

Robust H_∞ Control of a Class of Switching Nonlinear Systems with Time-Varying Delay Via T–S Fuzzy Model

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Abstract This paper considers H_∞ control of a class of switching nonlinear systems with time-varying delays via T–S fuzzy model based on piecewise fuzzy weighting-dependent Lyapunov–Krasovskii functionals (PFLKFs). The systems are switching among several nonlinear systems. The Takagi and Sugeno (T–S) fuzzy model is employed to approximate the sub-nonlinear dynamic systems. Thus, with two level functions, namely, crisp switching functions and local fuzzy weighting functions, we introduce a continuous-time switched fuzzy systems, which inherently contain the features of the switched hybrid systems and T–S fuzzy systems. Average dwell-time approach and PFLKFs methods are utilized for the stability analysis and controller design, and with free fuzzy weighting matrix scheme. Switching and control laws are obtained such that the H_∞ performance is satisfied. The conditions of stability and the control laws are given in the form of LMIs which can be obtained by solving a set of linear matrix inequalities (LMIs) that are numerically feasible. A numerical example and the control of an uncertain radio-controlled (R/C) hovercraft with time-varying delay are given to demonstrate the efficiency of the proposed method.

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1 Introduction

Switching systems are an important class of hybrid systems. Such systems can be described by a family of continuous-time subsystems (or discrete-time subsystems) and a rule that orchestrates the switching between them; for example, a given process exhibits a switching behavior caused by abrupt changes of the environment. This class of systems has numerous applications in the control of mechanical systems, the automotive industry, aircraft and air traffic control, switching power converters and many other fields. Although this class of systems can be seen as a particular case of linear parameter varying (LPV) systems, it has specific characteristics: the first is that the switch occurs between a finite number of subsystems; the second is that the switching sequence has to be taken into account in practical situations. For instance, one can only act using a certain sequence to stabilize the switched system. This class system has received great interest from researchers during the last decade [1, 10, 14].

The stability problem, caused by various switching, is a main concern in the field of switching systems [10]. So far, two stability issues have been addressed in literature, i.e., the stability under arbitrary switching and the stability under constrained switching. The former case is mainly investigated based on constructing a common Lyapunov function for all subsystems [10]. On the other hand, for switching systems under constrained switching, it is well known that the multiple Lyapunov-like function approach is more efficient in offering greater freedom for demonstrating stability of the system [14]. As a class of typical constrained switching signals, the average dwell-time switching means that the number of switches in a finite interval is bounded and the average time between consecutive switching is not less than a constant [10] and [14]. The average dwell-time switching can cover the dwell-time switching and its extreme case is actually the arbitrary switching. Therefore, it is of practical and theoretical significance to probe the stability of switching systems with average dwell time [14].

In recent years, switching linear systems have received a great deal of attention in continuous-time domain [1, 14]. However, there is less research in the field of continuous-time switching nonlinear systems [17, 24]. Since the introduction of T–S fuzzy models by Takagi and Sugeno [15] in 1985, fuzzy model control has been extensively studied because T–S fuzzy models provide an effective representation of complex nonlinear systems [3, 4, 8, 9, 16, 18]. The objective of this paper is to study switching nonlinear systems [11] with time-varying delay. Each subsystem is written as an equivalent T–S fuzzy model. Recently, there has been several literature in the field of switching fuzzy system [2, 17, 24]. However, they do not consider the average dwell time in stability analysis which is important for switching systems.

Generally speaking, real systems with time delays are common in biology, mechanics, society, and economics. Moreover, time-varying delay is more important and universal in real engineering processes and has more complex impacts on system

dynamics than constant delay. Since time delay is a main factor of instability of time-delay systems, the problem of stability analysis of time-delay systems has been one of the main concerns of researchers wishing to inspect the properties of such systems. Recently, the authors in [1] and [7, 20, 21, 25] considered switching linear systems with delays. As for switched nonlinear systems with delays we hope we can as get desirable result as switching linear systems such as exponential stability. Recently, there exist some advanced methods to deal with time delay in the T–S fuzzy systems [6, 13, 22, 23, 26], such as slack matrix, input-output method and fuzzy partitioning method and fuzzy weighting-dependent Lyapunov–Krasovskii functionals to deal with asymptotically stability. Based on these results, we can construct PFLKFs for the exponential stability of switched fuzzy systems with delays. On the other hand, due to modeling error or external disturbance, many practical systems are always subject to various kinds of uncertainties. One of the most important requirements for a control system is the so-called robustness. Since the pioneering work on the so-called H_∞ optimal control theory, there has been a considerable progress in H_∞ control theory [4, 18] and [24]. In fact, the switching fuzzy systems the best of our knowledge, the problems of stability analysis and robust H_∞ control of continuous-time switching fuzzy systems with both parametric uncertainties and time-varying delays has not been addressed, which is very challenging and remains open.

The main contribution of the paper is that we investigate the exponential stability of delayed switched nonlinear systems via T–S fuzzy model and get relaxed conditions. In addition, we present new H_∞ control design for uncertain continuous-time switched fuzzy systems with time-varying delay based on PFLKFs. Compared with the results based on PLKFs introduced in [14], the result based on PFLKFs is more relaxed. Moreover, the stability checking results, the switching laws and control laws can be obtained by solving a set of LMIs that are numerically tractable with commercially available software.

The rest of this paper is organized as follows. System descriptions and preliminaries are presented in Sect. 2. Stability analysis of uncertain continuous-time switching fuzzy systems with time-varying delays is presented in Sect. 3. H_∞ stability analysis and controller design for such systems is considered in Sect. 4. In Sect. 5, a numerical example and the application to the H_∞ control of uncertain radio-controlled (R/C) hovercraft with time-varying delays are provided to demonstrate effectiveness of our results. Finally, conclusions are given in Sect. 6.

Notations The notations used are fairly standard. We use $P > 0$ (\geq , $<$, ≤ 0) to denote a positive definite (semi-definite, negative definite, semi-negative definite) matrix P . \mathbb{R}^n denotes the n -dimensional Euclidean space and $L_2[0, \infty)$ is the space of square integrable functions on $[0, \infty)$. For $\tau > 0$, let $\mathbb{R}_+ = [0, +\infty]$ and $C_n = C^1([-\tau, 0], \mathbb{R}^n)$ be the Banach space of continuously differentiable mapping from $([-\tau, 0], \mathbb{R}^n)$ to \mathbb{R}^n with topology of uniform convergence. $\|\cdot\|$ denotes the usual 2-norm and $\|x(t + \theta)\|_d = \sup_{-\tau \leq \theta \leq 0} \{\|x(t + \theta)\|, \|\dot{x}(t + \theta)\|\}$. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of P . I and O represent the identity and zero matrices in the block matrix. The superscript ‘ T ’ stands for matrix transpose; and the symmetric terms in a matrix are denoted by $*$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 System Descriptions and Preliminaries

In this paper, we consider systems described by

$$\begin{aligned} \dot{x}(t) &= f_{\sigma(t)}(x(t)) + h_{\sigma(t)}(x(t - \tau_{\sigma(t)}(t))) + d_{\sigma(t)}(w(t)) + g_{\sigma(t)}(u(t)), \\ y(t) &= l_{\sigma(t)}(x(t)) + m_{\sigma(t)}(x(t - \tau_{\sigma(t)}(t))) + n_{\sigma(t)}(u(t)), \quad t \in [0, \infty), \\ x(t) &= \phi_{\sigma(t)}(t), \quad t \in [-\tau_{\sigma(t)}, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$, is the control, $w(t) \in \mathbb{R}^p$ is the exogenous disturbance which belong to $L_2[0, \infty)$ and $y(t) \in \mathbb{R}^q$ is the output. $\phi(t)$ is the continuous vector-value function specifying the initial state of the system and $\tau_{\sigma(t)}(t)$ is the continuous time-varying delay satisfying $0 \leq \tau_{\sigma(t)}(t) \leq \tau_{\sigma(t)}$, $\dot{\tau}_{\sigma(t)}(t) \leq \kappa_{\sigma(t)}$. $f_{\sigma(t)}(\cdot), h_{\sigma(t)}(\cdot), d_{\sigma(t)}(\cdot), g_{\sigma(t)}(\cdot), l_{\sigma(t)}(\cdot), m_{\sigma(t)}(\cdot), n_{\sigma(t)}(\cdot)$ are the Lipschitz functions.

The right continuous function $\sigma(t): [0, \infty) \rightarrow \underline{S} = \{1, 2, \dots, s\}$ is the switching signal, s is the number of switching regions. Corresponding to the switching signal $\sigma(t)$, we have the switching sequence $\{x_{i_0}; (i_0, t_0), \dots, (i_k, t_k), \dots, | i_k \in \underline{S}, k = 0, 1, \dots\}$, which means that the i_k th nonlinear subsystem is activated when $t \in [t_k, t_{k+1})$. In addition, we exclude Zeno behavior for all types of switching signals as commonly assumed in literature. We assume that the state of the switched system (1) does not jump at the switching instants, i.e., the trajectory $x(t)$ is everywhere continuous.

