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Relaxed H_∞ Controller Design for Continuous Markov Jump System with Incomplete Transition Probabilities

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Abstract This paper studies the H_∞ state feedback control of continuous-time Markov jump linear systems (MJLSs) with incomplete transition probabilities (TPs) which are allowed to be known, uncertain with known lower and upper bounds, and completely unknown. Combining the TP property and a matrix transformation technique, a new method for the H_∞ controller synthesis is proposed in terms of linear matrix inequalities (LMIs). The dominant feature of the proposed method is that two sets of slack variables without coupling relationship are introduced. It is shown that the proposed method is less conservative than the existing result. The effectiveness of the proposed method is further illustrated by numerical examples.

Keywords Markov jump linear system · H_∞ control · Parameter-dependent Lyapunov function · Linear matrix inequality

1 Introduction

In recent decades, much attraction has been drawn to control systems that must meet performance requirements and maintain acceptable behavior even in the presence

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of abrupt changes in their dynamics due to random component failures or repairs, abrupt environmental disturbances, changes in subsystem interconnections (an interconnected system is a class of systems consisting of similar units which directly interact with their nearest neighbors [7]), abrupt changes in the operating point of a nonlinear plant, etc. [23]. If these abrupt changes have only a small influence on system performance, classical sensitivity analysis methods may be enough. Otherwise, a stochastic model that gives a quantitative indication of the relative likelihood of various possible scenarios would be preferable [5]. Among the different ways to model the abrupt changes (such as impulse systems [3], Poisson processes [1], multi-modes [6], and so on), one of increasing interest is that of Markov jump linear systems (MJLSs). The literature of MJLSs on, e.g., to name just a few, stability and stabilization, H_∞ control and H_2 control, sampled-data control, and optimal estimation is extensive; some examples are [2, 4, 8, 10–22, 24–43].

It is worth noting that, among the references mentioned above, the transition probabilities (TPs) are assumed to be known [3–6, 8, 10–18, 20–25, 28–30, 32–36, 39]. Actually, this assumption may be restrictive in practice. The reason is that it is difficult or costly to measure Markov modes and TPs online exactly [2, 19, 37]. Taking networked control systems (NCSs) as an example, Markov chains are always utilized to model random network-induced packet dropout or time delay. During different running periods of the networks, the variation of the packet dropout or the time delay would be vague and random, with the result that all or part of the elements in the desired TP matrix may be inaccurate. With the help of robust control methodologies, the inaccurate TPs are presented by norm-bounded or polytopic-type uncertainties [19, 37]. Unlike the uncertainty method, a new approach is proposed in [40–43] in which the TPs are allowed to be known or unknown. By making full use of the boundary information of unknown TPs, the results proposed in [40, 43] are further improved by the work in [26, 27].

On the other hand, for results concerning H_∞ state feedback control, much attention has been devoted to discrete MJLSs, and the conditions for controller synthesis, based on the technique developed in [9], exhibit a kind of decoupling between the Lyapunov and the system matrices. Unfortunately, there is no parallel result for the continuous case. Though a new robust H_2 controller design method has been proposed in [12] using a parameter-dependent Lyapunov function approach, the method cannot be directly employed to solve the H_∞ state feedback control problem. Additionally, the TPs in [12] are still completely known.

Motivated by the above observations, we further consider the H_∞ state feedback control of continuous MJLSs with incomplete TPs for the cases of TPs that are known, uncertain with known bounds, and completely unknown. Employing the property of continuous TPs and a matrix transformation technique, a new method for H_∞ controller design is proposed in the framework of linear matrix inequalities (LMIs). The method has the following three new features. (1) Two sets of slack variables without a coupling relationship are introduced. Due to these variables, the proposed method can be readily extended to deal with the case of system matrices with norm-bounded or polytopic uncertainties. (2) A parameter-dependent Lyapunov function approach is employed to handle, in strict LMI form, TP matrices ranging from completely unknown to completely known. (3) It is demonstrated theoretically that the proposed method is less conservative than the existing results. Numerical examples are given to illustrate the effectiveness of the proposed design method.

The outline of this paper is as follows. The considered systems and some useful lemmas are stated in Sect. 2. In Sect. 3, the proposed approach is given, and the main result is established in LMI formulation. To show the effectiveness of the proposed method, two numerical examples are developed in Sect. 4, and the concluding remarks are presented in Sect. 5.

