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Robust Unknown Input Observer-Based Fault Estimation of Leader–Follower Linear Multi-agent Systems

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Abstract In this paper, the problem of the robust unknown input observer for a class of linear multi-agent systems and its application to fault estimation are considered. First, an undirected graph is used to represent the communication topology of a leader–follower linear multi-agent system. Then, using relative output estimation errors among agents, an unknown input observer is proposed to achieve fault estimation for the global augmented system in which the actuator or sensor fault vector is taken as an auxiliary state vector. A multi-constrained design algorithm based on linear matrix inequality technique is also designed to obtain gain matrices of unknown input observer. Simulation results show the effectiveness and advantages of the proposed robust unknown input observer method for fault estimation of multi-agent systems.

Keywords Multi-agent systems · Fault estimation · Unknown input observer · Linear matrix inequalities

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1 Introduction

Distributed networks of dynamic agents have gained increasing attention from researchers, partly because of broad applications of multi-agent systems in many fields, such as distributed sensor networks, automated highway systems, and unmanned aerial vehicles. In the past several years, various scientific communities in control and system fields have focused on a series of fundamental problems of multi-agent systems, including consensus problems, swarm problems, cooperative control, and formation control; these communities have achieved a considerable number of fruitful results, which have been included in several excellent books [7,15], an excellent survey [13] and references therein.

With the increasing scale and complexity of multi-agent systems, operating these systems autonomously, safely, and reliably is becoming increasingly important and urgent. Hence, the technology of fault diagnosis, including fault detection, fault isolation, and fault estimation (FE), is considered as an attractive topic for multi-agent systems and has elicited considerable attention. However, despite the abundant academic results on fault diagnosis with respect to an individual system have been achieved, the body of fault diagnosis research on multi-agent systems in terms of interaction topology, communication capability, and structural heterogeneity of the agents is relatively smaller because such research is more challenging [14,22].

In [10], an event-based fault detection filter of networked systems was proposed such that the error between the residual and fault signals was made as small as possible. Sufficient conditions for the existence of the desired fault detection filter were established in terms of linear matrix inequality. Based on a geometric fault detection and isolation (FDI) approach, [12] proposed a local/decentralized detection filter for spacecraft formation to detect faults by determining the required unobservable subspace of local systems. In [16], a new approach based on fractional subband Volterra series and the fractional correlation to extract the fault features was presented to diagnose incipient faults in nonlinear analog circuits. In [17,18], a dynamic neural network-based FDI scheme was studied to perform the formation flying mission of satellites.

In general, FDI is used to monitor the system and determine the location of the occurred fault because the final objective of fault diagnosis is to obtain fault information. Hu et al. [6] addressed the problem of FE for a class of Lipschitz nonlinear systems by proposing a second-order sliding mode observer based on the super-twisting algorithm to solve the chattering problem caused by the traditional sliding mode observer. In [11], a robust FE method based on sliding mode observer was investigated for a collection of agents that exchanged relative information over the communication network. A robust unknown input observer (UIO) approach was proposed in [23] to address fault diagnosis with respect to sampled-data control systems subject to the unknown input. An augmented form of a class of Lipschitz nonlinear systems was constructed by taking the sensor fault as an auxiliary state vector, and a robust UIO with H_∞ performance criterion was developed to estimate states and faults simultaneously in [21]. Similarly, a robust UIO-based FE scheme was designed and applied to a nonlinear multi-tank system in [19]. Particularly, a comparison of sliding mode observer and UIO for fault reconstruction was considered and an underlying link between the two approaches was investigated in [2]. The minimum variance unbiased estimation technique was used to address the problem of simultaneous input, and state estimation in the presence or absence of direct feedthrough was proposed with exhaustive stability analysis in [3]. In addition, based on the adaptive observer method, a fast fault estimator of linear multi-agent systems was designed in [9], which only considered a class of constant actuator faults via the input channel and did not consider further the unknown disturbance.

The model-based fault diagnosis is built upon the precise mathematical model, whereas the presence of uncertainties and external disturbances is not avoidable in practical systems [1,22]. Thus, to weaken or eliminate such unexpected effects of uncertainties, the method of UIO is considered immediately because of its insensitivity to the unknown input.

In this paper, motivated by the existing research situations, we concentrate on the problem of robust UIO-based FE of a class of leader–follower linear continuous-time multi-agent systems with an undirected topology. The main contributions of this study lie in three aspects: (1) for a class of multi-agent systems, a structure of augmented systems is derived by taking the actuator fault and sensor fault vector as an auxiliary state vector; (2) using the information of relative output estimation errors, an approach of robust UIO-based FE is proposed to utilize the topology feature for multi-agent systems and eliminate external the unknown input; (3) based on linear matrix inequality technique, a multi-constrained algorithm is designed to calculate the UIO gain matrices.

The remainder of this paper is organized as follows. Preliminaries and problem statement are presented in Sect. 2. In Sect. 3, based on graph theory, a unified global augmented structure of multi-agent systems is developed and an FE approach is proposed using relative output information. Simulation results of a linear multi-agent system are shown in Sect. 4, followed by concluding remarks in Sect. 5.

2 Preliminaries and System Description

2.1 Graph Theory

An undirected graph $\mathcal{G}$ [20] is a pair (υ, ε) in which $\upsilon = (1, \ldots, N)$ is the set of nodes and $\varepsilon \subseteq \upsilon \times \upsilon$ is the set of unordered pairs of nodes, called edges. Two nodes, i, j, are adjacent or neighboring if (i, j) is an edge of graph $\mathcal{G}$. The edges in the form of (i, i) are called loops. A graph with loops is called a multi-graph. Graphs without loops are known as a simple graph. A path on $\mathcal{G}$ from node i_1 to node i_N is a sequence of ordered edges of the form (i_k, i_{k+1}), $k = 1, \ldots, N-1$. An undirected graph is connected if a path exists between every pair of distinct nodes; otherwise, it is disconnected.

The adjacency matrix $\mathcal{A} \in \mathbb{R}^{N\times N}$ of graph $\mathcal{G}$ is defined by $\alpha_{ii} = 1$, if node i has a self loop but 0 otherwise, and $\alpha_{ij} = \alpha_{ji} = 1$ if the pair $(i, j) \in \varepsilon$ but 0 otherwise, which can represent whether information can flow from node i to node j or in reverse. The Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N\times N}$ can be defined as $\mathcal{L}_{ii} = \sum_{j=1}^{N} \alpha_{ij}$, $\mathcal{L}_{ij} = -\alpha_{ij}$, for $i \neq j$. The adjacency matrix $\mathcal{A}$ and Laplacian matrix $\mathcal{L}$ are both symmetric for an undirected graph.