Takagi and Sugeno [15] have proposed a fuzzy model to represent nonlinear systems. It is proved that the Takagi–Sugeno fuzzy model is a universal approximator. Then each nonlinear subsystem of (1) $i \in \underline{S}$ could be represented by a T–S fuzzy model described by r_i rules of the following uncertain form with time-varying delay:

Local Plant rule $k, k \in R_i \triangleq \{1, 2, \dots, r_i\}$

IF z_1^i is M_{k1}^i and ... and z_e^i is M_{ke}^i **THEN**

$$\begin{aligned} \dot{x}(t) &= (A_{ik} + \Delta A_{ik})x(t) + (A_{idk} + \Delta A_{idk})x(t - \tau_i(t)) + (B_{ik} + \Delta B_{ik})u(t) \\ &\quad + (D_{ik} + \Delta D_{ik})w(t), \\ y(t) &= E_{ik}x(t) + E_{idk}x(t - \tau_i(t)) + C_{ik}u(t), \\ x(t) &= \phi_i(t), \quad t \in [-\tau_i, 0], \end{aligned} \tag{2}$$

M_{kl}^i are fuzzy sets and $z_l^i (l = 1, 2, \dots, e)$ are the premise variables. $(A_{ik}, A_{idk}, B_{ik}, D_{ik}, E_{ik}, E_{idk}, C_{ik})$ is the k th local model in the i th switching region of the system, and $(\Delta A_{ik}, \Delta A_{idk}, \Delta B_{ik}, \Delta D_{ik})$ is the uncertainty terms of the k th local model in the i th switching region of the system. In this paper, the uncertainty terms are assumed to be of the form

$$[\Delta A_{ik}, \Delta A_{idk}, \Delta B_{ik}, \Delta D_{ik}] = M_{ik} F_i(t) [N_{i1k}, N_{i2k}, N_{i3k}, N_{i4k}], \tag{3}$$

where $M_{ik}, N_{i1k}, N_{i2k}, N_{i3k}$ and N_{i4k} are known real constant matrices and $F_i(t)$ is an unknown time-varying matrix function satisfying

$$F_i^T(t)F_i(t) \leq I, \quad t \geq 0. \tag{4}$$

From [3, 15], through the use of “fuzzy blending” the final switching fuzzy system (2) is inferred as follows:

$$\begin{aligned} \dot{x}(t) &= \left\{ \sum_{k=1}^{r_i} v_{ik}(t) [(A_{ik} + \Delta A_{ik})x(t) + (A_{idk} + \Delta A_{idk})x(t - \tau_i(t)) \right. \\ &\quad \left. + (B_{ik} + \Delta B_{ik})u(t) + (D_{ik} + \Delta D_{ik})w(t)] \right\} / \sum_{k=1}^{r_i} v_{ik}(t) \\ &= \sum_{k=1}^{r_i} h_{ik}(t) [(A_{ik} + \Delta A_{ik})x(t) + (A_{idk} + \Delta A_{idk})x(t - \tau_i(t)) \\ &\quad + (B_{ik} + \Delta B_{ik})u(t) + (D_{ik} + \Delta D_{ik})w(t)], \\ y(t) &= \frac{\sum_{k=1}^{r_i} v_{ik}(t) [E_{ik}x(t) + E_{idk}x(t - \tau_i(t)) + C_{ik}u(t)]}{\sum_{k=1}^{r_i} v_{ik}(t)} \\ &= \sum_{k=1}^{r_i} h_{ik}(t) [E_{ik}x(t) + E_{idk}x(t - \tau_i(t)) + C_{ik}u(t)], \end{aligned} \tag{5}$$

$$x(t) = \phi_i(t), \quad t \in [-\tau_i, 0],$$

with $v_{ik}(t) = \prod_{p=1}^{e_i} M_{kp}^i(z_p^i(t))$, $h_{ik}(t) = \frac{v_{ik}(t)}{\sum_{k=1}^{r_i} v_{ik}(t)}$, and $M_{kp}^i(z_p^i(t))$ is the grade of the membership function of z_p^i in M_{kp}^i . It is assumed that $v_{ik}(t) \geq 0$ for all $t \geq 0$, $i \in \underline{S}, k \in \underline{R}_i$. Therefore the normalized membership function $h_{ik}(t)$ satisfies $h_{ik}(t) \geq 0, \sum_{k=1}^{r_i} h_{ik}(t) = 1, t \geq 0$.

For convenient notation, we introduce $\bar{A}_{ik} = A_{ik} + \Delta A_{ik}, \bar{A}_{idk} = A_{idk} + \Delta A_{idk}, \bar{B}_{ik} = B_{ik} + \Delta B_{ik}, \bar{D}_{ik} = D_{ik} + \Delta D_{ik}$. Then, using this notation, the system model (5) can be expressed as

$$\begin{aligned} \dot{x}(t) &= \sum_{k=1}^{r_i} h_{ik}(t) [\bar{A}_{ik}x(t) + \bar{A}_{idk}x(t - \tau_i(t)) + \bar{B}_{ik}u(t) + \bar{D}_{ik}w(t)], \\ y(t) &= \sum_{k=1}^{r_i} h_{ik}(t) [E_{ik}x(t) + E_{idk}x(t - \tau_i(t)) + C_{ik}u(t)], \\ x(t) &= \phi_i(t), \quad t \in [-\tau_i, 0]. \end{aligned} \tag{6}$$

To end this section we state the following definitions and lemmas which will be used throughout the paper.

Definition 1 The equilibrium $x^* = 0$ of system (5) is said to be robust exponentially stable under control law $u(t)$ and switching signal $\sigma(t)$ if the solution $x(t)$ of system (1) with $w(t) = 0$ through $(t_0, \phi_{\sigma(t)}) \in \mathbb{R}_+ \times C_n$ satisfies $\|x(t)\| \leq K \|x(t_0)\|_d e^{-\lambda(t-t_0)}$, $\forall t \geq t_0$, for constant $K > 0$ and $\lambda > 0$.

Definition 2 [1] For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the number of switching of $\sigma(t)$ over (T_1, T_2) . If $N_\sigma(T_1, T_2) \leq N_0 + (T_2 - T_1)/T_\alpha$ holds for $T_\alpha > 0, N_0 \geq 0$, then T_α is called average dwell time.

Definition 3 For $\gamma > 0$, system (5) is said to have H_∞ performance γ , if it is exponentially stable and under zero initial condition $\phi_i(\theta) = 0, \theta \in [-\tau_i, 0], i \in \underline{S}_j$, we have $\int_0^\infty y^T(s)y(s) ds \leq \gamma^2 \int_0^\infty w^T(s)w(s) ds$.

Lemma 1 [5] For any real matrices X_i, X_{ij} for $i, j = 1, 2, \dots, r$, and $Q > 0$ with appropriate dimensions, we have

$$\sum_{i=1}^r \sum_{j=1}^r h_i(t)h_j(t)X_i^T Q X_j \leq \sum_{i=1}^r h_i(t)X_i^T Q X_i,$$

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r \sum_{l=1}^r h_i(t)h_j(t)h_k(t)h_l(t)X_{ij}^T Q X_{kl} \leq \sum_{i=1}^r \sum_{j=1}^r h_i(t)h_j(t)X_{ij}^T Q X_{ij},$$

where $h_i(t) \geq 0, \sum_{i=1}^r h_i(t) = 1 (1 \leq i \leq r)$.

Lemma 2 [12] Given matrices $Q = Q^T, H, E$ and $R = R^T > 0$ of appropriate dimensions, $Q + HFE + E^T F^T H^T < 0$ holds for all F satisfying $F^T F \leq R$, if and only if there exists scalar $\beta > 0$ such that $Q + \beta H^T R H + \beta^{-1} E^T R E < 0$.

3 Robust Stability

In this section, we consider the stability analysis of the systems (6) described in the last section. The stability condition for the system without control input and external disturbance can be summarized in the following theorem.

Theorem 3.1 The system (5), or equivalently (6) with $u(t) \equiv w(t) \equiv 0$, suppose that the time-varying delay $\tau_i(t)$ satisfies $0 \leq \tau_i(t) \leq \tau_i, \dot{\tau}_i(t) \leq \kappa_i (\tau_i > 0, \kappa_i < 1, i \in \underline{S})$. For given positive constants α and β , if there exist matrices $P_i^T = P_i > 0, Q_{ik}^T = Q_{ik} > 0, R_{ik}^T = R_{ik} > 0, X_{ik} = \begin{bmatrix} X_{ik11} & X_{ik12} \\ * & X_{ik22} \end{bmatrix} \geq 0$, and any matrices Y_{ik} and T_{ik} with appropriate dimensions such that

$$\Theta_{imkl} < 0, \quad k \leq l, m \in \underline{R}_j, i \in \underline{S} \tag{7}$$

$$\Phi_{ik} \geq 0, \quad k \in \underline{R}_j, i \in \underline{S} \tag{8}$$

where

$$\Phi_{ik} = \begin{bmatrix} X_{ik11} & X_{ik12} & Y_{ik} \\ * & X_{ik22} & T_{ik} \\ * & * & e^{-\alpha\tau_i} Q_{ik} \end{bmatrix},$$

$$\Theta_{imkl} = \begin{bmatrix} \Psi_{i11} & \Psi_{i12} & A_{ik}^T Q_{il} & A_{il}^T Q_{ik} & P_i M_{ik} & P_i M_{il} & N_{i1,k}^T & N_{i1l}^T \\ * & \Psi_{im22} & A_{idk}^T Q_{il} & A_{idl}^T Q_{ik} & 0 & 0 & N_{i2k}^T & N_{i2l}^T \\ * & * & -\tau_i^{-1} Q_{il} & 0 & Q_{il} M_{ik} & 0 & 0 & 0 \\ * & * & * & -\tau_i^{-1} Q_{ik} & 0 & Q_{ik} M_{il} & 0 & 0 \\ * & * & * & * & -\beta I & 0 & 0 & 0 \\ * & * & * & * & * & -\beta I & 0 & 0 \\ * & * & * & * & * & * & -\beta^{-1} I & 0 \\ * & * & * & * & * & * & * & -\beta^{-1} I \end{bmatrix},$$

with

$$\begin{aligned} \Psi_{i11} &= \varphi_{ik11} + \varphi_{il11}, \quad \Psi_{i12} = \varphi_{ik12} + \varphi_{il12}, \\ \Psi_{im22} &= \varphi_{ikm22} + \varphi_{ilm22}, \\ \varphi_{ik11} &= P_i A_{ik} + A_{ik}^T P_i + \alpha P_i + R_{ik} + Y_{ik} + Y_{ik}^T + \tau_i X_{ik11}, \\ \varphi_{ik12} &= P_i A_{idk} - Y_{ik} + T_{ik}^T + \tau_i X_{ik12}, \\ \varphi_{ikm22} &= -T_{ik}^T - T_{ik} + \tau_i X_{ik22} + (\kappa_i - e^{-\alpha\tau_i}) R_{im}. \end{aligned} \tag{9}$$

Then, system is exponentially stable for any switching signal with average dwell time satisfying

$$T_\alpha > T_\alpha^* = \frac{\ln \mu}{\alpha}. \tag{10}$$

Moreover, an estimate of state decay is given by

$$\|x(t)\| \leq \sqrt{\frac{b}{a}} \mu^{N_0/2} e^{-\lambda(t-t_0)} \|x(t_0)\|_d, \tag{11}$$

where

$$\begin{aligned} a &= \min_{i \in \underline{S}} \lambda_{\min}(P_i), \\ b &= \max_{i \in \underline{S}} \lambda_{\max}(P_i) + \max_{i \in \underline{S}, k \in \underline{R}_i} \frac{\tau_i^2}{2} \lambda_{\max}(Q_{ik}) + \max_{i \in \underline{S}, k \in \underline{R}_i} \tau_i \lambda_{\max}(R_{ik}) \end{aligned} \tag{12}$$

and $\lambda = 1/2(\alpha - \ln \mu / T_\alpha)$, $\mu \geq 1$ satisfies

$$P_i \leq \mu P_j, \quad Q_{ik} \leq \mu Q_{jn}, \quad R_{ik} \leq \mu R_{jn}, \quad \forall (i, j, k, n) \in \underline{S} \times \underline{S} \times \underline{R}_i \times \underline{R}_j. \tag{13}$$

Proof Consider the following PFLKFs for the system (6) with $u(t) \equiv w(t) \equiv 0$:

$$V_i(t) = V_{i1}(t) + V_{i2}(t) + V_{i3}(t), \quad i \in \underline{S}, \tag{14}$$

where $V_{i1}(t) = x^T(t)P_i x(t)$, $V_{i2}(t) = \int_{-\tau_i}^0 \int_{t+\theta}^t \dot{x}^T(s)e^{-\alpha(t-s)}Q_i(s)\dot{x}(s) ds d\theta$ with $Q_i(s) = \sum_{k=1}^{r_i} h_{ik}(s)Q_{ik}$, $V_{i3}(t) = \int_{t-\tau_i(t)}^t x^T(s)e^{-\alpha(t-s)}R_i(s)x(s) ds$ with $R_i(s) = \sum_{k=1}^{r_i} h_{ik}(s)R_{ik}$.

