Circuits, Systems, and Signal Processing (2020) 39:1335–1352
https://doi.org/10.1007/s00034-019-01218-7

Cooperative Adaptive Tracking Control for Unknown Nonlinear Multi-agent Systems with Signal Transmission Faults

Qilong Fu[1] · Qikun Shen[1] · Ziyang Jia[2]

Received: 18 April 2019 / Revised: 15 July 2019 / Accepted: 18 July 2019 / Published online: 25 July 2019

Abstract
In this paper, the cooperative adaptive fault-tolerant tracking control problem of high-order nonlinear multi-agent systems with signal transmission faults is studied. A neural network-based adaptive fault-tolerant control scheme is proposed, which guarantees that all followers asymptotically synchronize to a leader with tracking errors converging to a small neighborhood of the origin in spite of signal transmission faults. Based on algebraic graph theory and Lyapunov theory, the analysis of stability and parameter convergence of the proposed algorithm are conducted. Finally, an example is provided to validate the theoretical results.

Keywords Cooperative control · Fault-tolerant control · Multi-agent systems · Signal transmission faults · Nonlinear systems

1 Introduction

In the research field of multi-agent systems, due to the greater efficiency and operational capability, the distributed cooperative control has attracted extensive attention from a large number of domestic and foreign researchers in the past two decades [1,17]. Many results have been available for the consensus control problem of multi-agents systems [8,16,22,27]. The fundamental research on cooperative control of multi-agent

✉ Qikun Shen
qkshen@yzu.edu.cn

Qilong Fu
fuqilong1226@126.com

Ziyang Jia
zjia@umass.edu

1 College of Information Engineering, Yangzhou University, Yangzhou 225127, China

2 Isenberg School of Management, University of Massachusetts, Amherst 01003, USA

systems mainly included cooperative regulator problems and cooperative tracking problems. As the basis of distributed cooperative control of multi-agent systems, the consensus problem has attracted researchers' interests [8]. In [1], a method was proposed for the stability of the Vicsek model and proved that the first-order agent system can achieve consensus under the undirected communication topology. In addition, Jadbabaie et al. changed the Vicsek model and studied a model with leader–follower form in [9]. In [16], the consensus problem was investigated for single integrator multi-agent systems and proposed a basic framework for multi-agent system consensus algorithm. It turned out that balanced digraphs played a critical role in addressing consensus problems. Based on the convexity theory with the Lyapunov method, the authors solved the consensus problem for discrete multi-agent systems. It pointed out that as long as the coupling relationship between the agents meet certain convexity conditions and the communication structure is connected, the behavior of the multi-agent system will eventually become consensus [15]. In [18], the form of communication network was extended to weighted directed communication topology, and the authors proposed the sufficient condition for consensus. In [27], the synchronization problem was studied for networked higher-order nonlinear systems with an active leader, and a robust adaptive sliding mode control scheme was proposed. In [24], a distributed control method was proposed to solve the distributed consensus tracking problems of multi-agent systems. In [20], the consensus problem was investigated for uncertain second-order nonlinear multi-agent systems with unknown nonlinear dead zone. In [29], the finite-time consensus tracking control problem was studied for uncertain nonlinear multi-agent systems. However, the above studies do not take into account the situation, in which the system becomes faulty.

In general, actuators, sensors and components may become faulty in practical application and these faults may cause system instability, which can lead to catastrophic consequences. Therefore, many experts have proposed effective fault-tolerant control methods to improve the system reliability and ensure the stability of the controlled system in all situations [2–5,7,19,25,26,28]. In [21], a novel cooperative adaptive fuzzy tracking control scheme was proposed to guarantee that all followers asymptotically synchronize a leader in spite of actuator faults. The leader-following consensus problem was studied for multi-agent systems in [23], where it was assumed that there exist gain and bias transmission nonlinearities in the links in the communication network. Two distributed adaptive control schemes were designed to compensate for the faults. In [13], the finite-time fault-tolerant control problem was investigated for multiple-input multiple-output nonlinear systems. However, only actuator faults are considered in [3–5,12–14,21,30].

In this paper, we investigate the cooperative adaptive fault tolerant tracking control problem of high-order multi-agent systems and propose an active fault-tolerant control scheme against signal transmission faults. Compared with the results in [8,15,20,24, 27,29], signal transmission faults in communication network are considered, and a novel distributed adaptive control method is designed.

The rest of paper is organized as follows. In Sect. 2, the basic graph theory and radial basis function neural networks are introduced. The problem under study is formulated. In Sect 3, the distributed adaptive control scheme is designed. Based on Lyapunov theory, the closed-loop system stability analysis is developed in Sect 4.

Simulation results and discussion are reported in Sect. 5. The conclusion is drawn in Sect. 6.

Notations Throughout this paper, R, R^N denote the real numbers and the real n vectors, respectively; $|\cdot|$ is the absolute value of a real number; $||\cdot||$ is the Euclidean norm of a vector; $tr\{\cdot\}$ is the trace of a matrix; $\sigma(\cdot)$ is the set of singular values of a matrix; $\bar{\sigma}(\cdot)$ is the maximum singular value of a matrix; $(\cdot)$ is the minimum singular value of a matrix; matrix $P > 0$ means P is positive definite.

2 Preliminaries

2.1 Basic Graph Theory and Notations

Let $G = (\upsilon, E)$ be a weighted digraph, $\upsilon = (\upsilon_1, \ldots, \upsilon_N)$ is the nonempty set of nodes/agents, $E \subseteq \upsilon \times \upsilon$ is the set of edges, $(\upsilon_k, \upsilon_j) \in E$ means υ_j can obtain information from υ_k. Define an adjacency matrix $A = [a_{kj}] \in R^{N\times N}$ with $a_{kj} > 0$ if $(\upsilon_k, \upsilon_j) \in E$; otherwise, $a_{kj} = 0$. In this paper, it is assumed that $a_{kk} = 0$ and the topology is fixed, i.e., A is time invariant. Define $d_k = \sum_{j=1}^{N} a_{kj}$ as the weighted in-degree of node k and $D = \text{diag}(d_1, \ldots, d_N) \in R^{N\times N}$ as in-degree matrix. The graph Laplacian matrix is $L = [l_{kj}] = D - A \in R^{N\times N}$. Let $\underline{1} = [1, \ldots, 1]^{\mathrm{T}} \in R^{N\times 1}$ with appropriate dimension; then, $L\underline{1} = 0$. We use the set N_k to describe all neighboring agents of υ_k, i.e.,$N_k = \{j|(\upsilon_j, \upsilon_k) \in E\}$.

2.2 Problem Formulation

In this paper, we consider a team of $N + 1$ agents consisting of N followers and one leader. The dynamics of the kth follower agent is described as

$$\begin{cases} \dot{x}_{k,i}(t) = x_{k,i+1}(t), \quad i = 1, \ldots, n_k - 1 \\ \dot{x}_{k,n_k}(t) = f_k(\bar{x}_k) + u_k(t) + h_k(t) \end{cases} \tag{1}$$

where $k = 1, \ldots, N$, n_k denotes the order number, $x_{k,i} \in R$ denotes the ith state and $\bar{x}_k = [x_{k,1}, \ldots, x_{k,n_k}]^{\mathrm{T}} \in R^{n_k}$ denotes the state vector of node k; $f_k(\bar{x}_k) \in R$ is an unknown continuous function; $u_k \in R$ is the control input; $h_k(t) \in R$ is an external disturbance, which is unknown but bounded.

The dynamics of the leader is given by

$$\begin{cases} \dot{x}_{0,i}(t) = x_{0,i+1}(t) \\ \dot{x}_{0,n_0}(t) = f_0(\bar{x}_0, t) \end{cases}, \quad i = 1, \ldots, n_0 - 1 \tag{2}$$

where n_0 denotes the order number, $x_{0,i} \in R$ denotes the ith state and $\bar{x}_0 = [x_{0,1}, \ldots, x_{0,n}]^{\mathrm{T}} \in R^{n_0}$ is the state vector of the leader; $f_0(\bar{x}_0, t) \in R$ is piecewise continuous in time t and locally Lipschitz in $\bar{x}_0$ with $f_0(0, t) = 0$ for all $\forall t \geq 0$ and $\bar{x}_0 \in R^n$.

In this paper, we assumed $n_0 = n_1 = \cdots = n_k = n$.

