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H_∞ Static Output Control of Discrete-Time Networked Control Systems with an Event-Triggered Scheme

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Abstract This paper is devoted to the event-triggered H_∞ static output feedback control of linear discrete-time networked control systems. With the help of zero-holder, a time-delay formulation is adopted to describe the even-triggered output. Resorting to Finsler lemma and time-delay techniques, a co-design framework of event-triggering communication and static output controller is established in terms of linear matrix inequalities. Meanwhile, the required H_∞ performance could be ensured by the proposed framework. Two examples are supplied to verify the validity of the proposed method.

Keywords Networked control systems · H_∞ control · Output feedback control · Event-triggered control · Optimal control

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1 Introduction

As the rapid growing of digital and communication technologies, much interest and concern have been focused on networked control systems (NCSs) where network medium is used to communicate the data among the distributed sensors, controllers, actuators, and plants [24]. The advantages of low cost, flexibility and simple maintenance account for the wide applications in advanced aircraft, electrical power grid [21]. Moreover, a quantity of theoretical results about NCSs can be found in [9,14,16] and references therein. Considering the communication mechanism in these works, a periodical strategy also called time-triggering paradigm is executed. Under this scheme, the transmissions of small changed signals could reduce the utilization efficiency of the limited network bandwidth, particularly in wireless network environments [2]. Hence, it is of theoretical and practical significance to improve the efficiency of the shared network as well as maintain the system performance.

Alternatively, an aperiodic triggering method, called event-triggered scheme, has been presented in [19,33]. By pre-designing an appropriate triggering principle to avoid the transmission of unnecessary signals, more and more results based on event-triggered scheme have been derived in [4,19,22,37]. It is noteworthy that most of them need all system state to implement the event-triggered mechanism. Unfortunately, such requirement is difficult even impossible to be satisfied in practical cases [8,31]. To overcome this obstacle, output-based event-trigged control issues are addressed in [17,18,20,23,27,34,35]. [34] studies the design of event-triggered dynamic output feedback controller of continuous linear time invariant systems. For linear continuous-time systems, [17] copes with the dynamic output feedback event-triggered control problem with quantisation. By designing a Luenberger state estimator, [18] considers the robust event-triggered model predictive control for constrained linear systems with bounded disturbances. The event-triggered output feedback control of distributed NCSs is studied in [20], where a distributed observer is used to estimate the system state. Based on the available state, [35] investigates the design of observer-based event-triggered controller of linear continuous-time systems. As is known, dynamic and observer-based output control can be transformed to the framework of static output feedback (SOF) control, which is easy for implementing in engineering applications. Therefore, in [27], sufficient linear matrix inequalities (LMIs) conditions for event-triggered SOF control of continuous-time NCSs are addressed without H_∞ control synthesis. For continuous-time NCSs with time-varying sampling, the H_∞ SOF control problem is developed in [23], where the proposed conditions are non-convex. On the other hand, most aforementioned outcomes are considered for continuous-time cases, while the inherent nature of network-based communications is discrete. In NCSs, since communication protocols utilized to exchange signals between network nodes are usually based on data packet, a continuous manner to transmit information cannot be realized. Due to this fact, several results on event-triggered control problem for discrete-time systems have been reported in [10,15], in which the event-triggered schemes are dependent on system state. Note that few attention has been paid to study the design of the event-triggered H_∞ output feedback controller of discrete-time NCSs, especially for designing SOF controller via a convex method.

Inspired by the above observations, this paper aims to explore an event-based H_∞ SOF controller design method for discrete-time NCSs. Via the zero-holder, the event-triggered output is presented by a time-delay form. Based on this presentation and the event-triggering condition, an analysis condition for the closed-loop system to be asymptotically stable with the required H_∞ performance level is established via Finsler lemma which separates system matrices from Lyapunov matrix. Employing the obtained analysis condition, a co-design method of the event-triggering communication and static output feedback controller is realized by using Finsler lemma repeatedly. Finally, the validity of the provided approach is tested by a numerical example and an aircraft system.

This article is organized as follows. Section 2 provides the problem statement and preliminaries. In Sect. 3, sufficient conditions of event-triggered H_∞ SOF control are formulated by LMIs. The simulations are carried out to show the validity of the proposed method in Sect. 4. Finally, conclusions are produced in Sect. 5.

Notation Throughout this paper, a symmetric and positive (negative) definite matrix R is denoted by $R > 0$ (< 0). $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is used to show the set of all $n \times m$ real matrices. $\mathbb{Z}^+$ is the set of positive integers. $*$ refers to the symmetric entries of a symmetry matrix. The subscripts T and $\perp$ represent the transpose and the null space of a matrix, respectively. Moreover, $He(X)$ is defined to mean $(X + X^T)$.

