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Asynchronous H_∞ Control of Uncertain Switched Singular Systems with Time-Varying Delays

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Abstract

This paper is concerned with the problem of H_∞ control for a class of uncertain switched singular systems with time-varying state delays under asynchronous switching. The asynchronous phenomenon is caused by the choice of controller lagging behind the corresponding subsystem in practice. First, sufficient conditions by finding a novel piecewise Lyapunov–Krasovskii function combining with average dwell time technique are given to guarantee the exponential admissibility of the system. The algebraic equations and differential equations of the original system are proved to be exponentially stable. Then, a condition guaranteeing the H_∞ performance of the original system is derived based on the above analysis. Furthermore, strict LMI formulas for solving the state feedback controller are given. Finally, the effectiveness of the proposed methods is illustrated by numerical examples.

Keywords Switched singular systems · Asynchronous switching · Time-varying delay · Average dwell time · Linear matrix inequality

1 Introduction

In recent years, the research on switched singular systems has attracted much attention from many scholars [7,13,33]. This is mainly due to the fact that this form of the model is widely applied to many practical engineering problems, see [24–26] and the references therein. Compared with the switched nonsingular system, one of the biggest differences is that solutions of a singular system may contain instantaneous state jumping phenomena. Therefore, the conclusion about singular systems is not a simple generalization of the conclusion of conventional linear systems. Meanwhile,

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due to the delay in system information transmission and the insensitivity and aging of measuring devices, the system has time delay inevitably. The appearance of time delay usually changes the normal response of the control system, even makes it difficult to ensure the stability of the system, resulting in the decline of the system performance indicators [14,37]. System control problems with time delay have special difficulties in solving mathematical and engineering problems [3,12,41].

Not only the state time delay but also the presence of time delay in the controller is extremely important in the whole stabilization stability analysis. In practical engineering applications, accompanied by the occurrence of time delay and it takes time for the system to identify the controller that matches the current subsystem. Therefore, the research of asynchronous control has attracted the attention of many scholars [5,7,29,38]. Without emphasizing the existence of asynchronism, [37]'s research groups have analyzed the exponential H_∞ filtering problem for a class of discrete-time switched singular systems. The object studied by the authors in [4] is a class of switched neutral systems. However, the focus of our work is on asynchronous non-weighted H_∞ control of the uncertain switched singular systems. This requires us to consider the stability problem of algebraic subsystems under the control of the asynchronous controller. Besides, uncertainty is another reason for the instability of the system. This uncertainty mainly comes from the measurement error of parameters and the change in environment and operating conditions [19]. Because of the widespread existence of uncertainty, the stability of systems with uncertainties is studied in a large number of references [17,21,22]. In reference [38], the stability problem of a class of discrete-time switched singular systems with time delay was investigated. Finite-time control and asynchronous control for discrete-time switched singular systems with time delays were studied in references [1,7,15,20].

Previous studies are concerned about work taking account of some of the factors mentioned above [34,42,43]. The impulse-free property of the original system is not required in [11] and the global stability of the original system is controlled by limiting the upper and lower bounds of dwell time. In [8], a class of uncertain switched singular systems with time-varying delay by using the average dwell time approach was investigated. Asynchronous control and system performance analysis for switched singular systems become more complex and challenging when delay and uncertainty are both taken into account. Comparing with [28], not only the singular matrix E but also the disturbance variable and control output variable are adding to the model studied in our paper. Therefore, we need to consider the regularity and impulsivity of the system compared with the system in [28]. Second, the state variable $x(t)$ of singular systems can be divided into the slow subsystem variable and the fast subsystem variable. Uncertain switching singular systems with a time delay can be applied to describe the model of oil catalytic cracking and the typical DC chopper circuit in practical engineering. See the numerical examples in [27,30,39]. Now still less study has been made on asynchronous non-weighted H_∞ control of uncertain switching singular systems with time delay.

In this paper, the stabilization and non-weighted H_∞ performance for a class of uncertain switched singular systems are investigated via a new insight. It is remarkable that though state feedback controller has been widely used in switched singular systems; see [2,23,29], we use it to study the problem of asynchronous control for uncertain switched singular systems with time-varying delays. The main contributions are stated as follows.

(i) By constructing a more flexible Lyapunov function, a sufficient condition for the original system to be stable is obtained. The system instability caused by asynchrony is offset by a limit on the average dwell time of each subsystem. In the process of derivation, the time node has been reconstructed to facilitate the analysis and this is also the major contribution of our work.
(ii) The problem of asynchronous controller design for uncertain switched singular time-delay system is solved in this paper. By adding some free weight matrices, less conservation conditions are presented. A non-weighted H_∞ disturbance attenuation level for the considered system is obtained. The result has less conservation compared with [5,36,38].
(iii) In solving the controller gains, sufficient conditions in terms of strict linear matrix inequality (LMI) have been obtained by removing non-strict inequality constraints. Thus, the results are less conservative than those obtained by the approximate solution method.

The remaining parts of the paper are organized as follows. In Sect. 2, the system form to be studied and some useful lemmas are given. Section 3 embraces the main results. Taking the asynchronous situation into account, a state feedback controller is designed in this section such that the considered switched singular system is stabilization and the state solutions of the system have H_∞ performance. Specific examples along with numerical and simulation results are provided in Sect. 4. Section 5 is the conclusion of the work of this paper.

2 Problem Statement and Preliminaries

Consider a class of switched singular systems with time-varying delay described by the following equation

$$\begin{cases} E\dot{x}(t) = (A_{\sigma(t)} + \Delta A)x(t) + (B_{\sigma(t)} + \Delta B)x(t - d(t)) + G_{\sigma(t)}u(t) + E_{\sigma(t)}\omega(t) \\ z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}x(t - d(t)) + F_{\sigma(t)}\omega(t) \\ x(t) = \phi(t), t \in [-h, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $z(t) \in \mathbb{R}^l$ are the state vector, control input and channel output, respectively. $\omega(t) \in \mathbb{R}^h$ belonging to the space of square integrable denotes the disturbance. $\phi(t)$ is initial vector valued continuous function, and the switching signal $\sigma(t) : [0, \infty) \to \mathcal{I} = \{1, 2, \ldots, \mathbf{N}\}$ is a piecewise constant function of time t where $\mathbf{N}$ is the number of subsystems. $d(t)$ is a time delay that satisfies

$$d_1 \le d(t) \le d_2, \dot{d}(t) \le \mu_d < 1. \tag{2}$$

ΔA and ΔB are the uncertainties of the system and have the following form

$$[\Delta A \quad \Delta B] = MF(t)[N \quad N_d], \tag{3}$$

where M, N, N_d are known constant matrices with appropriate dimensions and $F(t)$ satisfies $F(t)^{\mathrm{T}}F(t) \le I, t \ge 0$ (the identity matrix of appropriate dimension). The forms of time-varying delays and uncertainties introduced in this paper exist widely in the previous literature on engineering system control. This just shows that this form of time delay and uncertainty is commonly used in practical application. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular and it is assumed that $\mathrm{rank}E = r \le n$. Since $\mathrm{rank}E = r \le n$, there exist nonsingular matrices $P, Q \in \mathbb{R}^{n\times n}$ such that $PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, then, without lose of generality, let $E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$.

The delay between the matched controller and corresponding subsystems is taken into account in this paper based on practical engineering application background. In other words, the switching sequence of the subsystems and the practical switching signal of the controllers can be described by

$$\begin{aligned} &\sigma : \{(0, \sigma(0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k)), \ldots\}, \\ &\tilde{\sigma} : \{(0, \sigma(0)), (t_1 + \Delta_1, \sigma(t_1)), \ldots, (t_k + \Delta_k, \sigma(t_k)), \ldots\}, \end{aligned}$$

respectively. Therefore, the form of the asynchronous controller is

$$u(t) = K_{\sigma(t-\Delta_k)}x(t),$$

where Δ_k is time-varying delay and conforms to $0 < \Delta_k \le \Delta_{\max} < t_{k+1} - t_k$.

The parametric equation after the decomposition of equation (1) has been founded.

$$\begin{aligned} &\tilde{A}_{\sigma(t)} = (A_{\sigma(t)} + G_{\sigma(t)}K_{\tilde{\sigma}(t)} + \Delta A) = \begin{bmatrix} A_{\sigma 1} & A_{\sigma 2} \\ A_{\sigma 3} & A_{\sigma 4} \end{bmatrix}, P_{\sigma(t)} = \begin{bmatrix} P_{\sigma 1} & P_{\sigma 2} \\ P_{\sigma 3} & P_{\sigma 4} \end{bmatrix}, \\ &\tilde{B}_{\sigma(t)} = (B_{\sigma(t)} + \Delta B) = \begin{bmatrix} B_{\sigma 1} & B_{\sigma 2} \\ B_{\sigma 3} & B_{\sigma 4} \end{bmatrix}. \end{aligned}$$

When $\omega(t) = 0$, the following closed-loop system can be obtained by adding the above asynchronous controller to the original system (1).

$$\begin{cases} \dot{x}_1(t) = \bar{A}_{\sigma 1}x_1(t) + \bar{A}_{d\sigma 1}x_1(t - d(t)) + \bar{A}_{d\sigma 2}x_2(t - d(t)) \\ x_2(t) = \bar{A}_{\sigma 3}x_1(t) + \bar{A}_{d\sigma 3}x_1(t - d(t)) + \bar{A}_{d\sigma 4}x_2(t - d(t)) \\ \ z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}x(t - d(t)) + F_{\sigma(t)}\omega(t) \\ \ x(t) = \phi(t), t \in [-h, 0] \end{cases} \tag{4}$$

where $\bar{A}_{\sigma 1} = A_{\sigma 1} - A_{\sigma 2}A_{\sigma 4}^{-1}A_{\sigma 3}$, $\bar{A}_{d\sigma 1} = B_{\sigma 1} - A_{\sigma 2}A_{\sigma 4}^{-1}B_{\sigma 3}$, $\bar{A}_{d\sigma 2} = B_{\sigma 2} - A_{\sigma 2}A_{\sigma 4}^{-1}B_{\sigma 4}$, $\bar{A}_{\sigma 3} = -A_{\sigma 4}^{-1}A_{\sigma 3}$, $\bar{A}_{d\sigma 3} = -A_{\sigma 4}^{-1}B_{\sigma 3}$, $\bar{A}_{d\sigma 4} = -A_{\sigma 4}^{-1}B_{\sigma 4}$.

Therefore, the stability problem of system (1) is equivalent to the stability problem of system (4).

Lemma 1 (Schur Complement) *For a given symmetric matrix* $\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^{\mathrm{T}} & \Phi_{22} \end{bmatrix}$, *the following conditions are equivalent:*

(i) $\Phi < 0$;
(ii) $\Phi_{11} < 0, \Phi_{22} - \Phi_{12}^{\mathrm{T}} \Phi_{11}^{-1} \Phi_{12} < 0$;
(iii) $\Phi_{22} < 0, \Phi_{11} - \Phi_{12} \Phi_{22}^{-1} \Phi_{12}^{\mathrm{T}} < 0$.

Some relative definitions and lemmas are listed as follows.