Notation Throughout this paper, M^T represents the transpose of matrix M. The notation $X \leq Y$ ($X < Y$), where X and Y are symmetric matrices, means that $X - Y$ is negative semidefinite (negative definite). I and 0 represent the identity matrix and zero matrix, respectively. $\mathcal{L}_2$ denotes the space of square integrable vector functions of a given dimension over $[0, \infty)$, with norm $\|x\|_2^2 = \{\int_0^\infty E\{x(t)^T x(t)\, dt\}\} < \infty$. $*$ denotes the entries of matrices implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate dimensions. Finally, the symbol $\mathrm{He}(X)$ is used to represent $(X + X^T)$.

2 Preliminaries and Problem Statement

Consider continuous-time MJLSs

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B_1(r(t))u(t) + B_2(r(t))w(t), \\ z(t) = C_1(r(t))x(t) + D_1(r(t))u(t) + D_2(r(t))w(t), \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state variable, $w(t) \in R^{n_w}$ is the disturbance input, which is assumed to be an arbitrary signal in $\mathcal{L}_2$, $z(t) \in R^p$ is the regulated output, and $r(t)$ is a time-homogeneous Markov process with right continuous trajectories and takes values on the finite set $\mathcal{I} = \{1, 2, \dots, N\}$ with stationary TPs

$$\Pr\big\{r(t+dt) = j \mid r(t) = i\big\} = \begin{cases} \pi_{ij} dt + o(dt), & i \neq j, \\ 1 + \pi_{ii} d_t + o(dt), & i = j, \end{cases}$$

where $dt > 0$, $\lim_{dt \to 0} \frac{o(dt)}{dt} = 0$. π_{ij} is the jump rate from mode i to mode j that satisfies the following relations:

$$\begin{cases} \pi_{ij} \geq 0, & \forall i \neq j \in \mathcal{I}, \\ \sum_{j=1, i\neq j}^{N} \pi_{ij} = -\pi_{ii}, & i = (1, \dots, N). \end{cases} \tag{2}$$

Unlike the existing results, the information for TPs of the jumping process $\{r(t), t \geq 0\}$ in this paper are assumed to be incomplete; namely, they are allowed to be known, uncertain with known lower and upper bounds, and completely unknown. For instance, for system (1) with four operation modes, the TP matrix may be expressed as:

$$\begin{bmatrix} \pi_{11} & ? & \pi_{13} & ? \\ ? & \pi_{22} & ? & \pi_{24} \\ \alpha & ? & \pi_{33} & ? \\ ? & ? & \beta & ? \end{bmatrix}, \tag{3}$$

where "?" represents the inaccessible elements, α and β are uncertain with known lower and upper bounds ($\underline{\alpha} \le \alpha \le \bar{\alpha}$ and $\underline{\beta} \le \beta \le \bar{\beta}$), and π_{ij} is completely known.

Therefore, the following three sets can be adopted to describe all possible cases to which the TPs may belong:

$$\begin{cases} \mathcal{R}^i_{\mathcal{K}} \triangleq \{j : \pi_{ij} \text{ is known}\}, \\ \mathcal{R}^i_{\mathcal{UK}1} \triangleq \{j : \text{lower and upper bounds of } \pi_{ij} \text{ are known}\}, \\ \mathcal{R}^i_{\mathcal{UK}2} \triangleq \{j : \text{there is no information available for } \pi_{ij}\}. \end{cases} \tag{4}$$

Although some elements are uncertain, their boundary information can be utilized. To make full use of the information of known and uncertain TPs, the above sets are further classified as follows:

$$\begin{cases} \mathcal{I}^i_k \triangleq \mathcal{R}^k_{\mathcal{K}} \cup \mathcal{R}^k_{\mathcal{UK}1}, \\ \mathcal{I}^i_{uk} \triangleq \mathcal{R}^k_{\mathcal{UK}2}. \end{cases} \tag{5}$$

Moreover, we employ $\mathcal{L}^i_k(\mathcal{L}^i_{uk}) \in \mathbb{N}^+$ to represent the index set of the mth known (unknown) element in the ith row of matrix π.

$$\mathcal{L}^i_k \triangleq \{m \mid m \in \mathcal{I}^i_k \text{ and } m \ne i\}, \qquad \mathcal{L}^i_{uk} \triangleq \{m \mid m \in \mathcal{I}^i_{uk} \text{ and } m \ne i\}.$$

Our aim is to design a state feedback controller

$$u(t) = K\big(r(t)\big)x(t) \tag{6}$$

such that the resulting closed-loop system

$$\begin{cases} \dot{x}(t) = (A(r(t)) + B_1(r(t))K(r(t)))x(t) + B_2(r(t))w(t), \\ z(t) = (C_1(r(t)) + D_1(r(t))K(r(t)))x(t) + D_2(r(t))w(t), \end{cases} \tag{7}$$

is stochastic stable (SS) and meets the prescribed H_∞ performance index.