2.2 System Description

We consider a linear continuous-time multi-agent system with N nodes, distributed on an undirected topology $\mathcal{G}$. The dynamics of the ith agent with the unknown input can be described by

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) + Dd_i(t) + Ef_{ai}(t) \\ y_i(t) = Cx_i(t) + Ff_{si}(t) \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ is the input vector, $y_i(t) \in \mathbb{R}^p$ is the measured output vector, and $d_i(t) \in \mathbb{R}^q$ is the unknown input vector. $f_{ai}(t) \in \mathbb{R}^{r_a}$ and $f_{si}(t) \in \mathbb{R}^{r_s}$ represent the actuator or sensor fault, respectively, with respect to $\|\dot{f}_{ai}(t)\| \leq \tilde{f}_1$ and $\|\dot{f}_{si}(t)\| \leq \tilde{f}_2$, where $\tilde{f}_1$ and $\tilde{f}_2$ are positive constant values. A, B, C, D, E, and F are known constant real matrices of appropriate dimensions, whereas matrices D, E, and F are full column rank. The pair (A, C) is observable. The number of unknown inputs is not more than the number of the measured output, i.e., $q \leq p$.

The system dynamics of the leader agent, labeled 0, is given by

$$\dot{x}_0(t) = Ax_0(t), \quad y_0(t) = Cx_0(t) \tag{2}$$

where $x_0(t) \in \mathbb{R}^n$ is the state vector and $y_0(t) \in \mathbb{R}^p$ is the output vector. The leader agent can be observed from a small subset of agents in graph $\mathcal{G}$. If the ith agent can obtain information from the leader, an edge (υ_0, υ_i) is said to existence with the weighting gain $g_i > 0$, and we refer to the ith agent with $g_i > 0$ as a pinned or controlled node. We denote the pinning matrix as $G = \text{diag}\{g_i\} \in \mathbb{R}^{N \times N}$.

Assumption 1 In this note, the undirected graph contains a spanning tree, and the root node (the ith agent) can observe information from the leader one, which is denoted as $g_i > 0$.

2.3 A Unified Global Augmented System

In this subsection, we provide an augmented structure by taking the actuator fault or sensor fault vector as an auxiliary state vector. A unified global form is also derived.

Actuator Fault Augmented Model

By considering only actuator faults, i.e., $F = 0$, the actuator fault vector can be treated as an auxiliary state vector; an augmented system with actuator faults is described by

$$\begin{cases} \dot{\bar{x}}_{ai}(t) = \bar{A}_a\bar{x}_{ai}(t) + \bar{B}_a u_i(t) + \bar{D}_a d_i(t) + \bar{I}_{r_a}\dot{f}_{ai}(t) \\ y_{ai}(t) = \bar{C}_a\bar{x}_{ai}(t) \end{cases} \tag{3}$$

where

$$\bar{x}_{ai}(t)=\begin{bmatrix} x_i(t) \\ f_{ai}(t) \end{bmatrix}, \bar{A}_a=\begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}, \bar{B}_a=\begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C}_a=\begin{bmatrix} C & 0 \end{bmatrix}, \bar{D}_a=\begin{bmatrix} D \\ 0 \end{bmatrix}, \bar{I}_{r_a}=\begin{bmatrix} 0 \\ I_{r_a} \end{bmatrix},$$

and $\bar{x}_{ai}(t) \in \mathbb{R}^{n+r_a}$ is the augmented state vector and $y_{ai}(t) \in \mathbb{R}^p$ is the output vector.

Sensor Fault Augmented Model

When the system only includes sensor faults, i.e., $E = 0$, considering the sensor fault vector as an auxiliary state vector, an augmented system with sensor faults is described by

$$\begin{cases} \dot{\bar{x}}_{si}(t) = \bar{A}_s\bar{x}_{si}(t) + \bar{B}_s u_i(t) + \bar{D}_s d_i(t) + \bar{I}_{r_s}\dot{f}_{si}(t) \\ y_{si}(t) = \bar{C}_s\bar{x}_{si}(t) \end{cases} \tag{4}$$

where

$$\bar{x}_{si}(t)=\begin{bmatrix} x_i(t) \\ f_{si}(t) \end{bmatrix}, \bar{A}_s=\begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \bar{B}_s=\begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C}_s=\begin{bmatrix} C & F \end{bmatrix}, \bar{D}_s=\begin{bmatrix} D \\ 0 \end{bmatrix}, \bar{I}_{r_s}=\begin{bmatrix} 0 \\ I_{r_s} \end{bmatrix},$$

and $\bar{x}_{si}(t) \in \mathbb{R}^{n+r_s}$ is the augmented state vector and $y_{si}(t) \in \mathbb{R}^p$ is the output vector.

The structures of (3) and (4) are similar; then, we can extend such augmented system dynamics to a unified form, which can be described as

$$\begin{cases} \dot{\bar{x}}_i(t) = \bar{A}\bar{x}_i(t) + \bar{B}u_i(t) + \bar{D}d_i(t) + \bar{I}_r\dot{f}_i(t) \\ y_i(t) = \bar{C}\bar{x}_i(t) \end{cases} \tag{5}$$

where

$$\bar{A}, \bar{B}, \bar{C}, \bar{D}, \bar{I}_r = \begin{cases} \bar{A}_a, \bar{B}_a, \bar{C}_a, \bar{D}_a, \bar{I}_{r_a} & \text{for the actuator fault} \\ \bar{A}_s, \bar{B}_s, \bar{C}_s, \bar{D}_s, \bar{I}_{r_s} & \text{for the sensor fault} \end{cases}$$

$$\bar{x}_i(t), \dot{f}_i(t), y_i(t) = \begin{cases} \bar{x}_{ai}(t) \in \mathbb{R}^{n+r_a}, \dot{f}_{ai}(t) \in \mathbb{R}^{r_a}, y_{ai}(t) \in \mathbb{R}^p & \text{for the actuator fault} \\ \bar{x}_{si}(t) \in \mathbb{R}^{n+r_s}, \dot{f}_{si}(t) \in \mathbb{R}^{r_s}, y_{si}(t) \in \mathbb{R}^p & \text{for the sensor fault} \end{cases}$$

Based on undirected graph theory, the unified global fault augmented multi-agent system dynamics can be represented as follows

$$\begin{cases} \dot{\bar{x}}(t) = (I_N \otimes \bar{A})\bar{x}(t) + (I_N \otimes \bar{B})u(t) + (I_N \otimes \bar{D})d(t) + (I_N \otimes \bar{I}_r)\dot{f}(t) \\ y(t) = (I_N \otimes \bar{C})\bar{x}(t) \end{cases} \tag{6}$$

where $\otimes$ denotes Kronecker product; $\bar{x}(t) = [\bar{x}_1^T, \bar{x}_2^T, \ldots, \bar{x}_N^T]^T \in \mathbb{R}^{(n+r)N}$ is the global state vector, $u(t) = [u_1^T, u_2^T, \ldots, u_N^T]^T \in \mathbb{R}^{mN}$ is the global input vector, $y(t) = [y_1^T, y_2^T, \ldots, y_N^T]^T \in \mathbb{R}^{pN}$ is the global output vector, $d(t) = [d_1^T, d_2^T, \ldots, d_N^T]^T \in \mathbb{R}^{qN}$ is the global unknown input vector; $\dot{f}(t) = [\dot{f}_1^T(t), \dot{f}_2^T(t), \ldots, \dot{f}_N^T(t)]^T \in \mathbb{R}^{rN}$ represents the derivative of the actuator or sensor fault vector in a unified form; r denotes r_a or r_s in the case considering actuator or sensor faults, respectively.