Definition 1 For the switching signal $\sigma(t)$, and any delay satisfying (2), the system (1) with $\omega(t) = 0$ is said to be

(i) regular if $\det(sE - A_i)$ is not identically zero for each $\sigma(t) = p, p \in \mathcal{I}$;
(ii) impulse free if $\deg(\det(sE - A_i)) = \operatorname{rank} E$ for each $\sigma(t) = p, p \in \mathcal{I}$;
(iii) exponentially stable under the switching signal $\sigma(t)$ if the solution $x(t)$ of the system satisfies $\|x(t)\| \le ce^{-\lambda(t-t_0)} \|x(t_0)\|_d$, $\forall t \ge t_0$;
(iv) exponentially admissible if it is regular, impulse free and exponentially stable under the switching signal $\sigma(t)$.

Definition 2 For the switching signal $\sigma(t)$ of system (1) and any $T_2 > T_1 \ge 0$, let $N_p(T_1, T_2)$ denotes the number of switching of subsystem p over (T_1, T_2), the total time for synchronization of the controller and subsystem p is represented by $T_p(T_1, T_2)$, if

$$N_p(T_1, T_2) \le N_0 + \frac{T_p(T_1, T_2)}{\tau_p} \tag{5}$$

holds for $\tau_p > 0$, $N_0 \ge 0$, then τ_p is called average dwell time of subsystem p and N_0 is called a chatter bound.

Definition 3 (i) For given $\alpha > 0, \gamma > 0$, system (1) with $\Delta A = \Delta B = 0$ is said to be uniformly asymptotically stable with γ-disturbance attenuation if (1) with $\omega(t) = 0$ is exponentially admissible and for a given scalar $\gamma > 0$, for any disturbance $\omega(t) \in L_2[0, \infty)$ and for the initial condition $\phi(t) = 0, t \in [-h, 0]$, the following H_∞ performance is satisfied:

$$\int_{t_0}^{\infty} \mathrm{e}^{-\alpha s} z^{\mathrm{T}}(s) z(s) \,\mathrm{d}s \le \gamma^2 \int_{t_0}^{\infty} \omega^{\mathrm{T}}(s) \omega(s) \,\mathrm{d}s.$$

(ii) System (1) is said to be uniformly robust asymptotically stable with γ-disturbance attenuation if it is uniformly asymptotically stable with γ-disturbance attenuation for all uncertainties satisfying (3).

Besides, if $\alpha = 0$, the switched system will be said to have a non-weighted (normal) L_2 gain. The condition (i) of non-weighted L_2 gain can be rewritten as $\int_{t_0}^{\infty} z^{\mathrm{T}}(s) z(s) \,\mathrm{d}s \le \gamma^2 \int_{t_0}^{\infty} \omega^{\mathrm{T}}(s) \omega(s) \,\mathrm{d}s$. Relatively speaking, as discussed in [6,16], the non-weighted L_2 gain is less conservative than the weighted one.

Lemma 2 [31] *For matrices M, N and Φ of appropriate dimensions and with Φ symmetric, then*

$$\Phi + MF(t)N + (MF(t)N)^{\mathrm{T}} < 0$$

for all $F(\sigma)$ satisfying $F(\sigma)^{\mathrm{T}} F(\sigma) \leq I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Phi + \varepsilon^{-1} M M^{\mathrm{T}} + \varepsilon N^{\mathrm{T}} N < 0.$$

Lemma 3 [10] *For a vector function $x(t)$ with first-order continuous-derivative entries, a scalar $d > 0$, and any matrices E and $R = R^{\mathrm{T}} > 0$. Then, the following descriptor integral inequality holds,*

$$\int_{t-d}^{t} (E\dot{x}(t))^{\mathrm{T}} RE\dot{x}(t)\,\mathrm{d}s \geq \frac{1}{d}\int_{t-d}^{t} (E\dot{x}(t))^{\mathrm{T}}\,\mathrm{d}s R \int_{t-d}^{t} E\dot{x}(t)\,\mathrm{d}s.$$

3 Main Results

Theorem 1 *Consider system (4) for prescribed scalars $d_1 \geq 0, d_2 > 0, \Delta_{\max} > 0, \mu_d < 1, \alpha_p > 0, \alpha_{pq} > 0, \hat{\mu} \geq 1$, and assume that there exist matrices $P_p, Q_p > 0, R_p > 0, P_{pq}, Q_{pq} > 0, R_{pq} > 0$, such that*

$$E^{\mathrm{T}} P_{\sigma(t)} = P_{\sigma(t)}^{\mathrm{T}} E \geq 0, \tag{6}$$

$$\begin{bmatrix} \Phi_{11} & P_p^{\mathrm{T}} B_p + \varepsilon^{-1} N^{\mathrm{T}} N_d & H_1^{\mathrm{T}}(A_p + G_p K_p) & 0 \\ * & \Phi_{22} & H_1^{\mathrm{T}} B_p & 0 \\ * & * & \Phi_{33} & 0 \\ * & * & * & -\mathrm{e}^{-\alpha_p d_2} \frac{R_p}{(d_2 - d_1)} \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} \Psi_{11} & P_{pq}^{\mathrm{T}} B_p + \varepsilon^{-1} N^{\mathrm{T}} N_d & H_1^{\mathrm{T}}(A_p + G_p K_q) & 0 \\ * & \Psi_{22} & H_1^{\mathrm{T}} B_p & 0 \\ * & * & \Psi_{33} & 0 \\ * & * & * & -\mathrm{e}^{\alpha_{pq} d_1} \frac{R_{pq}}{(d_2 - d_1)} \end{bmatrix} < 0, \tag{8}$$

$$\begin{aligned} &P_p \leq \hat{\mu} P_{pq}, Q_p \leq \hat{\mu} Q_{pq}, R_p \leq \hat{\mu} R_{pq}, \\ &P_{pq} \leq \hat{\mu} P_q, Q_{pq} \leq \hat{\mu} Q_q, R_{pq} \leq \hat{\mu} R_q, \end{aligned} \tag{9}$$

where

$$
\begin{aligned}
\Phi_{11} &= P_p^{\mathrm{T}}\left(A_p + G_pK_p\right) + \left(A_p + G_pK_p\right)^{\mathrm{T}} P_p + \alpha_p E^T P_p + Q_p + \varepsilon^{-1} N^{\mathrm{T}} N \\
&\quad + \varepsilon \left(P_p^{\mathrm{T}} M M^{\mathrm{T}} P_p + H_1^{\mathrm{T}} M M^{\mathrm{T}} H_1\right), \\
\Phi_{22} &= -\left(1-\mu_d\right) \mathrm{e}^{-\alpha_p d_2} Q_p + \varepsilon H_1^{\mathrm{T}} M M^{\mathrm{T}} H_1 + \varepsilon^{-1} N_d^{\mathrm{T}} N_d, \\
\Phi_{33} &= (d_2 - d_1) R_p - H_1 - H_1^{\mathrm{T}} + \varepsilon^{-1}\left(N^{\mathrm{T}} N + N_d^{\mathrm{T}} N_d\right), \\
\Psi_{11} &= P_{pq}^{\mathrm{T}}\left(A_p + G_pK_q\right) + \left(A_p + G_pK_q\right)^{\mathrm{T}} P_{pq} - \alpha_{pq} E^{\mathrm{T}} P_{pq} + Q_{pq} + \delta^{-1} N^{\mathrm{T}} N \\
&\quad + \delta \left(P_{pq}^{\mathrm{T}} M M^{\mathrm{T}} P_{pq} + H_1^{\mathrm{T}} M M^{\mathrm{T}} H_1\right), \\
\Psi_{22} &= -\left(1-\mu_d\right) \mathrm{e}^{\alpha_{pq} d_1} Q_{pq} + \delta H_1^{\mathrm{T}} M M^{\mathrm{T}} H_1 + \delta^{-1} N_d^{\mathrm{T}} N_d, \\
\Psi_{33} &= (d_2 - d_1) R_{pq} - H_1 - H_1^{\mathrm{T}} + \delta^{-1}\left(N^{\mathrm{T}} N + N_d^{\mathrm{T}} N_d\right).
\end{aligned}
$$

Then, the controller can guarantee that system is exponential admissibility for any switching signal with average dwell time satisfying

$$
\tau_p \geq \tau_p^* = \frac{\ln(\mu_{\mathrm{m}}\hat{\mu}^2) + \left(\alpha_{pq} + \alpha_p\right) \Delta_{\max}}{\alpha_p}, \tag{10}
$$

where $\mu_{\mathrm{m}} = \max\limits_{p,q \in \mathcal{I}} \mathrm{e}^{(\alpha_q + \alpha_{pq}) d_2}$.

Proof Firstly, the proof of the stability of the partial differential equations in the original system is given. The following piecewise Lyapunov function candidate for system (1) is designed. □

$$
\begin{aligned}
V(t, x(t)) = x^{\mathrm{T}}(t) E^{\mathrm{T}} P_{\sigma(t)} x(t) &+ \int_{t-d(t)}^{t} \mathrm{e}^{\nu(t-s)} x^{\mathrm{T}}(s) Q_{\sigma(t)} x(s) \mathrm{d}s \\
&+ \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} \mathrm{e}^{\nu(t-s)} \dot{x}^{\mathrm{T}}(s) E^{\mathrm{T}} R_{\sigma(t)} E \dot{x}(s) \mathrm{d}s \mathrm{d}\theta,
\end{aligned}
$$

where ν is

$$
\nu = \begin{cases} \alpha_{pq}, t \in \left[t_k, t_{k+\Delta k}\right) \\ -\alpha_p, t \in \left[t_{k+\Delta k}, t_{k+1}\right) \end{cases}.
$$

Then, when $t \in \left[t_k + \Delta_k, t_{k+1}\right)$, along the trajectories of system, we have

$$
\begin{aligned}
\dot{V}(t,x(t)) &= 2x^{\mathrm{T}}(t)P_{\sigma(t)}^{\mathrm{T}}E\dot{x}(t) + x^{\mathrm{T}}(t)Q_{\sigma(t)}x(t)\\
&\quad - (1-\dot{d}(t))\mathrm{e}^{\nu d(t)}x^{\mathrm{T}}(t-d(t))Q_{\sigma(t)}x(t-d(t))\\
&\quad + \nu\int_{t-d(t)}^{t}\mathrm{e}^{\nu(t-s)}x^{\mathrm{T}}(s)Q_{\sigma(t)}x(s)\mathrm{d}s + (d_2-d_1)\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}R_{\sigma(t)}E\dot{x}(t)\\
&\quad - \int_{-d_2}^{-d_1}\mathrm{e}^{-\nu\theta}\dot{x}^{\mathrm{T}}(t+\theta)E^{\mathrm{T}}R_{\sigma(t)}E\dot{x}(t+\theta)\mathrm{d}\theta\\
&\quad + \nu\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}\mathrm{e}^{\nu(t-s)}\dot{x}^{\mathrm{T}}(s)E^{\mathrm{T}}R_{\sigma(t)}E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\\
&= \nu V(t,x(t)) - \nu x^{\mathrm{T}}(t)E^{\mathrm{T}}P_{\sigma(t)}x(t) + 2x^{\mathrm{T}}(t)P_{\sigma(t)}^{\mathrm{T}}E\dot{x}(t)\\
&\quad + x^{\mathrm{T}}(t)Q_{\sigma(t)}x(t) - (1-\dot{d}(t))\mathrm{e}^{\nu d(t)}x^{\mathrm{T}}(t-d(t))Q_{\sigma(t)}x(t-d(t))\\
&\quad + (d_2-d_1)\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}R_{\sigma(t)}E\dot{x}(t)\\
&\quad - \int_{-d_2}^{-d_1}\mathrm{e}^{-\nu\theta}\dot{x}^{\mathrm{T}}(t+\theta)E^{\mathrm{T}}R_{\sigma(t)}E\dot{x}(t+\theta)\mathrm{d}\theta.
\end{aligned}
\tag{11}
$$

