

SHORT PAPER

# **Blind Equalization in the Presence of Co-channel Interference Based on Higher-Order Statistics**

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**Abstract** An improved blind channel equalization method is proposed based on the generalized eigenvector algorithm (GEA) in this paper. This new method can blindly equalize the desired signal in the presence of strong co-channel interference. The basic idea underlying the improved GEA method is that higher-order cumulants can be sensitive to frequency offset. By exploiting this property of higher-order statistics, blind equalizer can be designed to equalize the desired signal with known frequency offset while suppressing the interference with a different frequency offset. Simulation results are shown to demonstrate the effectiveness of the proposed method.

**Keywords** Channel equalization · Blind channel equalization · High-order statistics · Co-channel interference

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### **1 Introduction**

Channel equalization is an important method that is widely used to tackle the channel distortion effect faced by communication systems. As compared to the pilot data-based channel equalization method, blind channel equalization does not require repeated transmission of training signal. It is thus preferred in many applications where either data output rate is one main concern, or training signal is not available at all [\[9\]](#page-11-0). Blind channel equalization has been widely studied since 1970s, and many prominent works have been developed based on either high-order statistics (HOS) or second-order cyclostationary statistics (SOCS) (see, for example, [\[2](#page-11-1)[,10](#page-11-2)] and references therein). Among those methods, one class of blind equalization algorithms is based on the fourth-order statistics. For example, the constant modulus algorithm has been developed independently in [\[5](#page-11-3),[11](#page-11-4),[12\]](#page-11-5) in the 1980s. In general, HOS-based approaches can suffer from slow rate of convergence, sensitivity to timing jitter, and local minima, while SOCS can fail in the case of singular channel [\[6\]](#page-11-6). To solve these problems, a generalized eigenvector algorithm (GEA) was proposed that is able to perform almost as well as the recursive least square (RLS) equalizer [\[6](#page-11-6)]. In general, those blind equalization methods based on fourth-order statistics are not suitable to the applications where there is strong co-channel interference. That is because the co-channel interference can impair the computation of high-order statistics of the desired signal and consequently render the blind equalizer fail to extract the desired signal. Some attempts can be found to extend the single-source blind equalizer to multiple-source blind equalizer. However, these methods do not work well when co-channel signals are with inter-symbol interference [\[1](#page-11-7)].

This paper focuses on the problem of blind channel equalization in the presence of strong co-channel interference. To this end, an improved blind equalization method is developed based on the conventional GEA algorithm. The improved GEA method can extract desired signal in the presence of strong co-channel interference. In this study, we assume that the signal and the co-channel interference have different frequency offsets although they share the same frequency channel. This can be caused by the fact that the signal and interference are transmitted from different equipments that have distinct frequency offset with respect to the receiver, for example, in the context of co-channel signal separation  $[13]$ . Then, by exploiting higher-order statistics that is sensitive to frequency offset, new design criterion can be established and improved blind equalizer can be devised to enhance the desired signal with known frequency offset while suppressing the interference with a different frequency offset. Simulation results are given to demonstrate the effectiveness of the proposed method.

The rest of the paper is organized as follows. Section [2](#page-2-0) will present the signal model considered in this paper and a brief review of the conventional GEA method. Section [3](#page-3-0) will present the new GEA method followed by simulation studies in Sect. [4.](#page-7-0) Finally, Sect. [5](#page-10-0) will conclude this paper.

<span id="page-2-1"></span>

#### <span id="page-2-0"></span>**2 Signal Model and GEA Method**

In this paper, we consider the case where a signal is received in the presence of cochannel interference by a single channel receiver. The discrete time signal model can be written as below:

$$
v(k) = d_s(k) * h_s(k)e^{j\omega_s k} + d_i(k) * h_i(k)e^{j\omega_i k} + n(k)
$$
 (1)

where  $d_s(k)$ ,  $h_s(k)$ ,  $d_i(k)$ ,  $h_i(k)$ ,  $\omega_s$ ,  $\omega_i$  are signal waveform, signal channel, interference waveform, interference channel, signal frequency offset and interference frequency offset, respectively; ∗ stands for convolution operation; *n*(*k*) stands for white Gaussian noise of zero mean and variance  $\sigma^2$ , which is independent and identically distributed. The signal, interference and noise are independent to each other. This signal model is depicted in the block diagram of signal receiving model in Fig. [1.](#page-2-1) The objective of this paper is to design an improved GEA method that is able to equalize the signal and suppress the interference at the same time, which is especially crucial when both the signal and interference have similar channel response [\[14\]](#page-11-9).