Define $x_i = [x_{1,i}, \ldots, x_{N,i}]^{\mathrm{T}} \in R^N$, (1) can be written in the following compact form:

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) \\ \dot{x}_n(t) = f(\bar{x}) + u(t) + h(t) \end{cases}, \quad i = 1, \ldots, n-1 \tag{3}$$

where $f = [f_1, \ldots, f_N]^{\mathrm{T}} = [f_1(\bar{x}_1), \ldots, f_N(\bar{x}_N)]^{\mathrm{T}}$, $u = [u_1, \ldots, u_N]^{\mathrm{T}}$, $\bar{x} = [\bar{x}_1^{\mathrm{T}}, \ldots, \bar{x}_N^{\mathrm{T}}]^{\mathrm{T}}$, $h = [h_1, \ldots, h_N]^{\mathrm{T}} = [h_1(t), \ldots, h_N(t)]^{\mathrm{T}}$.

Under normal condition (no fault), define the ith tracking error for follower k as follows:

$$\delta_{k,i} = x_{k,i} - x_{0,i}, \quad i = 1, \ldots, n, k = 1, \ldots, N \tag{4}$$

Let $\delta_i = [\delta_{1,i}, \ldots, \delta_{N,i}]^{\mathrm{T}} \in R^N$, then

$$\delta_i = x_i - \underline{x}_{0,i} \tag{5}$$

where $\underline{x}_{0,i} = [x_{0,i}, \ldots, x_{0,i}]^{\mathrm{T}} \in R^N$.

The control objective of this paper is to design a distributed controller u for each follower such that each follower tracks the leader and the tracking error $\delta_i (i = 1, \ldots, n)$ converges to the small neighborhoods of the origin.

Define the neighborhood synchronization error

$$e_{k,i}(t) = \sum_{j \in N_k} a_{kj}(x_{j,i} - x_{k,i}) + b_k(x_{0,i} - x_{k,i}) \tag{6}$$

Note that if the node k can obtain the leader information, then $b_k > 0$; otherwise, $b_k = 0$;

Define the following notations: $e_i = [e_{1,i}, \ldots, e_{N,i}]^{\mathrm{T}} \in R^N$, $\underline{f}_0 = [f_0(x_0, t), \ldots, f_0(x_0, t)]^{\mathrm{T}} \in R^N$, $B = \mathrm{diag}\{b_1, \ldots, b_N\} \in R^{N \times N}$.

The above tracking error can be written as:

$$\begin{cases} \dot{e}_i(t) = e_{i+1}(t), \quad i = 1, \ldots, n-1 \\ \dot{e}_n(t) = -(L + B)(f + u(t) + h - \underline{f}_0) \end{cases} \tag{7}$$

Define the new augmented graph as $\bar{G} = \{\bar{\upsilon}, \bar{E}\}$, $\bar{\upsilon} = \{\upsilon_0, \upsilon_1, \ldots, \upsilon_N\}$ and $\bar{E} \subseteq \bar{\upsilon} \times \bar{\upsilon}$.

It is well known that information transmission in multi-agent systems is via communication network. There is a communication link between each two agents. For example, there are three agents k, j and h. Agents h and j are the neighbors of agent k, which can obtain the information from agent k, shown in Fig. 1. From Fig. 1, it is easily seen that there exist two communication links. One is used for the signal transmission between agents k and j, and the other one is used for the signal transmission between agents k and h.

In practical applications, communication links in the network may become faulty. From Fig. 1, when the state of node k is transmitted to node j, the polluted state

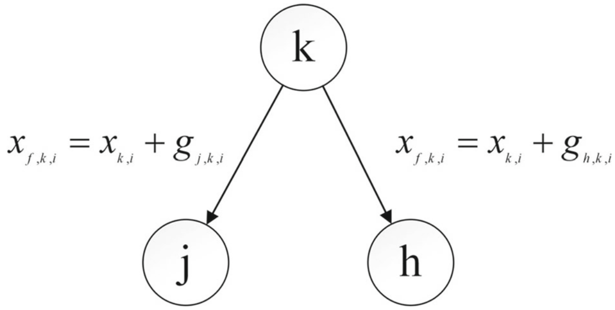

Fig. 1 Information transmission with faults

obtained by node j is

$$x_{f,k,i} = x_{k,i} + g_{j,k,i} \tag{8}$$

where $g_{j,k,i}$ is an unknown real constant, which denotes a bounded signal transmission fault. When the state of node k is transmitted to node h, the polluted state obtained by node h is

$$x_{f,k,i} = x_{k,i} + g_{h,k,i} \tag{9}$$

where $g_{h,k,i}$ is an unknown real constant, which denotes bounded a signal transmission fault.

In this paper, for convenience, let $g_{j,k,i} = g_{h,k,i}$. The following fault model is considered:

$$x_{f,k,i} = x_{k,i} + g_{k,i}, \quad x_{f,0,i} = x_{0,i} + g_{0,i}$$

where $g_{k,i}$ and $g_{0,i}$ are unknown bounded constant, which, respectively, denote follower and leader communication link faults.

Define the following symbols:

$$x_{f,k} = \bar{x}_k + g_k, \quad x_f = \bar{x} + g$$

where $x_{f,k} = [x_{f,k,1}, \ldots, x_{f,k,n_k}]^{\mathrm{T}}, g_k = [g_{k,1}, \ldots, g_{k,n_k}]^{\mathrm{T}}, x_f = [x_{f,1}^{\mathrm{T}}, \ldots, x_{f,N}^{\mathrm{T}}]^{\mathrm{T}}$, $g = [g_1^{\mathrm{T}}, \ldots, g_N^{\mathrm{T}}]^{\mathrm{T}}$. If signal transmission becomes faulty, the neighborhood synchronization error $e_{a,k,i}$ is

$$\begin{aligned} e_{a,k,i} &= \sum_{j \in N_i} a_{kj}(x_{f,j,i} - x_{f,k,i}) + b_k(x_{f,0,i} - x_{f,k,i}) \\ &= \sum_{j \in N_i} a_{kj}[(x_{j,i} + g_{j,i}) - (x_{k,i} + g_{k,i})] \\ &\quad + b_k[(x_{0,i} + g_{0,i}) - (x_{k,i} + g_{k,i})] \\ &= e_{k,i} + \sum_{j \in N_i} a_{kj}(g_{j,i} - g_{k,i}) + b_k(g_{0,i} - g_{k,i}) \end{aligned} \tag{10}$$

Let $\omega_k = \sum_{j\in N_1} a_{kj}(g_{j,1} - g_{k,1}) + b_k(g_{0,1} - g_{k,1})$, then

$$e_{a,k,1} = e_{k,1} + \omega_k \tag{11}$$

The following assumptions are made for cooperative tracking problems.

Assumption 1 The augmented graph $\bar{G}$ contains a spanning tree with the root node being the leader node 0.

Assumption 2 There exists a positive constant $M_0 \in R$ and $M_{f0} \in R$ such that $||\bar{x}_0(t)|| \le M_0$, $|f_0(\bar{x}_0, t)| \le M_{f0}$.

Assumption 3 There exists a positive constant $M_{h,k} > 0 \in R \quad k = 1, \ldots, N$ such that $|h_k(\bar{x}_k, t)| \le M_{h,k}$.

Assumption 4 There exists a positive constant $M_{\omega,k} > 0 \in R \quad k = 1, \ldots, N$ such that $|\omega_k| \le M_{\omega,k}$.

Lemma 1 [27] *Define* $q = [q_1, \ldots, q_N] = (L + B)^{-1}\underline{1}$, $\underline{1} = [1, \ldots, 1]^{\mathrm{T}} \in R^N$, $P = diag\{p_i\} = diag\{1/q_i\}$, $Q = P(L + B) + (L + B)^{\mathrm{T}}P$, *then* $P > 0$ *and* $Q > 0$.

Lemma 2 [27] $||\varphi_i|| \le ||e_{a,i}||/\underline{\sigma}(L+B)$, $i = 1, \ldots, n$, *where* $\underline{\sigma}(L+B)$ *is a minimum singular value of matrix* $L + B$.