In this paper, we aim to design a global fault estimator for the system dynamics (6) and eliminate the unknown input $d(t)$. To achieve these targets, the following procedures will be followed:

(1) To design a unified global augmented system (6), taking the actuator or sensor fault vector as an auxiliary state vector, respectively;

(2) To construct a global UIO and fault estimator using relative output estimation errors;

(3) To analyze the stability and robustness of the proposed FE algorithm.

3 Main Results

In this section, we design a global UIO to estimate actuator or sensor fault in a unified form. The UIO-based FE design can also eliminate the unknown input $d(t)$. Two assumptions are needed for the design of this global UIO [23].

Assumption 2 $(\bar{T}\bar{A}, \bar{C})$ is observable, where $\bar{T}$ will be illustrated in this section.

Assumption 3 rank $[B \quad D] = \text{rank}\ (B) + \text{rank}\ (D)$, rank $(\bar{C}\bar{D}) = \text{rank}\ (\bar{D})$

Remark 1 Assumption 2 ensures existence of the designed robust UIO for an individual system, and Assumption 3 represents that the unknown input and the fault can be decoupled totally.

3.1 UIO Design

The neighborhood output estimation error [8] for the ith agent is considered, which can be described as follows

$$\zeta_i(t) = \sum_{j \in N_i} a_{ij} \Big[(\hat{y}_i(t) - y_i(t)) - (\hat{y}_j(t) - y_j(t)) \Big] + g_i \Big[(\hat{y}_i(t) - y_i(t)) - (\hat{y}_0(t) - y_0(t)) \Big], \tag{7}$$

where $\hat{y}_j(t) \in \mathbb{R}^p$ is the estimate of the output vector $y_j(t)$ of the jth agent. N_i is the total number of other agents, which communicate with the ith one. a_{ij} represents the connection weight between the ith and jth agents, choosing $a_{ij} = 1$. g_i denotes the connection weight between the ith agent and the leader agent, choosing $g_i = 1$.

The UIO is constructed for the ith agent as follows:

$$\begin{cases} \dot{\bar{z}}_i(t) = \bar{T}\bar{A}\hat{\bar{x}}_i(t) + \bar{T}\bar{B}u_i(t) - \bar{K}\zeta_i(t) \\ \hat{\bar{x}}_i(t) = \bar{z}_i(t) + \bar{H}y_i(t) \\ \hat{f}_i(t) = \bar{I}_r^T \hat{\bar{x}}_i(t) \\ \hat{y}_i(t) = \bar{C}\hat{\bar{x}}_i(t) \end{cases} \tag{8}$$

where $\bar{z}_i(t)$ is the UIO state vector of the ith agent, $\hat{\bar{x}}_i(t) \in \mathbb{R}^{n+r}$ is the estimate of the state vector $\bar{x}_i(t)$, $\hat{f}_i(t) \in \mathbb{R}^r$ is the estimate of the fault vector $f_i(t)$, and $\hat{y}_i(t) \in \mathbb{R}^p$

is the estimate of the output vector $y_i(t)$. $\bar{T} \in \mathbb{R}^{(n+r)\times(n+r)}$, $\bar{K} \in \mathbb{R}^{(n+r)\times p}$, and $\bar{H} \in \mathbb{R}^{(n+r)\times p}$ are the UIO gain matrices to be designed.

Remark 2 We denote $e_{yi}(t) = \hat{y}_i(t) - y_i(t)$ as the output estimation error of the ith agent. The leader agent acts as a command generator, and the assumption that the leader's state is known is appropriate, i.e., $x_0(t) = \hat{x}_0(t)$. That is, $e_{y0}(t) = 0$ is tenable, which is an important feature of the considered leader–follower multi-agent systems.

The global neighborhood output estimation error for multi-agent systems is defined based on graph theory as

$$\zeta(t) = ((L+G)\otimes I_p)e_y(t) = ((L+G)\otimes \bar{C})(\hat{\bar{x}}(t) - \bar{x}(t)) \tag{9}$$

where $\zeta(t) = [\zeta_1^T(t), \zeta_2^T(t), \ldots, \zeta_N^T(t)]^T \in \mathbb{R}^{pN}$ refers to the global neighborhood output estimation error and $e_y(t) = [e_{y1}^T, e_{y2}^T, \ldots, e_{yN}^T]^T \in \mathbb{R}^{pN}$ refers to the global output estimation error. Matrix $(L+G)$ is symmetric because the undirected graph is considered.

A global UIO for the unified global augmented multi-agent system dynamics (6) can then be constructed further as follows

$$\begin{cases} \dot{\bar{z}}(t) = (I_N \otimes \bar{T}\bar{A})\hat{\bar{x}}(t) + (I_N \otimes \bar{T}\bar{B})u(t) - (I_N \otimes \bar{K})\zeta(t) \\ \hat{\bar{x}}(t) = \bar{z}(t) + (I_N \otimes \bar{H})y(t) \\ \hat{f}(t) = (I_N \otimes \bar{I}_r^T)\hat{\bar{x}}(t) \\ \hat{y}(t) = (I_N \otimes \bar{C})\hat{\bar{x}}(t) \end{cases} \tag{10}$$

where $\bar{z}(t) = [\bar{z}_1^T(t), \bar{z}_2^T(t), \ldots, \bar{z}_N^T(t)]^T \in \mathbb{R}^{(n+r)N}$, $\hat{\bar{x}}(t) = [\hat{\bar{x}}_1^T(t), \hat{\bar{x}}_2^T(t), \ldots, \hat{\bar{x}}_N^T(t)]^T \in \mathbb{R}^{(n+r)N}$, $\hat{f}(t) = [\hat{f}_1^T(t), \hat{f}_2^T(t), \ldots, \hat{f}_N^T(t)]^T \in \mathbb{R}^{rN}$, and $\hat{y}(t) = [\hat{y}_1^T(t), \hat{y}_2^T(t), \ldots, \hat{y}_N^T(t)]^T \in \mathbb{R}^{pN}$.

Remark 3 The design of this global UIO (10), which uses the neighborhood output estimation errors among agents, utilizes extensively the topology feature of a class of multi-agent systems with a undirected communication graph to design cooperative fault estimators. This approach is one of our innovations in this paper.