The set $\mathcal{I}$ comprises the operation modes of system (1), and for each possible value of $r(t) = i$, the system matrices are abbreviated as

$$\begin{aligned} &A_i = A\big(r(t) = i\big), \qquad B_{1i} = B_1\big(r(t) = i\big), \qquad B_{2i} = B_2\big(r(t) = i\big), \\ &C_i = C_1\big(r(t) = i\big), \qquad D_{1i} = D_1\big(r(t) = i\big), \qquad D_{2i} = D_2\big(r(t) = i\big), \\ &K_i = K\big(r(t)\big). \end{aligned}$$

Some useful definitions and lemmas are presented below.

Definition 1 System (1) is said to be SS if the following holds:

$$E\left\{\int_0^\infty \|x(t)\|^2 dt \mid x_0, r_0\right\} < \infty \tag{8}$$

for any initial condition x_0 and initial distribution r_0.

Definition 2 Given a positive scalar γ, system (7) is said to be SS and has an H_∞ noise attenuation performance index γ if it is SS and, under zero initial state, $\|z\|_2 \le \gamma\|w\|_2$ holds for all nonzero $w(t) \in L_2[0,\infty)$.

Lemma 1 [42] *Given a prescribed scalar γ, the nominal Markovian jump system (1) with $u(t) \equiv 0$ is SS and has H_∞ performance index γ, if and only if there exist matrices $P_i > 0$ such that the following coupled linear matrix inequalities hold*:

$$\begin{bmatrix} \mathrm{He}(P_iA_i) + \sum_{j=1}^{N} \pi_{ij}P_j & P_iB_{2i} & C_i^T \\ B_{2i}^T P_i & -\gamma^2 I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix} < 0. \tag{9}$$

Lemma 2 [42] *Consider system (1) with partly unknown TPs. There exists a controller (5) such that the resulting closed-loop system (6) is SS and has a prescribed H_∞ performance index γ if there exist matrices $X_i > 0$, Y_i such that*

$$\Lambda_i < 0, \tag{10}$$

$$\Omega_i + X_j \ge 0 \quad \left(j = i \in \mathcal{I}_{uk}^i\right), \tag{11}$$

$$\begin{bmatrix} \Omega_i & X_i \\ * & -X_j \end{bmatrix} \le 0 \quad \left(j \ne i,\ j \in \mathcal{I}_{uk}^i\right), \tag{12}$$

where

$$\Lambda_i = \begin{cases} \begin{bmatrix} (1 + \sum_{j\in\mathcal{I}_k^i} \pi_{ij})\Omega_i + \pi_{ii}X_i & B_{2i} & (C_{1i}X_i)^T & \mathcal{S}_k^i \\ * & -\gamma^2 I & D_{2i}^T & 0 \\ * & * & -I & 0 \\ * & * & * & -\mathcal{X}_k^i \end{bmatrix} & (i \in \mathcal{I}_k^i) \\ \begin{bmatrix} (1 + \sum_{j\ne i, j\in\mathcal{I}_k^i} \pi_{ij})\Omega_i & B_{2i} & (C_{1i}X_i)^T & \mathcal{S}_k^i \\ * & -\gamma^2 I & D_{2i}^T & 0 \\ * & * & -I & 0 \\ * & * & * & -\mathcal{X}_k^i \end{bmatrix} & (i \in \mathcal{I}_{uk}^i), \end{cases} \tag{13}$$

$$\begin{aligned} \Omega_i &= \mathrm{He}(A_iX_i + B_iY_i), \\ \mathcal{S}_k^i &= \begin{bmatrix} \sqrt{\pi_{i1}}X_i & \cdots & \sqrt{\pi_{i(i-1)}}X_i & \sqrt{\pi_{i(i+1)}}X_i & \cdots & \sqrt{\pi_{iN}}X_i \end{bmatrix}, \\ \mathcal{X}_k^i &= \mathrm{diag}\begin{bmatrix} X_1 & \cdots & X_{i-1} & X_{i+1} & \cdots & X_N \end{bmatrix}. \end{aligned}$$

Lemma 3 *If the following inequality holds*:

$$\Sigma = \begin{bmatrix} -\gamma^2 I & D^T \\ D & -I \end{bmatrix} < 0, \tag{14}$$

then one has

$$\begin{bmatrix} -\gamma^2 I & D^T \\ D & -I \end{bmatrix}^{-1} = \begin{bmatrix} R^{-1} & R^{-1}D^T \\ DR^{-1} & I + DR^{-1}D^T \end{bmatrix}, \tag{15}$$

where $R^{-1} = (\gamma^2 I - D^T D)^{-1}$.