2.3 Neural Networks

Neural networks have been widely used in modeling and controlling of nonlinear systems. The feasibility of applying neural networks to unknown dynamic systems control has been demonstrated in many studies [10,11]. As can be seen from Fig. 2, radial basis function (RBF) neural networks (NNs) is presented in

$$f_k(\bar{x}_k) = \theta_k^{\mathrm{T}} \xi_k(\bar{Z}_k) + \varepsilon_k(\bar{Z}_k)$$

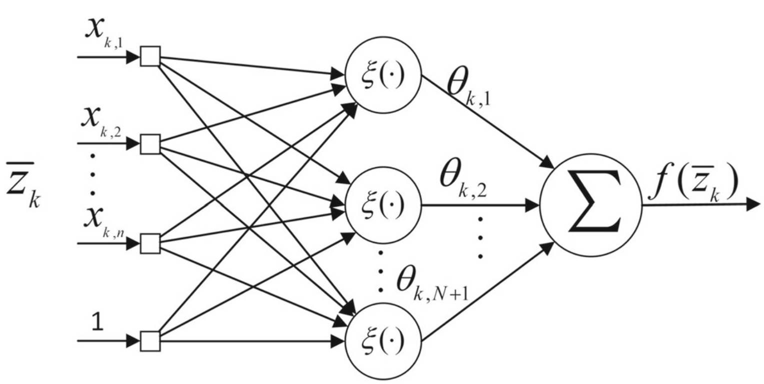

Fig. 2 RBF neural networks (in normal)

where $\varepsilon_k(\bar{Z}_k)$ denotes the optimal approximation error, $\bar{Z}_k = [z_{k,1}, \ldots, z_{k,p_k}]^{\mathrm{T}} = [\bar{x}_k^{\mathrm{T}}, 1]^{\mathrm{T}}$, $\xi_k(\bar{Z}_k) = \left[\zeta_{k,1}(\bar{Z}_k), \ldots \zeta_{k,N_W}(\bar{Z}_k)\right]^{\mathrm{T}}$, N_W is the number of the NNs.

$\zeta_{k,i}(\bar{Z}_k) = \exp(-\sum_{j=1}^{\psi_k} \frac{(z_{k,j} - q_{k,i,j})^2}{{c_{k,i}}^2})$, ψ_k is the dimension of Z_k, where $c_{k,i} > 0$ is the center of the receptive field, and $q_{k,i,j}$ is the width of the Gaussian function. Let

$$\theta_k^* = \arg\min_{\theta \in \Omega_\theta} \left[\sup_{z \in \Omega_z} \left| \theta_k^{\mathrm{T}} \xi_k(\bar{Z}_k) - f_k(\bar{x}_k) \right| \right]$$
$$\Omega_\theta = \{\theta_k | ||\theta_k|| \le \beta_\theta\}$$

with a constant $\beta_\theta > 0$, Ω_z denotes an enough large compact set.

From [6], we know NNs can approximate any continuous function to any accuracy on a compact set.

From Fig. 2, we know the input of NNs contain the state. In this paper, signal transmission faults are considered, the state $\bar{x}_k$ cannot taken as the input of NNs, and only $x_{f,k}$ is accessed. Therefore, we use the neural network to approximate the unknown smooth function $f_k(\bar{x}_k)$ in system (1). From Fig. 3, we know

$$\begin{aligned} f_k(\bar{x}_k) &= \theta_k^{*\mathrm{T}} \xi_k(\bar{Z}_k) + \varepsilon_k \\ &= \theta_k^{*\mathrm{T}} \xi_k(\bar{Z}_{f,k}) - \theta_k^{*\mathrm{T}} \xi_k(\bar{Z}_{f,k}) + \theta_k^{*\mathrm{T}} \xi_k(\bar{Z}_k) + \varepsilon_k \\ &= \theta_k^{*\mathrm{T}} \xi_k(\bar{Z}_{f,k}) - \theta_k^{*\mathrm{T}} [\xi_k(\bar{Z}_{f,k}) - \xi_k(\bar{Z}_k)] + \varepsilon_k \end{aligned} \tag{12}$$

Assumption 5 There exists a positive constant $M_{\varepsilon,k} > 0 \in R \quad k = 1, \ldots, N$ such that$|\varepsilon_k| \le M_{\varepsilon,k}$.

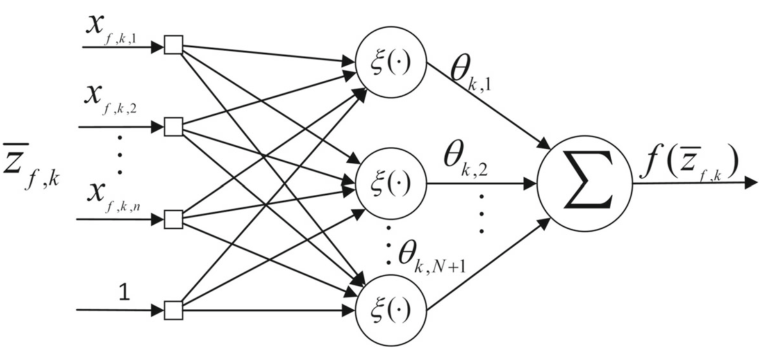

Fig. 3 RBF neural networks (with faults)

3 Main Results

Define the sliding mode error for the kth follower as follows:

$$s_k = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right)^{n-1} e_{k,1}(t) = \sum_{i=1}^{n-1} c_{k,i} e_{k,i}(t) + e_{k,n}(t), k = 1, \ldots, N \tag{13}$$

where $c_{k,i} = C_{n-1}^{i-1}\lambda^{n-i}$, $\lambda_k > 0$. Let $\bar{e}_k(t) = [e_{k,1}(t), \ldots, e_{k,n}(t)]^{\mathrm{T}}$.

Lemma 3 [21] *Let s_k be defined by (13), and then*

(1) *if $s_k = 0$, then $\lim_{t\to\infty}\bar{e}_k(t) = 0$;*
(2) *if $|s_k| \le \alpha_k$, $\bar{e}_k(0) \in \Omega_{\alpha_k}$, then $e_k(t) \in \Omega_{\alpha_k}$, $\forall t \ge 0$;*
(3) *if $|s_k| \le \alpha_k$, $\bar{e}_k(0) \notin \Omega_{\alpha_k}$, then $\exists T_k = (m_k - 1)/\lambda_k$, $\forall t \ge T_k$, $\bar{e}_k(t) \in \Omega_{\alpha_k}$; where $\Omega_{\alpha_k} = \{\bar{e}_k(t) || e_{k,i}| \le 2^{(j-1)}\lambda_k^{j-m_k}\alpha_k, i = 1, 2, \ldots, n, j = 1, 2, \ldots m_k\}$.*

Let $c_{1,i} = \cdots = c_{N,i} = \lambda_i$, $\lambda_n = 1$, then

$$s_k = \lambda_1 e_{k,1} + \cdots + \lambda_n e_{k,n} \tag{14}$$

Define the global sliding mode error $s = [s_1, \ldots, s_N]^{\mathrm{T}}$, then

$$s = \lambda_1 e_1 + \cdots + \lambda_n e_n \tag{15}$$

Differentiating s, we have

$$\begin{aligned} \dot{s} &= \lambda_1 \dot{e}_1 + \cdots + \lambda_n \dot{e}_n = \sum\nolimits_{i=1}^{n-1} \lambda_i e_i + \dot{e}_n \\ &= \gamma - (L + B)(f(\bar{x}) + u + h - \underline{f}_0) \end{aligned} \tag{16}$$

where $\gamma = \sum_{i=1}^{n-1} \lambda_i e_{i+1}$.

Let $\xi_{f,k} = \theta_k^{*\mathrm{T}}[\xi(\bar{Z}_{f,k}) - \xi(\bar{Z}_k)]$, $\xi_f = [\xi_{f,1}, \ldots, \xi_{f,N}]^{\mathrm{T}}$, $\varepsilon = [\varepsilon_1, \ldots, \varepsilon_N]^{\mathrm{T}}$, $\theta^{*\mathrm{T}} = \mathrm{diag}(\theta_1^{*\mathrm{T}}, \ldots, \theta_N^{*\mathrm{T}})$, $\xi = [\xi_1^{\mathrm{T}}(\bar{Z}_{f,1}), \ldots, \xi_N^{\mathrm{T}}(\bar{Z}_{f,N})]^{\mathrm{T}}$, then

$$f(\bar{x}) = \theta^{*\mathrm{T}}\xi - \xi_f + \varepsilon$$

where ξ_f is a bounded vector function.

If the signal transmission becomes faulty, s_k is not obtained and only $s_{f,k}$ is accessed which is defined as follows:

$$\begin{aligned} s_{f,k} &= \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right)^{n-1} e_{a,k,1}(t) = \sum_{i=1}^{n-1} c_{k,i} e_{k,i}(t) + e_{k,n}(t) \\ &\quad + \lambda_1 \left[\sum_{j\in N_i} a_{kj}(g_{f,j,1} - g_{f,k,1}) + b_k(g_{f,0,1} - g_{f,k,1})\right] \\ &= s_k + \lambda_1 \omega_k \end{aligned} \tag{17}$$

Let $s_f = [s_{f,1}, \ldots, s_{f,n}]^{\mathrm{T}}, \omega = [\lambda_1\omega_1, \ldots, \lambda_1\omega_n]^{\mathrm{T}}$, then

$$s_f = s + \omega \tag{18}$$

Differentiating s_f with respect to time t , we have

$$\dot{s}_f = \dot{s} + \dot{\omega} = \gamma - (L + B)(f(\bar{x}) + u + h - \underline{f}_0) \tag{19}$$

Define the following Lyapunov function

$$V_s = s^{\mathrm{T}} P s/2 \tag{20}$$

where $P = P^{\mathrm{T}} > 0 \in R^{n\times n}$.