Since the method developed in this paper is based on the GEA algorithm, we briefly review this algorithm as follows. The implementation of the original GEA method in the absence of interference is illustrated in the block diagram of equalization model in Fig. [1.](#page-2-1) The original GEA method consists of one equalizer and one reference system, as depicted in Fig. [1,](#page-2-1) and relies on the use of fourth-order cumulant for the blind equalization. In particular, it estimates equalization filter coefficients by iteratively maximizing  $C_{4,2}^{xy}$ , which is the fourth-order cross-cumulant between the equalizer output  $x(k)$  and the reference system  $y(k)$  with two conjugate terms. When only signal of interest is considered,  $x(k)$  and  $y(k)$  can be written as

$$
x(k) = xs(k)ej\omegask, y(k) = ys(k)ej\omegask
$$
 (2)

Then, following the theoretical development on HOS in [\[7](#page-11-10),[8\]](#page-11-11), the cumulant  $C_{4,2}^{xy}$  can be defined by:

<span id="page-3-1"></span>
$$
C_{4,2}^{xy} = \text{Cum}\left(x_s(k)e^{j\omega_s k}, x_s^*(k)e^{-j\omega_s k}, y_s(k)e^{j\omega_s k}, y_s^*(k)e^{-j\omega_s k}\right)
$$
  
=  $E\left\{|x_s(k)^2| |y_s(k)^2|\right\} - E\left\{|x_s(k)^2|\right\} E\left\{|y_s(k)^2|\right\}$   
-  $|E\{x_s(k)y_s(k)\}|^2 - |E\{x_s(k)y_s^*(k)\}|^2$  (3)

The cumulant in [\(3\)](#page-3-1) can be estimated from the sample estimates of the corresponding moments, as follows:

$$
\hat{C}_{4,2}^{xy} = \frac{1}{L} \sum_{k=1}^{L} |x(k)^2| |y(k)^2|
$$
  
 
$$
- \left\{ \frac{1}{L} \sum_{k=1}^{L} |x(k)^2| \right\} \left\{ \frac{1}{L} \sum_{k=1}^{L} |y(k)^2| \right\}
$$
  
 
$$
- \left| \frac{1}{L} \sum_{k=1}^{L} x(k) y(k) \right|^2 - \left| \frac{1}{L} \sum_{k=1}^{L} x(k) y^*(k) \right|^2
$$
 (4)

where *L* is the total number of samples.

When only one signal is present, the GEA method works well regardless of the frequency offset. However, this method may break down when there are two cochannel signals. This is because that the fourth-order cross-cumulants are frequencyinsensitive and consequently both the signal and the interference contribute to the cumulant computation, which may diverge the convergence of GEA algorithm. To see this, we may decompose the equalizer output of  $x(k)$  and  $y(k)$  first, as:

<span id="page-3-2"></span>
$$
x(k) = xs(k)ej\omegask + xi(k)ej\omegaik
$$
  

$$
y(k) = ys(k)ej\omegask + yi(k)ej\omegaik
$$
 (5)

Then, applying the additivity property of cumulants, it yields:

$$
C_{4,2}^{x,y} = C_{4,2}^{x_s, y_s} + C_{4,2}^{x_i, y_i} = \text{Cum}\left(x_s(k)e^{j\omega_s k}, x_s^*(k)e^{-j\omega_s k}, y_s(k)e^{j\omega_s k}, y_s^*(k)e^{-j\omega_s k}\right) + \text{Cum}\left(x_i(k)e^{j\omega_i k}, x_i^*(k)e^{-j\omega_i k}, y_i(k)e^{j\omega_i k}, y_i^*(k)e^{-j\omega_i k}\right)
$$
(6)

where we can see that the equalizer is affected by both signal and interference and thus the convergence to the signal of interest may not be guaranteed.

#### <span id="page-3-0"></span>**3 Improved GEA Method**

Considering that interference and signal usually have different frequency offsets in practice, we in this paper propose an improved GEA method by exploiting the higherorder cumulants that are frequency-sensitive. The basic idea underlying this method is that some higher-order statistics of signal are sensitive to signal frequency offset, and by exploiting this property blind channel equalizer can be designed to extract signal of interest in the presence of co-channel interference which has different frequency offset. In general, the improved GEA method follows the same methodology and implementation structure of the original GEA method. The main difference is that it adopts different higher-order statistics to exploit the sensitivity of cumulants to frequency offset. Without loss of generality, we assume the signal of interest is with zero frequency offset and the interference is with frequency offset of  $\omega$ . In the case there is phase noise in the receiver leading to frequency offset drifting, we assume that in the data block of interest the frequency offset is almost constant and thus it can be compensated to zero so that the assumption still holds.