2 Problem Statement and Preliminaries

Consider the linear discrete-time system captured by

$$\begin{cases} x(k+1) = Ax(k) + B_1u(k) + B_2\omega(k) \\ z(k) = C_1x(k) + D_1u(k) + D_2\omega(k) \\ y(k) = C_2x(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state, $u(k) \in \mathbb{R}^m$ means control input, $\omega(k) \in \mathbb{R}^p$ represents the disturbance in $\mathcal{L}_2\ [0\ \ \infty)$, $z(k) \in \mathbb{R}^q$ denotes control output, $y(k) \in \mathbb{R}^s$ is measured output and n, m, p, q, s belong to $\mathbb{Z}^+$.

Taking into account the limited network bandwidth and unavailable system state, a data triggering scheme, as drawn in Fig. 1, is described as below

$$k_{s+1} = k_s + \min_k \left\{k|e^T(k)\Phi e(k) \geqslant \delta y^T(k)\Phi y(k)\right\} \tag{2}$$

where Φ is a weighting matrix, $\delta \in [0, 1)$ is a prescribed scalar, and $e(k) = y(k)-y(k_s)$ is the error between the present sampled signal $y(k)$ and the latest triggered one $y(k_s)$, $k, k_s \in \mathbb{Z}^+$.

To hold the input signal $u(k)$ of the actuator by the last released data $y(k_s)$ before the new one arrives, a zero-order-holder (ZOH) is adopted, that is

$$u(k) = Ky(k_s), \quad k \in \left[k_s + \tau_{k_s}, \quad k_{s+1} + \tau_{k_s+1} - 1\right] \tag{3}$$

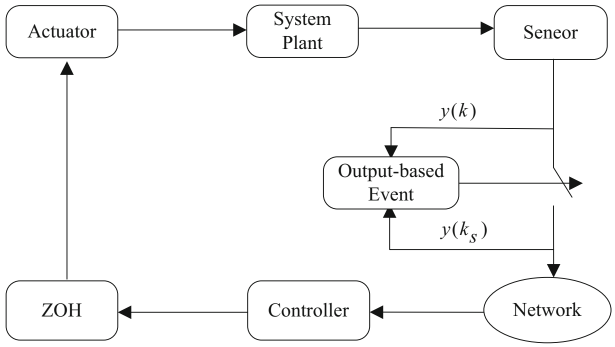

Fig. 1 Framework of an event-triggered NCS

where K is the controller gain to be designed, $\tau_k \in [0, \ \tau_M]$ is the communication delay. The time interval $\Omega = [k_s + \tau_{k_s}, \quad k_{s+1} + \tau_{k_s+1} - 1]$ of ZOH is represented as $\Omega = \cup \Omega_r$, where $\Omega_r = [k_s + n + \tau_{k_s+n}, \quad k_s + n + 1 + \tau_{k_s+n+1}]$, $n = 0, 1, 2, \ldots, k_{s+1} - k_s - 1$.

Define $d_k \triangleq k - k_s - n$, $k \in \Omega_r$. It is obvious that $0 \leqslant d_k \leqslant \tau_m + 1 \triangleq d_m$, where d_m denotes an artificial delay containing the impact of both communication delay and event-triggered scheme. Then, the actual control input of the actuator is written as

$$u(k) = KC_2 x(k - d_k) - Ke(k), \quad k \in \Omega_r \tag{4}$$

Remark 1 When choosing $C_2 = I$, (2) reduces to the state-based event-triggered scheme proposed in [15]. Considering the function of a logical zero-order holder, the system control signal $u(k)$ is expressed by network induced delay and the error information of event-triggered scheme in (4).

Taking (4) into (1), the resulting closed-loop system is transformed into the following delay system

$$\begin{cases} x(k+1) = Ax(k) + B_1 KC_2 x(k - d_k) - B_1 Ke(k) + B_2 \omega(k) \\ z(k) = C_1 x(k) + D_1 KC_2 x(k - d_k) - D_1 Ke(k) + D_2 \omega(k) \\ x(\vartheta) = \phi(\vartheta) = x(0), \quad \vartheta \in [-\tau_m, \quad 0], \quad k \in \Omega_r \end{cases} \tag{5}$$

To design an event-based SOF controller such that the closed-loop system (5) with required disturbance attenuation performance is asymptotically stable, the definition of H_∞ performance for (5) and several technical lemmas are shown as follows.