By considering $t \in \left[t_k + \Delta_k, t_{k+1}\right)$, according to Jensen's inequality in Lemma 3, the following inequalities can be obtained,

$$
\begin{aligned}
&-\int_{-d_2}^{-d_1}\mathrm{e}^{\alpha_p\theta}\dot{x}^{\mathrm{T}}(t+\theta)E^{\mathrm{T}}R_pE\dot{x}(t+\theta)\mathrm{d}\theta\\
&\quad\le -\mathrm{e}^{-\alpha_p d_2}\left(\int_{-d_2}^{-d_1}E\dot{x}(t+\theta)\mathrm{d}\theta\right)^{\mathrm{T}}\frac{R_p}{(d_2-d_1)}\left(\int_{-d_2}^{-d_1}E\dot{x}(t+\theta)\mathrm{d}\theta\right),
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&-(1-\dot{d}(t))\mathrm{e}^{-\alpha_p d(t)}x^{\mathrm{T}}(t-d(t))Q_px(t-d(t))\\
&\quad\le -(1-\mu_d)\,\mathrm{e}^{-\alpha_p d_2}x^{\mathrm{T}}(t-d(t))Q_px(t-d(t)).
\end{aligned}
\tag{13}
$$

Thus, Eq. (11) can be transformed into

$$
\begin{aligned}
&\dot{V}(t,x(t)) + \alpha_pV(t,x(t))\\
&\quad\le \alpha_px^{\mathrm{T}}(t)E^{\mathrm{T}}P_px(t) + 2x^{\mathrm{T}}(t)P_p^{\mathrm{T}}\left(\tilde{A}_px(t) + \tilde{B}_px(t-d(t))\right)\\
&\qquad + x^{\mathrm{T}}(t)Q_px(t) + (d_2-d_1)\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}R_pE\dot{x}(t)\\
&\qquad - \mathrm{e}^{-\alpha_pd_2}\left(\int_{-d_2}^{-d_1}E\dot{x}(t+\theta)\mathrm{d}\theta\right)^{\mathrm{T}}\frac{R_p}{(d_2-d_1)}\left(\int_{-d_2}^{-d_1}E\dot{x}(t+\theta)\mathrm{d}\theta\right)\\
&\qquad - (1-\mu_d)\,\mathrm{e}^{-\alpha_pd_2}x^{\mathrm{T}}(t-d(t))Q_px(t-d(t))
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&= \alpha_p x^{\mathrm{T}}(t) E^{\mathrm{T}} P_p x(t) + 2x^{\mathrm{T}}(t) P_p^{\mathrm{T}} \left(\left(A_p + G_p K_p + \varDelta A \right) x(t) \right) \\
&\quad + \left(B_p + \varDelta B \right) x(t - d(t)) + x^{\mathrm{T}}(t) Q_p x(t) \\
&\quad + (d_2 - d_1) \dot{x}^{\mathrm{T}}(t) E^{\mathrm{T}} R_p E \dot{x}(t) - \mathrm{e}^{-\alpha_p d_2} \left(\int_{-d_2}^{-d_1} E \dot{x}(t+\theta) \mathrm{d}\theta \right)^{\mathrm{T}} \\
&\quad \times \frac{R_p}{(d_2 - d_1)} \left(\int_{-d_2}^{-d_1} E \dot{x}(t+\theta) \mathrm{d}\theta \right) \\
&\quad - (1 - \mu_d) \mathrm{e}^{-\alpha_p d_2} x^{\mathrm{T}}(t - d(t)) Q_p x(t - d(t)).
\end{aligned}
$$

By noting free matrices, for any matrices H_1 with appropriate dimensions such that

$$
0 = 2 \left(E \dot{x}(t) \right)^{\mathrm{T}} H_1^{\mathrm{T}} \left[-E \dot{x}(t) + \tilde{A}_p x(t) + \tilde{B}_p x(t - d(t)) \right] \tag{15}
$$

and also taking (12) into consideration, inequality (14) can be rewritten as

$$
\dot{V}(t) + \alpha_p V(t) \leq \xi^{\mathrm{T}} \tilde{\varPhi} \xi, \tag{16}
$$

with $\xi^{\mathrm{T}} = \left[x^{\mathrm{T}}(t) \; x^{\mathrm{T}}(t - d(t)) \; (E \dot{x}(t))^{\mathrm{T}} \; \left(\int_{-d_2}^{-d_1} E \dot{x}(t+\theta) \mathrm{d}\theta \right)^{\mathrm{T}} \right]$. We denote

$$
Y = \begin{bmatrix}
\tilde{\varPhi}_{11} & P_p^{\mathrm{T}} B & H_1^{\mathrm{T}} \left(A_p + G_p K_p \right) & 0 \\
* & \tilde{\varPhi}_{22} & H_1^{\mathrm{T}} B & 0 \\
* & * & \tilde{\varPhi}_{33} & 0 \\
* & * & * & -\mathrm{e}^{-\alpha_p d_2} \frac{R}{d_2 - d_1}
\end{bmatrix},
$$

where

$$
\begin{aligned}
\tilde{\varPhi}_{11} &= P_p^{\mathrm{T}} \left(A_p + G_p K_p \right) + \left(A_p + G_p K_p \right)^{\mathrm{T}} P_p + \alpha_p E^{\mathrm{T}} P_p + Q_p, \\
\tilde{\varPhi}_{22} &= -(1 - \mu_d) \mathrm{e}^{-\alpha_p d_2} Q_p, \\
\tilde{\varPhi}_{33} &= (d_2 - d_1) R_p - H_1 - H_1^{\mathrm{T}}.
\end{aligned}
$$

And

$$
Y_1 = \begin{bmatrix}
P_p^{\mathrm{T}} M & H_1^{\mathrm{T}} M & 0 \\
0 & 0 & H_1^{\mathrm{T}} M \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \; Y_2 = \begin{bmatrix}
N & N_d & 0 & 0 \\
0 & 0 & N & 0 \\
0 & 0 & N_d & 0
\end{bmatrix},
$$

$Y_3 = \mathrm{diag}\{F(\sigma), F(\sigma), F(\sigma)\}$. By Lemma 2, the inequality $\tilde{\varPhi} = Y + Y_1 Y_3 Y_2 + (Y_1 Y_3 Y_2)^{\mathrm{T}} < 0$ is established if and only if there exists a scalar $\varepsilon > 0$ such that $\varPhi = Y + \varepsilon Y_1 Y_1^{\mathrm{T}} + \varepsilon^{-1} Y_2^{\mathrm{T}} Y_2 < 0$. From condition (7), we have $\xi^{\mathrm{T}}(t) \varPhi \xi(t) \leq 0$ which implies $\xi^{\mathrm{T}} \tilde{\varPhi} \xi < 0$, then $\dot{V}(t) \leq -\alpha_p V(t)$. As for $t \in [t_k, t_k + \varDelta_k)$, it is similar to the result of the above deduction, then we have

$$\dot{V}(t) \leq \nu V(x) = \begin{cases} \alpha_{pq} V(t), t \in [t_k, t_k + \Delta_k) \\ -\alpha_p V(t), t \in \left[t_k + \Delta_k, t_{k+1}\right). \end{cases} \tag{17}$$

From condition (9), we can get

$$\begin{aligned} V(t_k + \Delta_k) &= x^{\mathrm{T}}(t) E^{\mathrm{T}} P_p x(t) + \int_{t-d(t)}^{t} \mathrm{e}^{-\alpha_p (t-s)} x^{\mathrm{T}}(s) Q_p x(s) \mathrm{d}s \\ &\quad + \int_{t-d_1}^{t} \mathrm{e}^{-\alpha_p (t-s)} x^{\mathrm{T}}(s) R_p x(s) \mathrm{d}s \\ &\leq \hat{\mu} V((t_k + \Delta_k)^-), \end{aligned} \tag{18}$$

$$\begin{aligned} V(t_k) &= x^{\mathrm{T}}(t) E^{\mathrm{T}} P_{pq} x(t) + \int_{t-d(t)}^{t} \mathrm{e}^{\alpha_{pq} (t-s)} x^{\mathrm{T}}(s) Q_{pq} x(s) \mathrm{d}s \\ &\quad + \int_{t-d_1}^{t} \mathrm{e}^{\alpha_{pq} (t-s)} x^{\mathrm{T}}(s) R_{pq} x(s) \mathrm{d}s \\ &\leq \hat{\mu} \mathrm{e}^{(\alpha_q + \alpha_{pq}) d_2} V(t_k^-) \\ &\leq \mu_{\mathrm{m}} \hat{\mu} V(t_k^-), \end{aligned} \tag{19}$$

where $\mu_{\mathrm{m}} = \max\limits_{p,q \in \mathcal{I}} \mathrm{e}^{(\alpha_q + \alpha_{pq}) d_2}$.

We note $T_p(s, t)$ as the total time for synchronization of the controller and subsystems in $[s, t]$. For simplicity of notation, let $T_{pq}(s, t)$ stand for total time of mismatch between controller and subsystem. Considering $t \in [t_k, t_k + \Delta_k)$ and combining (18) with (19), we have

$$\begin{aligned} V(t) &\leq \mathrm{e}^{\alpha_{pq} T_{pq}(t_k, t)} V(t_k) \\ &\leq \mu_{\mathrm{m}} \hat{\mu} \mathrm{e}^{\alpha_{pq} T_{pq}(t_k, t)} V\left(t_k^-\right) \\ &\leq \mu_{\mathrm{m}} \hat{\mu} \mathrm{e}^{\alpha_{pq} T_{pq}(t_k, t) - \alpha_p T_p(t_{k-1}, t)} V(t_{k-1} + \Delta_{k-1}) \\ &\leq \mu_{\mathrm{m}} \hat{\mu}^2 \mathrm{e}^{\alpha_{pq} T_{pq}(t_k, t) - \alpha_p T_p(t_{k-1}, t)} V\left((t_{k-1} + \Delta_{k-1})^-\right) \\ &\leq \cdots \\ &\leq \mathrm{e}^{N_0 \left(\ln \mu_{\mathrm{m}} \hat{\mu}^2 + (\alpha_{pq} + \alpha_p) \Delta_{\max}\right)} \times \mathrm{e}^{\left(\frac{\ln \mu_{\mathrm{m}} \hat{\mu}^2 + (\alpha_{pq} + \alpha_p) \Delta_{\max}}{\tau_p} - \alpha_p\right)(t - t_0)} V(t_0). \end{aligned} \tag{20}$$