Differentiating V_s with respect to time t, we have

$$\begin{aligned}\dot{V}_s &= s^{\mathrm{T}} P \left[\gamma - (L + B)(\theta^{*\mathrm{T}}\xi - \xi_f + \varepsilon + u + h - \underline{f}_0)\right] \\ &= s^{\mathrm{T}} P\gamma - s^{\mathrm{T}} P(L + B)u - s^{\mathrm{T}} P(D + B)\theta^{*\mathrm{T}}\xi \\ &\quad + s^{\mathrm{T}} PA\theta^{*\mathrm{T}}\xi + s^{\mathrm{T}} P(D + B)\xi_f - s^{\mathrm{T}} PA\xi_f \\ &\quad - s^{\mathrm{T}} P(D + B)(\varepsilon + h - \underline{f}_0) + s^{\mathrm{T}} PA(\varepsilon + h - \underline{f}_0)\end{aligned} \tag{21}$$

Define the following control law:

$$u = (D + B)^{-1}\gamma - \hat{f} - \mathrm{sgn}((s_f - \hat{\omega})^{\mathrm{T}} P(D + B))\hat{\bar{M}}_{\varepsilon hf} + m(s_f - \hat{\omega}) \tag{22}$$

where $\hat{f} = [\hat{f}_1, \ldots, \hat{f}_N]^{\mathrm{T}}$, $\hat{f}_k = \hat{\theta}_k^{\mathrm{T}}\xi_k(\bar{Z}_{f,k})$ is the estimate of $f_k(\bar{Z}_{f,k})$. $P_{s,k}$ is the kth element of $(s_f - \hat{\omega})^{\mathrm{T}} P(D + B)$, $\hat{\bar{M}}_{\varepsilon hf} = [\hat{\bar{M}}_{\varepsilon hf,1}, \ldots, \hat{\bar{M}}_{\varepsilon hf,N}]^{\mathrm{T}}$, $\hat{M}_{\varepsilon hf,k}$ is the estimate of $M_{\varepsilon hf} = M_{\varepsilon,k} + M_{h,k} + M_{f0}$. $m > 0 \in R$ is a design parameter, which satisfies

$$m\underline{\sigma}(Q)/2 - (5r + 2rm + \bar{\lambda}^2/4r\underline{\lambda}^2)\bar{\sigma}(A)\bar{\sigma}(P) - r\bar{\sigma}(P) > 0$$

Let $\hat{\theta}^{\mathrm{T}} = \mathrm{diag}(\hat{\theta}_1^{\mathrm{T}}, \ldots, \hat{\theta}_N^{\mathrm{T}})$, which $\hat{\theta}_k^{\mathrm{T}}$ is the estimates of $\theta_k^{*\mathrm{T}}$.

Define the notations as follows:

$$\tilde{\theta} = \theta^* - \hat{\theta}, \tilde{\bar{M}}_{\varepsilon hf} = \bar{M}_{\varepsilon hf} - \hat{\bar{M}}_{\varepsilon hf}, \tilde{\omega} = \omega - \hat{\omega}$$

Substituting control law (22) in (21), one has

$$\begin{aligned}\dot{V}_s =& -ms^{\mathrm{T}}P(D+B)s + ms^{\mathrm{T}}PAs - s^{\mathrm{T}}P(D+B)\tilde{\theta}^{\mathrm{T}}\xi \\ &- s^{\mathrm{T}}P(D+B)\,\mathrm{sgn}(s^{\mathrm{T}}P(D+B))\tilde{\bar{M}}_{\varepsilon hf} + s^{\mathrm{T}}PA\bar{M}_{\varepsilon hf} \\ &+ ms^{\mathrm{T}}PA\omega - ms^{\mathrm{T}}PA\hat{\omega} + s^{\mathrm{T}}PA\tilde{\theta}^{\mathrm{T}}\xi - s^{\mathrm{T}}PA\xi_f \\ &- ms^{\mathrm{T}}P(D+B)\tilde{\omega} - s^{\mathrm{T}}PA\,\mathrm{sgn}(s^{\mathrm{T}}P(D+B))\hat{\bar{M}}_{\varepsilon hf} \\ &+ s^{\mathrm{T}}PA(D+B)^{-1}\gamma + s^{\mathrm{T}}P(D+B)\xi_f\end{aligned} \tag{23}$$

Since

$$\begin{aligned}&-s^{\mathrm{T}}PA\xi_f \le \bar{\sigma}(P)\bar{\sigma}(A)rs^{\mathrm{T}}s + \bar{\sigma}(P)\bar{\sigma}(A)/4r\xi_f^{\mathrm{T}}\xi_f \\ &s^{\mathrm{T}}PA\tilde{\theta}^{\mathrm{T}}\xi \le \bar{\sigma}(P)\bar{\sigma}(A)rs^{\mathrm{T}}s + \bar{\sigma}(P)\bar{\sigma}(A)/4r\xi^{\mathrm{T}}\tilde{\theta}\tilde{\theta}^{\mathrm{T}}\xi \\ &ms^{\mathrm{T}}PA\omega \le \bar{\sigma}(P)\bar{\sigma}(A)rms^{\mathrm{T}}s + \bar{\sigma}(P)\bar{\sigma}(A)/4rm\omega^{\mathrm{T}}\omega \\ &-ms^{\mathrm{T}}PA\hat{\omega} \le \bar{\sigma}(P)\bar{\sigma}(A)rms^{\mathrm{T}}s + \bar{\sigma}(P)\bar{\sigma}(A)/4rm\beta_\omega^2 \\ &s^{\mathrm{T}}PA\bar{M}_{\varepsilon hf} \le \bar{\sigma}(P)\bar{\sigma}(A)rs^{\mathrm{T}}s + \bar{\sigma}(P)\bar{\sigma}(A)/4r\bar{M}_{\varepsilon hf}^{\mathrm{T}}\bar{M}_{\varepsilon hf} \\ &s^{\mathrm{T}}PA(D+B)^{-1}\gamma \le \bar{\sigma}(P)\bar{\sigma}(A)rs^{\mathrm{T}}s + \bar{\sigma}(P)\bar{\sigma}(A)/4r\gamma^{\mathrm{T}}\gamma \\ &s^{\mathrm{T}}P(D+B)\xi_f \le \bar{\sigma}(P)rs^{\mathrm{T}}s + \bar{\sigma}(P)/4r\xi_f^{\mathrm{T}}(D+B)^{\mathrm{T}}(D+B)\xi_f \\ &-s^{\mathrm{T}}PA\mathrm{sgn}(s^{\mathrm{T}}P(D+B))\hat{\bar{M}}_{\varepsilon hf} \le \bar{\sigma}(P)\bar{\sigma}(A)rs^{\mathrm{T}}s + \bar{\sigma}(P)\bar{\sigma}(A)/4r\beta_M^2\end{aligned}$$

where $r > 0 \in R$ is a design parameter, and $\bar{M}_{\varepsilon hf} = [M_{\varepsilon hf,1}, \ldots, M_{\varepsilon hf,N}]^{\mathrm{T}}$, one has

$$\begin{aligned}\dot{V}_s \le& -ms^{\mathrm{T}}P(L+B)s - s^{\mathrm{T}}P(D+B)\tilde{\theta}^{\mathrm{T}}\xi - ms^{\mathrm{T}}P(D+B)\tilde{\omega} \\ &- s^{\mathrm{T}}P(D+B)\,\mathrm{sgn}(s^{\mathrm{T}}P(D+B))\tilde{\bar{M}}_{\varepsilon hf} + \bar{\sigma}(P)\bar{\sigma}(A)/4r\gamma^{\mathrm{T}}\gamma \\ &+ r\bar{\sigma}(P)(5\bar{\sigma}(A) + 2m\bar{\sigma}(A) + 1)s^{\mathrm{T}}s + \bar{\sigma}(P)/4r\xi_f^{\mathrm{T}}(D+B)^{\mathrm{T}}(D+B)\xi_f \\ &+ \bar{\sigma}(P)\bar{\sigma}(A)/4r(\xi^{\mathrm{T}}\tilde{\theta}\tilde{\theta}^{\mathrm{T}}\xi + \xi_f^{\mathrm{T}}\xi_f + \beta_M^2 + m\beta_\omega^2 + m\omega^{\mathrm{T}}\omega + \bar{M}_{\varepsilon hf}^{\mathrm{T}}\bar{M}_{\varepsilon hf})\end{aligned} \tag{24}$$

Since θ_k^* and $\hat{\theta}_k$ are bounded, $\tilde{\theta}^{\mathrm{T}}\xi$ is bounded. From Assumptions 2, 3 and 5, $\bar{M}_{h\varepsilon f}$ is bounded. Adaptive law (31) ensures that $||\hat{\bar{M}}_{h\varepsilon f}|| \le \beta_M$. Further, $\tilde{\bar{M}}_{h\varepsilon f}$ is bounded as well.