Definition 1 [5] Assume that the system (5) is asymptotically stable and the following inequality

$$\sum_{i=0}^{\infty} z^T(k) z(k) < \gamma^2 \sum_{i=0}^{\infty} \omega^T(k) \omega(k) \tag{6}$$

holds for all $\omega(k) \in \mathbb{R}^q$, then the H_∞ norm of the system (5) is less than γ.

Lemma 1 *Finsler Lemma* [1]. *Let* $\mathcal{P} = \mathcal{P}^T \in \mathbb{R}^{n\times n}$*, and* $\mathcal{B} \in \mathbb{R}^{m\times n}$ *be given matrices, then the following statements are equivalent:*

(a) $\upsilon^T \mathcal{P}\upsilon < 0$*, for all* $\upsilon \neq 0$*,* $\mathcal{H}\upsilon = 0$*;*
(b) $\mathcal{B}^{\perp T}\mathcal{P}\mathcal{B}^{\perp} < 0$*;*
(c) $\exists \mathcal{S} \in \mathbb{R}^{n\times m}$ *such that* $\mathcal{P} + He(\mathcal{S}\mathcal{B}) < 0$.

Lemma 2 [26] *For arbitrarily vector* ϖ*, matrices* R*,* M_1*,* M_2 *and a positive scalar* $\alpha \in [0,\ 1]$*, define the function* $\aleph(\alpha, R)$ *given by:*

$$\aleph(\alpha, R) = \frac{1}{\alpha}\ \varpi^T M_1^T R M_1 \varpi + \frac{1}{1-\alpha}\ \varpi^T M_2^T R M_2 \varpi \tag{7}$$

Then, if there exists a matrix X *such that* $\begin{bmatrix} R & X \\ * & R \end{bmatrix} > 0$*, the following inequality holds*

$$\min_{\alpha\in(0,1)} \aleph(\alpha, R) \geqslant \begin{bmatrix} M_1\varpi \\ M_2\varpi \end{bmatrix}^T \begin{bmatrix} R & X \\ * & R \end{bmatrix} \begin{bmatrix} M_1\varpi \\ M_2\varpi \end{bmatrix} \tag{8}$$

3 Main Results

This section provides novel sufficient conditions of event-based H_∞ stability and controller design for linear discrete-time systems.

Theorem 1 *For given scalars* d_m*,* δ*,* γ*, under the event-triggered scheme (2), the asymptotical stability of closed-loop system (5) with prescribed disturbance attenuation performance* γ *is satisfied if there exist symmetric matrices* $P > 0$*,* $Q > 0$*,* $S > 0$*,* $R > 0$*, matrices* X*,* G*,* F *satisfying*

$$\begin{bmatrix} R & X \\ * & R \end{bmatrix} > 0 \tag{9}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & GB_1KC_2 & 0 & -GB_1K & 0 & GB_2 \\ * & \Pi_{22} & \Pi_{23} & X^T & -FB_1K & C_1^T & FB_2 \\ * & * & \Pi_{33} & R-X^T & \delta C_2^T\Phi & (D_1KC_2)^T & 0 \\ * & * & * & -R-Q & 0 & 0 & 0 \\ * & * & * & * & (\delta-1)\Phi & -(D_1K)^T & 0 \\ * & * & * & * & * & -I & D_2 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{10}$$

where $\Pi_{11} = d_m^2R + P - He(G)$*,* $\Pi_{12} = GA - F^T - d_m^2R$*,* $\Pi_{22} = Q + S + d_m^2R - P - R + He(FA)$*,* $\Pi_{23} = R - X^T + FB_1KC_2$*,* $\Pi_{33} = -S - 2R + He(X) + \delta C_2^T\Phi C_2$.

Proof Choose the Lyapunov–Krasovskii functional as

$$V(x(k)) = x^T(k)Px(k) + \sum_{s=k-d_m}^{k-1} x^T(s)Qx(s) + \sum_{s=k-d_k}^{k-1} x^T(s)Sx(s) + d_m \sum_{l=-d_m+1}^{0} \sum_{s=k+l-1}^{k-1} \sigma^T(s)R\sigma(s) \tag{11}$$

where $\sigma(s) = x(s+1) - x(s)$.