Similarly, we can also get when $t \in \left[t_k + \Delta_k, t_{k+1}\right)$,

$$
\begin{aligned}
V(t) &\leq \mathrm{e}^{-\alpha_p T_p(t_k+\Delta_k,t)} V\left(t_k+\Delta_k\right) \\
&\leq \hat{\mu}\mathrm{e}^{-\alpha_p T_p(t_k+\Delta_k,t)} V\left(\left(t_k+\Delta_k\right)^-\right) \\
&\leq \hat{\mu}\mathrm{e}^{\alpha_{pq} T_{pq}(t_k,t_k+\Delta_k)-\alpha_p T_p(t_k+\Delta_k,t)} V\left(t_k\right) \\
&\leq \mu_{\mathrm{m}}\hat{\mu}^2\mathrm{e}^{\alpha_{pq} T_{pq}(t_k,t_k+\Delta_k)-\alpha_p T_p(t_k+\Delta_k,t)} V\left(t_k^-\right) \\
&\cdots \\
&\leq \frac{1}{\hat{\mu}}\mathrm{e}^{N_0\left(\ln\mu_{\mathrm{m}}\hat{\mu}^2+(\alpha_{pq}+\alpha_p)\Delta_{\max}\right)} \times \mathrm{e}^{\left(\frac{\ln\mu_{\mathrm{m}}\hat{\mu}^2+(\alpha_{pq}+\alpha_p)\Delta_{\max}}{\tau_p}-\alpha_p\right)(t-t_0)} V\left(t_0\right).
\end{aligned}
\tag{21}
$$

Let

$$
\begin{aligned}
\lambda_1 &= \min_{i,j}\lambda_{\min}(P_{\tilde{\sigma}11}), \\
\lambda_2 &= \max_{i,j}\left\{\lambda_{\max}(P_{\sigma(t)})\right\} + d_2\max\left\{\lambda_{\max}(Q_{\sigma(t)})\right\} + \tfrac{(d_2-d_1)^2}{2}\max\left\{\lambda_{\max}(R_{\sigma(t)})\right\},
\end{aligned}
$$

and set $\delta = \sqrt{\frac{\lambda_2}{\lambda_1}}\mathrm{e}^{\frac{1}{2}N_0\left(\ln\mu_{\mathrm{m}}\hat{\mu}^2+(\alpha_{pq}+\alpha_p)\Delta_{\max}\right)}$, $\eta = \frac{1}{2}\left\{\alpha_p - \frac{\ln\mu_{\mathrm{m}}\hat{\mu}^2+(\alpha_{pq}+\alpha_p)\Delta_{\max}}{\tau_p}\right\}$. Thus, we can get

$$
\lambda_1\left\|x_1(t)\right\|^2 \leq V(t),\; V(t_0) \leq \lambda_2\left\|x(t_0)\right\|_d^2.
$$

That is $\|x_1(t)\| \leq \delta e^{-\eta(t-t_0)}\|x(t_0)\|$. Here, we have proved the stability of the differential equations contained in the original system.

Secondly, the proof of the stability of the algebraic equations in the original system is given. Considering the case of $t \in [t_i, t_i+\Delta_i)$, inspired by the literature [35], some new variables are used to characterize the effects of time delay in $x_2(t)$. Define

$$
k_0 = t, k_1 = t-d(t) = k_0-d(k_0), k_2 = k_1-d(k_1), \ldots, k_i = k_{i-1}-d(k_{i-1}).
$$

It is known from the deformed system (4) that

$$
\begin{aligned}
x_2(k_0) &= \bar{A}_{pq3}x_1(k_0)+\bar{A}_{dpq3}x_1(k_1)+\bar{A}_{dpq4}x_2(k_1), \\
x_2(k_1) &= \bar{A}_{pq3}x_1(k_1)+\bar{A}_{dpq3}x_1(k_2)+\bar{A}_{dpq4}x_2(k_2), \\
x_2(k_0) &= \bar{A}_{dpq4}^3x_2(k_3)+\sum_{m=0}^{2}\bar{A}_{dpq4}^m\left[\bar{A}_{pq3}x_1(k_m)+\bar{A}_{dpq3}x_1(k_{m+1})\right].
\end{aligned}
$$

There exists an integer $N_{i,i-1}$ such that $k_{N_{i,i-1}} \in \left[t_{i-1}+\Delta_{i-1}, t_i\right), k_{N_{i,i-1}-1} \in [t_i, t_i+\Delta_i)$. By iteration, we have

$$
x_2(k_0) = \bar{A}_{dpq4}^{N_{i,i-1}}x_2(k_{N_{i,i-1}})+\sum_{m=0}^{N_{i,i-1}-1}\bar{A}_{dpq4}^m\left[\bar{A}_{pq3}x_1(k_m)+\bar{A}_{dpq3}x_1(k_{m+1})\right].
$$

Then, following a similar procedure as above, there exists integer $N_{i-1,i-1}$ such that

$$k_{N_{i-1,i-1}} \in \left[t_{i-1}, t_{i-1} + \Delta_{i-1}\right), k_{N_{i-1,i-1}-1} \in \left[t_{i-1} + \Delta_{i-1}, t_i\right).$$

By iteration, we have

$$\begin{aligned} x_2(k_{N_{i,i-1}+N_{i-1,i-1}}) = \; & \bar{A}_{dpq4}^{N_{i-1,i-2}} x_2(k_{N_{i,i-1}+N_{i-1,i-1}+N_{i-1,i-2}}) \\ & + \sum_{m_{i-1}=0}^{N_{i-1,i-2}-1} \bar{A}_{dpq4}^{m_{i-1}} \left[\bar{A}_{pq3} x_1\left(k_{m_{i-1}}\right) + \bar{A}_{dpq3} x_1\left(k_{m_{i-1}+1}\right)\right]. \end{aligned}$$

There exists an integer $k_{N_{i,i-1}+N_{i-1,i-1}+\cdots+N_0} \in [-d_2, 0)$ such that

$$\begin{aligned} x_2(k_0) = & \prod_i \bar{A}_{dpq4}^{N_{i,i-1}} \bar{A}_{dp4}^{N_{i-1,i-1}} x_2(k_{N_{i,i-1}+N_{i-1,i-1}+\cdots+N_0}) \\ & + \bar{A}_{dpq4}^{N_{i,i-1}} \bar{A}_{dp4}^{N_{i-1,i-1}} \cdots \bar{A}_{dpq4}^{N_{1,0}} \sum_{n_0=0}^{N_0-1} \bar{A}_{dp4}^{n_0} [\bar{A}_{p3} x_1(k_{N_{i,i-1}+N_{i-1,i-1}+\cdots+N_{1,1}+N_{1,0}+n_0}) \\ & + \bar{A}_{dp3} x_1(k_{N_{i,i-1}+N_{i-1,i-1}+\cdots+N_{1,1}+N_{1,0}+n_0+1})] \\ & + \cdots \\ & + \bar{A}_{dpq4}^{N_{i,i-1}} \sum_{n_{i-1}=0}^{N_{i-1,i-1}-1} \bar{A}_{dp4}^{n_{i-1}} \left[\bar{A}_{p3} x_1(k_{N_{i,i-1}+n_{i-1}}) + \bar{A}_{dp3} x_1(k_{N_{i,i-1}+n_{i-1}+1})\right] \\ & + \sum_{m_i=0}^{N_{i,i-1}-1} \bar{A}_{dpq4}^{m_i} \left[\bar{A}_{pq3} x_1\left(k_{m_i}\right) + \bar{A}_{dpq3} x_1\left(k_{m_i+1}\right)\right]. \end{aligned}$$

Let $\hat{A}_{pq3} = \max_{p,q \in \mathrm{I}} \left\|\bar{A}_{pq3}\right\|$, $\hat{A}_{dpq3} = \max_{p,q \in \mathrm{I}} \left\|\bar{A}_{dpq3}\right\|$. Inequality (7) implies that

$$\begin{bmatrix} P_{p22}^{\mathrm{T}} + P_{p22} + Q_{p22} & P_{p22}^{\mathrm{T}} \bar{A}_{dp4} \\ * & -(1-\mu)\, \mathrm{e}^{-\alpha_p d_2} Q_{p22} \end{bmatrix} < 0.$$

Then, from [8], there exist constants $\hbar_i > 1$ and $\ell_i > 0$ such that

$$\left\| \left(\mathrm{e}^{\frac{1}{2}\alpha_p d_2} \bar{A}_{dp4} \right)^N \right\| \leq \hbar_i \mathrm{e}^{-l_i N}.$$

Therefore,

$$
\begin{aligned}
\|x_2(k_0)\| \leq & \left[\mathop{\mathit{\Pi}}_{i=0}^{k} \hbar_i \mathrm{e}^{-l_i N_{i,i-1}}\right] \mathrm{e}^{-\frac{1}{2}\left(\alpha_p - \frac{\ln(\mu_{\mathrm{m}}\hat{\mu}^2)}{\tau_p}\right)(t-t_0)} \|x(t_0)\| \\
& + \hbar_i \hat{A}_{pq3}\sqrt{\frac{\lambda_2}{\lambda_1}} \frac{\mathrm{e}^{l_i}}{\mathrm{e}^{l_i}-1} \mathrm{e}^{-\frac{1}{2}\left(\alpha_p - \frac{\ln(\mu_{\mathrm{m}}\hat{\mu}^2)}{\tau_p}\right)(t-t_0)} \|x(t_0)\| \\
& + \hbar_i \mathrm{e}^{\frac{1}{2}\alpha_p d_2} \hat{A}_{dpq3} \frac{\mathrm{e}^{l_i}}{\mathrm{e}^{l_i}-1} \mathrm{e}^{-\frac{1}{2}\left(\alpha_p - \frac{\ln(\mu_{\mathrm{m}}\hat{\mu}^2)}{\tau_p}\right)(t-t_0)} \|x(t_0)\| \\
& + \cdots \\
& + \hat{A}_{pq3}\sqrt{\frac{\lambda_2}{\lambda_1}} \left(\sum_{i=1}^{k} \hbar_i \left[\mathop{\mathit{\Pi}}_{j=i}^{k} \hbar_j \mathrm{e}^{-l_j N_{j,j-1}}\right] \frac{\mathrm{e}^{l_i}}{\mathrm{e}^{l_i}-1}\right) \mathrm{e}^{-\frac{1}{2}\left(\alpha_p - \frac{\ln}{\tau_p}\right)(t-t_0)} \|x(t_0)\| \\
& + \mathrm{e}^{\frac{1}{2}\alpha_p d_2} \hat{A}_{dpq3}\sqrt{\frac{\lambda_2}{\lambda_1}} \left(\sum_{i=1}^{k} \hbar_i \left[\mathop{\mathit{\Pi}}_{j=i}^{k} \hbar_j \mathrm{e}^{-l_j N_{j,j-1}}\right] \frac{\mathrm{e}^{l_i}}{\mathrm{e}^{l_i}-1}\right) \\
& \mathrm{e}^{-\frac{1}{2}\left(\alpha_p - \frac{\ln(\mu_{\mathrm{m}}\hat{\mu}^2)}{\tau_p}\right)(t-t_0)} \|x(t_0)\| .
\end{aligned}
$$

Then, we have

$$
\|x_2(t)\| = \|x_2(k_0)\| \leq \hat{c}\mathrm{e}^{-\frac{1}{2}\bar{\lambda}(t-t_0)} \|x(t_0)\| ,
$$

which confirms exponential stability of the algebraic subsystems.