We construct transformations for (6) and (10), and then define the designed matrices $\bar{T}$ and $\bar{H}$, which are chosen to satisfy

$$\bar{T} = I - \bar{H}\bar{C}.$$

Given that $\dot{y}(t) = (I_N \otimes \bar{C})\dot{\bar{x}}(t)$, (6) indicates that

$$\begin{aligned} \dot{\bar{x}}(t) &= (I_N \otimes \bar{A})\bar{x}(t) + (I_N \otimes \bar{B})u(t) + (I_N \otimes \bar{D})d(t) + (I_N \otimes \bar{I}_r)\dot{f}(t) + (I_N \otimes \bar{H})\dot{y}(t) \\ &\quad - (I_N \otimes H\bar{C})\dot{\bar{x}}(t) \\ &= (I_N \otimes \bar{A})\bar{x}(t) + (I_N \otimes \bar{B})u(t) + (I_N \otimes \bar{D})d(t) + (I_N \otimes \bar{I}_r)\dot{f}(t) + (I_N \otimes \bar{H})\dot{y}(t) \\ &\quad - (I_N \otimes \bar{H}\bar{C})\big[(I_N \otimes \bar{A})\bar{x}(t) + (I_N \otimes \bar{B})u(t) + (I_N \otimes \bar{D})d(t) + (I_N \otimes \bar{I}_r)\dot{f}(t)\big] \end{aligned}$$

$$= (I_N \otimes \bar{T}\bar{A})\bar{x}(t) + (I_N \otimes \bar{T}\bar{B})u(t) + (I_N \otimes \bar{T}\bar{D})d(t) + (I_N \otimes \bar{T}\bar{I}_r)\dot{f}(t) + (I_N \otimes \bar{H})\dot{y}(t) \tag{11}$$

Define the global augmented state estimation error to be

$$\bar{e}_x(t) = \hat{\bar{x}}(t) - \bar{x}(t). \tag{12}$$

We then obtain the global error dynamics:

$$\begin{aligned}\dot{\bar{e}}_x(t) &= \dot{\hat{\bar{x}}}(t) - \dot{\bar{x}}(t) \\ &= \dot{\bar{z}}(t) + (I_N \otimes \bar{H})\dot{y}(t) - \dot{\bar{x}}(t) \\ &= (I_N \otimes \bar{T}\bar{A} - (L+G) \otimes \bar{K}\bar{C})\bar{e}_x(t) - (I_N \otimes \bar{T}\bar{D})d(t) - (I_N \otimes \bar{T}\bar{I}_r)\dot{f}(t)\end{aligned} \tag{13}$$

Under $\bar{T}\bar{D} = 0$, the unknown input $d(t)$ is eliminated and we derive the global error dynamics

$$\begin{cases}\dot{\bar{e}}_x(t) = (I_N \otimes \bar{T}\bar{A} - (L+G) \otimes \bar{K}\bar{C})\bar{e}_x(t) - (I_N \otimes \bar{T}\bar{I}_r)\dot{f}(t) \\ e_f(t) = (I_N \otimes \bar{I}_r^T)\bar{e}_x(t)\end{cases} \tag{14}$$

3.2 Calculation of UIO Gain Matrices

In this subsection, we present a multi-constrained design to calculate the UIO gain matrices. Before main results are presented, a lemma is first given.

Lemma 1 [5] *The eigenvalues of a given matrix $\mathcal{A} \in \mathbb{R}^{n\times n}$ belong to the circular region $\mathcal{D}(\alpha, \tau)$ with the center $\alpha + j0$ and the radius τ if and only if there exists a symmetric positive definite matrix $\mathcal{P} \in \mathbb{R}^{n\times n}$ such that the following condition holds*

$$\begin{bmatrix} -\mathcal{P} & \mathcal{P}(\mathcal{A} - \alpha\mathcal{I}_n) \\ * & -\tau^2\mathcal{P}\end{bmatrix} < 0 \tag{15}$$

Theorem 1 *Let a circular region $\mathcal{D}(\alpha, \tau)$ and a prescribed H_∞ performance level $\gamma > 0$ be given. If there exist a symmetric positive definite matrix $\bar{P} \in \mathbb{R}^{(n+r)\times(n+r)}$ and matrices $\bar{Y} \in \mathbb{R}^{(n+r)\times p}$, $\bar{W} \in \mathbb{R}^{(n+r)\times p}$ satisfying the following linear matrix inequalities*

$$\begin{bmatrix} \varphi & I_N \otimes (\bar{P}\bar{I}_r - \bar{P}\bar{U}\bar{C}\bar{I}_r - \bar{W}\bar{V}\bar{C}\bar{I}_r) & I_N \otimes \bar{I}_r \\ * & -\gamma I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} -I_N \otimes \bar{P} & I_N \otimes (\bar{P}\bar{A} - P\bar{U}\bar{C}\bar{A} - \bar{W}\bar{V}\bar{C}\bar{A}) + (L+G) \otimes \bar{Y}\bar{C} - \alpha(I_N \otimes \bar{P}) \\ * & -\tau^2(I_N \otimes \bar{P}) \end{bmatrix} < 0 \tag{17}$$

where $\varphi = I_N \otimes (\bar{P}\bar{A} - \bar{P}\bar{U}\bar{C}\bar{A} - \bar{W}\bar{V}\bar{C}\bar{A} + \bar{A}^T\bar{P} - (\bar{U}\bar{C}\bar{A})^T\bar{P} - (\bar{V}\bar{C}\bar{A})^T\bar{W}^T) - (L+G) \otimes (\bar{Y}\bar{C} + \bar{C}^T\bar{Y}^T)$, $\bar{U} = \bar{D}(\bar{C}\bar{D})^+$ *and* $\bar{V} = I - \bar{C}\bar{D}(\bar{C}\bar{D})^+$*; then, the eigenvalues of* $(I_N \otimes \bar{T}\bar{A} + (L+G) \otimes \bar{C})$ *belong to* $\mathcal{D}(\alpha, \tau)$ *and the global error dynamics* (14) *satisfies the* H_∞ *performance* $\|e_f(t)\|_2 < \gamma\|\dot{f}(t)\|_2$*. The UIO gain matrices are given by* $\bar{K} = \bar{P}^{-1}\bar{Y}$, $\bar{H} = \bar{U} + \bar{P}^{-1}\bar{W}\bar{V}$ *and* $\bar{T} = I - \bar{H}\bar{C}$.

Proof Condition (16): Since matrix $\bar{P}$ is symmetric positive definite, we choose Lyapunov matrix $(I_N \otimes \bar{P})$ for the proof of condition (16). Based on bounded real lemma of continuous-time systems [4], it follows from the global error dynamics (14) that

$$\begin{bmatrix} I_N \otimes (\bar{P}\bar{T}\bar{A} - (L+G) \otimes \bar{P}\bar{K}\bar{C}) + (I_N \otimes (\bar{P}\bar{T}\bar{A} - (L+G) \otimes \bar{P}\bar{K}\bar{C})^T & -I_N \otimes \bar{P}\bar{T}\bar{I}_r & I_N \otimes \bar{I}_r \\ * & -\gamma I & 0 \\ * & * & -\gamma I \end{bmatrix} < 0 \tag{18}$$

where matrix $(L+G)$ is symmetric because of the undirected graph considered in this paper.