Now, calculating the time-derivative of $V(k)$, one has

$$\begin{aligned}\Delta V(k) = {} & x^T(k+1)Px(k+1) - x^T(k)Px(k) + x^T(k)Qx(k) \\ & - x^T(k-d_m)Qx(k-d_m) + x^T(k)Sx(k) - x^T(k-d_k)Sx(k-d_k) \\ & + d_m^2\sigma^T(k)R\sigma(k) - d_m \sum_{k-d_m}^{k-1} \sigma^T(s)R\sigma(s) + e^T(k)\Phi e(k) - e^T(k)\Phi e(k)\end{aligned} \tag{12}$$

According to Jensen inequality, $-d_m \sum_{k-d_m}^{k-1} \sigma^T(s)R\sigma(s)$ is relaxed as

$$-d_m \sum_{k-d_m}^{k-1} \sigma^T(s)R\sigma(s) \leqslant -\frac{d_m}{d_m - d_k}\eta^T(k)e_1^T Re_1\eta(k) - \frac{d_m}{d_k}\eta^T(k)e_2^T Re_2\eta(k) \tag{13}$$

where

$$\begin{aligned}\eta^T(k) &= \begin{bmatrix} x^T(k) & x^T(k-d_k) & x^T(k-d_m) & e^T(k) & \omega^T(k) \end{bmatrix}, \\ e_1 &= \begin{bmatrix} 0 & I & -I & 0 & 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} I & -I & 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

Resorting to Lemma 2, (13) becomes

$$-d_m \sum_{k-d_m}^{k-1} \sigma^T(s)R\sigma(s) \leqslant -\eta^T(k) \begin{bmatrix} e_1^T & e_2^T \end{bmatrix} \begin{bmatrix} R & X \\ * & R \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \eta(k) \tag{14}$$

where

$$\begin{bmatrix} R & X \\ * & R \end{bmatrix} > 0$$

Recalling the definition of H_∞ performance, define the following index

$$J = \sum_{k=0}^{\infty} \left(\Delta V(k) + z^T(k)z(k) - \gamma^2\omega^T(k)\omega(k) \right) \tag{15}$$

From (2), (13) and (15), it yields

$$J \leqslant \sum_{k=0}^{\infty} \eta^T(k)\Lambda_1^T \Psi_1 \Lambda_1 \eta(k) \tag{16}$$

where

$$\Lambda_1 = \begin{bmatrix} A & B_1KC_2 & 0 & -B_1K & B_2 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ C_1 & D_1KC_2 & 0 & -D_1K & D_2 \\ 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} d_m^2R+P & -d_m^2R & 0 & 0 & 0 & 0 & 0 \\ * & \Psi_1(2,2) & R-X^T & X^T & 0 & 0 & 0 \\ * & * & \Psi_1(3,3) & R-X^T & \delta C_2^T\Phi & 0 & 0 \\ * & * & * & -R-Q & 0 & 0 & 0 \\ * & * & * & * & (\delta-1)\Phi & 0 & 0 \\ * & * & * & * & * & I & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$\Psi_1(2,2) = Q + S + d_m^2R - P - R,\ \ \Psi_1(3,3) = -S - 2R + He(X) + \delta C_2^T\Phi C_2$

To guarantee the closed-loop system to be asymptotically stable with given H_∞ performance, the following inequality should be satisfied

$$J \leqslant \sum_{k=0}^{\infty} \eta^T(k)\Lambda_1^T \Psi_1 \Lambda_1 \eta(k) < 0 \tag{17}$$

which is equivalent to

$$\Lambda_1^T \Psi_1 \Lambda_1 < 0 \tag{18}$$

Thus, motivated by [28], according to the description of system (5) and Lemma 1, one has

$$\Psi_1 + He\left(\mathcal{H}\Lambda_1^{\perp}\right) < 0 \tag{19}$$

where

$$\Lambda_1^{\perp} = \begin{bmatrix} -I & A & B_1KC_2 & 0 & -B_1K & 0 & B_2 \\ 0 & C_1 & D_1KC_2 & 0 & -D_1K & -I & D_2 \end{bmatrix},$$

$$\mathcal{H} = \begin{bmatrix} G^T & F^T & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \end{bmatrix}^T.$$

Then, (19) can also be expressed as

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & GB_1KC_2 & 0 & -GB_1K & 0 & GB_2 \\ * & \Xi_{22} & \Xi_{23} & X^T & -FB_1K & C_1^T & FB_2 \\ * & * & \Xi_{33} & R-X^T & \delta C_2^T\Phi & (D_1KC_2)^T & 0 \\ * & * & * & -R-Q & 0 & 0 & 0 \\ * & * & * & * & (\delta-1)\Phi & -(D_1K)^T & 0 \\ * & * & * & * & * & -I & D_2 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (20)$$

which is ensured by (10) with $\Xi_{11} = \Pi_{11}$, $\Xi_{12} = \Pi_{12}$, $\Xi_{22} = \Pi_{22}$, $\Xi_{23} = \Pi_{23}$, $\Xi_{33} = \Pi_{33}$. □

Remark 2 Taking into account the event-triggering condition (2), a co-analysis framework of event-triggering communication and static output feedback is established in Theorem 1. During the derivation, Finsler lemma is employed to separate the coupling of system matrices and Lyapunov matrix P.