Since $\tilde{\theta}^{\mathrm{T}}\xi$, $\bar{M}_{h\varepsilon f}$ and ω are bounded, there exists an appropriate parameter r , then

$$\begin{aligned}&\bar{\sigma}(P)\bar{\sigma}(A)/4r(\xi^{\mathrm{T}}\tilde{\theta}\tilde{\theta}^{\mathrm{T}}\xi + \xi_f^{\mathrm{T}}\xi_f + \beta_M^2 + m\beta_\omega^2 + m\omega^{\mathrm{T}}\omega + \bar{M}_{\varepsilon hf}^{\mathrm{T}}\bar{M}_{\varepsilon hf}) \le \mu_{s1} \\ &\bar{\sigma}(P)/4r\xi_f^{\mathrm{T}}(D+B)^{\mathrm{T}}(D+B)\xi_f \le \mu_{s2}\end{aligned}$$

Since

$$\gamma_k^2 = \sum_{i=1}^{n-1}\lambda_i^2 e_{k,i+1}^2 \le \bar{\lambda}^2 s_k^2/\underline{\lambda}^2$$

$$\gamma^{\mathrm{T}}\gamma = \sum_{k=1}^{N}\gamma_k^2 \le \sum_{k=1}^{N}\frac{\bar{\lambda}^2}{\underline{\lambda}^2}s_k^2 = \frac{\bar{\lambda}^2}{\underline{\lambda}^2}\sum_{k=1}^{N}s_k^2 = \frac{\bar{\lambda}^2}{\underline{\lambda}^2}s^{\mathrm{T}}s$$

where $\bar{\lambda} = \max\{\lambda_1, \ldots, \lambda_n\}$, $\underline{\lambda} = \min\{\lambda_1, \ldots, \lambda_n\}$. Further, one has

$$\begin{aligned}\dot{V}_s \le & -ms^{\mathrm{T}}P(L+B)s + ((5r+2rm+\bar{\lambda}^2/4r\underline{\lambda}^2)\bar{\sigma}(A)+r)\bar{\sigma}(P)s^{\mathrm{T}}s \\ & - s^{\mathrm{T}}P(D+B)\tilde{\theta}^{\mathrm{T}}\xi - s^{\mathrm{T}}P(D+B)\operatorname{sgn}(s^{\mathrm{T}}P(D+B))\tilde{\bar{M}}_{\varepsilon hf} \\ & - ms^{\mathrm{T}}P(D+B)\tilde{\omega} + \mu_s\end{aligned} \tag{25}$$

where $\mu_s = \mu_{s1} + \mu_{s2}$, μ_s is a design parameter.

Define the Lyapunov function

$$V_\theta = tr\{\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\}/2\eta_1 + \tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\tilde{\bar{M}}_{\varepsilon hf}/2\eta_2 + \tilde{\omega}^{\mathrm{T}}\tilde{\omega}/2\eta_3 \tag{26}$$

where $\eta_1 > 0 \in R$, $\eta_2 > 0 \in R$, $\eta_3 > 0 \in R$ are design parameters.

Differentiating V_θ with respect to time t, we have

$$\dot{V}_\theta = -tr\{\tilde{\theta}^{\mathrm{T}}\dot{\hat{\theta}}\}/\eta_1 - \tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\dot{\hat{\bar{M}}}_{\varepsilon hf}/\eta_2 - \tilde{\omega}^{\mathrm{T}}\dot{\hat{\omega}}/\eta_3 \tag{27}$$

Define Lyapunov function

$$V_0 = V_s + V_\theta \tag{28}$$

Differentiating V_0 with respect to time t, we have

$$\begin{aligned}\dot{V}_0 \le & -ms^{\mathrm{T}}P(L+B)s + ((5r+2rm+\bar{\lambda}^2/4r\underline{\lambda}^2)\bar{\sigma}(A)+r)\bar{\sigma}(P)s^{\mathrm{T}}s \\ & - s^{\mathrm{T}}P(D+B)\tilde{\theta}^{\mathrm{T}}\xi - s^{\mathrm{T}}P(D+B)\operatorname{sgn}(s^{\mathrm{T}}P(D+B))\tilde{\bar{M}}_{\varepsilon hf} \\ & - ms^{\mathrm{T}}P(D+B)\tilde{\omega} - \tilde{\theta}^{\mathrm{T}}\dot{\hat{\theta}}/\eta_1 - \tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\dot{\hat{\bar{M}}}_{\varepsilon hf}/\eta_2 - \tilde{\omega}^{\mathrm{T}}\dot{\hat{\omega}}/\eta_3 + \mu_s\end{aligned} \tag{29}$$

Define the following adaptive laws:

$$\dot{\hat{\theta}} = \begin{cases}\tau_\theta, & \text{if } ||\hat{\theta}|| < \beta_\theta \text{ or } ||\hat{\theta}|| = \beta_\theta \text{ and } \hat{\theta}^{\mathrm{T}}\tau_\theta \le 0 \\ \tau_\theta - \frac{\hat{\theta}\hat{\theta}^{\mathrm{T}}}{||\hat{\theta}||^2}\tau_\theta, & \text{if } ||\hat{\theta}|| = \beta_\theta \text{ and } \hat{\theta}^{\mathrm{T}}\tau_\theta > 0\end{cases} \tag{30}$$

$$\dot{\hat{\bar{M}}}_{\varepsilon hf} = \begin{cases}\tau_M, & \text{if } ||\hat{\bar{M}}_{\varepsilon hf}|| < \beta_M \text{ or } ||\hat{\bar{M}}_{\varepsilon hf}|| = \beta_M \text{ and } \hat{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\tau_M \le 0 \\ \tau_M - \frac{\hat{\bar{M}}_{\varepsilon hf}\hat{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}}{||\hat{\bar{M}}_{\varepsilon hf}||^2}\tau_M, & \text{if } ||\hat{\bar{M}}_{\varepsilon hf}|| = \beta_M \text{ and } \hat{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\tau_M > 0\end{cases} \tag{31}$$

$$\dot{\hat{\omega}} = \begin{cases}\tau_\omega, & \text{if } ||\hat{\omega}|| < \beta_\omega \text{ or } ||\hat{\omega}|| = \beta_\omega \text{ and } \hat{\omega}^{\mathrm{T}}\tau_\omega \le 0 \\ \tau_\omega - \frac{\hat{\omega}\hat{\omega}^{\mathrm{T}}}{||\hat{\omega}||^2}\tau_\omega, & \text{if } ||\hat{\omega}|| = \beta_\omega \text{ and } \hat{\omega}^{\mathrm{T}}\tau_\omega > 0\end{cases} \tag{32}$$

where $\tau_\theta = -\eta_1\xi(s_f - \hat{\omega})^{\mathrm{T}}P(D+B) + \eta_\theta\hat{\theta}$, $\tau_M = -\eta_2\operatorname{sgn}((s_f - \hat{\omega})^{\mathrm{T}}P(D+B))(s_f - \hat{\omega})^{\mathrm{T}}P(D+B) + \eta_M\hat{\bar{M}}_{\varepsilon hf}$, $\tau_\omega = -\eta_3 m\lambda_1(s_f - \hat{\omega})^{\mathrm{T}}P(D+B) + \eta_\omega\hat{\omega}$.

$\eta_\theta > 0 \in R$, $\eta_M > 0 \in R$, $\eta_\omega > 0 \in R$ are design parameters. Note that project operators are adopted to ensure that $\hat{\theta}$, $\hat{\bar{M}}_{\varepsilon hf}$ and $\hat{\omega}$ are bounded.

