $P_{ii}(t)$, the first-order partial derivative about $\bar{K}_i(t+1)$ of $\mathrm{Tr}\{\tilde{P}_{ii}(t+1)\}$ is given as follows:

$$
\begin{aligned}
\frac{\partial \mathrm{Tr}\{\tilde{P}_{ii}(t+1)\}}{\partial \bar{K}_i(t+1)} &= 2\bar{K}_i(t+1)\Theta_i(t+1)C_iAS_{ii1}(t)A^{\mathrm{T}}C_i^{\mathrm{T}}\Theta_i(t+1)^{\mathrm{T}} \\
&\quad -2AS_{ii1}(t)A^{\mathrm{T}}C_i^{\mathrm{T}}\Theta_i(t+1)^{\mathrm{T}} + 2\alpha_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i(t)^{\mathrm{T}} \\
&\quad +2\bar{K}_i(t+1)\Theta_i(t+1)C_iBS_{ii2}(t)B^{\mathrm{T}}C_i^{T}\Theta_i(t+1)^{\mathrm{T}} \\
&\quad -2BS_{ii2}(t)B^{\mathrm{T}}C_i^{\mathrm{T}}\Theta_i(t+1)^{\mathrm{T}} + 2\beta_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i(t)^{\mathrm{T}} \\
&\quad +2\bar{K}_i(t+1)\Theta_i(t+1)S_{ii3}(t)\Theta_i(t+1)^{\mathrm{T}} + 2\gamma_{ii}^{-1}\tilde{H}_i(t)\tilde{H}_i(t)^{\mathrm{T}}
\end{aligned} \tag{40}
$$

When (40) equals zero, the optimal Kalman gain $K_i(t+1)$ can be derived in the linear minimum variance sense as (31), and the minimum upper bound $P_{ii}(t+1)$ can be obtained by substituting the optimal Kalman gain $K_i(t+1)$ into (39).

On the other hand, when the FC receives the quantized innovations, (21) is exploited to calculate the local estimate $\hat{x}_{fi}(t+1)$. According to Remark 3, the local estimates are relatively invariant during the transmission process. Thus, $P_{ij}(t+1)$, the minimum upper bound of cross-covariance and the composite covariance matrix $P(t+1)$ can be calculated by (35) and (28). Finally, the optimal weight matrix $W(t+1)$ in the linear unbiased minimum variance sense is derived by (25), and the optimal fusion estimate is calculated by (22). This completes the proof. □

Based on Theorem 1, the computation procedures for distributed dimensionality reduction fusion Kalman filtering with quantized innovations are summarized as Algorithm 1.

Algorithm 1 Distributed dimensionality reduction fusion Kalman filtering algorithm

1: For given $\alpha_{ij}, \beta_{ij}, \gamma_{ij} (i = 1, 2, 3; j = 1, 2, 3)$, $P_{ii}(0)$, $\hat{x}_i(0)$, $P(0)$, $\hat{x}(0)$.
2: Calculate $\tilde{A}_{i1}(t)$, $\tilde{A}_{i2}(t)$, $\tilde{A}_{i3}(t)$, S_{ii1}, S_{ii2} and S_{ii3} by (33) and (34).
3: Calculate $K_i(t+1)$, $P_{ii}(t+1)$ and $\hat{x}_i(t+1)$ by (31), (32) and (18).
4: Calculate $\hat{x}_{fi}(t+1)$ and $P_{ij}(t+1)$ by (21) and (35).
5: Calculate $P(t+1)$, $W(t+1)$ and $\hat{x}_m(t+1)$ by (25) to (28) and (22).
6: Return to Step 2 and implement Step 2-5 for calculating $\hat{x}_m(t+2)$.

4 Simulation Examples

Consider a target tracking system with two sensors, and the system parameters in (1) and (2) are given by [1]:

$$A = \begin{bmatrix} 1 & T_0 & \frac{T_0^2}{2} \\ 0 & 1 & T_0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{T_0^2}{2} \\ T_0 \\ 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \tag{41}$$

where $T_0 = 0.8s$ is the sampling period. Denote the state vector $x(t) = \mathrm{col}\{X(t), \dot{X}(t), \ddot{X}(t)\}$, where $X(t), \dot{X}(t), \ddot{X}(t)$ are the position, velocity and acceleration of moving target at time t, respectively. $w(t)$, $v_1(t)$ and $v_2(t)$ are respective uncorrelated white Gaussian noises with covariances

$$Q = 1, R_1 = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.7 \end{bmatrix}, R_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}. \tag{42}$$

These random variables can be generated by the function "rand()" of MATLAB.