The H_∞ stability analysis conditions given in Theorem 1 are non-convex because of the coupling among the slack variables G, F and system matrices. To handle this, novel LMI conditions for event-triggered H_∞ SOF controller design are proposed in Theorem 2 by using Lemma 1.

Theorem 2 *For given scalars d_m, δ, γ, b_1, b_2, b_3, b_4, under the event-triggered scheme (2), the asymptotical stability of closed-loop system (5) with required H_∞ performance γ is satisfied if there exist symmetric matrices $P > 0$, $Q > 0$, $S > 0$, $R > 0$, matrices X, G, F, W, N such that (9) and*

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & 0 & -b_1B_1N & 0 & GB_2 & \Sigma_{18} \\ * & \Sigma_{22} & \Sigma_{23} & X^T & -b_2B_1N & C_1^T & FB_2 & \Sigma_{28} \\ * & * & \Sigma_{33} & R-X^T & \delta C_2^T\Phi & b_3(D_1NC_2)^T & 0 & \Sigma_{38} \\ * & * & * & -R-Q & 0 & 0 & 0 & 0 \\ * & * & * & * & (\delta-1)\Phi & -b_3(D_1N)^T & 0 & \Sigma_{58} \\ * & * & * & * & * & -I & D_2 & \Sigma_{68} \\ * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & \Sigma_{88} \end{bmatrix} < 0 \quad (21)$$

hold, where $\Sigma_{11} = \Pi_{11}$, $\Sigma_{12} = \Pi_{12}$, $\Sigma_{13} = b_1B_1NC_2$, $\Sigma_{18} = GB_1 - b_1B_1W$, $\Sigma_{22} = \Pi_{22}$, $\Sigma_{23} = R - X^T + b_2B_1NC_2$, $\Sigma_{28} = FB_1 - b_2B_1W$, $\Sigma_{33} = \Pi_{33}$, $\Sigma_{38} = b_4(NC_2)^T$, $\Sigma_{58} = -b_4N^T$, $\Sigma_{68} = D_1 - b_3D_1W$, $\Sigma_{88} = -b_4He(W)$. Moreover, the SOF controller gain is computed by $K = W^{-1}N$.

Proof Reformulate (10) in Theorem 1 as

$$
\mathcal{B}^{\perp T}\mathcal{P}\mathcal{B}^{\perp} < 0
$$

$$
\mathcal{P} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & GB_1KC_2 & 0 & -GB_1K & 0 & GB_2 & 0 \\ * & \Pi_{22} & \Pi_{23} & R - X^T & -FB_1K & C_1^T & FB_2 & 0 \\ * & * & -S-R & R-X & \delta C_2^T\Phi & (D_1KC_2)^T & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & (\delta-1)\Phi & -(D_1K)^T & 0 & 0 \\ * & * & * & * & * & -I & D_2 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix},
$$

$$
\mathcal{B}^{\perp} = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & -KC_2 & 0 & K & 0 & 0 & 0 \end{bmatrix}. \tag{22}
$$

By employing Lemma 1 to (22) once again, it leads to

$$
\mathcal{P} + He(\mathcal{S}\mathcal{B}) < 0 \tag{23}
$$

where

$$
\mathcal{S} = \begin{bmatrix} (b_1B_1W - GB_1)^T & (b_2B_1W - FB_1)^T & 0 & 0 & 0 & (b_3D_1W - D_1)^T & 0 & b_4W^T \end{bmatrix}^T,
$$
$$
\mathcal{B} = \begin{bmatrix} 0 & 0 & KC_2 & 0 & -K & 0 & 0 & -I \end{bmatrix}.
$$

Thereby, (21) is satisfied by (23), which completes the proof. □

Remark 3 Constrast to a scaling technique to deal with the coupling term PB_1KC_2, Finsler lemma is employed to manage the coupling terms GB_1KC_2 and FB_1KC_2, which could make the obtained result be less conservative.