4 Stability Analysis

Theorem 1 *Consider multi-agent system (1) and leader node (2) under Assumptions 1–5. Using distributed control law (22) and adaptive laws (30)–(32), the tracking errors $\delta_i (i = 1, \ldots, n)$ are cooperative uniformly ultimately bounded and the tracking error δ_i converges to the small neighborhoods of the origin.*

Proof Define the Lyapunov function

$$V_0 = s^{\mathrm{T}} Ps/2 + tr\{\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\}/2\eta_1 + \tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\tilde{\bar{M}}_{\varepsilon hf}/2\eta_2 + \tilde{\omega}^{\mathrm{T}}\tilde{\omega}/2\eta_3$$

Differentiating V_0 with respect to time t, we have

$$\begin{aligned}\dot{V}_0 = {} & s^{\mathrm{T}} P\gamma - s^{\mathrm{T}} P(L + B)u - s^{\mathrm{T}} P(D + B)\theta^{*\mathrm{T}}\xi \\ & + s^{\mathrm{T}} PA\theta^{*\mathrm{T}}\xi + s^{\mathrm{T}} P(D + B)\xi_f - s^{\mathrm{T}} PA\xi_f \\ & - s^{\mathrm{T}} P(D + B)(\varepsilon + h - \underline{f}_0) + s^{\mathrm{T}} PA(\varepsilon + h - \underline{f}_0) \\ & - tr\{\tilde{\theta}^{\mathrm{T}}\dot{\hat{\theta}}\}/\eta_1 - \tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\dot{\hat{\bar{M}}}_{\varepsilon hf}/\eta_2 - \tilde{\omega}^{\mathrm{T}}\dot{\hat{\omega}}/\eta_3\end{aligned} \tag{33}$$

Substituting control law (22) and adaptive law (30)–(32) in (33), one has

$$\begin{aligned}\dot{V}_0 \le & - ms^{\mathrm{T}} P(L + B)s + ((5r + 2rm + \bar{\lambda}^2/4r\underline{\lambda}^2)\bar{\sigma}(A) + r)\bar{\sigma}(P)s^{\mathrm{T}}s \\ & - \frac{\eta_\theta}{\eta_1} tr\{\tilde{\theta}^{\mathrm{T}}\hat{\theta}\} - \frac{\eta_M}{\eta_2}\tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\hat{\bar{M}}_{\varepsilon hf} - \frac{\eta_\omega}{\eta_3}\tilde{\omega}^{\mathrm{T}}\hat{\omega} + \mu_s\end{aligned} \tag{34}$$

Since

$$-\frac{\eta_\theta}{\eta_1} tr\{\tilde{\theta}^{\mathrm{T}}\hat{\theta}\} \le -\frac{\eta_\theta}{2\eta_1} tr\{\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\} + \frac{\eta_\theta}{2\eta_1} tr\{\theta^{*\mathrm{T}}\theta^*\}$$

$$-\frac{\eta_M}{\eta_2}\tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\hat{\bar{M}}_{\varepsilon hf} \le -\frac{\eta_M}{2\eta_2}\tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\tilde{\bar{M}}_{\varepsilon hf} + \frac{\eta_M}{2\eta_2}\bar{M}_{\varepsilon hf}^{\mathrm{T}}\bar{M}_{\varepsilon hf}$$

$$-\frac{\eta_\omega}{\eta_3}\tilde{\omega}^{\mathrm{T}}\hat{\omega} \le -\frac{\eta_\omega}{2\eta_3}\tilde{\omega}^{\mathrm{T}}\tilde{\omega} + \frac{\eta_\omega}{2\eta_3}\omega^{\mathrm{T}}\omega$$

one has

$$\begin{aligned}\dot{V}_0 \le & - \frac{1}{2} ms^{\mathrm{T}} Qs + ((5r + 2rm + \bar{\lambda}^2/4r\underline{\lambda}^2)\bar{\sigma}(A) + r)\bar{\sigma}(P)s^{\mathrm{T}}s \\ & - \frac{\eta_\theta}{2\eta_1} tr\{\tilde{\theta}^{\mathrm{T}}\tilde{\theta}\} - \frac{\eta_M}{2\eta_2}\tilde{\bar{M}}_{\varepsilon hf}^{\mathrm{T}}\tilde{\bar{M}}_{\varepsilon hf} - \frac{\eta_\omega}{2\eta_3}\tilde{\omega}^{\mathrm{T}}\tilde{\omega} \\ & + \frac{\eta_\theta}{2\eta_1} tr\{\theta^{*\mathrm{T}}\theta^*\} + \frac{\eta_M}{2\eta_2}\bar{M}_{\varepsilon hf}^{\mathrm{T}}\bar{M}_{\varepsilon hf} + \frac{\eta_\omega}{2\eta_3}\omega^{\mathrm{T}}\omega + \mu_s\end{aligned} \tag{35}$$

From the previous analysis, we know θ^*, $\bar{M}_{\varepsilon hf}$, ω are bounded. If η_θ, η_M, η_ω and η_1, η_2, η_3 are chosen appropriately, then

$$\frac{\eta_\theta}{2\eta_1} tr\{\theta^{*\mathrm{T}}\theta^*\} + \frac{\eta_M}{2\eta_2}\bar{M}_{\varepsilon hf}^{\mathrm{T}}\bar{M}_{\varepsilon hf} + \frac{\eta_\omega}{2\eta_3}\omega^{\mathrm{T}}\omega \leq \mu_{\theta M}$$

where $\mu_{\theta M} > 0 \in R$ is a design parameter.

Let $\mu_0 = \mu_s + \mu_{\theta M}$, then

$$\dot{V}_0 \leq -\lambda_0 V_0 + \mu_0$$

Further, one has

$$0 \leq V_0(t) \leq \frac{\mu_0}{\lambda_0} + \left(V_0(t_0) - \frac{\mu_0}{\lambda_0}\right) e^{-\lambda_0(t-t_0)}$$

where $\lambda_0 = \min\left\{\frac{1/2m\underline{\sigma}(Q)}{\bar{\sigma}(P)} - (5r + 2rm + \bar{\lambda}^2/4r\underline{\lambda}^2)\bar{\sigma}(A) - r,\ \frac{\eta_\theta}{2\eta_1}, \frac{\eta_M}{2\eta_2}, \frac{\eta_\omega}{2\eta_3}\right\}$.

Let $V_0 = \frac{1}{2}\underline{\sigma}(P)M^2$, $M > 0 \in R$ is a design parameter. Since $\underline{\sigma}(P)||s(t)||^2 \leq 2V_s(t) \leq 2V_0(t)$, then $||s(t)|| \leq \sqrt{2V_0(t)/\underline{\sigma}(P)}$, $||s(t)|| \leq M$. Similarly $||\tilde{\theta}(t)|| \leq \sqrt{\eta_1}M$, $||\tilde{\bar{M}}_{\varepsilon hf}(t)|| \leq \sqrt{\eta_2}M$, $||\tilde{\omega}|| \leq \sqrt{\eta_3}M$. According to the previous analysis, we know s and ω are bounded, because $s_f = s + \omega$, so s_f also is bounded. Since $|e_{k,i}| \leq 2^{i-1}\lambda_k^{i-n}M$, one has $||e_i|| \leq \sqrt{N(2^{i-1}\lambda_k^{i-n}M)^2}$. According to (16), we can know $||e_{a,i}|| \leq \sqrt{N(2^{i-1}\lambda_k^{i-n}M)^2} + \beta_\omega$ $i = 1, \ldots, n$ $k = 1, \ldots, N$. From Lemma 2, we can get the following result: from the previous analysis, $s_f(t)$ is cooperative uniform ultimate boundedness. Further, $s_{f,k}(t)$ also are cooperative uniform ultimate boundedness. Because $e_{a,i}(t)$ is bounded, from Lemma 2, one has $\delta_{k,i}$ are bounded. And since the state of the leader is bounded, the state $x_{f,k}$ are bounded as well.