Finally, we show that the switched singular system is regular and impulse free. Equation (6) implies that $P_{\sigma 2} = 0$. From inequalities (7) and (8), $\Phi_{11} = P_{\sigma(t)}^{\mathrm{T}} \tilde{A}_{\sigma(t)} + \tilde{A}_{\sigma(t)}^{\mathrm{T}} P_{\sigma(t)} + \nu E^{\mathrm{T}} P_{\sigma(t)} + Q_{\sigma(t)} < 0$. Thus, $P_{\sigma(t)}^{\mathrm{T}} \tilde{A}_{\sigma(t)} + \tilde{A}_{\sigma(t)}^{\mathrm{T}} P_{\sigma(t)} + \nu E^{\mathrm{T}} P_{\sigma(t)} < 0$, $P_{\sigma 4}^{\mathrm{T}} A_{\sigma 4} + A_{\sigma 4}^{\mathrm{T}} P_{\sigma 4} < 0$, which implies that $A_{\sigma 4}$ is nonsingular. By [32], closed-loop system (4) is regular and impulse free. Therefore, there exists a continuous solution of the original system. This completes the proof.

Remark 1 When the controller selection lags behind the switching of subsystems, that is, the asynchronous control problem is considered in the above theorem. On the contrary, if we take $\Delta_{\max} = 0$, the theorem in this paper can be applied to general switched singular system under synchronous control.

Remark 2 When we construct the Lyapunov function, the parameters α_p and α_{pq} are model-dependent. So the dwell time that we get is also model-dependent. It is more flexible than the average dwell time obtained in [18], which is to be met by all subsystems. The multi-parameter selection is also beneficial to the feasibility solution of linear matrix inequality. In the matching period time between the subsystem and controller, the energy of the Lyapunov function corresponding to subsystem decreases. During the mismatching period between the subsystem and controller, the energy of corresponding the Lyapunov function is allowed to increase. By constraining the average dwell time, the energy of the Lyapunov function is reduced as a whole, thus ensuring the stability of the differential equations.

Theorem 2 *For the switched singular system* (1)*, let* $\gamma > 0, d_1 \geq 0, d_2 > 0, \Delta_{\max} > 0, \mu_d < 1, \alpha_p > 0, \alpha_{pq} > 0, \hat{\mu} > 1$*, if there exist matrices* $P_p, Q_p > 0, R_p > 0, P_{pq}, Q_{pq} > 0, R_{pq} > 0, \varepsilon > 0, \delta > 0$*, such that* (6)*,* (9) *and the following inequalities hold*

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & H_1^{\mathrm{T}}(A_p + G_p K_p) & 0 & \Theta_{15} \\ * & \Theta_{22} & H_1^{\mathrm{T}} B_p & 0 & D_p^{\mathrm{T}} F_p \\ * & * & \Theta_{33} & 0 & H_1^{\mathrm{T}} E_p \\ * & * & * & -\mathrm{e}^{-\alpha_p d_2} \frac{R_p}{(d_2-d_1)} & 0 \\ * & * & * & * & -\gamma^2 I + F_p^{\mathrm{T}} F_p \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & H_1^{\mathrm{T}}(A_p + G_p K_q) & 0 & \Sigma_{15} \\ * & \Sigma_{22} & H_1^{\mathrm{T}} B_p & 0 & D_p^{\mathrm{T}} F_p \\ * & * & \Sigma_{33} & 0 & H_1^{\mathrm{T}} E_p \\ * & * & * & -\mathrm{e}^{\alpha_{pq} d_2} \frac{R_{pq}}{(d_2-d_1)} & 0 \\ * & * & * & * & -\gamma^2 I + F_p^{\mathrm{T}} F_p \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned}
\Theta_{11} &= P_p^{\mathrm{T}} \left(A_p + G_p K_p\right) + \left(A_p + G_p K_p\right)^{\mathrm{T}} P_p + \alpha_p E^{\mathrm{T}} P_p + Q_p + \varepsilon^{-1} N^{\mathrm{T}} N \\
&\quad + \varepsilon \left(P_p^{\mathrm{T}} M M^{\mathrm{T}} P_p + H_1^{\mathrm{T}} M M^{\mathrm{T}} H_1\right) + C_p^{\mathrm{T}} C_p, \\
\Theta_{12} &= P_p^{\mathrm{T}} B_p + \varepsilon^{-1} N^{\mathrm{T}} N_d + C_p^{\mathrm{T}} D_p, \Theta_{15} = C_p^{\mathrm{T}} F_p + P_p^{\mathrm{T}} E_p, \\
\Theta_{22} &= -\left(1 - \mu_d\right) \mathrm{e}^{-\alpha_p d_2} Q_p + \varepsilon H_1^{\mathrm{T}} M M^{\mathrm{T}} H_1 + \varepsilon^{-1} N_d^{\mathrm{T}} N_d + D_p^{\mathrm{T}} D_p, \\
\Theta_{33} &= (d_2 - d_1) R_p - H_1 - H_1^{\mathrm{T}} + \varepsilon^{-1} \left(N^{\mathrm{T}} N + N_d^{\mathrm{T}} N_d\right), \\
\Sigma_{11} &= P_{pq}^{\mathrm{T}} \left(A_p + G_p K_q\right) + \left(A_p + G_p K_q\right)^{\mathrm{T}} P_{pq} - \alpha_{pq} E^{\mathrm{T}} P_{pq} + Q_{pq} + \delta^{-1} N^{\mathrm{T}} N \\
&\quad + \delta \left(P_{pq}^{\mathrm{T}} M M^{\mathrm{T}} P_{pq} + H_1^{\mathrm{T}} M M^{\mathrm{T}} H_1\right) + C_p^{\mathrm{T}} C_p, \\
\Sigma_{12} &= P_{pq}^{\mathrm{T}} B_p + \varepsilon^{-1} N^{\mathrm{T}} N_d + C_p^{\mathrm{T}} D_p, \Sigma_{15} = C_p^{\mathrm{T}} F_p + P_{pq}^{\mathrm{T}} E_p, \\
\Sigma_{22} &= -\left(1 - \mu_d\right) \mathrm{e}^{\alpha_{pq} d_1} Q_{pq} + \delta H_1^{\mathrm{T}} M M^{\mathrm{T}} H_1 + \delta^{-1} N_d^{\mathrm{T}} N_d + D_p^{\mathrm{T}} D_p, \\
\Sigma_{33} &= (d_2 - d_1) R_{pq} - H_1 - H_1^{\mathrm{T}} + \delta^{-1} \left(N^{\mathrm{T}} N + N_d^{\mathrm{T}} N_d\right).
\end{aligned}$$

Then, the looped system is exponential admissibility with H_∞ *performance level* $\hat{\gamma}$ *under the switching signal with average dwell time satisfying* (10)*, where* $\hat{\gamma} = \sqrt{\mathrm{e}^{(\alpha_{pq}+\alpha_p)\Delta_{\max}}}\gamma$.

Proof Inequality constraints (7) and (8) can be obtained from (22) and (23). Thus, by Theorem 1, switched singular system (1) is exponentially admissible with $\omega(t) = 0$. Then, the analysis of the H_∞ performance of the system (1) is given.

When $t \in \left[t_{k+\Delta k}, t_{k+1}\right)$, constructing the Lyapunov function as above and using the same method in Theorem 1, we have

$$\dot{V}(t, x(t)) \leq -\alpha_p V\left(t, x(t)\right) - \Gamma\left(t\right) + \eta^{\mathrm{T}} \Lambda \eta,$$

where

$$\begin{aligned}
&\eta = \left[x^{\mathrm{T}}\left(t\right) \; x^{\mathrm{T}}\left(t-d(t)\right) \; \left(E\dot{x}\left(t\right)\right)^{\mathrm{T}} \; \left(\int_{-d_2}^{-d_1} E\dot{x}(t+\theta)d\theta\right)^{\mathrm{T}} \; \omega^{T}\left(t\right) \right]^{\mathrm{T}}, \\
&\Gamma\left(t\right) = z^{\mathrm{T}}\left(t\right) z\left(t\right) - \gamma^2 \omega^{\mathrm{T}}\left(t\right) \omega\left(t\right).
\end{aligned}$$

From the condition (22), we have $\eta^{\mathrm{T}} \Lambda \eta < 0$, then

$$\dot{V}(t, x(t)) \leq -\alpha_p V\left(t, x(t)\right) - \Gamma\left(t\right), t \in \left[t_{k+\Delta k}, t_{k+1}\right). \tag{24}$$

Similarly, we also have

$$\dot{V}(t, x(t)) \leq \alpha_{pq} V\left(t, x(t)\right) - \Gamma\left(t\right), t \in \left[t_k, t_{k+\Delta k}\right). \tag{25}$$

Integrating both sides of (24) and (25), it holds that

$$\begin{aligned}
V\left(t\right) &\leq \mathrm{e}^{-\alpha_p\left(t-t_{k+\Delta k}\right)} V\left(t_{k+\Delta k}\right) - \int_{t_{k+\Delta k}}^{t} \mathrm{e}^{-\alpha_p(t-s)} \Gamma\left(s\right) \mathrm{d}s, \\
V\left(t_{k+\Delta k}\right) &\leq \mathrm{e}^{\alpha_{pq}\Delta_k} V\left(t_k\right) - \int_{t_k}^{t_{k+\Delta k}} \mathrm{e}^{\alpha_{pq}\left(t_{k+\Delta k}-s\right)} \Gamma\left(s\right) \mathrm{d}s.
\end{aligned} \tag{26}$$

After many iterations, we have

$$\begin{aligned}
V\left(t\right) \leq\; & \mathrm{e}^{-\alpha_p\left(t-t_{k+\Delta k}\right)} \left[\mathrm{e}^{\alpha_{pq}\Delta_k} V\left(t_k\right) - \int_{t_k}^{t_{k+\Delta k}} \mathrm{e}^{\alpha_{pq}\left(t_{k+\Delta k}-s\right)} \Gamma\left(s\right) \mathrm{d}s \right] \\
& - \int_{t_{k+\Delta k}}^{t} \mathrm{e}^{-\alpha_p(t-s)} \Gamma\left(s\right) \mathrm{d}s \\
\leq\; & \mathrm{e}^{-\alpha_p\left(t-t_k\right)+\left(\alpha_p+\alpha_{pq}\right)\Delta_k} \mu_{\mathrm{m}} \hat{\mu} V\left(t_k^{-}\right) \\
& - \int_{t_k}^{t_{k+\Delta k}} \mathrm{e}^{-\alpha_p(t-s)+\left(\alpha_p+\alpha_{pq}\right)\left(t_{k+\Delta k}-s\right)} \Gamma\left(s\right) \mathrm{d}s \\
& - \int_{t_{k+\Delta k}}^{t} \mathrm{e}^{-\alpha_p(t-s)} \Gamma\left(s\right) \mathrm{d}s \\
\leq\; & \cdots
\end{aligned}$$

$$
\begin{aligned}
&\leq \mu_{\mathrm{m}}^{k} \hat{\mu}^{2k} \mathrm{e}^{\alpha_{pq} T_{pq}(0,t) - \alpha_p T_p(0,t)} V(t_0) \\
&\quad - \int_{t_0}^{t} \mathrm{e}^{\alpha_{pq} T_{pq}(s,t) - \alpha_p T_p(s,t) + N_\sigma(s,t) \ln \mu_{\mathrm{m}} \hat{\mu}^2} \Gamma(s)\, \mathrm{d}s \\
&\leq \mu_{\mathrm{m}}^{k} \hat{\mu}^{2k} \mathrm{e}^{\alpha_{pq} T_{pq}(0,t) - \alpha_p T_p(0,t)} V(t_0) \\
&\quad - \int_{t_0}^{t} \mathrm{e}^{(\alpha_{pq}+\alpha_p) N_\sigma(s,t) \Delta_{\max} - \alpha_p (t-s) + N_\sigma(s,t) \ln \mu_{\mathrm{m}} \hat{\mu}^2} \Gamma(s)\, \mathrm{d}s.
\end{aligned} \tag{27}
$$