#### **3.1 Impact of Frequency Offset on Cumulant**

The key idea is that by using odd number of conjugates, the impact of frequency offset on cumulants can be retained. Let us consider  $C_{4,1}^{xy}$  for illustration. First, following [\(5\)](#page-3-2), we decompose the equalizer output of  $x(k)$  and  $y(k)$  as:

$$
x(k) = x_s(k) + x_i(k)e^{j\omega k}, \quad y(k) = y_s(k) + y_i(k)e^{j\omega k}
$$
 (7)

After that, we apply the additive property of cumulants and arrive at:

$$
C_{4,1}^{x,y} = C_{4,1}^{x_s, y_s} + C_{4,1}^{x_i, y_i}
$$
  
= Cum(x<sub>s</sub>(k), x<sub>s</sub><sup>\*</sup>(k), y<sub>s</sub>(k), y<sub>s</sub>(k))  
+ Cum (x<sub>i</sub>(k)e<sup>j\omega k</sup>, x<sub>i</sub><sup>\*</sup>(k)e<sup>-j\omega k</sup>, y<sub>i</sub>(k)e<sup>j\omega k</sup>, y<sub>i</sub>(k)e<sup>j\omega k</sup>) (8)

For the interference term in the summation, we further write it as

<span id="page-4-0"></span>
$$
C_{4,1}^{x_i, y_i} = \text{Cum}(x_i(k)e^{j\omega k}, x_i^*(k)e^{-j\omega k}, y_i(k)e^{j\omega k}, y_i(k)e^{j\omega k})
$$
  
=  $E\left[|x_i(k)|^2 y_i(k)^2 e^{2j\omega k}\right] - E\left[|x_i(k)|^2\right]E[y_i(k)^2 e^{2j\omega k}\right]$  (9)  
-  $2E\left[x_i(k)y_i(k)e^{2j\omega k}\right]E\left[x_i^*(k)y_i(k)\right]$ 

This cumulant can also be evaluated with the estimates of respective moments in practice. Meanwhile, from [\(9\)](#page-4-0), we may observe that when the data length and the frequency offset are sufficiently large, the terms with frequency offset inside the expectation operation are almost uniformly distributed over the complex plane so that the moment estimates when evaluated with sample average tend to zero. This can be verified by numerical simulation indeed.

In addition, we hereby provide a brief mathematical proof on the aforementioned claim that sample-averaged cumulant  $C_{4,1}^{x_i y_i}$  is zero in the presence of frequency offset. Considering the case of linear modulation, the cumulant in [\(9\)](#page-4-0) can be derived from the time-variant cumulant when it is sampled at symbol rate. The time-variant cumulant at time *k* can be written as [\[3](#page-11-12)]

$$
c_{4,1}^{x_i, y_i}(k) = \text{Cum}\left(x_i(k)e^{j\omega k}, x_i^*(k)e^{-j\omega k}, y_i(k)e^{j\omega k}, y_i(k)e^{j\omega k}\right)
$$
  
\n
$$
= e^{j2\omega k} \text{Cum}(x_i(k), x_i^*(k), y_i(k), y_i(k))
$$
  
\n
$$
= e^{j2\omega k} \text{Cum}\left(\sum_m a_{i,m}g_e(k - mT), \sum_m a_{i,m}^*g_e^*(k - mT), \sum_m a_{i,m}g_f(k - mT), \sum_m a_{i,m}g_f(k - mT)\right)
$$
  
\n
$$
= e^{j2\omega k} C_{4,1}^{c_i} \sum_m g_e(k - mT)g_e^*(k - mT)
$$
  
\n
$$
g_f(k - mT)g_f(k - mT)
$$
  
\n
$$
g_f(kT - mT)g_f(kT - mT)
$$
  
\n
$$
\triangleq e^{j2\omega k} C_{4,1}^{c_i} \sum_m g(k - mT)
$$

where  $C_{4,1}^{c_i}$  stands for the cumulant of the symbols  $a_{i,m}$ ,  $g_e(k)$  stands for the channel impulse response function of the equalizer system, and  $g_f(k)$  stands for the channel impulse response function of the reference system. Then, the sample-averaged cumulant value should be equal to 0 due to the oscillation effect introduced by frequency offset:

$$
C_{4,1}^{x_i, y_i} = \left\langle c_{4,1}^{x_i, y_i} (k) \right\rangle = \frac{C_{4,1}^{c_i}}{L} \sum_{k=1T}^{LT} e^{j2\omega k} \sum_m g(k - mT)
$$
  

$$
= \frac{C_{4,1}^{c_i}}{L} \sum_{k=1T}^{LT} e^{j2\omega k} \left( \frac{1}{T} \sum_n \tilde{g} \left( \frac{n}{T} \right) e^{j2\pi \frac{n}{T} k} \right)
$$
  

$$
= \frac{C_{4,1}^{c_i}}{LT} \sum_n \tilde{g} \left( \frac{n}{T} \right) \sum_{k=1T}^{LT} e^{j2\omega k} e^{j2\pi \frac{n}{T} k}
$$
  

$$
= \frac{C_{4,1}^{c_i}}{LT} \sum_n \tilde{g} \left( \frac{n}{T} \right) \sum_{k=1T}^{LT} e^{j2\omega k} \approx 0
$$
 (11)

where Poisson summation formula is applied from step 2 to step 3, and  $\tilde{g}(\cdot)$  stands for the Fourier transform of  $g(k)$ . Meanwhile, since the signal of interest is with zero frequency offset, its time-averaged cumulant remains unchanged. Thus, we can have

$$
C_{4,1}^{x,y} \approx C_{4,1}^{x_s, y_s} \tag{12}
$$

This property lays the foundation for the new GEA method.

#### **3.2 Improved GEA Method**

In this way, when a new GEA method is designed with the new cumulants, the effect of interference can be eliminated. For many modulation types, e.g. QPSK and high-

order QAM signals, the value of  $C_{4,1}^{xy}$  is zero [\[3](#page-11-12)]; we thus propose to use higherorder cumulant in the design of improved GEA method. We take  $C_{6,1}^{xy}$ , for example, in the following development and its extension to even higher-order cumulants is straightforward.

First, we define  $C_{6,1}^{xy}$  as

<span id="page-6-0"></span>
$$
C_{6,1}^{xy} = \text{Cum}(x(k), x^*(k), y(k), y(k), y(k), y(k))
$$
\n(13)

Defining  $\mathbf{v}_k = [v(k) \dots v(k - L_c)]^T$ , and  $\mathbf{e} = [e(k) \dots e(k - L_c)]^T$  with  $L_c$  being the equalizer length, substituting  $x(k) = e(k) * v(k) = e^H v_k$  in [\(13\)](#page-6-0), and applying the moment to cumulant formula by considering all the partitions [\[3](#page-11-12)], we arrive at

$$
C_{6,1}^{xy} = C(e^{H}v_{k}, v_{k}^{H}e, y(k), y(k), y(k), y(k))
$$
  
= c<sub>1</sub> - c<sub>21</sub> - 4c<sub>22</sub> - 4c<sub>23</sub> + 2(3c<sub>31</sub> + 12c<sub>32</sub>)  
= e<sup>H</sup>(**R**<sub>1</sub> - **R**<sub>21</sub> - 4**R**<sub>22</sub> - 4**R**<sub>23</sub> + 2(3**R**<sub>31</sub> + 12**R**<sub>32</sub>))**e**  

$$
\triangleq e^{H}C_{6,1}^{yv}e
$$
 (14)

where,

$$
c_1 = e^H E \left\{ \mathbf{v}_k \mathbf{v}_k^H y(k) y(k) y(k) y(k) \right\} e = e^H \mathbf{R}_1 e
$$
  
\n
$$
c_{21} = e^H E \left\{ \mathbf{v}_k \mathbf{v}_k^H \right\} E \left\{ y(k) y(k) y(k) y(k) \right\} e = e^H \mathbf{R}_{21} e
$$
  
\n
$$
c_{22} = e^H E \left\{ \mathbf{v}_k y(k) \right\} E \left\{ \mathbf{v}_k^H y(k) y(k) y(k) \right\} e = e^H \mathbf{R}_{22} e
$$
  
\n
$$
c_{23} = e^H E \left\{ \mathbf{v}_k^H y(k) \right\} E \left\{ \mathbf{v}_k y(k) y(k) y(k) \right\} e = e^H \mathbf{R}_{23} e
$$
  