Remark 4 It is noted that the scalar variables b_1, b_2, b_3 and b_4 render the derived conditions non-convex. To solve this difficulty, a line search algorithm in [25] over four parameters can be utilized to optimize the H_∞ performance γ by LMIs. If these tuning parameters are equal, the algorithm is with respect to only one scalar variable, which decreases the amount of computation.

Table 1 The optimal γ for different δ

δ	0.1	0.3	0.5
Φ	68.2723	40.3924	77.6123
K	−1.2437	−0.7548	−0.4915
γ	2.2970	3.4776	17.2499
Computation time	1.8750	1.9220	1.8130

4 Numerical Example

Example 1 Consider system (1) with the following parameters

$$A = \begin{bmatrix} 1.1 & 0 \\ 1 & 0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.8 & 0.1 \end{bmatrix}.$$

For $\tau_m = 0.6$, $b_1 = b_2 = b_3 = b_4 = 1$, sampling interval $h = 0.25$, the weighting matrices Φ, control law gains K, the optimal H_∞ gains γ and computation time calculated by Theorem 2 are given in Table 1.

From the results shown in Table 1, one can see that the smaller the event trigger parameter is, the better disturbance attenuation performance will be. Thus, for the above obtained controller gains, $\omega(k) = sin(2\pi kh)$ for $1.5 \leqslant kh \leqslant 2.5s$ (otherwise $\omega(k) = 0$) and zero initial condition, the simulations executed by $\delta = 0.1$, $\delta = 0.3$ and $\delta = 0.5$ are drawn in Figs. 2, 3 and 4, respectively. Figures 5 and 6 show the compared state response curves for different δ. The compared transmitting time intervals of $\delta = 0.1, 0.3, 0.5$ are illustrated in Fig. 7.

According to Figs. 2, 3 and 4, one can see that the H_∞ asymptotical stability of closed-loop system is ensured by the designed controller. From Figs. 5, 6 and 7, it is noted that as the event threshold parameter decreases, the amount of the transmitted signals increases as well as the system performance is improved. However, more transmitted information also means consuming more cost or energy. Consequently, a trade-off between the cost and system performance should be considered.

Example 2 An aircraft system borrowed from [36] is given as:

$$\begin{bmatrix} \dot{\alpha}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} -1.175 & 0.9871 \\ -8.458 & -0.8776 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} -0.194 & -0.03593 \\ -19.29 & -3.803 \end{bmatrix} \begin{bmatrix} \delta_E(t) \\ \delta_{PTV}(t) \end{bmatrix}.$$

For $h = 0.05$, the above continuous-time system is discretized as

$$A = \begin{bmatrix} 0.9331 & 0.0467 \\ -0.4004 & 0.9471 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.0324 & -0.0063 \\ -0.9384 & -0.1850 \end{bmatrix}.$$

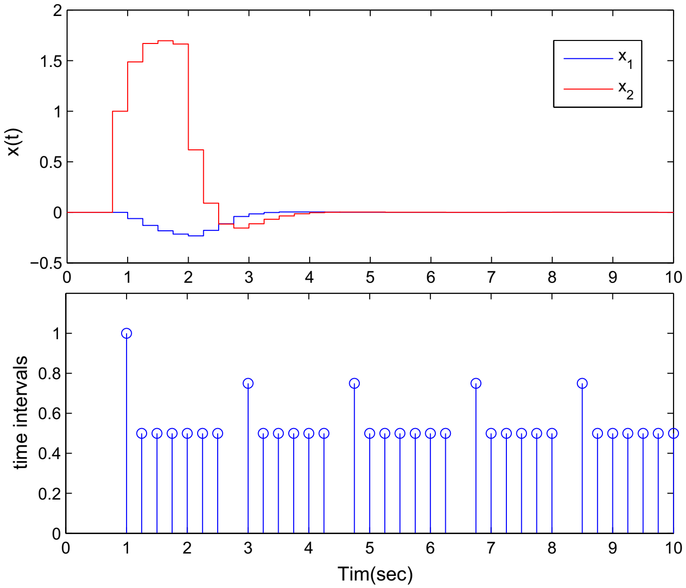

Fig. 2 The trajectories of system state $x(k)$ and transmitting time intervals under $\delta = 0.1$