Under the zero initial condition,

$$
\int_{t_0}^{t} \mathrm{e}^{\sum_{p=1}^{N} [(\alpha_{pq}+\alpha_p) N_\sigma(s,t) \Delta_{\max} - \alpha_p (t-s) + N_\sigma(s,t) \ln \mu_{\mathrm{m}} \hat{\mu}^2]} \Gamma(s)\, \mathrm{d}s \leq 0. \tag{28}
$$

Then, multiplying both sides by $\mathrm{e}^{\sum_{p=1}^{N} [-(\alpha_{pq}+\alpha_p) N_\sigma(0,t) \Delta_{\max} - N_\sigma(0,t) \ln \mu_{\mathrm{m}} \hat{\mu}^2]}$,

$$
\int_{t_0}^{t} \mathrm{e}^{\sum_{p=1}^{N} [-(\alpha_{pq}+\alpha_p) N_\sigma(0,s) \Delta_{\max} - \alpha_p (t-s) - N_\sigma(0,s) \ln \mu_{\mathrm{m}} \hat{\mu}^2]} \Gamma(s)\, \mathrm{d}s \leq 0.
$$

Combining (5) and (10) leads to

$$
\int_{t_0}^{t} \mathrm{e}^{\sum_{p=1}^{N} [-\alpha_p t - (\alpha_{pq}+\alpha_p) \Delta_{\max} - N_0 \ln \mu_{\mathrm{m}} \hat{\mu}^2]} z^{\mathrm{T}}(s) z(s)\, \mathrm{d}s \leq \int_{t_0}^{t} \mathrm{e}^{-\sum_{p=1}^{N} \alpha_p (t-s)} \gamma^2 \omega^{\mathrm{T}}(s) \omega(s)\, \mathrm{d}s.
$$

Integrating both sides of it from $t = t_0$ to ∞ yields

$$
\begin{aligned}
&\int_{t_0}^{\infty} \int_{t_0}^{t} \mathrm{e}^{-\sum_{p=1}^{N} \alpha_p t} z^{\mathrm{T}}(s) z(s)\, \mathrm{d}s \mathrm{d}t \leq \hat{\gamma}^2 \int_{t_0}^{\infty} \int_{t_0}^{t} \mathrm{e}^{-\sum_{p=1}^{N} \alpha_p (t-s)} \omega^{\mathrm{T}}(s) \omega(s)\, \mathrm{d}s \mathrm{d}t, \\
&\int_{t_0}^{\infty} \mathrm{e}^{-\sum_{p=1}^{N} \alpha_p s} z^{\mathrm{T}}(s) z(s)\, \mathrm{d}s \leq \hat{\gamma}^2 \int_{t_0}^{\infty} \omega^{\mathrm{T}}(s) \omega(s)\, \mathrm{d}s,
\end{aligned}
$$

where $\hat{\gamma} = \mathrm{e}^{\frac{1}{2}[\sum_{p=1}^{N} (\alpha_{pq}+\alpha_p) \Delta_{\max} + N_0 \ln(\mu_{\mathrm{m}} \hat{\mu}^2)]} \gamma$. This means that system (4) achieves H_∞ performance level. The proof is completed. □

Corollary 1 *Given the same conditions as Theorem 2, then the looped system (1) is exponential admissibility with non-weighted H_∞ performance level $\tilde{\gamma}$ under the switching signal with average dwell time satisfying (10), where*

$$
\tilde{\gamma}^2 = \sum_{p=1}^{N} \alpha_p \mathrm{e}^{\sum_{p=1}^{N} [(\alpha_{pq}+\alpha_p) \Delta_{\max} + \ln \mu_{\mathrm{m}} \hat{\mu}^2] N_0} \gamma^2. \tag{29}
$$

Proof From (28), we have

$$
\begin{aligned}
&\int_{t_0}^{t} e^{\sum\limits_{p=1}^{N}[(\alpha_{pq}+\alpha_p)N_\sigma(s,t)\Delta_{\max}-\alpha_p(t-s)+N_\sigma(s,t)\ln\mu_{\mathrm{m}}\hat{\mu}^2]} z^{\mathrm{T}}(s)\,z(s)\,\mathrm{d}s \\
&\le \int_{t_0}^{t} e^{\sum\limits_{p=1}^{N}[(\alpha_{pq}+\alpha_p)N_\sigma(s,t)\Delta_{\max}-\alpha_p(t-s)+N_\sigma(s,t)\ln\mu_{\mathrm{m}}\hat{\mu}^2]} \gamma^2\omega^{\mathrm{T}}(s)\,\omega(s)\,\mathrm{d}s.
\end{aligned}
$$

Integrating the left side from t_0 to ∞, we can obtain

$$
\begin{aligned}
&\int_{t_0}^{\infty}\int_{t_0}^{t} e^{\sum\limits_{p=1}^{N}[(\alpha_{pq}+\alpha_p)N_\sigma(s,t)\Delta_{\max}-\alpha_p(t-s)+N_\sigma(s,t)\ln\mu_{\mathrm{m}}\hat{\mu}^2]} z^{\mathrm{T}}(s)\,z(s)\,\mathrm{d}s\mathrm{d}t \\
&\quad\ge \int_{t_0}^{\infty}\int_{t_0}^{t} e^{\sum\limits_{p=1}^{N}-\alpha_p(t-s)} z^{\mathrm{T}}(s)\,z(s)\,\mathrm{d}s\mathrm{d}t \\
&\quad\ge \int_{t_0}^{\infty} z^{\mathrm{T}}(s)\,z(s)\left(\int_{s}^{\infty} e^{\sum\limits_{p=1}^{N}-\alpha_p(t-s)}\mathrm{d}t\right)\mathrm{d}s \\
&\quad\ge \frac{1}{\sum\limits_{p=1}^{N}\alpha_p}\int_{t_0}^{\infty} z^{\mathrm{T}}(s)\,z(s)\,\mathrm{d}s.
\end{aligned}
$$

At the same time, the integral result of the right term is

$$
\begin{aligned}
&\int_{t_0}^{\infty}\int_{t_0}^{t} e^{\sum\limits_{p=1}^{N}[(\alpha_{pq}+\alpha_p)N_\sigma(s,t)\Delta_{\max}-\alpha_p(t-s)+N_\sigma(s,t)\ln\mu_{\mathrm{m}}\hat{\mu}^2]} \gamma^2\omega^{\mathrm{T}}(s)\,\omega(s)\,\mathrm{d}s\mathrm{d}t \\
&\le \gamma^2\int_{t_0}^{\infty}\int_{t_0}^{t} e^{\sum\limits_{p=1}^{N}[(\alpha_{pq}+\alpha_p)\Delta_{\max}+\ln\mu_{\mathrm{m}}\hat{\mu}^2](N_0+\frac{t-s}{\tau_p})-\alpha_p(t-s)} \omega^{\mathrm{T}}(s)\,\omega(s)\,\mathrm{d}s\mathrm{d}t \\
&\le e^{\sum\limits_{p=1}^{N}[(\alpha_{pq}+\alpha_p)\Delta_{\max}+\ln\mu_{\mathrm{m}}\hat{\mu}^2]N_0} \gamma^2\int_{t_0}^{\infty}\int_{t_0}^{t} e^{\sum\limits_{p=1}^{N}[\frac{(\alpha_{pq}+\alpha_p)\Delta_{\max}+\ln\mu_{\mathrm{m}}\hat{\mu}^2}{\tau_p}-\alpha_p](t-s)} \\
&\quad\omega^{\mathrm{T}}(s)\,\omega(s)\,\mathrm{d}s\mathrm{d}t \\
&\qquad\le e^{\sum\limits_{p=1}^{N}[(\alpha_{pq}+\alpha_p)\Delta_{\max}+\ln\mu_{\mathrm{m}}\hat{\mu}^2]N_0} \gamma^2\int_{t_0}^{\infty}\omega^{\mathrm{T}}(s)\,\omega(s)
\end{aligned}
$$

$$\left(\int_s^\infty \mathrm{e}^{\sum_{p=1}^{N}\left[\frac{(\alpha_{pq}+\alpha_p)\Delta_{\max}+\ln\mu_{\mathrm{m}}\hat{\mu}^2}{\tau_p}-\alpha_p\right](t-s)}\mathrm{d}t\right)\mathrm{d}s$$
$$\le \mathrm{e}^{\sum_{p=1}^{N}[(\alpha_{pq}+\alpha_p)\Delta_{\max}+\ln\mu_{\mathrm{m}}\hat{\mu}^2]N_0}\gamma^2\int_{t_0}^\infty \omega^{\mathrm{T}}(s)\,\omega(s)\,\mathrm{d}s.$$

Accordingly, we can get $\int_{t_0}^{\infty} z^{\mathrm{T}}(s)\, z(s)\,\mathrm{d}s \le \tilde{\gamma}^2 \int_{t_0}^{\infty} \omega^{\mathrm{T}}(s)\,\omega(s)\,\mathrm{d}s$, which implies that the system (1) has a non-weighted L_2 gain as (29). Then, this corollary is proved. □

Theorem 3 *For the switched singular system* (1)*, let* $\gamma > 0, d_1 \ge 0, d_2 > 0, \mu_d < 1, \alpha_p > 0, \alpha_{pq} > 0, \hat{\mu} > 1$*, if there exist matrices* $\bar{P}_p > 0, \bar{Q}_p > 0, \bar{R}_p > 0, W_p, \mathscr{Q}_p, \bar{P}_{pq} > 0, \bar{Q}_{pq} > 0, \bar{R}_{pq} > 0, \mathscr{Q}_{pq}$ *and* $\varepsilon_p > 0, \varepsilon_{pq} > 0$*, such that* (9) *and the following inequalities hold*

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & E_p\Omega(\bar{P}_p,\mathscr{Q}_p) & \Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}}C_p^{\mathrm{T}} & \Xi_{17} \\ * & \Xi_{22} & \Omega(\bar{P}_p,\mathscr{Q}_p)B_p^{\mathrm{T}} & 0 & 0 & \Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}}D_p^{\mathrm{T}} & \Xi_{27} \\ * & * & \Xi_{33} & 0 & E_p\Omega(\bar{P}_p,\mathscr{Q}_p) & 0 & \Xi_{37} \\ * & * & * & \Xi_{44} & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & \Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}}F_p^{\mathrm{T}} & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -\varepsilon_p I \end{bmatrix} < 0 \quad (30)$$