Then, one has

$$\begin{aligned} \dot{V}_{i1}(t) &= 2x^T(t)P_i\dot{x}(t) \\ &= \sum_{k=1}^{r_i} h_{ik}(t)[x^T(t)(2P_i\bar{A}_{ik})x(t) + x^T(t)(2P_i\bar{A}_{idk})x(t - \tau_i(t))], \end{aligned} \tag{15}$$

$$\dot{V}_{i2}(t) \leq -\alpha V_{i2}(t) + \tau_i \dot{x}^T(t)Q_i(t)\dot{x}(t) - \int_{t-\tau_i(t)}^t \dot{x}(s)e^{-\alpha\tau_i}Q_i(s)\dot{x}(s) ds, \tag{16}$$

$$\begin{aligned} \dot{V}_{i3}(t) &\leq -\alpha V_{i3}(t) + x^T(t)R_i(t)x(t) \\ &\quad + x^T(t - \tau_i(t))[(\kappa_i - e^{-\alpha\tau_i})R_i(t - \tau_i(t))]x(t - \tau_i(t)). \end{aligned} \tag{17}$$

Let $\eta^T(t) = [x^T(t), x^T(t - \tau_i(t))]$, $\hat{X}_{ik} = [\bar{A}_{ik}, \bar{A}_{idk}]$ and by Lemma 1 we have

$$\begin{aligned} \tau_i \dot{x}^T(t)Q_i(t)\dot{x}(t) &= \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} h_{ik}(t)h_{il}(t)[\eta^T(t)\hat{X}_{ik}^T\tau_i Q_i(t)\hat{X}_{il}\eta(t)] \\ &\leq \sum_{k=1}^{r_i} h_{ik}(t)\{\eta^T(t)\hat{X}_{ik}^T\tau_i Q_i(t)\hat{X}_{ik}\eta(t)\} \\ &= \sum_{k=1}^{r_i} h_{ik}(t) \left\{ \eta^T(t) \begin{bmatrix} \bar{A}_{ik}^T\tau_i Q_i(t)\bar{A}_{ik} & \bar{A}_{ik}^T\tau_i Q_i(t)\bar{A}_{idk} \\ * & \bar{A}_{idk}^T\tau_i Q_i(t)\bar{A}_{idk} \end{bmatrix} \eta(t) \right\}. \end{aligned} \tag{18}$$

From Leibniz–Newton, we obtain

$$2[x^T(t)Y_i(t) + x^T(t - \tau_i(t))T_i(t)]\left[x(t) - x(t - \tau_i(t)) - \int_{t-\tau_i(t)}^t \dot{x}(s) ds\right] = 0 \tag{19}$$

with $Y_i(t) = \sum_{k=1}^{r_i} h_{ik}(t)Y_{ik}$ and $T_i(t) = \sum_{k=1}^{r_i} h_{ik}(t)T_{ik}$.

It holds that

$$\tau_i \eta^T(t)X_{ik}\eta(t) - \int_{t-\tau_i(t)}^t \eta^T(s)X_{ik}\eta(s) ds \geq 0. \tag{20}$$

Combining (14)–(20), $\dot{V}_i(t) + \alpha V_i(t)$ can be presented totally as follows:

$$\begin{aligned} \dot{V}_i(t) + \alpha V_i(t) &\leq \sum_{m=1}^{r_i} h_{im}(t) \left\{ \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} h_{ik}(t) h_{il}(t) \eta^T(t) \Xi_{imkl} \eta(t) \right\} \\ &\quad - \sum_{k=1}^{r_i} h_{ik}(t) \int_{t-\tau_i(t)}^t \eta^T(t, s) \Phi_{ik} \eta(t, s) ds \\ &= \sum_{m=1}^{r_i} h_{im}(t) \left\{ \sum_{k=1}^{r_i} h_{ik}^2(t) \eta^T(t) [(\Xi_{imkk} + \Xi_{imkk})/2] \eta(t) \right. \\ &\quad \left. + \sum_{k=1}^{r_i-1} \sum_{l>k}^{r_i} h_{ik}(t) h_{il}(t) \eta^T(t) (\Xi_{imkl} + \Xi_{imlk}) \right\} \\ &\quad - \sum_{k=1}^{r_i} h_{ik}(t) \int_{t-\tau_i(t)}^t \eta^T(t, s) \Phi_{ik} \eta(t, s) ds, \end{aligned} \tag{21}$$

where

$$\Xi_{imkl} = \begin{bmatrix} \phi_{ik11} + \tau_i \bar{A}_{ik}^T Q_{il} \bar{A}_{ik} & \phi_{ik12} + \tau_i \bar{A}_{ik}^T Q_{il} \bar{A}_{idk} \\ * & \phi_{ikm22} + \tau_i \bar{A}_{ik}^T Q_{il} \bar{A}_{idk} \end{bmatrix},$$

$$\eta^T(t, s) = [\eta^T(t), \dot{x}^T(s)],$$

$$\phi_{ik11} = P_i \bar{A}_{ik}^T + \bar{A}_{ik} P_i + R_{ik} + \alpha P_i + Y_{ik}^T + Y_{ik} + \tau_i X_{ik11},$$

$$\phi_{ik12} = P_i \bar{A}_{idk}^T - Y_{ik} + T_{ik}^T + \tau_i X_{ik12},$$

$$\phi_{ikm22} = -T_{ik} - T_{ik}^T + (\kappa_i - e^{-\alpha \tau_i}) R_{im} + \tau_i X_{ik22}.$$

Next, let

$$\hat{M}_{ikl}^T = \begin{bmatrix} M_{ik}^T P_i & 0 & M_{ik}^T Q_{il} & 0 \\ M_{il}^T P_i & 0 & 0 & M_{il}^T Q_{ik} \end{bmatrix}, \quad \hat{N}_{ikl} = \begin{bmatrix} N_{i1k} & N_{i2k} & 0 & 0 \\ N_{i1l} & N_{i2l} & 0 & 0 \end{bmatrix},$$

then by Schur complement, conditions (7) is equivalent to the following inequalities:

$$\underbrace{\begin{bmatrix} \phi_{ik11} + \phi_{il11} & \phi_{ik12} + \phi_{il12} & A_{ik}^T Q_{il} & A_{il}^T Q_{ik} \\ * & \phi_{ikm22} + \phi_{ilm22} & A_{idk}^T Q_{il} & A_{idl}^T Q_{ik} \\ * & * & -\tau_i^{-1} Q_{il} & 0 \\ * & * & * & -\tau_i^{-1} Q_{ik} \end{bmatrix}}_{(:=\nabla)} + \beta^{-1} \hat{M}_{ikl} \hat{M}_{ikl}^T + \beta \hat{N}_{ikl} \hat{N}_{ikl}^T < 0. \tag{22}$$

From Lemma 2 and (4) we know that when (22) is satisfied, the following inequalities hold:

$$\nabla + \hat{M}_{ikl} F_i^T(t) \hat{N}_{ikl}^T + \hat{N}_{ikl} F_i(t) \hat{M}_{ikl}^T = \begin{bmatrix} \phi_{ik11} + \phi_{il11} & \phi_{ik12} + \phi_{il12} & \bar{A}_{ik}^T Q_{il} & \bar{A}_{il}^T Q_{ik} \\ * & \phi_{ikm22} + \phi_{ilm22} & \bar{A}_{idk}^T Q_{il} & \bar{A}_{idl}^T Q_{ik} \\ * & * & -\tau_i^{-1} Q_{il} & 0 \\ * & * & * & -\tau_i^{-1} Q_{ik} \end{bmatrix} < 0. \tag{23}$$

Then using Schur complement again, inequalities in (23) are equivalent to the following condition:

$$\begin{bmatrix} \phi_{ik11} + \phi_{il11} & \phi_{ik12} + \phi_{il12} \\ * & \phi_{ikm22} + \phi_{ilm22} \end{bmatrix} + \begin{bmatrix} \bar{A}_{ik}^T Q_{il} & \bar{A}_{il}^T Q_{ik} \\ \bar{A}_{idk}^T Q_{il} & \bar{A}_{idl}^T Q_{ik} \end{bmatrix} \cdot \begin{bmatrix} \tau_i Q_{il}^{-1} & 0 \\ * & \tau_i Q_{ik}^{-1} \end{bmatrix} \cdot \begin{bmatrix} \bar{A}_{ik}^T Q_{il} & \bar{A}_{il}^T Q_{ik} \\ \bar{A}_{idk}^T Q_{il} & \bar{A}_{idl}^T Q_{ik} \end{bmatrix}^T = \mathcal{E}_{imkl} + \mathcal{E}_{imlk} < 0, \quad k \leq l, m \in \underline{R}_i, i \in \underline{S}. \tag{24}$$