5 Simulation Results

Consider a 5-node digraph G and a leader node described in Fig. 4. The dynamics of the leader node is described as follows:

$$\begin{cases} \dot{x}_{0,1}(t) = x_{0,2}(t) \\ \dot{x}_{0,2}(t) = x_{0,3}(t) \\ \dot{x}_{0,3}(t) = -x_{0,1}(t) - 1.5x_{0,2}(t) - 2x_{0,3}(t) + 2\sin(2t) + 4\cos(2t) \end{cases}$$

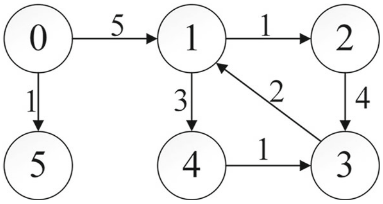

Fig. 4 Topology of the communication

The follower nodes are described by third-order nonlinear systems in the form of (1) with

$$
\begin{aligned}
\dot{x}_{1,3}(t) &= -x_{1,1}x_{1,2} + \sin(x_{1,3}) + u_1 + h_1\\
\dot{x}_{2,3}(t) &= x_{2,1}\cos(x_{2,2}) + 2x_{2,3} + u_2 + h_2\\
\dot{x}_{3,3}(t) &= -x_{3,1} + \sin(x_{3,2}) + u_3 + h_3\\
\dot{x}_{4,3}(t) &= (x_{4,1} + x_{4,2})^2 + 3x_{4,3} + \cos(2t) + u_4 + h_4\\
\dot{x}_{5,3}(t) &= -2x_{5,1} + x_{5,2} + u_5 + h_5
\end{aligned}
$$

In this paper, the disturbance h_k is random constant and bounded by $|h_k| \leq 1$. Choose the following initial states: $x_0 = [0, 1, 1]^{\mathrm{T}}, x_1 = [1, -1, 0]^{\mathrm{T}}, x_2 = [0, -1, -2]^{\mathrm{T}}, x_3 = [0, 1, 0]^{\mathrm{T}}, x_4 = [1.5, 0, 0]^{\mathrm{T}}, x_5 = [0, 1, -1]^{\mathrm{T}}$.

From Fig. 1 and Lemma 1, we can know

$$
A = \begin{bmatrix} 0 & 0 & 2 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\\ 0 & 4 & 0 & 1 & 0\\ 3 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
B = \begin{bmatrix} 5 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 5 & 0 & 0\\ 0 & 0 & 0 & 3 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
L = \begin{bmatrix} 2 & 0 & -2 & 0 & 0\\ -1 & 1 & 0 & 0 & 0\\ 0 & -4 & 5 & -1 & 0\\ -3 & 0 & 0 & 3 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
L + B = \begin{bmatrix} 7 & 0 & -2 & 0 & 0\\ -1 & 1 & 0 & 0 & 0\\ 0 & -4 & 5 & -1 & 0\\ -3 & 0 & 0 & 3 & 0\\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
q = \begin{bmatrix} 0.62\\ 1.62\\ 1.69\\ 0.96\\ 1.00 \end{bmatrix}
P = \begin{bmatrix} 1.59 & 0 & 0 & 0 & 0\\ 0 & 0.61 & 0 & 0 & 0\\ 0 & 0 & 0.59 & 0 & 0\\ 0 & 0 & 0 & 1.04 & 0\\ 0 & 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

$$
Q = \begin{bmatrix} 11.16 & 0 & -1.18 & 0 & 0\\ -1.59 & 0.61 & 0 & 0 & 0\\ 0 & -2.45 & 2.95 & -1.04 & 0\\ -4.78 & 0 & 0 & 3.12 & 0\\ 0 & 0 & 0 & 0 & 1.00 \end{bmatrix}
$$

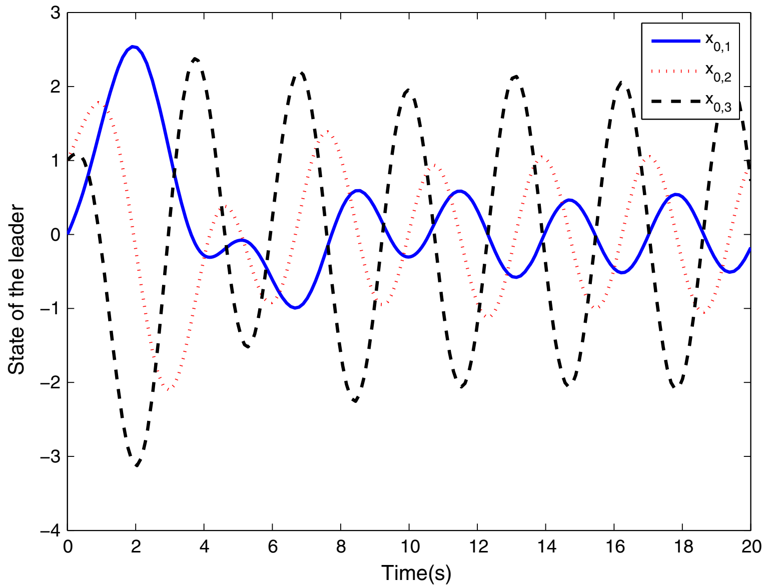

Fig. 5 The state of the leader 0

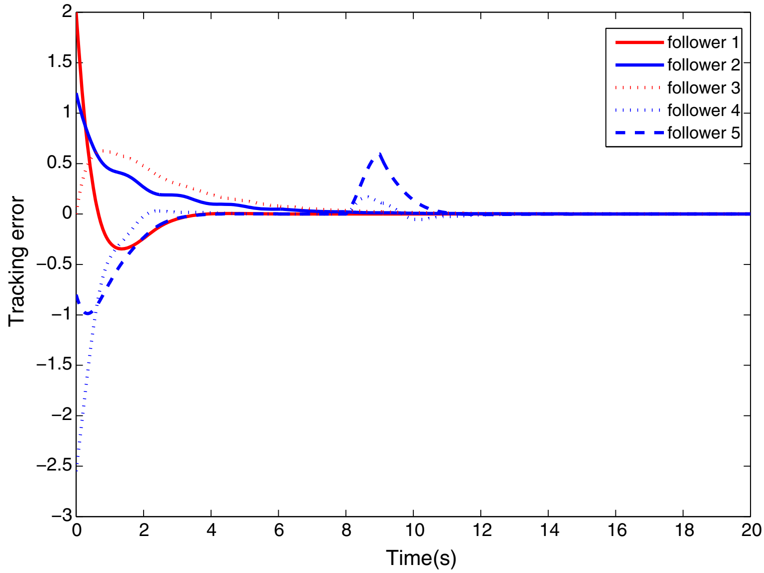

Fig. 6 Tracking error δ_1

Further, we have the singular values of: 11.01, 4.03, 0.42, 2.36, 1.00. $\bar{\sigma}(A) = 2.41$, $\bar{\sigma}(P) = 1.59$, $\bar{\sigma}(Q) = 11.01$, $\bar{\sigma}(D + B) = 1.00$. In this simulation, we choose $m = 1$, $\lambda_1 = 2$, $\lambda_2 = 2$, $\lambda_3 = 1$, $\eta_1 = \eta_2 = \eta_3 = 100$, $\eta_\theta = \eta_M = \eta_\omega = 0.01$, $\mu_s = \mu_{\theta M} = 0.1$, $r = 100$.