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & E_p\Omega(\bar{P}_{pq},\mathscr{Q}_{pq}) & \Omega(\bar{P}_{pq},\mathscr{Q}_{pq})^{\mathrm{T}}C_p^{\mathrm{T}} & \Pi_{17} \\ * & \Pi_{22} & \Pi_{23} & 0 & 0 & \Pi_{26} & \Pi_{27} \\ * & * & \Pi_{33} & 0 & E_p\Omega(\bar{P}_{pq},\mathscr{Q}_{pq}) & 0 & \Pi_{37} \\ * & * & * & \Pi_{44} & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & \Omega(\bar{P}_{pq},\mathscr{Q}_{pq})^{\mathrm{T}}F_p^{\mathrm{T}} & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -\varepsilon_p I \end{bmatrix} < 0 \quad (31)$$

where

$$\Xi_{11} = \Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}}A_p^{\mathrm{T}} + A_p\Omega(\bar{P}_p,\mathscr{Q}_p) + W_p^{\mathrm{T}}G_p^{\mathrm{T}} + G_pW_p + \alpha_pE^{\mathrm{T}}\Omega(\bar{P}_p,\mathscr{Q}_p) + \bar{Q}_p,$$
$$\Xi_{12} = B_p\Omega(\bar{P}_p,\mathscr{Q}_p),\ \Xi_{13} = A_p\Omega(\bar{P}_p,\mathscr{Q}_p) + G_pW_p,\ \Xi_{22} = -(1-\mu_d)\,\mathrm{e}^{-\alpha_pd_2}\bar{Q}_p,$$
$$\Xi_{33} = (d_2-d_1)\,\bar{R}_p - \Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}} - \Omega(\bar{P}_p,\mathscr{Q}_p),\ \Xi_{44} = -\mathrm{e}^{-\alpha_pd_2}\frac{\bar{R}_p}{d_2-d_1},$$
$$\Xi_{17} = \left[\Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}}N^{\mathrm{T}}\ 0\ 2M\ 0\right],\ \Xi_{27} = \left[\Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}}N_d^{\mathrm{T}}\ 0\ 0\ M\right],$$
$$\Xi_{37} = \left[\Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}}N^{\mathrm{T}}\ \Omega(\bar{P}_p,\mathscr{Q}_p)^{\mathrm{T}}N_d^{\mathrm{T}}\ 0\ 0\right].$$

$$\begin{aligned}
\varPi_{11} &= \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq})^{\mathrm{T}}(A_p + G_p K_q)^{\mathrm{T}} + (A_p + G_p K_q)\varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq}) \\
&\quad - \alpha_{pq} E^{\mathrm{T}} \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq}) \\
&\quad + \bar{Q}_{pq}, \varPi_{12} = B_p \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq}), \varPi_{13} = A_p \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq}) + G_p W_p, \\
\varPi_{22} &= -(1-\mu_d)\,\mathrm{e}^{\alpha_{pq} d_1}\,\bar{Q}_p, \varPi_{23} = \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq}) B_p^{\mathrm{T}}, \varPi_{26} = \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq})^{\mathrm{T}} D_p^{\mathrm{T}}, \\
\varPi_{33} &= (d_2 - d_1)\,\bar{R}_{pq} - \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq})^{\mathrm{T}} - \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq}), \varPi_{44} = -\mathrm{e}^{\alpha_{pq} d_2} \frac{\bar{R}_{pq}}{d_2 - d_1}, \\
\varPi_{17} &= \left[\, \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq})^{\mathrm{T}} N^{\mathrm{T}} \;\; 0 \;\; 2M \;\; 0 \,\right], \varPi_{27} = \left[\, \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq})^{\mathrm{T}} N_d^{\mathrm{T}} \;\; 0 \;\; 0 \;\; M \,\right], \\
\varPi_{37} &= \left[\, \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq})^{\mathrm{T}} N^{\mathrm{T}} \;\; \varOmega(\bar{P}_{pq}, \mathscr{Q}_{pq})^{\mathrm{T}} N_d^{\mathrm{T}} \;\; 0 \;\; 0 \,\right].
\end{aligned}$$

Then, the looped system (4) *is exponential admissibility with H_∞ performance level $\hat{\gamma}$ under the switching signal with average dwell time satisfying* (10), *where $\hat{\gamma} = \sqrt{e^{(\alpha_{pq}+\alpha_p)\Delta_{\max}}}\gamma$. And the feedback controller can be chosen as*

$$u(t) = W_p \varOmega(\bar{P}_p, \mathscr{Q}_p)^{-1} x(t). \tag{32}$$

Here, $\varOmega(\bar{P}_p, \mathscr{Q}_p) = \bar{P}_p E^{\mathrm{T}} + S\mathscr{Q}_p$, $\bar{P}_p > 0$, $S \in \mathbb{R}^{n-r}$ is any matrix with full column rank and satisfies $ES = 0$.

Proof In the process of proving Theorem 1, H_1 is an arbitrary matrix with suitable dimensions. So Theorem 1 is still established when we select $H_1 = P_p$. Pre-multiply and post-multiply the matrix (22) by diag$\{\bar{P}_p^{\mathrm{T}}, \bar{P}_p^{\mathrm{T}}, \bar{P}_p^{\mathrm{T}}, \bar{P}_p^{\mathrm{T}}, \bar{P}_p^{\mathrm{T}}\}$ and it's transposition, and denote $\bar{P}_p = P_p^{-1}$, $\bar{Q}_p = \bar{P}_p^{\mathrm{T}} Q_p \bar{P}_p$, $\bar{R}_p = \bar{P}_p^{\mathrm{T}} R_p \bar{P}_p$, and using Schur complement the following inequality can be obtained

$$\begin{bmatrix}
\tilde{\varXi}_{11} & \tilde{\varXi}_{12} & (A_p + G_p K_p)\bar{P}_p & 0 & E_p \bar{P}_p & \bar{P}_p^{\mathrm{T}} C_p^{\mathrm{T}} & \tilde{\varXi}_{17} \\
* & \tilde{\varXi}_{22} & B_p \bar{P}_p & 0 & 0 & \bar{P}_p^{\mathrm{T}} D_p^{\mathrm{T}} & \tilde{\varXi}_{27} \\
* & * & \tilde{\varXi}_{33} & 0 & E_p \bar{P}_p & 0 & \tilde{\varXi}_{37} \\
* & * & * & -\mathrm{e}^{-\alpha_p d_2} \frac{\bar{R}_p}{(d_2 - d_1)} & 0 & 0 & 0 \\
* & * & * & * & -\gamma^2 I & \bar{P}_p^{\mathrm{T}} F_p^{\mathrm{T}} & 0 \\
* & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & -\varepsilon I
\end{bmatrix} < 0, \tag{33}$$

$$\begin{aligned}
\tilde{\varXi}_{11} &= \bar{P}_p^{\mathrm{T}} \left(A_p + G_p K_p\right)^{\mathrm{T}} + \left(A_p + G_p K_p\right) \bar{P}_p + \alpha_p E^{\mathrm{T}} \bar{P}_p + \bar{Q}_p, \tilde{\varXi}_{12} = B_p \bar{P}_p, \\
\tilde{\varXi}_{22} &= -(1-\mu_d)\,\mathrm{e}^{-\alpha_p d_2}\,\bar{Q}_p, \tilde{\varXi}_{33} = (d_2 - d_1)\bar{R}_p - \bar{P}_p - \bar{P}_p^{\mathrm{T}}, \\
\tilde{\varXi}_{17} &= \left[\, \bar{P}_p^{\mathrm{T}} N^{\mathrm{T}} \;\; 0 \;\; 2M \;\; 0 \,\right], \tilde{\varXi}_{27} = \left[\, \bar{P}_p^{\mathrm{T}} N_d^{\mathrm{T}} \;\; 0 \;\; 0 \;\; M \,\right], \\
\tilde{\varXi}_{37} &= \left[\, \bar{P}_p^{\mathrm{T}} N^{\mathrm{T}} \;\; \bar{P}_p^{\mathrm{T}} N_d^{\mathrm{T}} \;\; 0 \;\; 0 \,\right].
\end{aligned}$$

It is noted that in Theorem 2, conditions (6) and (33) are non-strict LMI due to the existence of equality (6). By following [32], replacing $\bar{P}_p$ in (33) with $\varOmega(\bar{P}_p, \mathscr{Q}_p)$, here $\varOmega(\bar{P}_p, \mathscr{Q}_p) = \bar{P}_p E^{\mathrm{T}} + S\mathscr{Q}_p$, $\bar{P}_p > 0$, and $S \in \mathbb{R}^{n-r}$ is any matrix with full column rank and satisfies $ES = 0$. Let $W_p = K_p \varOmega(\bar{P}_p, \mathscr{Q}_p)$ and using Schur complement, it can be easily found that condition (30) is equivalent to (33), inequality

(30) is equivalent to combination of (6) and (22). Similarly, formula (31) is equivalent to formula (23). The proof is completed. □

Remark 3 The singular system is more extensive than the general system because it contains algebraic constraints. In the problem statement section, we give the condition that rank $E = r \leq n$. When $r = n$, the switched singular system naturally degenerates into a switched general system and the proof of Theorems 2 and 3 is also tenable.

Remark 4 With the restrictive condition (6), instead of the exact solution, an approximate solution is obtained. In order to solve the controller more conveniently, sufficient conditions for solving the controller are transformed into strict linear matrix inequalities in this paper.

4 Numerical Example

In this section, two numerical examples are given to illustrate the correctness and validity of the theorems in this paper.