According to equalities $\bar{T} = I - \bar{H}\bar{C}$ and $\bar{T}\bar{D} = 0$, one gets

$$(I - \bar{H}\bar{C})\bar{D} = 0 \tag{19}$$

Since $(\bar{C}\bar{D})$ is of full column rank, the solutions of $\bar{D} = \bar{H}\bar{C}\bar{D}$ have the following form

$$\bar{H} = \bar{D}(\bar{C}\bar{D})^+ + H_0(I - \bar{C}\bar{D}(\bar{C}\bar{D})^+) \tag{20}$$

where $(\bar{C}\bar{D})^+ = \left((\bar{C}\bar{D})^T(\bar{C}\bar{D})\right)^{-1}(\bar{C}\bar{D})^T$ and H_0 is an arbitrary matrix with appropriate dimension.

Let

$$\bar{U} = \bar{D}(\bar{C}\bar{D})^+, \quad \bar{V} = I - \bar{C}\bar{D}(\bar{C}\bar{D})^+$$

then we have

$$\bar{H} = \bar{U} + H_0\bar{V}, \quad \bar{T} = I - \bar{U}\bar{C} - H_0\bar{V}\bar{C} \tag{21}$$

By substituting matrix $\bar{T}$ into (18) and making $\bar{P}\bar{K} = \bar{Y}$, $\bar{P}H_0 = \bar{W}$, condition (16) is derived.

Condition (17): Given Lyapunov matrix $(I_N \otimes \bar{P})$ and a circular region $\mathcal{D}(\alpha, \tau)$, it follows from the global error dynamics (14) that

$$\begin{bmatrix} -I_N \otimes \bar{P} & I_N \otimes (\bar{P}\bar{T}\bar{A}) - (L+G) \otimes (\bar{P}\bar{K}\bar{C}) - \alpha(I_N \otimes \bar{P}) \\ * & -\tau^2(I_N \otimes \bar{P}) \end{bmatrix} < 0 \tag{22}$$

using Lemma 1. We obtain condition (17) directly based on definitions of matrices $\bar{T}$, $\bar{Y}$ and $\bar{W}$. □

Remark 4 Based on Theorem 1, we can determine that the proposed FE design with H_∞ performance can restrain the effects of the term $\dot{f}(t)$ on the FE error $e_f(t)$ and improve the estimation performance of time-varying faults, which is not considered effectively in [9]. H_∞ performance can also ensure system stability [4]. Considering

H_∞ performance can guarantee stability and robustness of the error dynamics. The regional pole constraint (17) is introduced to improve FE transient performance [5]. The unified form of actuator and sensor fault estimators makes the presented approach more comprehensive and convincing.

After that, we can rewrite the FE algorithm as follows

$$\hat{f}(t) = \left(I_N \otimes \bar{I}_r^T\right)\hat{\bar{x}}(t) = \left(I_N \otimes \bar{I}_r^T\right)\left(\bar{z}(t) + (I_N \otimes \bar{H} y(t)\right) \tag{23}$$

so we can obtain the FE with improved transient performance.

3.3 Existence of Global UIO Design

The global error dynamics (14) indicates that the existence condition of the proposed UIO is that $(I_N \otimes \bar{T}\bar{A},\ (L+G)\otimes\bar{C})$ is observable. That is, following modern control theory, the condition that $(I_N \otimes \bar{T}\bar{A},\ (L+G)\otimes\bar{C})$ is observable means

$$\text{rank}\left(\begin{bmatrix} sI_{(n+r)N} - I_N \otimes \bar{T}\bar{A} \\ (L+G)\otimes\bar{C} \end{bmatrix}\right) = (n+r)N, \quad \forall s \in \mathbb{C} \tag{24}$$

Matrix $(L+G)$ is a symmetric positive definite, and hence, we can obtain $(L+G) = \Gamma^T \Lambda \Gamma$, where the orthogonal matrix $\Gamma \in \mathbb{R}^{N\times N}$ constitutes the eigenvectors of $(L+G)$ and $\Gamma^T\Gamma = I$, $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\} \in \mathbb{R}^{N\times N}$ and λ_i $(i = 1, \ldots, N)$ are eigenvalues of $(L+G)$.

Matrix $\begin{bmatrix} sI_{(n+r)N} - I_N \otimes \bar{T}\bar{A} \\ (L+G)\otimes\bar{C} \end{bmatrix}$ can be rewritten as $\begin{bmatrix} I_N \otimes (sI_{n+r} - \bar{T}\bar{A}) \\ (L+G)\otimes\bar{C} \end{bmatrix}$, the pre-multiplying full-rank matrix $\begin{bmatrix} (\Gamma^T \otimes I_{n+r}) & 0 \\ 0 & (\Gamma^T \otimes I_p) \end{bmatrix}$ and post-multiplying full-rank matrix $\begin{bmatrix} \Gamma \otimes I_{n+r} \end{bmatrix}$, we can obtain

$$\begin{aligned}
&\text{rank}\left(\begin{bmatrix} I_N \otimes (sI_{n+r} - \bar{T}\bar{A}) \\ (L+G)\otimes\bar{C} \end{bmatrix}\right) \\
&= \text{rank}\left(\begin{bmatrix} (\Gamma^T \otimes I_{n+r}) & 0 \\ 0 & (\Gamma^T \otimes I_p) \end{bmatrix}\begin{bmatrix} I_N \otimes (sI_{n+r} - \bar{T}\bar{A}) \\ (L+G)\otimes\bar{C} \end{bmatrix}\begin{bmatrix} \Gamma \otimes I_{n+r} \end{bmatrix}\right) \\
&= \text{rank}\left(\begin{bmatrix} I_N \otimes (sI_{n+r} - \bar{T}\bar{A}) \\ \Lambda \otimes \bar{C} \end{bmatrix}\right) \\
&= \sum_{i=1}^{N} \text{rank}\left(\begin{bmatrix} sI_{n+r} - \bar{T}\bar{A} \\ \lambda_i \bar{C} \end{bmatrix}\right)
\end{aligned} \tag{25}$$

Given that all the eigenvalues of $(L+G)$ are positive, the existence condition of the proposed UIO design is that the pair $(\bar{T}\bar{A}, \bar{C})$ is observable.

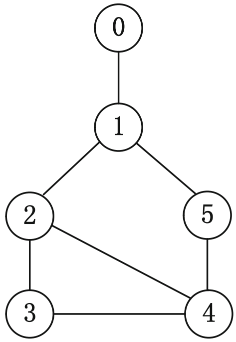

Fig. 1 Communication topology

4 Simulation Results and Analysis

In this section, an example is given to illustrate the validity of our theoretical results for a linear multi-agent system. A network of five aircrafts is presented and model parameters of each aircraft system are given as follows:

$$A = \begin{bmatrix} -9.9477 & -0.7476 & 0.2632 & 5.0337 \\ 52.1659 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$D = 0.01 \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$$

where state vector $x_i(t) = [V_h, V_v, q, \theta]^T$ includes horizontal velocity V_h, vertical velocity V_v, pith rate q, and pitch angle θ. The input vector $u_i(t) = [\delta_c, \delta_l]^T$ is collective pith control δ_c and longitudinal cyclic pitch control δ_l.