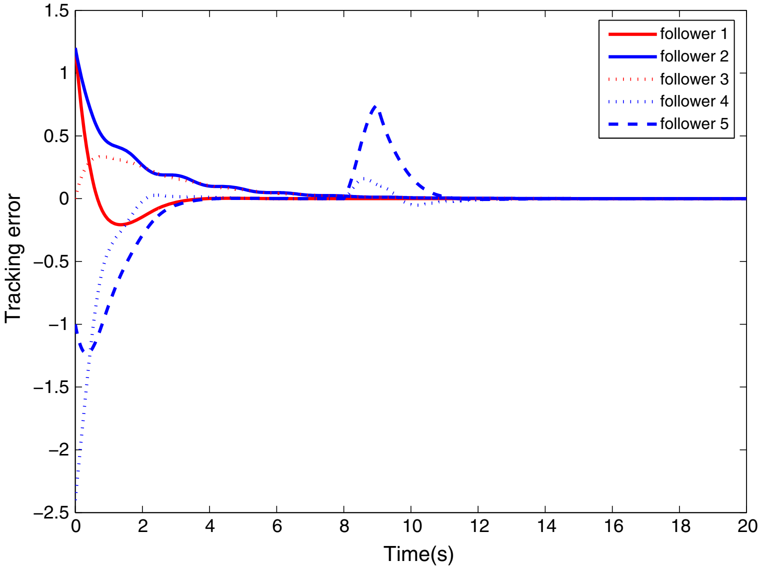

Fig. 7 Tracking error δ_2

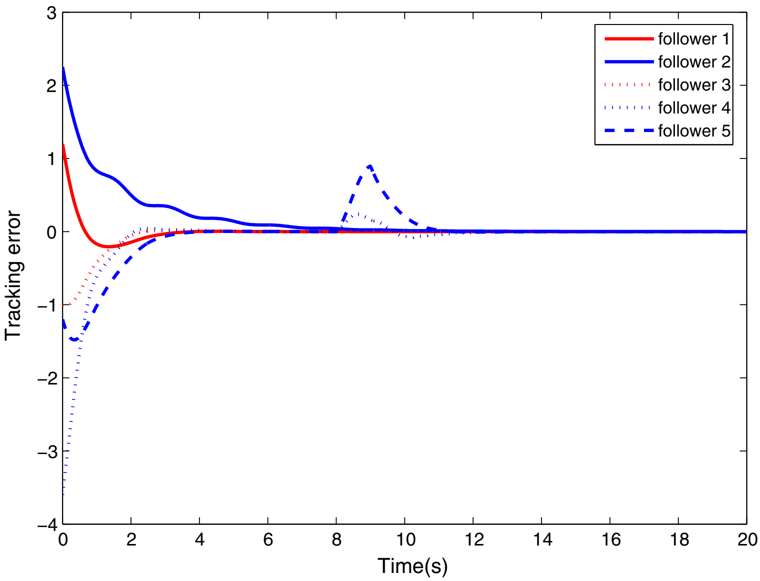

Fig. 8 Tracking error δ_3

The simulation results are presented in Figs. 5, 6, 7 and 8. From Fig. 5, we can find that the state of the leader is bound. From Figs. 6, 7 and 8, at the beginning, under the normal controller, the tracking errors converge to the neighborhood of the origin. When signal transmission faults occurred, we can find the tracking errors deviate from the neighborhood of the origin. However, if the signal transmission faults are compensated for proposed fault tolerant controllers (22), we can obtain better tracking control performance again. The simulation results have illustrated the effectiveness of the proposed scheme.

6 Conclusions

In this paper, the cooperative adaptive fault-tolerant tracking control problem of high-order nonlinear multi-agent systems with signal transmission faults is studied. Based on the approximation capability of neural networks, an adaptive fault-tolerant control scheme is proposed, which guarantees that all followers asymptotically synchronize to a leader with tracking errors converging to a small adjustable neighborhood of the origin. However, the topology among nodes considered in this paper is fixed. In practical applications, the topology may be variable. For example, a new node is added to or removed from communication network. In the case, how to handle the cooperative control problem of nonlinear multi-agent systems is important and challenging, which is studied in our further research.

Acknowledgements The work is partly supported by the National Natural Science Foundation of China (61873229, 61473250).

References

1. D.P. Bertsekas, J.N. Tsitsiklis, Comments on “coordination of groups of mobile autonomous agents using nearest neighbor rules”. IEEE Trans. Autom. Control **52**(5), 968–969 (2007)
2. X.X. Chen, K. Zhang, B. Jiang, Finite-time unknown input observer-based distributed fault diagnosis for multi-agent systems with disturbances. Circuits Syst. Signal Process. **37**(10), 4215–4233 (2018)
3. C. Deng, G.H. Yang, Distributed adaptive fault-tolerant containment control for a class of multi-agent systems with non-identical matching non-linear functions. IET Control Theory Appl. **10**(3), 273–281 (2016)
4. C. Deng, G.H. Yang, Cooperative adaptive output regulation for linear multi-agent systems with actuator faults. IET Control Theory Appl. **11**(14), 2396–2402 (2017)
5. C. Deng, G.H. Yang, Distributed adaptive fuzzy control for nonlinear multi-agent systems under directed graphs. IEEE Trans. Fuzzy Syst. **26**(3), 1356–1366 (2018)
6. S.S. Ge, F. Hong, T.H. Lee, Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients. IEEE Trans. Syst. Man Cybern. B Cybern. **34**(1), 499–516 (2004)
7. Y.Z. Hua, X.W. Dong, Q.D. Li, Z. Ren, Distributed fault-tolerant time-varying formation control for second-order multi-agent systems with actuator failures and directed topologies. IEEE Trans. Circuits Syst. II Exp. Briefs **65**(6), 774–778 (2018)
8. C.C. Hua, K. Li, X.P. Guan, Semi-global/global output consensus for nonlinear multiagent systems with time delays. Automatica **103**, 480–489 (2019)
9. A. Jadbabaie, J. Lin, A.S. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans. Autom. Control **48**(6), 988–1001 (2003)
10. H.M. Jiao, M. Shi, Q.K. Shen, J.W. Zhu, P. Shi, Filter design with adaptation to time-delay parameters for genetic regulatory networks. IEEE/ACM Trans. Comput. Biol. Bioinform. **15**(1), 323–329 (2018)
11. H.M. Jiao, L.P. Zhang, Q.K. Shen, J.W. Zhu, P. Shi, Robust gene circuit control design for time-delayed genetic regulatory networks without SUM regulatory logic. IEEE/ACM Trans. Comput. Biol. Bioinform. **15**(6), 2086–2093 (2018)

12. X.Z. Jin, S.F. Wang, J.H. Qin, W.X. Zheng, Y. Kang, Adaptive fault-tolerant consensus for a class of uncertain nonlinear second-order multi-agent systems with circuit implementation. IEEE Trans. Circuits Syst. I Regul. Pap. **65**(7), 2243–2255 (2018)
13. Z.Y. Ma, H.J. Ma, Adaptive finite-time dynamic output-feedback FTC design for MIMO nonlinear systems with actuator and sensor faults. IEEE Access **6**, 44824–44834 (2018)
14. H.J. Ma, G.H. Yang, Adaptive fault tolerant control of cooperative heterogeneous systems with actuator faults and unreliable interconnections. IEEE Trans. Autom. Control **61**(11), 3240–3255 (2016)
15. L. Moreau, Stability of multiagent systems with time-dependent communication links. IEEE Trans. Autom. Control **50**(2), 169–182 (2005)
16. R. Olfati-Saber, R.M. Murray, Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. Autom. Control **49**(9), 1520–1533 (2004)
17. W. Ren, Y.C. Cao, *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues* (Springer, London, 2011)
18. W. Ren, R.W. Beard, E.M. Atkins, Information consensus in multivehicle cooperative control. IEEE Control Syst. Mag. **27**(2), 71–82 (2007)
19. E. Semsar-Kazerooni, K. Khorasani, Optimal consensus algorithms for cooperative team of agents subject to partial information. Automatica **44**(11), 2766–2777 (2008)
20. Q.K. Shen, P. Shi, Output consensus control of multiagent systems with unknown nonlinear dead zone. IEEE Trans. Syst. Man Cybern. Syst. **46**(10), 1329–1337 (2016)
21. Q.K. Shen, B. Jiang, P. Shi, J. Zhao, Cooperative adaptive fuzzy tracking control for networked unknown nonlinear multiagent systems with time-varying actuator faults. IEEE Trans. Fuzzy Syst. **22**(3), 494–504 (2014)
22. Q.K. Shen, P. Shi, Y. Shi, J.H. Zhang, Adaptive output consensus with saturation and dead-zone and its application. IEEE Trans. Ind. Electron. **64**(6), 5025–5034 (2017)
23. Q.K. Shen, P. Shi, J.W. Zhu, L.P. Zhang, Adaptive consensus control of leader-following systems with transmission nonlinearities. Int. J. Control **92**(2), 317–328 (2019)
24. P. Shi, Q.K. Shen, Observer-based leader-following consensus of uncertain nonlinear multi-agent systems. Int. J. Robust Nonlinear Control **27**(17), 3794–3811 (2017)
25. D. Ye, X.G. Zhao, B. Cao, Distributed adaptive fault-tolerant consensus tracking of multi-agent systems against time-varying actuator faults. IET Control Theory Appl. **10**(5), 554–563 (2016)
26. D. Ye, M.M. Chen, H.J. Yang, Distributed adaptive event-triggered fault-tolerant consensus of multi-agent systems with general linear dynamics. IEEE Trans. Cybern. **49**(3), 757–767 (2019)
27. H.W. Zhang, F.L. Lewis, Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics. IEEE Trans. Autom. Control **48**(7), 1432–1439 (2012)
28. K. Zhang, G.S. Liu, B. Jiang, Robust unknown input observer-based fault estimation of leader–follower linear multi-agent. Circuits Syst. Signal Process. **36**(2), 525–542 (2017)
29. L. Zhao, J.P. Yu, C. Lin, Y.M. Ma, Adaptive neural consensus tracking for nonlinear multiagent systems using finite-time command filtered backstepping. IEEE Trans. Syst. Man Cybern. Syst. **48**(11), 2003–2012 (2018)
30. Z.Q. Zuo, J. Zhang, Y.J. Wang, Adaptive fault tolerant tracking control for linear and lipschitz nonlinear multi-agent systems. IEEE Trans. Ind. Electron. **62**(2), 3923–3931 (2015)