\n
$$
c_{31} = e^H E \left\{ \mathbf{v}_k \mathbf{v}_k^H \right\} E \left\{ y(k) y(k) \right\} E \left\{ y(k) y(k) \right\} e = e^H \mathbf{R}_{31} e
$$
  
\n
$$
c_{32} = e^H E \left\{ y(k) y(k) \right\} E \left\{ \mathbf{v}_k y(k) \right\} E \left\{ \mathbf{v}_k^H y(k) \right\} e = e^H \mathbf{R}_{32} e
$$
  
\n(15)

To solve the equalizer **e**, we resort to:

$$
\max_{\mathbf{S} \in \mathbf{R}} \|\mathbf{e}^H \mathbf{C}_{6,1}^{xy} \mathbf{e}\|
$$
  
s.t. 
$$
\mathbf{e}^H \mathbf{R}_{vv} \mathbf{e} = \sigma^2
$$
 (16)

In [\[6](#page-11-6)], the solution to this optimization problem is achieved by performing eigenvalue decomposition first and then selecting the principle eigenvector. However, in this paper, by examining the property of the matrix  $\mathbf{C}^{xy}_{6,1}$ , one may find that the matrix is not Hermitian, which may give rise to a computation problem if the eigenvalue decomposition-based optimization is performed directly on it. Instead, we propose to apply the singular value decomposition to obtain the principle singular vector to maximize the objective function, which can be seen as a generalization of the solution in [\[6](#page-11-6)]. Finally, it results in,

<span id="page-6-1"></span>
$$
\mathbf{e}_{\text{EVA}} = \mathcal{P}\left\{ \mathbf{R}_{vv}^{-1} \mathbf{C}_{6,1}^{yv} \right\}
$$
 (17)

where  $\mathcal{P}\{\cdot\}$  means the principle singular vector. From [\(17\)](#page-6-1) the singular vector corresponding to the maximum singular value of the matrix is the solution  $e_{EVA}$  of the current iteration which will be used to initialize the  $f(k)$  for the next iteration. The full implementation of the improved GEA method based on [\(17\)](#page-6-1) can then directly follow that of original GEA method. The same equalizer model in Fig. [1](#page-2-1) is applicable to improved GEA method as well. Finally, the improved GEA method can be summarized as follows:

- *Step-1* Set the reference system to  $f^0(k) = \delta(k [L_c/2])$  and begin the iteration with iteration counter  $i = 0$ . From  $v(1), ..., v(l)$  calculate the matrix  $\mathbf{R}_{vv} \to \hat{\mathbf{R}}_{vv}$ .
- − *Step*-2 Determine  $y(k) = v(k) * f<sub>i</sub>(k)$  and estimate the matrix  $\mathbf{C}_{6,1}^{yv} \Rightarrow \mathbf{C}_{6,1}^{yv}$
- *Step-3* With  $\hat{\mathbf{R}}_{vv}$  and  $\hat{\mathbf{C}}_{6,1}^{yv}$  substituted for  $\mathbf{R}_{vv}$  and  $\mathbf{C}_{6,1}^{yv}$  in Eq. [\(17\)](#page-6-1), calculate the **e***EV A* by choosing the singular vector corresponding to the maximum singular value of the matrix  $\hat{\mathbf{R}}_{vv}^{-1} \hat{\mathbf{C}}_{6,1}^{y\bar{v}}$ .
- $-$  *Step-4* Load  $e_{EVA}^{(i)}(k)$  in to the reference system, i.e. let  $f^{(i+1)}(k) = e_{EVA}^{(i)}(k)$ . Increment the iteration counter:  $i \rightarrow i + 1$ ; if  $i < I$ , go to step 2.

Here the parameter *l* is the number of samples of  $v(k)$  used to determine the matrices  $\hat{\mathbf{R}}_{vv}$  and  $\hat{\mathbf{C}}_{6,1}^{yv}$ , and *I* denotes number of total iterations to estimate the equalizer coefficients.

#### **3.3 Remarks on Other Modulation Types**

Since cumulant is dependent on the modulation type [\[3](#page-11-12)], the improved GEA method may be tailored to a desired modulation by choosing a proper cumulant. For example, for BPSK modulation, instead of using sixth-order cumulant, we can use fourth-order cumulant such as  $C_{4,1}$ , where the sensitivity to frequency offset can be achieved using one conjugate item. As for 8PSK signals, we may use eighth-order cumulant such as  $C_{8,1}$ . As for the general class of QAM signals, sixth-order cumulant can also be used. When using sixth- or higher-order cumulant, the computation load will be higher than using fourth-order cumulant.