The communication topology is shown in Fig. 1. The figure shows that the graph is an undirect graph, and hence, we can conclude that the Laplacian matrix L and pinning matrix G are

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 3 & -1 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

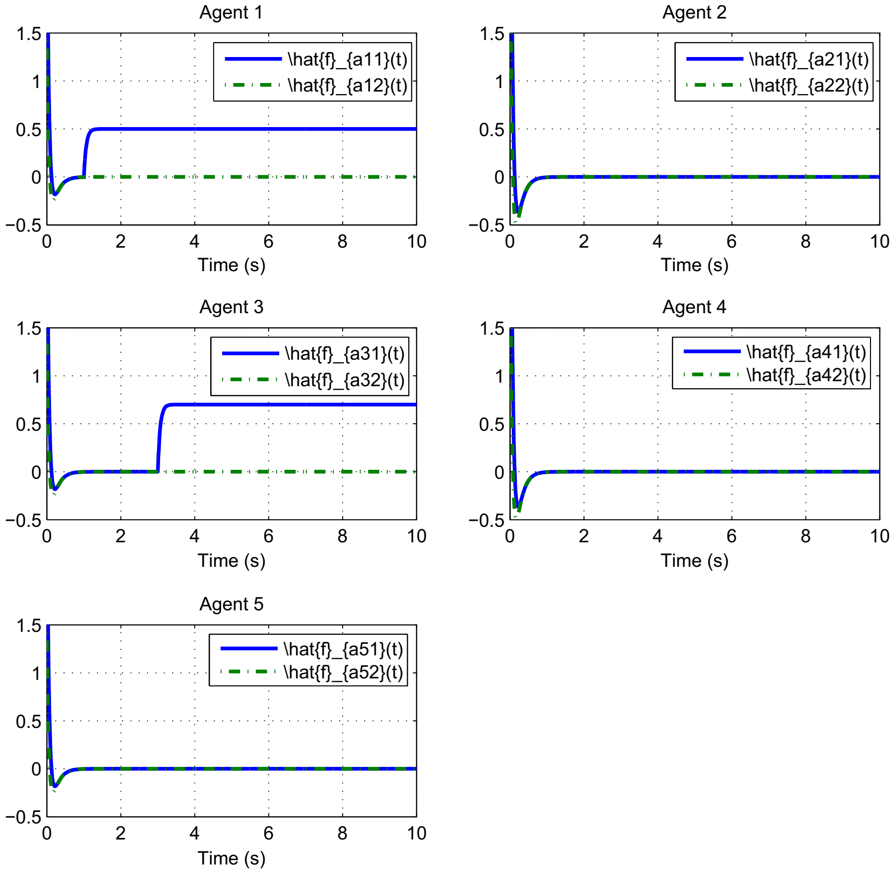

Fig. 2 Simulation results of constant actuator faults

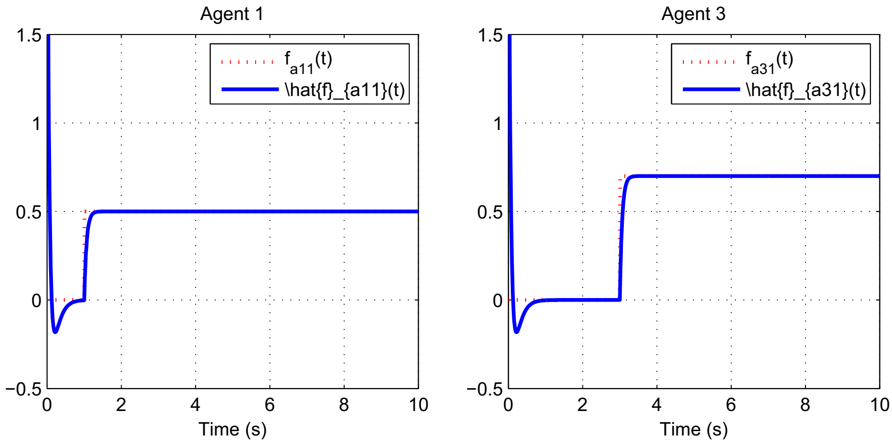

Fig. 3 FE of $f_{a11}(t)$ and $f_{a31}(t)$ (*dotted* real faults, *solid* fault estimates)

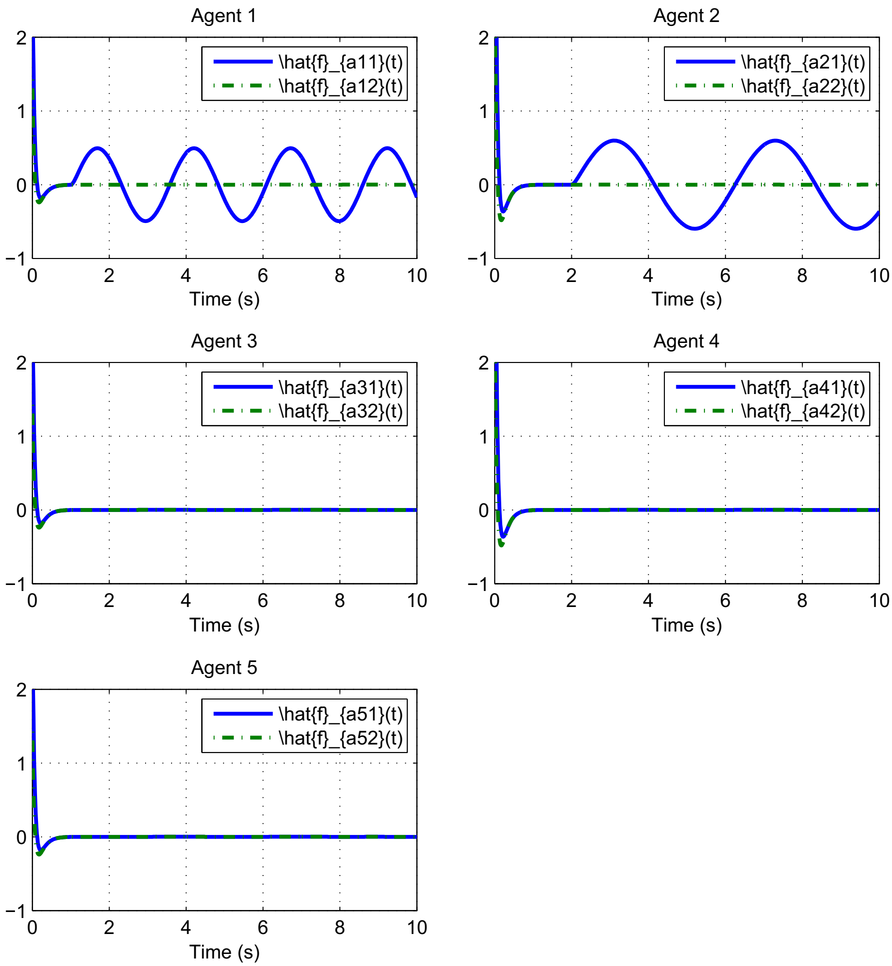

Fig. 4 Simulation results of time-varying actuator faults

Matrix $(L + G)$ can be verified to be nonsingular and its eigenvalues are positive, i.e., 0.1414, 1.5713, 2.7995, 3.6728, and 4.8150. We take sampling time $T = 0.01s$, reference input $u_i(t) = [0.5, 0.5]^T$, and unknown input $d(t) = 0.1\sin(t)$. The initial values of multi-agent systems are set to be nonzero for the all simulations.