Thus from (24) and (21) we conclude that (7) and (8) imply $\dot{V}_i(t) + \alpha V_i(t) < 0$, then integrating $\dot{V}_i(t) + \alpha V_i(t) < 0$ from t_k to t gives

$$V_{\sigma(t)}(t) \leq e^{-\alpha(t-t_k)} V_{\sigma(t_k)}(t_k), \quad t \in [t_k, t_{k+1}), i \in \underline{S}. \tag{25}$$

There exists matrix Q_0 such that $\mu Q_{jl} \geq Q_0 \geq Q_{ik}$, then we have

$$\sum_{l=1}^{r_j} h_{jl}(k) \mu Q_{jl} \geq \sum_{l=1}^{r_j} h_{jl}(k) Q_0 = Q_0, \quad Q_0 = \sum_{k=1}^{r_i} h_{ik}(t) Q_0 \geq \sum_{k=1}^{r_i} h_{ik}(t) Q_{ik}.$$

Obviously, from (14) we have $V_{i2}(t) \leq \mu V_{j2}(t)$. Similarly, $V_{i3}(t) \leq \mu V_{j3}(t)$. Finally, using (13) and (14) at switching time t_i , we have

$$V_{\sigma(t_i)}(t_i) \leq \mu V_{\sigma(t_i^-)}(t_i^-), \quad i \in \underline{S} \tag{26}$$

Therefore, it follows from (10), (25) and (26) and the relation $k = N_{\sigma(t)} \leq N_0 + (t - t_0)/T_\alpha$ with $N_0 \geq 0$ that

$$\begin{aligned} V_{\sigma(t_i)}(t_i) &\leq e^{-\alpha(t-t_k)} \mu V_{\sigma(t_k^-)}(t_k^-) \leq \dots \leq e^{-\alpha(t-t_0)} \mu^k V_{\sigma(t_0)}(t_0) \\ &\leq \mu^{N_0} e^{-(\alpha - \ln \mu / T_\alpha)(t-t_0)} V_{\sigma(t_0)}(t_0). \end{aligned} \tag{27}$$

According (12) and (14) we have

$$a \|x(t)\|^2 \leq V_{\sigma(t)}(t), \quad V_{\sigma(t_0)}(t_0) \leq b \|x(t_0)\|_d^2. \tag{28}$$

Let $\lambda = 1/2(\alpha - \ln \mu / T_\alpha)$, combining (27) and (28) gives rise to

$$\|x(t)\|^2 \leq \frac{1}{a} V_{\sigma(t)}(t) \leq \frac{b}{a} \mu^{N_0} e^{-(\alpha - \ln \mu / T_\alpha)(t-t_0)} \|x(t_0)\|_d^2. \tag{29}$$

Therefore $\|x(t)\| \leq \sqrt{\frac{b}{a}} \mu^{N_0/2} e^{-\lambda(t-t_0)} \|x(t_0)\|_d$, thus the proof is completed. \square

Corollary 3.1 For $\dot{d}(t)$ does not exist or is unknown, when (7) with $R_{ik} = 0$ and (8) hold, the system (5), or equivalently (6) with $u(t) \equiv w(t) \equiv 0$ is exponentially stable for any switching signal with average dwell time satisfying (10). Moreover, an estimate of state decay is given by (11), where $\lambda = 1/2(\alpha - \ln \mu / T_\alpha)$, a and b are given in (12) with $R_{ik} = 0$, and $\mu \geq 1$ satisfies (13) with $R_{ik} = R_{jn} = 0$.

Proof The proof is similar to that of Theorem 3.1. \square

Remark 3.1 PLKFs used in [3] to study switching linear system with delays can also be extended to switching fuzzy system with $Q_{ik} = Q_i, R_{ik} = R_i, X_{ik11} = X_{i11}, X_{ik12} = X_{i12}, X_{i22} = X_{i22}, Y_{ik} = Y_i$ and $T_{ik} = T_i$. But the results summarized as follows are conservative compared with the result in Theorem 3.1 based on PFLKFs.

Corollary 3.2 The system (5), or equivalently (6) with $u(t) \equiv w(t) \equiv 0$, suppose that the time-varying delay $\tau_i(t)$ satisfies $0 \leq \tau_i(t) \leq \tau_i$ and $\dot{\tau}_i(t) \leq \kappa_i (\tau_i > 0, i \in \underline{S})$. For given positive constants α and β , if there exist matrices $P_i^T = P_i > 0, Q_i^T = Q_i > 0, R_i^T = R_i > 0, X_i = \begin{bmatrix} X_{i11} & X_{i12} \\ * & X_{i22} \end{bmatrix} \geq 0$, and any matrices Y_i and T_i with appropriate dimensions such that

$$\begin{bmatrix} X_{i11} & X_{i12} & Y_i \\ * & X_{i22} & T_i \\ * & * & e^{-\alpha \tau_i} Q_i \end{bmatrix} \geq 0, \quad i \in \underline{S}, \tag{30}$$

$$\begin{bmatrix} \omega_{ik11} & \omega_{ik12} & A_{ik}^T Q_i & P_i M_{ik} & N_{i1k}^T \\ * & \omega_{i22} & A_{idk}^T Q_i & 0 & N_{i2k}^T \\ * & * & -\tau_i^{-1} Q_i & Q_i M_{ik} & 0 \\ * & * & * & -\beta I & 0 \\ * & * & * & * & -\beta^{-1} I \end{bmatrix} < 0, \quad k \in \underline{R}, i \in \underline{S} \tag{31}$$

with

$$\begin{aligned} \omega_{ik11} &= P_i A_{ik} + A_{ik}^T P_i + \alpha P_i + R_i + Y_i + Y_i^T + \tau_i X_{i11}, \\ \omega_{ik12} &= P_i A_{idk} - Y_i + T_i^T + \tau_i X_{i12}, \\ \omega_{i22} &= -T_i^T - T_i + \tau_i X_{i22} + (\kappa_i - e^{-\alpha \tau_i}) R_i. \end{aligned} \tag{32}$$

Then, system is exponentially stable for any switching signal with average dwell time satisfying (10). Moreover, an estimate of state decay is given by (11) where

$$a = \min_{i \in \underline{S}} \lambda_{\min}(P_i), \quad b = \max_{i \in \underline{S}} \lambda_{\max}(P_i) + \max_{i \in \underline{S}} \frac{\tau_i^2}{2} \lambda_{\max}(Q_i) + \max_{i \in \underline{S}} \tau_i \lambda_{\max}(R_i), \tag{33}$$

$\lambda = 1/2(\alpha - \ln \mu/T_\alpha)$, and $\mu \geq 1$ satisfies

$$P_i \leq \mu P_j, \quad Q_i \leq \mu Q_j, \quad R_i \leq \mu R_j, \quad \forall (i, j) \in \underline{S} \times \underline{S}. \tag{34}$$

Proof The proof is similar to that of Theorem 3.1, here it is omitted. □

4 H_∞ Analysis and Controller Design

In this section, we first analyze the H_∞ disturbance attenuation performance for the open loop continuous-time system. Consider the continuous-time system with time-varying delay as in (6) without control input, then we are ready to present the following H_∞ performance analysis result.

Theorem 4.1 Consider continuous-time system as in (6) with $u(t) \equiv 0$, suppose that the time-varying delay $\tau_i(t)$ satisfies $0 \leq \tau_i(t) \leq \tau_i$ and $\dot{\tau}_i \leq \kappa_i(\tau_i > 0, i \in \underline{S})$. Given positive constants α, β, γ and there exist matrices

$$P_i = P_i^T > 0, \quad Q_{ik} = Q_{ik}^T > 0, \quad R_{ik} = R_{ik}^T > 0, \\ \Lambda_{ik} = \begin{bmatrix} X_{ik11} & X_{ik12} & X_{ik13} \\ * & X_{ik22} & X_{ik23} \\ * & * & X_{ik33} \end{bmatrix} \geq 0,$$

and any matrices $Y_{ik}, T_{ik}, L_{ik}, k \in \underline{R}_i, i \in \underline{S}$, with appropriate dimensions such that

$$\begin{bmatrix} \hat{E}_{imkl} & \hat{A}_{ikl} & \hat{A}_{ilk} & \hat{E}_{ikl} & \hat{M}_{ik} & \hat{M}_{il} & \hat{N}_{ikl} \\ * & -\tau_i^{-1} Q_{il} & 0 & 0 & Q_{il} M_{ik} & 0 & 0 \\ * & * & -\tau_i^{-1} Q_{ik} & 0 & 0 & Q_{ik} M_{il} & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -\beta I & 0 & 0 \\ * & * & * & * & * & -\beta I & 0 \\ * & * & * & * & * & * & -\beta^{-1} I \end{bmatrix} < 0, \\ k \leq l, m, \in \underline{R}_i, i \in \underline{S}, \tag{35}$$

$$\begin{bmatrix} X_{ik11} & X_{ik12} & X_{ik13} & Y_{ik} \\ * & X_{ik22} & X_{ik23} & T_{ik} \\ * & * & X_{ik33} & L_{ik} \\ * & * & * & e^{-\alpha \tau_i} Q_{ik} \end{bmatrix} \geq 0, \quad k \in \underline{R}_i, i \in \underline{S}, \tag{36}$$

where

$$\hat{\Theta}_{imkl} = \begin{bmatrix} \varpi_{ik11} + \varpi_{il11} & \varpi_{ik12} + \varpi_{il12} & \varpi_{ik13} + \varpi_{il13} \\ * & \varpi_{ikm22} + \varpi_{ilm22} & \varpi_{ik23} + \varpi_{il23} \\ * & * & \varpi_{ik33} + \varpi_{il33} \end{bmatrix},$$

$$\hat{A}_{ikl} = \begin{bmatrix} A_{ik}^T Q_{il} \\ A_{idk}^T Q_{il} \\ D_{ik}^T Q_{il} \end{bmatrix}, \quad \hat{E}_{ikl} = \begin{bmatrix} E_{ik}^T & E_{il}^T \\ E_{idk}^T & E_{idl}^T \\ 0 & 0 \end{bmatrix}, \quad \hat{M}_{ik} = \begin{bmatrix} P_i M_{ik} \\ 0 \\ 0 \end{bmatrix}, \tag{37}$$

$$\hat{N}_{ikl} = \begin{bmatrix} N_{i1k}^T & N_{i1l}^T \\ N_{i2k}^T & N_{i2l}^T \\ N_{i4k}^T & N_{i4l}^T \end{bmatrix}, \quad \varpi_{ik11} = \varphi_{ik11}, \quad \varpi_{ik12} = \varphi_{ik12},$$

$$\varpi_{ikm22} = \varphi_{ikm22}, \quad \varpi_{ik33} = \tau_i X_{ik33} - \gamma^2 I,$$

$$\varpi_{ik13} = P_i D_{ik} + L_{ik}^T + \tau_i X_{ik13}, \quad \varpi_{ik23} = -L_{ik}^T + \tau_i X_{ik23},$$

and $\varphi_{ik11}, \varphi_{ik12}, \varphi_{ikm22}$ are defined in (9).