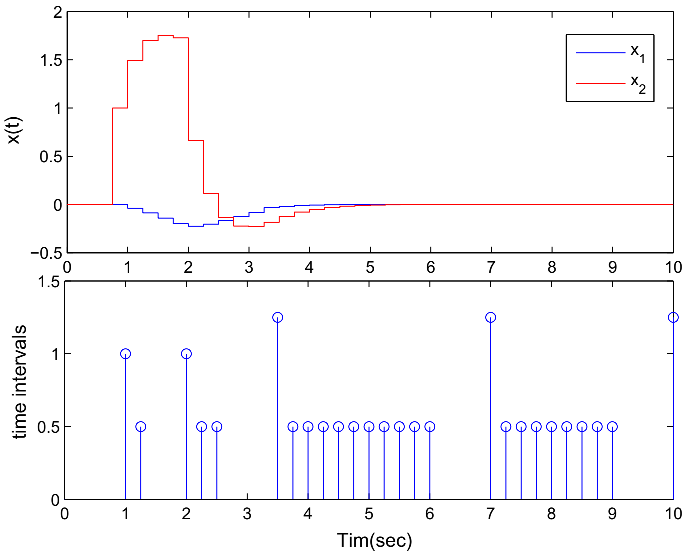

Fig. 3 The trajectories of system state $x(k)$ and transmitting time intervals under $\delta = 0.3$

The other corresponding system matrices are chosen as

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

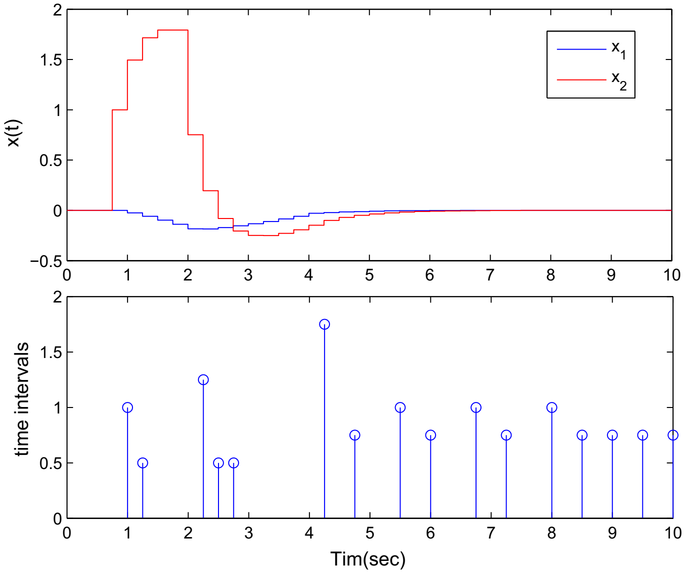

Fig. 4 The trajectories of system state $x(k)$ and transmitting time intervals under $\delta = 0.5$

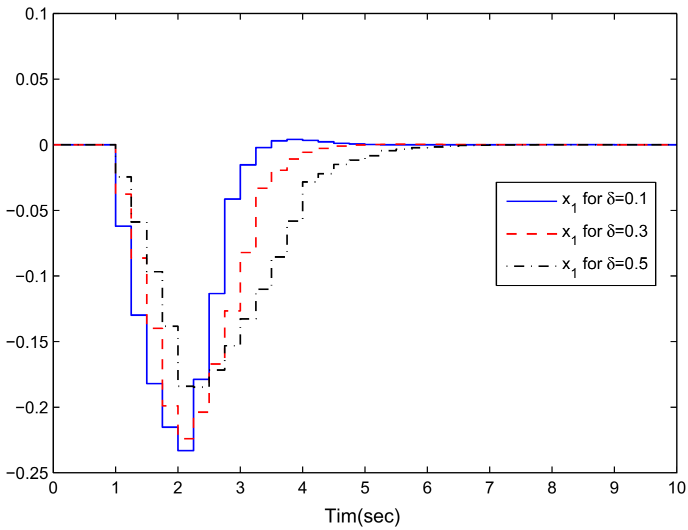

Fig. 5 The trajectories of $x_1(k)$ under $\delta = 0.1$, $\delta = 0.3$ and $\delta = 0.5$

By taking $b_1 = b_2 = b_3 = 1$, $b_4 = 10$, $\tau_m = 0.1$ and $\delta = 0.1$, solving Theorem 2 gives $\gamma = 5.5773$, $\Phi = 2.1336$, $K = \begin{bmatrix} 0.2159 \\ 0.0443 \end{bmatrix}$ and computation time is 1.8440. The initial condition of the considered system is taken as $x_0 = [1 \ -0.5]$, and the simulation