Suppose that each sensor has enough processing capabilities to compute local Kalman filter. It is considered for DDCS that only $r_1 = 1$ components of $\varepsilon_1(t)$ are allowed to be transmitted to the FC at time t, and the quantization parameter is given by $l = 0.01$. For example, the original innovation $\varepsilon_1(t)$ is col$\{-0.6412, -0.7150\}$ which can be represented by twelve decimal bits. After dimensionality reduction strategy with $(\Theta_1(t) = \mathrm{diag}\{1, 0\})$, the RI $\varepsilon_1^r(t)$ becomes col$\{-0.6412, 0\}$. Then, after quantization, the QI $\varepsilon_1^q(t)$ becomes col$\{-0.64, 0\}$ which can only be represented by four decimal bits. Therefore, the transmission amount is greatly reduced.

To demonstrate Remark 1, the following figures are presented. Figure 3a shows the local estimates and innovations in sensor 1. It can be seen from this figure that in unstable systems, the state estimates diverge, while the innovations are bounded. Meanwhile, it is the same in Fig. 3b which shows the quantized value of local estimates and innovations. Accordingly, this result proves the reason why quantizing innovations instead of states in unstable NMFSs.

To demonstrate the effectiveness of the designed fusion estimation algorithm, the following figures are presented. Figure 4a shows the trajectories of target $x(t)$ and fusion tracking $\hat{x}_m(t)$, and it illustrates the designed fusion estimate can catch up with the target well under bandwidth constraints. Due to the existence of stochastic noises, the performance is assessed by its mean square errors (MSEs) over an average of 100 runs Monte Carlo method. It is obviously seen in Fig. 4b that the MSE of fusion

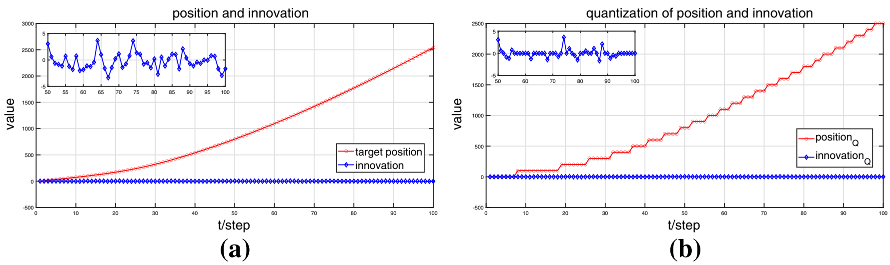

Fig. 3 **a** The position and innovation in unstable systems. **b** The quantization of position and innovation in unstable systems

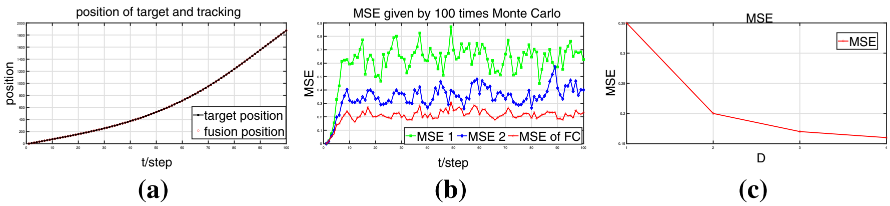

Fig. 4 **a** The trajectories of target and fusion tracking. **b** The MSEs of local and fusion estimates. **c** The relationship between the MSE and the dimension of innovation

estimator is lower than that of any local estimator, and this means the fusion estimate performance better than any local estimate. Moreover, Fig. 4c shows that the MSE decreases with the increase in the dimension of innovation. According to the accuracy variation shown in this figure, the dimension r_i can be determined suitably.

5 Conclusions

In this paper, the distributed fusion Kalman filtering problem has been studied for a class of unstable NMFSs with communication constraints, where the DDCS was introduced to deal with the innovations. By using the uncertainty processing method, the optimal local recursive form of minimum upper bound of covariance and Kalman filter gain were obtained. Then, the distributed dimensionality reduction fusion Kalman filter with quantized innovations was designed by utilizing the optimal weighted matrix fusion criterion. Finally, it was illustrated from the simulation examples that the proposed fusion estimation algorithm can efficiently solve the estimator design problem for unstable NMFSs.

Data Availability All data included in this study are available upon request by contacting the corresponding author.

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