Then the system is exponentially stable and has H_∞ γ performance as in Definition 3 for any switching signal with average dwell time satisfying (10) and $\mu \geq 1$ satisfies (13).

Proof It is easy to see that the LMIs (35) and (36) imply LMIs (7) and (8), respectively; therefore, it follows from Theorem 3.1 that the system as in (6) with $u(t) \equiv 0$ is exponentially stable. Now, we show that the system have H_∞ performance by Definition 3.

Under zero initial condition, for system (6) considering PFKLFs (14) and by Lemma 1 we have (16), (17) and the following results:

$$\begin{aligned} & y^T(t)y(t) - \gamma^2 w^T(t)w(t) \\ & \leq \sum_{k=1}^{r_i} h_{ik}(t)\eta^T(t) \begin{bmatrix} E_{ik}^T \\ E_{idk}^T \end{bmatrix} \begin{bmatrix} E_{ik}^T \\ E_{idk}^T \end{bmatrix}^T \eta(t) - \gamma^2 w^T(t)w(t) \\ & = \sum_{k=1}^{r_i} h_{ik}(t)\xi^T(t) \begin{bmatrix} E_{ik}^T E_{ik} & E_{ik}^T E_{idk} & 0 \\ * & E_{idk}^T E_{idk} & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix} \xi(t), \end{aligned} \tag{38}$$

$$\begin{aligned} \dot{V}_{i1}(t) &= \sum_{k=1}^{r_i} h_{ik}(t) [x^T(t)(2P_i \bar{A}_{ik})x(t) + x^T(t)(2P_i \bar{A}_{idk})x(t - \tau_i(t)) \\ & \quad + x^T(t)(2P_i \bar{D}_{ik})w(t)], \end{aligned} \tag{39}$$

$$\tau_i \dot{x}^T(t) Q_i(t) \dot{x}(t) \leq \sum_{k=1}^{r_i} h_{ik}(t) \left\{ \xi^T(t) \begin{bmatrix} \bar{A}_{ik}^T \tau_i Q_i(t) \bar{A}_{ik} & \bar{A}_{ik}^T \tau_i Q_i(t) \bar{A}_{idk} & \bar{A}_{ik}^T \tau_i Q_i(t) \bar{D}_{ik} \\ * & \bar{A}_{idk}^T \tau_i Q_i(t) \bar{A}_{idk} & \bar{A}_{idk}^T \tau_i Q_i(t) \bar{D}_{ik} \\ * & * & \bar{D}_{ik}^T Q_i(t) \bar{D}_{ik} \end{bmatrix} \xi(t) \right\}, \tag{40}$$

where $\eta^T(t) = [x^T(t), x^T(t - \tau_i(t))]$, $\xi^T(t) = [x^T(t), x^T(t - \tau_i(t)), w^T(t)]$.

From Leibniz–Newton, we obtain

$$2\xi^T(t) \begin{bmatrix} Y_i^T(t) & T_i^T(t) & L_i^T(t) \end{bmatrix}^T \left[x(t) - x(t - \tau_i(t)) - \int_{t-\tau_i(t)}^t \dot{x}(s) ds \right] = 0, \tag{41}$$

with $Y_i(t) = \sum_{k=1}^{r_i} h_{ik}(t) Y_{ik}$, $T_i(t) = \sum_{k=1}^{r_i} h_{ik}(t) T_{ik}$ and $L_i(t) = \sum_{k=1}^{r_i} h_{ik}(t) L_{ik}$. For $A_{ik} \geq 0$ given in Theorem 4.1, we have

$$\tau_i \xi^T(t) A_{ik} \xi(t) - \int_{t-\tau_i(t)}^t \xi^T(s) A_{ik} \xi(s) ds \geq 0. \tag{42}$$

Define $\zeta^T(t, s) = [\xi^T(t), \dot{x}^T(s)]$, then combine (14), (16)–(17) and (38)–(42) yields

$$\begin{aligned} & \dot{V}_i(t) + \alpha V_i(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) \\ & \leq \sum_{m=1}^{r_i} \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} h_{im}(t) h_{ik}(t) h_{il}(t) \xi^T(t) \Omega_{imkl} \xi(t) \\ & \quad - \sum_{k=1}^{r_i} h_{ik}(t) \int_{t-\tau_i(t)}^t \zeta^T(t, s) A_{ik} \zeta(t, s) ds \\ & = \sum_{m=1}^{r_i} h_{im}(t) \left[\sum_{k=l=1}^{r_i} h_{ik}^2(t) \xi^T(t) \frac{\Omega_{imkk} + \Omega_{imkk}}{2} \xi(t) \right. \\ & \quad \left. + \sum_{k=1}^{r_i-1} \sum_{l>k}^{r_i} h_{ik}(t) h_{il}(t) \xi^T(t) (\Omega_{imkl} + \Omega_{imlk}) \xi(t) \right] \\ & \quad - \sum_{k=1}^{r_i} h_{ik}(t) \int_{t-\tau_i(t)}^t \zeta^T(t, s) A_{ik} \zeta(t, s) ds, \tag{43} \end{aligned}$$

where

$$\Omega_{imkl} = \begin{bmatrix} \mu_{ik11} + \tau_i \bar{A}_{ik}^T Q_{il} \bar{A}_{ik} + E_{ik}^T E_{ik} & \mu_{ik12} + \tau_i \bar{A}_{ik}^T Q_{il} \bar{A}_{idk} + E_{ik}^T E_{idk} & \mu_{ik13} + \tau_i \bar{A}_{ik}^T Q_{il} \bar{D}_{ik} \\ * & \mu_{imk22} + \tau_i \bar{A}_{idk}^T Q_{il} \bar{A}_{idk} + E_{idk}^T E_{idk} & \mu_{ik23} + \tau_i \bar{A}_{idk}^T Q_{il} \bar{D}_{ik} \\ * & * & \mu_{ik33} \end{bmatrix},$$

$$\begin{aligned} \mu_{ik11} &= \phi_{ik11}, & \mu_{ik12} &= \phi_{ik12}, & \mu_{imk22} &= \phi_{imk22}, & \mu_{ik23} &= \tau_i X_{ik23} - L_{ik}^T, \\ \mu_{ik33} &= \tau_i X_{ik33} - \gamma^2 I, & \mu_{ik13} &= \tau_i X_{ik13} + P_i \bar{D}_{ik} + L_{ik}^T \end{aligned}$$

with $\phi_{ik11}, \phi_{ik12}, \phi_{imk22}$ are defined in (21).

By Schur complement, conditions (35) are equivalent to the following inequalities:

$$\hat{\Upsilon}_{imkl} + \beta^{-1} \tilde{M}_{ikl} \tilde{M}_{ikl}^T + \beta \tilde{N}_{ikl} \tilde{N}_{ikl}^T < 0, \tag{44}$$

with

$$\begin{aligned} \tilde{N}_{ikl}^T &= \begin{bmatrix} \hat{N}_{ikl}^T & 0 & 0 & 0 \end{bmatrix}, & \tilde{M}_{ikl}^T &= \begin{bmatrix} \hat{M}_{ik}^T & M_{ik}^T Q_{il} & 0 & 0 \\ \hat{M}_{il}^T & 0 & M_{il}^T Q_{ik} & 0 \end{bmatrix}, \\ \hat{\Upsilon}_{imkl} &= \begin{bmatrix} \hat{\Theta}_{imkl} & \hat{A}_{ikl} & \hat{A}_{ilk} & \hat{E}_{ikl} \\ * & -\tau_i^{-1} Q_{il} & 0 & 0 \\ * & * & -\tau_i^{-1} Q_{ik} & 0 \\ * & * & * & -I \end{bmatrix}. \end{aligned}$$

By Lemma 2 we know that when (44) are satisfied the following inequalities hold:

$$\begin{aligned} &\hat{\Upsilon}_{imkl} + \tilde{M}_{ikl} F_i(t) \tilde{N}_{ikl}^T + \tilde{N}_{ikl} F_i(t) \tilde{M}_{ikl}^T \\ &= \begin{bmatrix} \hat{\Theta}_{imkl} & \hat{A}_{ikl} & \hat{A}_{ilk} & \hat{E}_{ikl} \\ * & -\tau_i^{-1} Q_{il} & 0 & 0 \\ * & * & -\tau_i^{-1} Q_{ik} & 0 \\ * & * & * & -I \end{bmatrix} < 0, \end{aligned} \tag{45}$$

where $\hat{A}_{ikl}^T = [Q_{il} \bar{A}_{ik} \quad Q_{il} \bar{A}_{idk} \quad Q_{il} \bar{D}_{ik}]$.