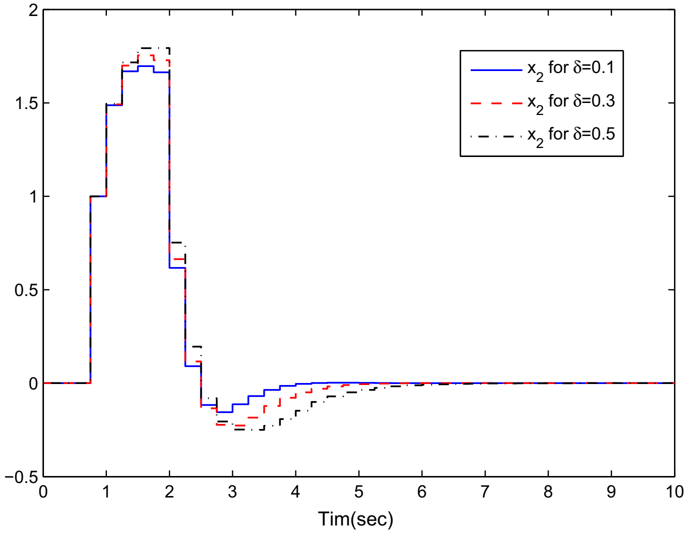

Fig. 6 The trajectories of $x_2(k)$ under $\delta = 0.1$, $\delta = 0.3$ and $\delta = 0.5$

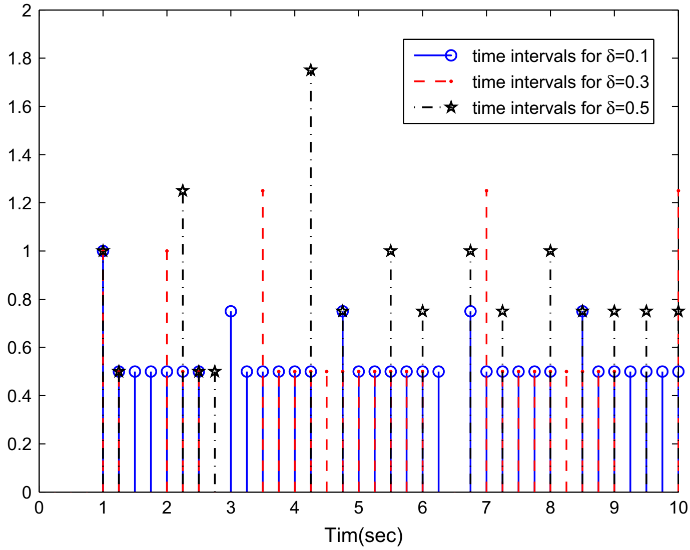

Fig. 7 Time intervals under $\delta = 0.1$, $\delta = 0.3$ and $\delta = 0.5$

curves are shown in Fig. 8. From Fig. 8, one can observe that the effectiveness of this proposed method for designing the SOF controller for the considered aircraft system is also verified.

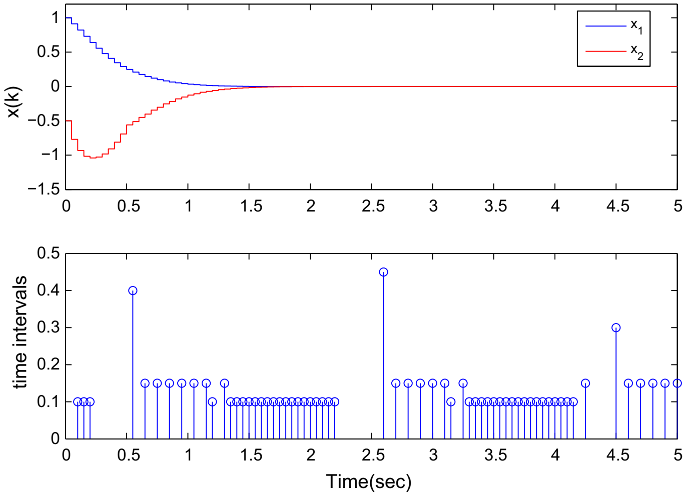

Fig. 8 The trajectories of system state $x(k)$ and transmitting time intervals under $\delta = 0.1$

5 Conclusions

This paper investigates the event-triggered H_∞ SOF control of discrete-time NCSs. Resorting to a separation strategy, sufficient conditions for ensuring the closed-loop system to be asymptotically stable with prescribed H_∞ index are formed via LMIs. Two examples are given to illustrate the effectiveness of the proposed method. Then, the developed event-triggered output feedback control approach will be extended to deal with the output feedback control of neural networks [11], T–S fuzzy systems [6,7,12], Markov jump systems [13,29] and vehicle active suspension systems [30,32].

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