Example 1 Consider the uncertain switched singular system (1), where the parameter matrices of each subsystem are: Subsystem 1:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0.2 \\ 0.7 & -1.2 \end{bmatrix}, B_1 = \begin{bmatrix} -0.9 & 0 \\ 0.2 & -0.3 \end{bmatrix}, C_1 = \begin{bmatrix} 0.4 & 0.1 \\ 0.3 & -0.6 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 1.1 & 1.2 \\ -0.4 & 0.1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.02 \end{bmatrix}, F_1 = \begin{bmatrix} -0.1 & 0 \\ -0.6 & 0.5 \end{bmatrix}, G_1 = \begin{bmatrix} -1.2 \\ 0.1 \end{bmatrix};$$

Subsystem 2:

$$A_2 = \begin{bmatrix} -1.4 & 0.2 \\ 0.9 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 0.6 \\ 0 & -0.1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & -0.6 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0.4 & 0.9 \\ -0.7 & 0.2 \end{bmatrix}, E_2 = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.03 \end{bmatrix}, F_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.3 \end{bmatrix}, G_2 = \begin{bmatrix} -1.1 \\ 0.4 \end{bmatrix}.$$
$$M = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.04 \end{bmatrix}, N = \begin{bmatrix} -0.04 & 0 \\ 0 & -0.03 \end{bmatrix}, N_d = \begin{bmatrix} -0.05 & 0 \\ 0 & 0.02 \end{bmatrix}.$$

Let $d(t) = 0.3 + 0.2\sin(t), \alpha_1 = 0.6, \alpha_2 = 0.3, \gamma = 0.5, S = [0; 0.6]$, then $d_1 = 0.1, d_2 = 0.5, \mu_d = 0.2$. According to Theorem 1, we can obtain matrices $\bar{P}_1, W_1, \mathcal{Q}_1, \bar{P}_2, W_2, \mathcal{Q}_2$ by solving linear matrix inequality (30).

$$\bar{P}_1 = \begin{bmatrix} 0.6184 & 0 \\ 0 & 1.1885 \end{bmatrix}, W_1 = \begin{bmatrix} 1.1886 \\ -0.0105 \end{bmatrix}^{\mathrm{T}}, \mathcal{Q}_1 = \begin{bmatrix} 0.5627 \\ 1.8514 \end{bmatrix}^{\mathrm{T}},$$
$$\bar{P}_2 = \begin{bmatrix} 0.9290 & 0 \\ 0 & 2.3771 \end{bmatrix}, W_2 = \begin{bmatrix} 3.4468 \\ -0.9077 \end{bmatrix}^{\mathrm{T}}, \mathcal{Q}_2 = \begin{bmatrix} 1.7973 \\ 3.4963 \end{bmatrix}^{\mathrm{T}}.$$

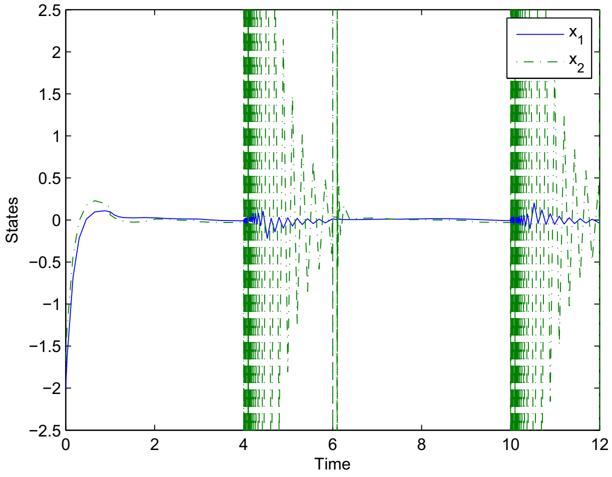

Fig. 1 State response of the closed-loop system with $\tau_1 = 1$, $\tau_2 = 2$, $\Delta_{\max} = 0.1$

Then, the controllers corresponding to each subsystem can be calculated by (32), the results are

$$K_1 = \begin{bmatrix} 1.9270 & -0.0095 \end{bmatrix}, \; K_2 = \begin{bmatrix} 4.2125 & -0.4327 \end{bmatrix}, \tag{34}$$

respectively. By selecting parameters $\alpha_{12} = 0.4$, $\alpha_{21} = 0.2$, $\Delta_{\max} = 0.1$, $\hat{\mu} = 9.7$, then we can calculate $\mu_m = 1.4918$, $\tau_1^* = 8.41619$, $\tau_2^* = 16.6657$ through condition (10). First, we do not follow the result of Theorem 1 and choose $\tau_1 = 1$, $\tau_2 = 2$. Pulse phenomenon of state response is discovered at switching time in Fig. 1. The result of Fig. 1 shows that a smaller dwell time may be impracticable for asynchronous switching. Then, use the result of Theorem 1, select initial state $x(0) = [-2, -1.5]^{\mathrm{T}}$, $F(t) = \mathrm{diag}\{\sin t, \cos t\}$, $\tau_1 = 8.5$, $\tau_2 = 16.7$, $\omega(t) = [\sin(t), \cos(t)]^{\mathrm{T}}$. Then, the state response of the closed-loop system can be simulated as shown in Fig. 2. In order to illustrate the proposed results, the comparison between synchronous switching signal and asynchronous switching signal is given. Let $\Delta_{\max} = 0$, we generate the dwell time $\tau_1^* = 3.6$, $\tau_2^* = 7.2$, this means that the switching of controller and subsystem is synchronous. The corresponding state responses of the linear switched singular systems are shown in Fig. 3. Obviously, the state of the system is not asymptotically stable under the control of synchronous switching signal. The result indicates that the switched singular system exhibits stability when the switching sequences are selected appropriately. At the same time, the performance index of the system is $\hat{\gamma} = 1.08$.

Example 2 Delay-dependent switching system can be applied to the problem of river pollution control [9,28,40]. We consider a set of numerical simulations according to the practical significance of coefficient matrices, where $E = I$ and the parameter matrices of each subsystem are:

$$A_1 = \begin{bmatrix} -1.5 & 0 \\ -2.7 & -3.4 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0.5 & 1.2 \\ 0 & 0.5 \end{bmatrix}, \; G_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix};$$

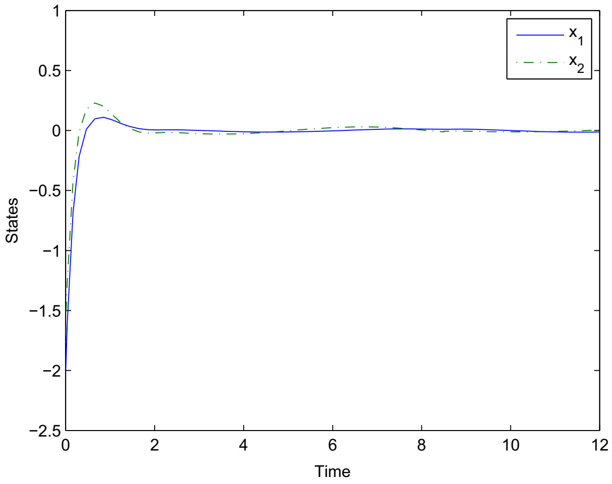

Fig. 2 State response of the closed-loop system with $\tau_1 = 8.5, \tau_2 = 16.7, \Delta_{\max} = 0.1$ and controller gains (34)

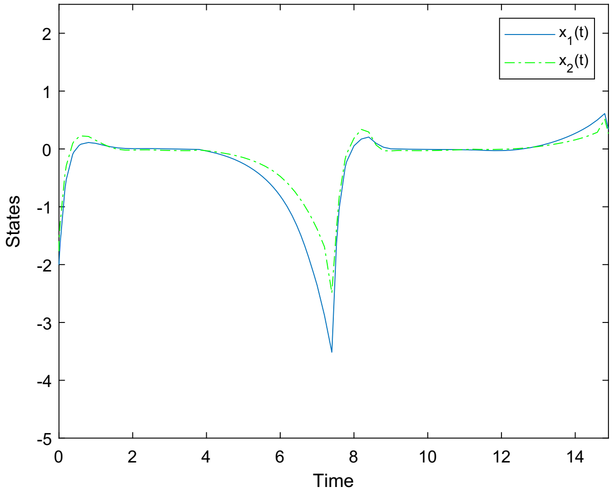

Fig. 3 State response of the closed-loop system with $\tau_1 = 3.6, \tau_2 = 7.2, \Delta_{\max} = 0$

$$A_2 = \begin{bmatrix} -2 & 0 \\ -3.2 & -1.6 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, \; G_2 = \begin{bmatrix} 0.9 & 0 \\ 0 & 1 \end{bmatrix};$$

Let $d(t) = 0.1 + 0.1 \sin t, d_1 = 0, d_2 = 0.2, \Delta_{\max} = 0.5, \alpha_1 = \alpha_2 = 0.8, \alpha_{12} = \alpha_{21} = 1.5, \hat{\mu} = 3, x_0 = [1.6 - 0.9]^{\mathrm{T}}$. Then, we can obtain controller gains by employing Theorem 0.3,

$$K_1 = \begin{bmatrix} -0.0963 & -0.7819 \\ 2.8122 & 1.9542 \end{bmatrix}, \; K_2 = \begin{bmatrix} 0.4307 & -0.0000 \\ 3.2000 & -0.0123 \end{bmatrix}.$$

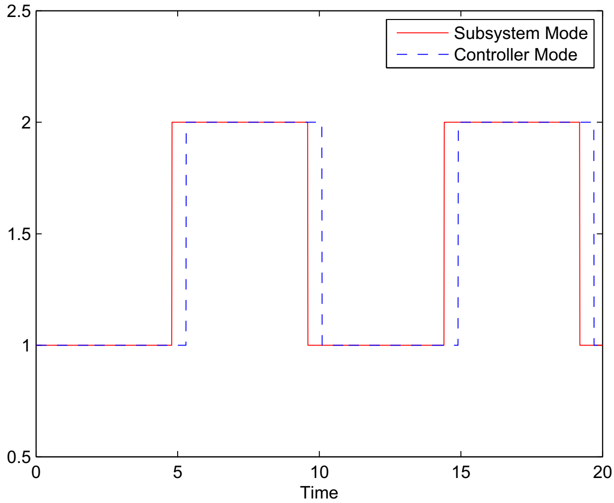

Fig. 4 Switching signals of subsystems and controllers

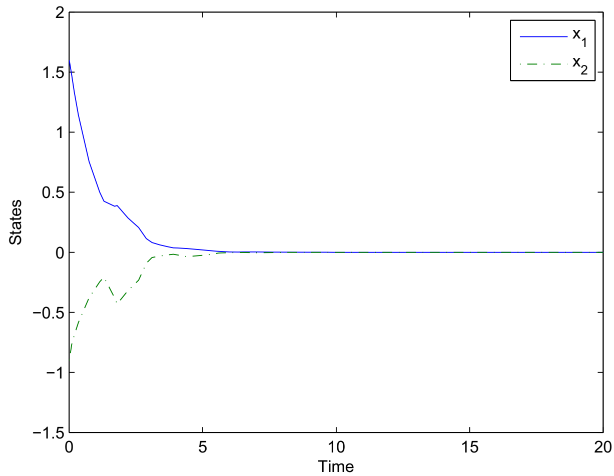

Fig. 5 State response of the closed-loop system with $\tau_1 = \tau_2 = 4.8$, $\Delta_{\max} = 0.5$

The lower bound of residence time of each subsystem $\tau^* = 4.7683$ can be obtained by substituting known values into formula (10). We choose $\tau_1 = \tau_2 = 4.8$, then two lines in Fig. 4 describe the switching signals of the subsystems and the controllers, respectively. As seen in Fig. 5, under the action of the switching signal designed by us, the exponentially stable state trajectory of the system is depicted. This shows that the results of this paper can be applied to simplified engineering models.

5 Conclusions

In this paper, we have investigated the asynchronous H_∞ performance of uncertain switched singular systems with interval time-varying delays, which is a more general class of switched systems. By constructing a new piecewise Lyapunov–Krasovskii functional, delay-dependent stability conditions have been derived for the closed-loop system to be regular, impulse free and exponentially stable in the presence of asynchronous switching. Furthermore, with the help of the average dwell time approach, a class of switching signals has been found under which the system has non-weighted H_∞ performance and strict LMIs are given to solve controllers. By limiting the residence time of each subsystem, the increased energy caused by the asynchronous phenomenon of the system is offset. Two numerical examples are given to illustrate the feasibility and effectiveness of the theorem.

Data Availability Statement The data sets generated during and analyzed during the current study are available from the corresponding author on reasonable request.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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