Then using Schur complement again, (45) are equivalent to the following condition with Ω_{imkl} given in (43):

$$\begin{aligned} &\hat{\Theta}_{imkl} + \begin{bmatrix} \hat{A}_{ikl} & \hat{A}_{ilk} \end{bmatrix} \begin{bmatrix} \tau_i Q_{il}^{-1} & 0 \\ * & \tau_i Q_{ik}^{-1} \end{bmatrix} \begin{bmatrix} \hat{A}_{ikl}^T \\ \hat{A}_{ilk}^T \end{bmatrix} + \hat{E}_{ikl} \hat{E}_{ikl}^T = \Omega_{imkl} < 0, \\ &k \leq l, m \in \underline{R}_i, i \in \underline{S}. \end{aligned} \tag{46}$$

Thus from (43) and (46) we conclude that when (35) and (36) hold the inequality as follows is satisfied:

$$\dot{V}_i(t) + \alpha V_i(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) < 0. \tag{47}$$

Let $X(t) = y^T(t)y(t) - \gamma^2 w^T(t)w(t)$. Using (13) and (14) at switching time t_i , we have (26). Let $t_0 = 0$, therefore it follows from (26) and (47) and the relation

$k = N_\sigma(0, t)$, for any $t \in [t_k, t_{k+1})$ we have

$$\begin{aligned} V_{\sigma(t_i)}(t_i) &\leq e^{-\alpha(t-t_k)} V_{\sigma(t_k)}(t_k) - \int_{t_k}^t e^{-\alpha(t-s)} X(s) ds \\ &\leq \mu^k e^{-\alpha t} V_{\sigma(0)}(0) - \int_{t_k}^t e^{-\alpha(t-s)} X(s) ds \\ &= e^{-\alpha t + N_\sigma(0,t) \ln \mu} V_{\sigma(0)}(0) - \int_{t_k}^t e^{-\alpha(t-s)} X(s) ds. \end{aligned} \tag{48}$$

Under zero initial condition, (48) gives $-\int_{t_k}^t e^{-\alpha(t-s)} X(s) ds \geq 0$. Similarly, we have

$$-\int_0^t e^{-\alpha(t-s) + N_\sigma(s,t) \ln \mu} \Upsilon(s) ds \geq 0. \tag{49}$$

Multiplying both sides of (32) by $e^{-N_\sigma(0,t) \ln \mu}$ yields

$$\int_0^t e^{-\alpha(t-s) - N_\sigma(0,s) \ln \mu} y^T(s) y(s) ds \leq \int_0^t e^{-\alpha(t-s) - N_\sigma(0,s) \ln \mu} \gamma^2 w^T(s) w(s) ds. \tag{50}$$

Noticing that $N(0, s) \leq N_0 + s/T_\alpha$, $N_0 > 0$ and $T_\alpha > \ln \mu / \alpha$, we have

$$N_\sigma(0, s) \ln \mu \leq N_0 \ln \mu + \alpha s. \tag{51}$$

Thus, it follows from (33) and (34) that

$$\int_0^t e^{-\alpha t} y^T(s) y(s) ds < \int_0^t e^{-\alpha(t-s)} \gamma^2 w^T(s) w(s) ds.$$

Then multiplying both sides of (49) by $e^{\alpha(t-s)}$ yields

$$\int_0^t y^T(s) y(s) ds \leq \int_0^t \gamma^2 w^T(s) w(s) ds, \tag{52}$$

integrating both sides of this inequality from $t = 0$ to ∞ leads to H_∞ performance by Definition 3. Thus this complete the proof. \square

Remark 4.1 When we get lower bound μ , the lower bound T_α^* by (10) can be obtained. Then, the search problem of lower bound μ can be formulated as the following GEVP problem to obtain

$$\begin{aligned} \mu_{\min} : & \text{ minimize } \mu > 0, \\ \text{s.t. } & \begin{cases} P_i = P_i^T > 0, Q_{ik} = Q_{ik}^T > 0, R_{ik} = R_{ik}^T > 0, A_{ik} \geq 0, \\ \text{inequalities: (35),(36),(13), } 1 < \mu. \end{cases} \end{aligned}$$

Meanwhile for a given $\mu > \mu_{min}$ the search problem of lower bound γ can be formulated as the following optimal problem to obtain

$$\begin{aligned} \gamma_{\min} : \quad & \text{minimize } \gamma > 0, \\ \text{s.t.} \quad & \begin{cases} P_i = P_i^T > 0, Q_{ik} = Q_{ik}^T > 0, R_{ik} = R_{ik}^T > 0, \Lambda_{ik} \geq 0, \\ \text{inequalities: (13), (35), (36).} \end{cases} \end{aligned}$$

Corollary 4.1 For $\dot{d}(t)$ does not exist or is unknown, when (35) with $R_{ik} = 0$ and (36) hold, the system (6) with $u(t) = 0$ is stable and has H_∞ performance for any switching signal with average dwell time satisfying (10).

Proof Its proof is similar to that of Theorem 4.1, it is omitted here. □

Next we consider H_∞ controller problems. Recall that the PDC technique was presented by [18, 19], the control law can be given as follows:

Switching Region $i, i \in \underline{S}$

Local Plant Rule $k, k \in \underline{R}_i$

IF z_1^i is M_{k1}^i and ... and z_e^i is M_{ke}^i THEN

$$u(t) = F_{ik}x(t), t \geq 0.$$

Then the closed-loop system (6) is rewritten as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} h_{ik}(t)h_{il}(t) [(\bar{A}_{ik} + \bar{B}_{ik}F_{il})x(t) + \bar{A}_{idk}x(t - \tau_i(t)) + \bar{D}_{ik}w(t)], \\ y(t) &= \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} h_{ik}(t)h_{il}(t) [(E_{ik} + C_{ik}F_{il})x(t) + E_{idk}x(t - \tau_i(t))], \end{aligned} \tag{53}$$

$$x(t) = \phi_i(t), \quad t \in [-\tau_i, 0].$$

Theorem 4.2 Consider the closed-loop system (53) with time-varying delays $\tau_i(t)$ satisfies $0 < \tau_i(t) \leq \tau_i$ and $\dot{\tau}_i(t) \leq \kappa_i(\tau_i > 0, i \in \underline{S})$. For given positive constants α and γ , if there exist scalar $\beta > 0$ and matrices

$$\begin{aligned} \tilde{P}_i = \tilde{P}_i^T > 0, \quad \tilde{Q}_{ik} = \tilde{Q}_{ik}^T > 0, \quad \tilde{R}_{ik} = \tilde{R}_{ik}^T > 0, \\ \tilde{X}_{ik} = \begin{bmatrix} \tilde{X}_{ik11} & \tilde{X}_{ik12} & \tilde{X}_{ik13} \\ * & \tilde{X}_{ik22} & \tilde{X}_{ik23} \\ * & * & \tilde{X}_{ik33} \end{bmatrix} \geq 0, \end{aligned}$$

and any matrices $\check{Y}_{ik}, \check{T}_{ik}, \check{L}_{ik}$ with appropriate dimensions such that

$$\mathcal{E}_{imnkl} = \begin{bmatrix} \Sigma_{inkl} & \check{K}_{ikl} & \check{K}_{ilk} & \check{E}_{ikl} & \check{M}_{ik} & \check{M}_{il} & \check{N}_{ikl} \\ * & -\tau_i^{-1} \check{Q}_{im} & 0 & 0 & M_{ik} & 0 & 0 \\ * & * & -\tau_i^{-1} \check{Q}_{im} & 0 & 0 & M_{il} & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -\beta I & 0 & 0 \\ * & * & * & * & * & -\beta I & 0 \\ * & * & * & * & * & * & -\beta^{-1} I \end{bmatrix} < 0,$$

$$k \leq l, m, n \in \underline{R}_i, i \in \underline{S}, \tag{54}$$

$$\begin{bmatrix} \check{X}_{ik11} & \check{X}_{ik12} & \check{X}_{ik13} & \check{Y}_{ik} \\ * & \check{X}_{ik22} & \check{X}_{ik23} & \check{T}_{ik} \\ * & * & \check{X}_{ik33} & \check{L}_{ik} \\ * & * & * & e^{-\alpha\tau_i} \check{Q}_{ik} \end{bmatrix} \geq 0, \quad k \in \underline{R}_i, i \in \underline{S}, \tag{55}$$

where

$$\Sigma_{inkl} = \begin{bmatrix} \pi_{ikl11} + \pi_{ilk11} & \pi_{ik12} + \pi_{il12} & \pi_{ik13} + \pi_{il13} \\ * & \pi_{ink22} + \pi_{inl22} & \pi_{ik23} + \pi_{il23} \\ * & * & \pi_{ik33} + \pi_{il33} \end{bmatrix},$$

$$\check{K}_{ikl} = \begin{bmatrix} \check{P}_i A_{ik}^T + K_{il}^T B_{ik}^T \\ \check{P}_i A_{idk}^T \\ D_{ik}^T \end{bmatrix}, \quad \check{E}_{ikl} = \begin{bmatrix} \check{P}_i E_{ik}^T + K_{il}^T C_{ik}^T & \check{P}_i E_{il}^T + K_{ik}^T C_{il}^T \\ \check{P}_i E_{idk}^T & \check{P}_i E_{idl}^T \\ 0 & 0 \end{bmatrix},$$

$$\check{N}_{ikl} = \begin{bmatrix} \check{P}_i N_{i1k}^T + K_{il}^T N_{i3k}^T & \check{P}_i N_{i1l}^T + K_{ik}^T N_{i3l}^T \\ \check{P}_i N_{i2k}^T & \check{P}_i N_{i2l}^T \\ N_{i4k}^T & N_{i4l}^T \end{bmatrix}, \quad \check{M}_{ik}^T = [M_{ik}^T \quad 0 \quad 0],$$

$$\pi_{ikl11} = A_{ik} \check{P}_i + \check{P}_i A_{ik}^T + B_{ik} K_{il} + (B_{ik} K_{il})^T + \alpha \check{P}_i + \check{R}_{ik} + \check{Y}_{ik}^T + \check{Y}_{ik} + \tau_i \check{X}_{ik11},$$

$$\pi_{ik12} = A_{idk} \check{P}_i - \check{Y}_{ik}^T + \check{T}_{ik}^T + \tau_i \check{X}_{ik12},$$

$$\pi_{ink22} = -\check{T}_{ik}^T - \check{T}_{ik} + (\kappa_i - e^{-\alpha\tau_i}) \check{R}_{in} + \tau_i \check{X}_{ik22},$$

$$\pi_{ik13} = D_{ik} + \check{L}_{ik}^T + \tau_i \check{X}_{ik13}, \quad \pi_{ik23} = -\check{L}_{ik}^T + \tau_i \check{X}_{ik23},$$

$$\pi_{ik33} = -\gamma^2 I + \tau_i \check{X}_{ik33}.$$

Then the system (53) is exponentially stable with γ -disturbance attenuation H_∞ performance under the control law for any switching signal with average dwell satisfying

(10), and $\mu \geq 1$ satisfies

$$\tilde{P}_i \leq \mu \tilde{P}_j, \quad \tilde{Q}_{ik} \leq \mu \tilde{Q}_{jn}, \quad \tilde{R}_{ik} \leq \mu \tilde{R}_{jn}, \quad \forall (i, j, k, n) \in \underline{S} \times \underline{S} \times \underline{R}_i \times \underline{R}_j. \quad (56)$$

Moreover, the feedback gain is given by $F_{ik} = K_{ik} \tilde{P}_i^{-1}, k \in \underline{R}_i, i \in \underline{S}$.