4.1 Actuator FE Results

We first consider an actuator fault occurring in the input channel, where the actuator fault distribution matrix is $E = B$. By solving conditions (16) and (17) with the circular region $\mathcal{D}(-8, 8)$, we can obtain the minimum H_∞ performance index $\gamma = 0.0626$ and the designed matrices $\bar{K}$, $\bar{H}$, and $\bar{T}$ as follows

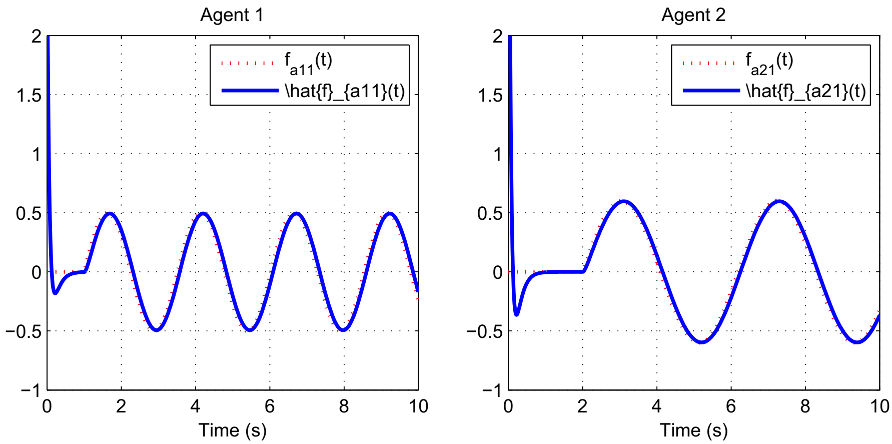

Fig. 5 FE of $f_{a11}(t)$ and $f_{a21}(t)$ (*dotted* real faults, *solid* fault estimates)

$$\bar{K} = \begin{bmatrix} 1.8723 & -0.0556 & 0.4161 \\ -0.0677 & 2.8116 & 0.0317 \\ -0.3372 & -0.1001 & -1.6004 \\ 0.4633 & 0.0271 & 2.7207 \\ 19.3810 & 1.6622 & -10.5803 \\ 16.9576 & 1.0541 & -8.5820 \end{bmatrix}, \quad \bar{H} = \begin{bmatrix} 1.0011 & -0.0000 & -0.0011 \\ 0.0001 & 1.0000 & -0.0001 \\ -6.5283 & -0.7428 & 8.2712 \\ -0.0006 & 0.0000 & 1.0006 \\ 30.5269 & 0.7066 & -31.2335 \\ 14.2667 & -1.7766 & -12.4901 \end{bmatrix},$$

$$\bar{T} = \begin{bmatrix} -0.0011 & 0.0000 & 0 & 0.0011 & 0 & 0 \\ -0.0001 & 0.0000 & 0 & 0.0001 & 0 & 0 \\ 6.5283 & 0.7428 & 1.0000 & -8.2712 & 0 & 0 \\ 0.0006 & -0.0000 & 0 & -0.0006 & 0 & 0 \\ -30.5269 & -0.7066 & 0 & 31.2335 & 1.0000 & 0 \\ -14.2667 & 1.7766 & 0 & 12.4901 & 0 & 1.0000 \end{bmatrix}$$

The pair $(\bar{T}\bar{A}, \bar{C})$ is verified to be observable, and Assumptions 2 and 3 are satisfied. We denote the actuator fault vector of the ith aircraft as $f_{ai}(t) = [f_{ai1}, f_{ai2}]^T$.

First, we assume that constant actuator faults occur in the first and third agents, which are described as follows:

$$f_{a11}(t) = \begin{cases} 0 & 0s \le t \le 1s \\ 0.5 & 1s < t \le 10s \end{cases}, \qquad f_{a12}(t) = 0$$

$$f_{a31}(t) = \begin{cases} 0 & 0s \le t \le 3s \\ 0.7 & 3s < t \le 10s \end{cases}, \qquad f_{a32}(t) = 0$$

The others are fault-free, i.e., $f_{a2} = f_{a4} = f_{a5} = 0$. The simulation results of the robust UIO-based method are shown in Fig. 2. FE results of $f_{a11}(t)$ and $f_{a31}(t)$ are zoomed in and illustrated in Fig. 3.

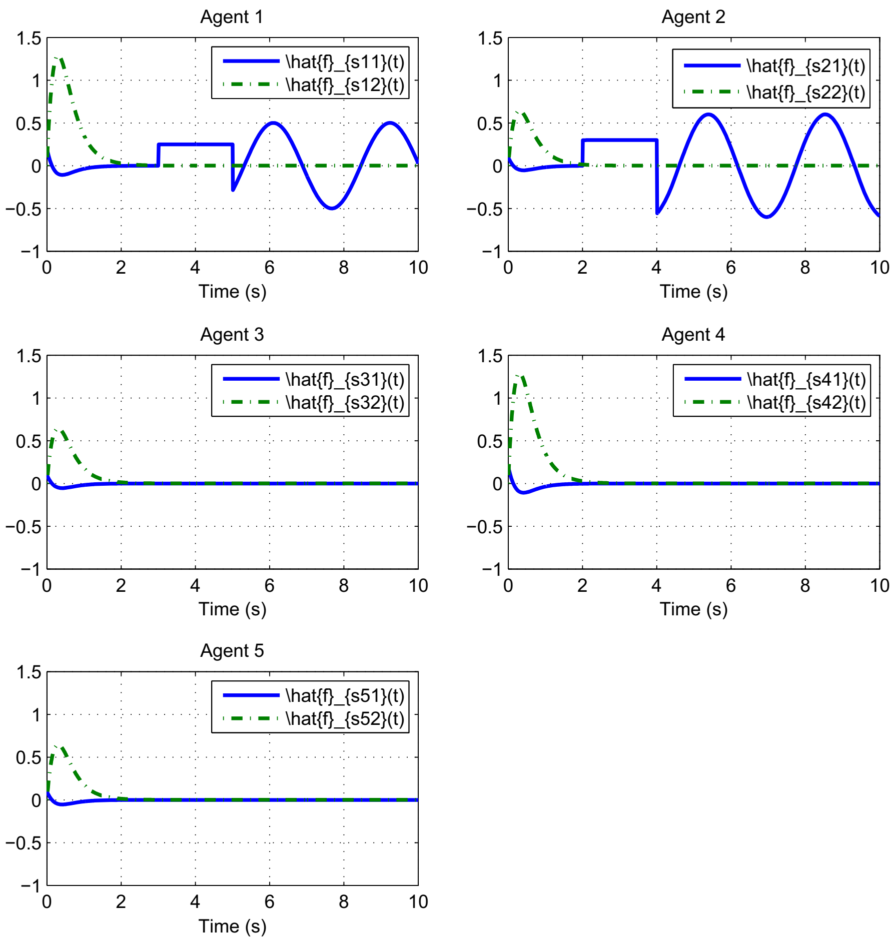

Fig. 6 Simulation results of time-varying sensor faults

Second, we assume that time-varying actuator faults occur in the first and second agents to be as follows:

$$f_{a11}(t) = \begin{cases} 0 & 0s \le t \le 1s \\ 0.5\sin(2.5t - 2.5) & 1s < t \le 10s \end{cases}, \qquad f_{a12}(t) = 0$$

$$f_{a21}(t) = \begin{cases} 0 & 0s \le t \le 2s \\ 0.6\sin(1.5t - 3.0) & 2s < t \le 10s \end{cases}, \qquad f_{a22}(t) = 0$$

The others are fault-free, i.e., $f_{a3} = f_{a4} = f_{a5} = 0$. The simulation results of the robust UIO-based method are shown in Fig. 4, while the FE results of $f_{a11}(t)$ and $f_{a21}(t)$ are zoomed in and illustrated in Fig. 5.

4.2 Sensor FE Results

We consider the case of sensor FE. The sensor fault distribution matrix is assumed to be $F = [1, 0; 0, 1; 0, 0]$. By solving conditions (16) and (17) with the circular region $\mathcal{D}(-8, 8)$, we can obtain the minimum H_∞ performance index $\gamma = 2.2656 \times 10^{-5}$ and designed matrices $\bar{K}$, $\bar{H}$, $\bar{T}$ as follows

$$\bar{K} = \begin{bmatrix} -0.3575 & -0.1722 & 2.0609 \\ 4.4586 & 2.2003 & -5.4479 \\ 1.3710 & 0.5848 & -4.1441 \\ -0.0637 & -0.0136 & 2.9200 \\ 3.3093 & 0.1659 & -2.1860 \\ -4.4619 & 0.6316 & 5.4181 \end{bmatrix}, \ \bar{H} = \begin{bmatrix} -0.0000 & 0.0000 & 1.0000 \\ 0.0000 & -0.0000 & 1.0000 \\ -0.0000 & 0.0000 & 1.0000 \\ -0.0000 & 0.0000 & 1.0000 \\ 1.0000 & -0.0000 & -1.0000 \\ -0.0000 & 1.0000 & -1.0000 \end{bmatrix}$$

$$\bar{T} = \begin{bmatrix} 1.0000 & -0.0000 & 0 & -1.0000 & 0.0000 & -0.0000 \\ -0.0000 & 1.0000 & 0 & -1.0000 & -0.0000 & 0.0000 \\ 0.0000 & -0.0000 & 1.0000 & -1.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0 & 0.0000 & 0.0000 & -0.0000 \\ -1.0000 & 0.0000 & 0 & 1.0000 & -0.0000 & 0.0000 \\ 0.0000 & -1.0000 & 0 & 1.0000 & 0.0000 & -0.0000 \end{bmatrix}$$

The pair $(\bar{T}\bar{A}, \bar{C})$ is verified to be observable, and Assumptions 2 and 3 are also satisfied. We denote the ith sensor fault vector as $f_{si}(t) = [f_{si1}, f_{si2}]^T$ and time-varying sensor faults are assumed as follows:

$$f_{s11}(t) = \begin{cases} 0 & 0s \le t \le 3s \\ 0.25 & 3s < t \le 5s \\ 0.5\sin(2t - 30 + \frac{\Pi}{6}) & 5s < t \le 10s \end{cases}, \qquad f_{s12}(t) = 0$$

$$f_{s21}(t) = \begin{cases} 0 & 0s \le t \le 2s \\ 0.3 & 2s < t \le 4s \\ 0.6\sin(2t - 30 + \frac{\Pi}{6}) & 4s < t \le 10s \end{cases}, \qquad f_{s22}(t) = 0$$

The others are fault-free, i.e., $f_{s3} = f_{s4} = f_{s5} = 0$. The simulation results of the robust UIO-based method are shown in Fig. 6. FE results of $f_{s11}(t)$ and $f_{s21}(t)$ are zoomed in and illustrated in Fig. 7.

Simulation results of both actuator and sensor faults show that as expected, the proposed robust UIO-based FE approach can obtain high rapidity and accuracy; this approach can also eliminate effects of unknown inputs. The simulation shows that the condition of nonzero initial values does not affect the FE of multi-agent systems.

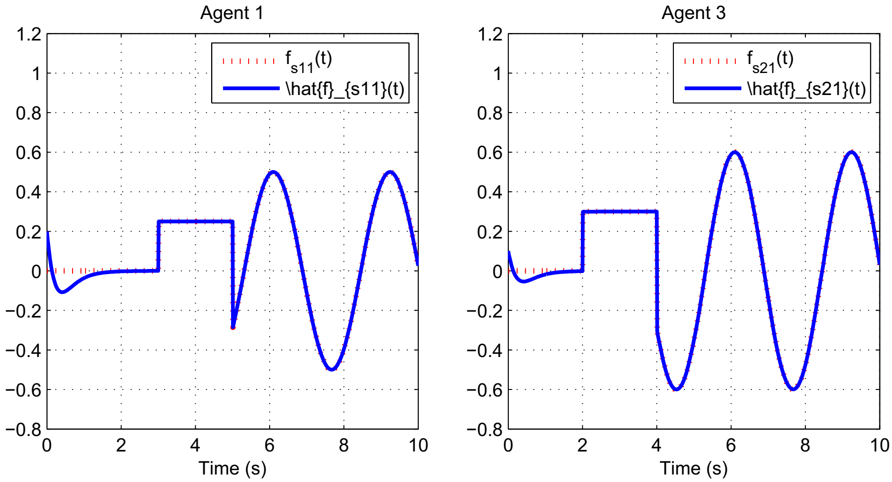

Fig. 7 FE of $f_{s11}(t)$ and $f_{s21}(t)$ (*dotted* real faults, *solid* fault estimates)

5 Conclusions

In this paper, a robust UIO-based FE that uses the relative output information has been proposed to utilize the communication topology for multi-agent systems with undirected graphs. A global augmented system is first derived by taking the actuator fault and sensor fault vector as an auxiliary state vector. Then the cooperative fault estimators are designed. Next, a multi-constrained design based on linear matrix inequality technique is provided to calculate the UIO gain matrices. Finally, simulation results show the effectiveness and advantages of the designed robust UIO-based FE approach. In our future works, FE methods of heterogeneous multi-agent systems and nonlinear multi-agent systems will be studied.

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