#### <span id="page-7-0"></span>**4 Simulations Studies**

In this section, we conduct simulation studies to validate the proposed method with QPSK modulation. Root-raised cosine filter with coefficient of 0.35 and truncated length of 5 is taken to generate both signal and interference at symbol rate of 400 K symbols per second. Time offsets are 0 and 0.2 for signal and interference, respectively. Those simulation setups can be seen in the context of co-channel MPSK signals [\[13,](#page-11-8) [14\]](#page-11-9). For the signal, unless otherwise stated, we set frequency offset to be 0 Hz, and signal-to-noise ratio (SNR) 15 dB in all the simulations. The SNR in this paper means the SNR of single signal or interference.

In the first example, we illustrate the effect of frequency offset on the cumulant as developed from (9) to (11) in section III , which is the basis of the improved GEA method. Here, we take the interference signal for study. We evaluate the  $C_{6,1}$  with 2000

<span id="page-8-0"></span>

<span id="page-8-1"></span>symbols at 15 dB SNR for a range of frequency offset. The result is shown in Fig. [2,](#page-8-0) indicating that when the frequency offset is not zero, the cumulant goes to almost zero. Then, we compare the clustering property of outputs from both the conventional GEA and the proposed GEA. The frequency offset of the interference is set to 1000 Hz. The SNR of interference is also 15 dB. Data length is 2000 symbols. Figure [3](#page-8-1) shows the signal cluster after equalization with the original GEA, and Fig. [4](#page-9-0) shows the signal cluster after equalization with the improved GEA. It can be clearly seen that the improved GEA method produced better clustering result, i.e. equalization, than the original GEA method. In order to compare the two GEA methods in a quantitative way, we compute the symbol error rate (SER) in different simulation scenarios. Hundred Monte Carlo trials were conducted to generate the average SER for the following simulations. First, in Fig. [5,](#page-9-1) we show the results of SERs of both methods with respect to the SNR of the interference. Signal SNR is 15 dB, and data length is kept as 2000 symbols. Frequency offset is 1000 Hz for the interference. From this result, we can see that although both methods prefer low SNR of interference, the proposed method is more robust to strong interference. Second, in Fig. [6,](#page-10-1) we study the effect of data length

<span id="page-9-0"></span>

<span id="page-9-1"></span>on the SER performance, where SNR of interference is 15 dB, and frequency offset is 1000 Hz for the interference. From this figure, we can see that the new GEA method enjoys longer data for better convergence. Finally, in Fig. [7](#page-10-2) we investigate the SER performance versus the frequency offset, where the data length is 2000 symbols and SNR of interference is 15 dB, showing that the proposed new GEA method requires certain amount of frequency offset in interference to work. In addition, from the results, we may see that at 0 dB signal-to-interference noise ratio, the improved GEA method can still achieve roughly 0.1 SER, while the original GEA totally fails. Although this SER is relatively high, channel coding techniques and nonlinear equalization methods can be applied to further improve the performance in some practical communication systems, which is very difficult, if not possible, to be achieved if the conventional GEA method is used. Furthermore, when the interference power goes lower, the two GEA methods eventually converge to a similar SER, which means that when the desired signal has considerably higher SNR than the interference, both methods work with comparable performance. Moreover, the improved GEA method requires sufficient frequency offset and/or data length to perform well as shown in Figs. [6](#page-10-1) and [7.](#page-10-2)

0 200 400 600 800 1000

frequency offset, Hz

<span id="page-10-1"></span>

# **Fig. 6** SER of the desired signal versus number of symbols (frequency offset =  $[0, 1000 \text{ Hz}]$ ,

<span id="page-10-0"></span>**5 Conclusions**

In this paper, we have proposed an improved GEA method for blind channel equalization in the presence of strong co-channel interference. This was achieved by exploiting the fact that higher-order cumulant, when properly chosen, can be dependent on frequency offset. Thus, in the case that signal and interference have different frequency offsets, the cumulant of signal can be retained while that of interference be suppressed. Therefore, when designed with such cumulants, the improved GEA method is able to equalize the signal while suppressing the interference. Simulation results were presented to illustrate the effectiveness of the proposed method. In practical applications, when it is not possible to achieve good equalization performance in some extreme cases like the simulation in this paper, this method can be used as an initialization method that is applied jointly with the signal cancellation scheme [\[4](#page-11-13)] to achieve single channel source separation.

<span id="page-10-2"></span> $0$ <sub>0</sub> 0.1

## **References**

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