Proof Consider the PFLKFs (14), one has (16), (17), and

$$\dot{V}_{i2}(t) \leq -\alpha V_{i2}(t) + \tau_i \dot{x}(t) Q_i(t) \dot{x}(t) - \int_{t-\tau_i(t)}^t \dot{x}^T(s) e^{-\alpha \tau_i} Q_i(s) \dot{x}(s) ds. \quad (57)$$

Let $\vartheta^T(t) = [x^T(t), x^T(t - \tau_i(t)), w^T(t)]$, $\Delta_{ikl}^T = [\bar{A}_{ik} + \bar{B}_{ik} F_{il} \quad \bar{A}_{idk} \quad \bar{D}_{ik}]$, $\tilde{\Lambda}_{ikl}^T = [E_{ik} + C_{idk} F_{il} \quad E_{idk} \quad 0]$, and by Lemma 1 and from system (51) one has

$$\begin{aligned} & \tau_i \dot{x}(t) Q_i(t) \dot{x}(t) \\ &= \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \sum_{n=1}^{r_i} h_{ik}(t) h_{il}(t) h_{im}(t) h_{in}(t) [v^T(t) \Delta_{ikl} \tau_i Q_i(t) \Delta_{imn}^T v(t)] \\ &\leq \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} h_{ik}(t) h_{il}(t) [v^T(t) \Delta_{ikl} \tau_i Q_i(t) \Delta_{ikl}^T v(t)], \end{aligned} \quad (58)$$

$$\begin{aligned} y^T(t) y(t) &= \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} \sum_{m=1}^{r_i} \sum_{n=1}^{r_i} h_{ik}(t) h_{il}(t) h_{im}(t) h_{in}(t) [v^T(t) \tilde{\Lambda}_{ikl} \tilde{\Lambda}_{imn}^T v(t)] \\ &\leq \sum_{k=1}^{r_i} \sum_{l=1}^{r_i} h_{ik}(t) h_{il}(t) [v^T(t) \tilde{\Lambda}_{ikl} \tilde{\Lambda}_{ikl}^T v(t)]. \end{aligned} \quad (59)$$

Then the rest of the proof is similar to the proof of H_∞ performance analysis in Theorem 4.1, here it is omitted.

Based on the result in Theorem 4.1, we get (36) and the following inequalities which correspond to (54) with $m, n \in \underline{R}_i, i \in \underline{S}$:

$$\left[\begin{array}{ccccccc} \prod_{i,n,k,l} & \check{K}_{imkl} & \check{K}_{imlk} & \check{E}_{ikl} & \check{M}_{ik} & \check{M}_{il} & \check{N}_{ikl} \\ * & -\tau_i^{-1} Q_{im} & 0 & 0 & Q_{im} M_{ik} & 0 & 0 \\ * & * & -\tau_i^{-1} Q_{im} & 0 & 0 & Q_{im} M_{il} & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -\beta I & 0 & 0 \\ * & * & * & * & * & -\beta I & 0 \\ * & * & * & * & * & * & -\beta^{-1} I \end{array} \right] < 0,$$

$$k \leq l \in \underline{R}_i, i \in \underline{S}, \quad (60)$$

where

$$\prod_{inkl} = \begin{bmatrix} \pi_{ikl11} + \pi_{ilk11} & \pi_{ik12} + \pi_{il12} & \pi_{ik13} + \pi_{il13} \\ * & \pi_{ink22} + \pi_{inl22} & \pi_{ik23} + \pi_{il23} \\ * & * & \pi_{ik33} + \pi_{il33} \end{bmatrix},$$

$$\check{K}_{imkl} = \begin{bmatrix} (A_{ik} + B_{ik}F_{il})^T Q_{im} \\ A_{idk}^T Q_{im} \\ D_{ik}^T Q_{im} \end{bmatrix},$$

$$\check{E}_{ikl} = \begin{bmatrix} (E_{ik} + C_{ik}F_{il})^T & (E_{il} + C_{il}F_{ik})^T \\ E_{idk}^T & E_{idl}^T \\ 0 & 0 \end{bmatrix},$$

$$\check{N}_{ikl} = \begin{bmatrix} (N_{i1k} + N_{i3k}F_{il})^T & (N_{i1l} + N_{i3l}F_{ik})^T \\ N_{i2k}^T & N_{i2l}^T \\ N_{i2k}^T & N_{i2l}^T \end{bmatrix}, \quad \check{M}_{ik}^T = \begin{bmatrix} M_{ik}^T P_i & 0 & 0 \end{bmatrix},$$

$$\pi_{ikl11} = P_i(A_{ik} + B_{ik}F_{il}) + (A_{ik} + B_{ik}F_{il})^T P_i + \alpha P_i + R_{ik} + Y_{ik}^T + Y_{ik} + \tau_i X_{ik11},$$

$$\pi_{ik12} = P_i A_{idk}^T - Y_{ik}^T + L_{ik}^T + \tau_i X_{ik12},$$

$$\pi_{ink22} = -T_{ik}^T - T_{ik} + (\kappa_i - e^{-\alpha\tau_i})R_{in} + \tau_i X_{ik22},$$

$$\pi_{ik13} = P_i D_{ik} + L_{ik}^T + \tau_i X_{ik13}, \quad \pi_{ik23} = -L_{ik}^T + \tau_i X_{ik23},$$

$$\pi_{ik33} = -\gamma^2 I + \tau_i X_{ik33}.$$

Then pre- and post-multiplying (60) by $\text{diag}\{P_i^{-1}, P_i^{-1}, I, Q_{im}^{-1}, Q_{im}^{-1}, I, I, I, I, I\}$ and its transposed matrix, (36) by $\text{diag}\{P_i^{-1}, P_i^{-1}, I, Q_{ik}^{-1}\}$, respectively, and applying the change of variable such that $\tilde{P}_i = P_i^{-1}$, $\tilde{Q}_{ik} = Q_{ik}^{-1}$, $\tilde{R}_{ik} = \tilde{P}_i R_{ik} \tilde{P}_i$, $\tilde{Y}_{ik} = \tilde{P}_i Y_{ik} \tilde{P}_i$, $\tilde{T}_{ik} = \tilde{P}_i T_{ik} \tilde{P}_i$, $\tilde{L}_{ik} = L_{ik} \tilde{P}_i$, $K_{iv} = F_{iv} \tilde{P}_i$, $\tilde{X}_{ik11} = \tilde{P}_i X_{ik11} \tilde{P}_i$, $\tilde{X}_{ik12} = \tilde{P}_i X_{ik12} \tilde{P}_i$, $\tilde{X}_{ik13} = \tilde{P}_i X_{ik13}$, $\tilde{X}_{ik22} = \tilde{P}_i X_{ik22} \tilde{P}_i$, $\tilde{X}_{ik23} = \tilde{P}_i X_{ik23}$, $\tilde{X}_{ik33} = X_{ik33}$, we get inequalities (13), (54), and (55); this completes the proof. \square

5 Simulation examples

In this section, two simulation examples are given to illustrate the effectiveness of the proposed approach.

Example 1 (H_∞ performance analysis) Let $x^T(t) = [x_1^T(t), x_2^T(t)]$, consider the following two uncertain switching fuzzy time-varying delays system (5) with $u(t) \equiv 0$.

Switching Region 1:

$$A_{11} = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.16 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix},$$

$$A_{1d1} = A_{1d2} = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad D_{11} = D_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$N_{111} = N_{112} = N_{121} = N_{122} = 0, \quad N_{141} = N_{142} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$E_{11} = E_{12} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \quad E_{1d1} = E_{1d2} = 0.$$

Switching Region 2:

$$A_{21} = \begin{bmatrix} -0.5 & 0.1 \\ 0.01 & -a \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -b \end{bmatrix},$$

$$A_{2d1} = A_{2d2} = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad D_{21} = D_{22} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$N_{211} = N_{212} = N_{221} = N_{222} = 0, \quad N_{241} = N_{242} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$E_{21} = E_{22} = \begin{bmatrix} 0 & 0 \\ 0.2 & 1 \end{bmatrix}, \quad E_{2d1} = E_{2d2} = 0.$$

Membership functions: $h_{i1}(t) = \sin^2(t)$, $h_{i2} = 1 - \sin^2(t)$, $i = 1, 2$.

First of all we will compare the feasible regions of the system $w(t) \equiv 0$ for the results in Theorem 4.1 (PFLKFs) and the results in Corollary 4.1 (PLKFs) for given $\tau_i = 0.29$, $\kappa_i = 3$, $\beta = 0.8$, $\alpha = 0.63$, $\mu = 1.60$ and $\gamma = 1.7321$ by changing a and b , by changing a and b , where a takes value between -0.2 and 0.2 by step of 0.05 and b takes value between 0.3 and 0.8 by step of 0.05 . The simulation in Fig. 1 show the result by Theorem 4.1 covers bigger regions than the one by Corollary 4.1, which means conditions in Theorem 4.1 is more relaxed.

Next, using PLKFs and PFLKFs, respectively, the achievable minimum H_∞ attenuation level γ_{min} for the robust H_∞ stability analysis can be obtained and is summarized in Table 1 for different τ_i , κ_i , μ and α . From it we can see that the result we obtain by PFLKFs is smaller than PLKFs.

Fig. 1 Feasible area for PLKFs and PFLKFs

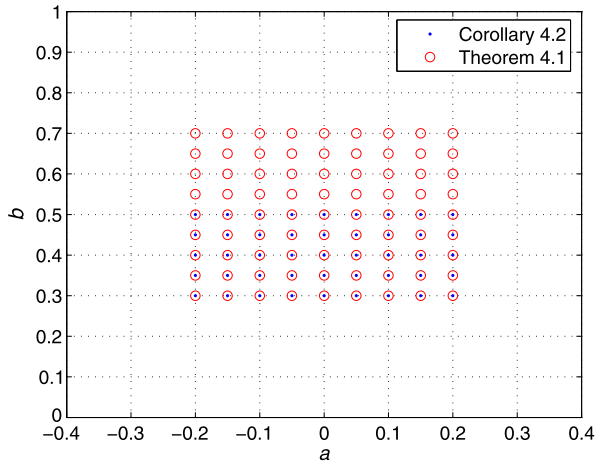


Table 1 γ_{\min} obtained by different approach

τ_i	0.29	0.29	0.29	0.1	0.07
κ_i	3	3	3	1.0374	0.7241
μ	1.5	1.5	1.8	1.5	1.5
α	0.05	0.65	0.65	0.65	0.65
γ_{\min} by PLKFs	0.9820	1.6946	1.6940	1.3068	1.2969
γ_{\min} by PFLKFs	0.9814	1.6486	1.6464	1.3028	1.2946

From Fig. 1 and Table 1, it can be seen that the PFLKFs based approach produces less conservative results than the PLKFs (widely used in [14]) based approach.

Example 2 (R/C Hovercraft [17]) The controlled object of hovercraft type vehicle (HTV) dynamics is represented as

$$\ddot{y}_c(t) = \frac{\sin \theta(t)}{M} f_1(t), \tag{61}$$

$$\ddot{\theta}(t) = \frac{l \sin \phi(t)}{I} f_2(t), \tag{62}$$

where $f_1(t) = f_R(t) + f_L(t)$, $f_2(t) = f_R(t) - f_L(t)$; θ is the angle of the vehicle; l is the distance between the gravity position and fans; ϕ is the angle between the gravity position and fans; f_R is the force generate by right side fan; f_L is the force generated by left side fan; M is the mass of the hovercraft; I is the inertia of the hovercraft. In this simulation, $\phi = \pi/4$, $M = 0.1$. The control purpose is $\lim_{t \rightarrow \infty} y_c(t) = 0$ and $\lim_{t \rightarrow \infty} \theta(t) = 0$ by manipulating $f_1(t)$ and $f_2(t)$. Due to modeling error or external disturbance, the practical system are always subject to various kinds of uncertainties and time-varying delays are universal by various of factors. To make a switching fuzzy model for (61) and (62), assume that $\theta(t) \in [-179.427^\circ \ 179.427^\circ]$. Thus we can construct the following uncertain switching fuzzy model of system (61) and (62)

with time-varying delays. The parameter matrices are given as follows:

$$\begin{aligned}
 A_{ik} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, & B_{lk} &= \begin{bmatrix} \frac{1}{M}a_{ki} & 0 \\ 0 & 0 \\ 0 & \frac{l \sin \phi}{T} \\ 0 & 0 \end{bmatrix}, \\
 B_{2k} &= \begin{bmatrix} 0 & 0 & 0 & \frac{C}{M}a_{2i} \\ 0 & 0 & 0 & 0 \\ \frac{l \sin \phi}{T} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, & A_{idk} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
 E_{ik} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, & D_{ik} &= \begin{bmatrix} 0 & 0 \\ 0.8 & 0 \\ 0 & 0 \\ 0 & 0.8 \end{bmatrix}, & N_{i4} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
 N_{i1} = N_{i2} = N_{i3} = 0, & M_{ik} &= \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.8 \end{bmatrix},
 \end{aligned}$$

$$C_{ik} = E_{idk} = 0, \quad i = 1, 2, 3, \quad k = 1, 2, \quad l = 1, 3.$$

Here,

$$x^T(t) = [\dot{y}_c^T(t) \quad y_c^T(t) \quad \dot{\theta}^T(t) \quad \theta^T(t)], \quad u^T(t) = [f_1^T(t) \quad f_2^T(t)]$$

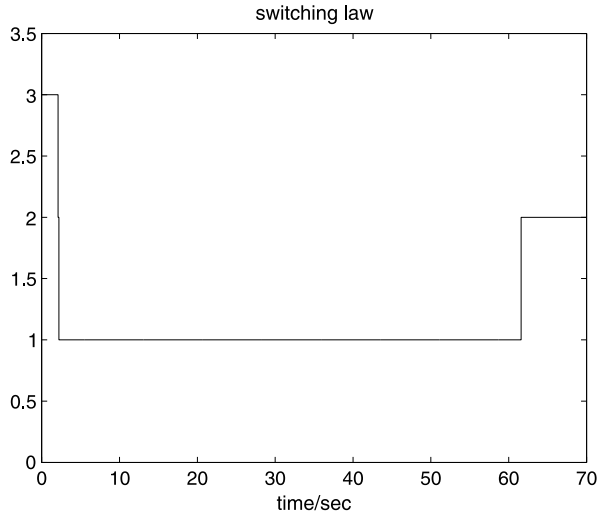
and $w^T(t) = [e^{-0.8t}, e^{-0.1t}]$ is external disturbance with $F_i(t) = \sin(t)$.

Its the membership functions are given as:

$$\begin{aligned}
 h_{11}(\theta(t)) &= \frac{\sin \theta(t) - a_{12}}{a_{11} - a_{12}}, & h_{12}(\theta(t)) &= \frac{a_{11} - \sin \theta(t)}{a_{11} - a_{12}}, \\
 h_{21}(\theta(t)) &= \frac{\frac{\sin \theta(t)}{\theta(t)} - a_{22}}{a_{21} - a_{22}}, & h_{22}(\theta(t)) &= \frac{a_{21} - \frac{\sin \theta(t)}{\theta(t)}}{a_{21} - a_{22}}, \\
 h_{31}(\theta(t)) &= \frac{\sin \theta(t) - a_{32}}{a_{31} - a_{32}}, & h_{32}(\theta(t)) &= \frac{a_{31} - \sin \theta(t)}{a_{31} - a_{32}},
 \end{aligned}$$

with $a_{11} = 1, a_{12} = \sin(179.427^\circ) \simeq 0.01, a_{21} = 1, a_{22} = \sin(d)/d, a_{31} = -1$ and $a_{32} = \sin(-179.427^\circ) \simeq -0.01$.

For $\gamma = 2, \alpha = 0.1, \beta = 0.8, \tau_i = 2, \kappa_i = 0.001, \mu = 4$ and $d = \pi/50, C = 0.5$. Thus $T_\alpha^* = \ln \mu / \alpha = 13.863$, the switching law in Fig. 2 (here, ‘1’, ‘2’ and ‘3’ represent the first, second, and third switching region, respectively) shows that average

Fig. 2 Switching signal

dwell time $T_\alpha = 17.5 > T_\alpha^*$ does satisfy (10). Under the switching law, using Theorem 4.2 we can get the feedback gains as

$$\begin{aligned}
 F_{11} &= \begin{bmatrix} -9.4105 & -0.2980 & 0.0060 & 0 \\ -0.1298 & 0.1646 & -699.0 & -81.5619 \\ 0.0017 & -5.2260 & 0 & 0 \\ 0 & -0.1131 & 0 & 0 \end{bmatrix}, \\
 F_{12} &= \begin{bmatrix} -9.4107 & -0.2980 & 0.0060 & 0 \\ -0.1298 & 0.1646 & -699.5861 & -81.5619 \\ 0.0017 & -5.2260 & 0 & 0 \\ 0 & -0.1131 & 0 & 0 \end{bmatrix}, \\
 F_{21} &= \begin{bmatrix} 0 & 0.0003 & -383.9211 & -0.1444 \\ 0 & 0 & 0 & -0.0007 \\ 5.4981 & 0 & 0 & 0 \\ -7.1658 & -0.0457 & 0 & 0 \end{bmatrix}, \\
 F_{22} &= \begin{bmatrix} 0 & 0.0003 & -383.9211 & -0.1444 \\ 0 & 0 & 0 & -0.0007 \\ 5.4980 & 0 & 0 & 0 \\ -7.1656 & -0.0459 & 0 & 0 \end{bmatrix}, \\
 F_{31} &= \begin{bmatrix} 3.2300 & 0.1108 & 0.0032 & 0 \\ 0.0505 & -0.0613 & -239.9262 & -28.4264 \\ 0.0013 & -1.7246 & 0 & 0 \\ 0 & -0.0412 & 0 & 0 \end{bmatrix},
 \end{aligned}$$

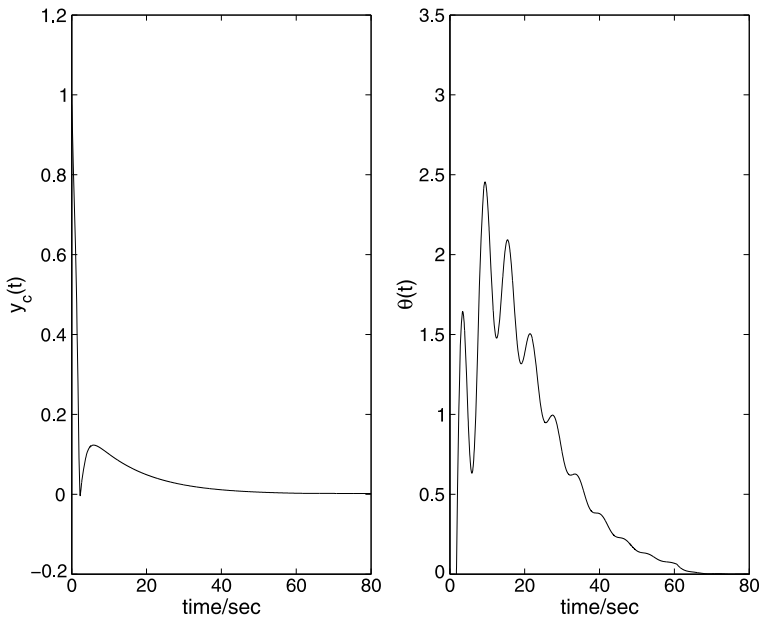


Fig. 3 Output $y(t)$

$$F_{32} = \begin{bmatrix} 3.1844 & 0.0627 & 0.0003 & 0 \\ 0.0236 & -0.0301 & -240.9449 & -28.4572 \\ -0.0002 & -1.7230 & 0 & 0 \\ 0 & -0.0411 & 0 & 0 \end{bmatrix}.$$

Figure 3 shows the output $y_1(t) = y_c(t)$, $y_2(t) = \theta(t)$ responses of the closed-loop system in the presence of disturbances. It can be observed that the controller proposed in this paper based on PFLKs not only stabilizes the system, but also effectively attenuates the disturbances.

6 Conclusions

In this paper, a H_∞ controller design method is developed for uncertain switching fuzzy systems with time-varying delays based on PFLKFs. It is shown that the stability and control synthesis results based on the PFLKFs are less conservative than those based on the PLKFs. A numerical example and a real plant are presented to demonstrate the advantages of the proposed approach. As for switched systems, if there exist some unstable subsystems the systems may be still be stable. However, the controllers designed in this paper and others all require the controlled subsystems are stable, not allow to have unstable ones. Thus, the controlled systems which is allowed to have unstable systems will be our future interesting work. An asynchronous controller may be a solution.

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