Proof From (14), we have the fact that Σ is nonsingular and symmetric. Therefore, there exists a matrix

$$M = \begin{bmatrix} M_{11} & M_{12}^T \\ M_{12} & M_{22} \end{bmatrix}$$

satisfying

$$(-\Sigma)M = M(-\Sigma) = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}, \tag{16}$$

where I_1 and I_2 are identity positive definite matrices with appropriate dimensions. Then, one has

$$\begin{bmatrix} M_{11} \times \gamma^2 I_1 - M_2 D^T & M_{11} \times (-D)^T + M_2 \\ M_{12}^T \times \gamma^2 I_1 - M_3 D & M_{12}^T \times (-D)^T + M_2 \end{bmatrix} = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}. \tag{17}$$

Solving the above equation (17) leads to the equality given in (15). □

Before ending this section, some abbreviations are introduced to facilitate the subsequent discussion.

$$\lambda_k^i = -\pi_{ii} - \sum_{j\in\mathcal{L}_k^i} \pi_{ij}, \qquad \delta_k^i = -\sum_{j\in\mathcal{L}_k^i} \pi_{ij}, \qquad \mathcal{P}_k^i = \sum_{j\in\mathcal{L}_k^i} \pi_{ij} P_j,$$

$$\bar{\lambda}_k^i = -\underline{\pi}_{ii} - \sum_{j\in\mathcal{L}_k^i} \underline{\pi}_{ij}, \qquad \bar{\delta}_k^i = -\sum_{j\in\mathcal{L}_k^i} \underline{\pi}_{ij}, \qquad \bar{\mathcal{P}}_k^i = \sum_{j\in\mathcal{L}_k^i} \bar{\pi}_{ij} P_j.$$

3 Main Results

In this section, a new method, based on the parameter-dependent Lyapunov function approach, is proposed to deal with the H_∞ state feedback control problem. The proposed method can be employed to handle, in strict LMI form, TPs ranging from completely unknown to completely known. Moreover, it is shown that the method is less conservative than the existing result.

Theorem 1 *Consider the system* (1) *with incomplete TPs, for a prescribed positive scalar* γ. *If there exist* $P_i > 0$, V_i, T_i *satisfying the following*:

for $\pi_{ii} \in \mathcal{I}_k^i$

$$\begin{bmatrix} \mathrm{He}(-V_i) & * & * & * & * & * & * \\ A_i V_i + Q_i & \bar{\pi}_{ii} Q_i + T_i - 2Q_i & * & * & * & * & * \\ 0 & B_{2i}{}^T & -\gamma^2 I & * & * & * & * \\ C_i V_i & 0 & D_{2i} & -I & * & * & * \\ V_i & 0 & 0 & 0 & -T_i & * & * \\ \mathcal{C}_k^i & 0 & 0 & 0 & 0 & -\mathcal{D}_k^i & * \\ \sqrt{\bar{\lambda}_k^i} V_i & 0 & 0 & 0 & 0 & 0 & -Q_l \end{bmatrix} < 0, \tag{18}$$

for $\pi_{ii} \in \mathcal{I}_{uk}^{i}$

$$\begin{cases} \begin{bmatrix} \mathrm{He}(-V_i) & * & * & * & * & * \\ A_i V_i + Q_i & \bar{\delta}_i Q_i + T_i - 2Q_i & * & * & * & * \\ 0 & B_{2i} & -\gamma^2 I & * & * & * \\ C_i V_i & 0 & D_{2i} & -I & * & * \\ V_i & 0 & 0 & 0 & -T_i & * \\ \mathfrak{C}_k^i & 0 & 0 & 0 & 0 & -\mathfrak{D}_k^i \end{bmatrix} < 0, \\ Q_i \leq Q_l \quad (l \in \mathcal{L}_{uk}^i), \end{cases} \tag{19}$$

where

$$\mathfrak{C}_k^i = \left[(\sqrt{\bar{\pi}_{i1}} V_i)^T \quad \cdots \quad (\sqrt{\bar{\pi}_{iK_i}} V_i)^T \right]^T,$$
$$\mathfrak{D}_k^i = \mathrm{diag}\{Q_1, \ldots, Q_{K_i}\},$$

where K_i *is the maximal number in* $\mathcal{L}_k^i$, *then the considered autonomous system* (1) *is SS and has the prescribed* H_∞ *performance index* γ.

Proof The incomplete TPs considered in this paper cause the condition in Lemma 1 to be nonconvex. In order to overcome this difficulty, property (2) is employed as follows. If $\pi_{ii} \in \mathcal{I}_k^i$, we keep it. Otherwise, it is replaced with $\pi_{ii} = -\sum_{j \in \mathcal{L}_k^i} \pi_{ij} - \sum_{l \in \mathcal{L}_{uk}^i} \pi_{il}$. Applying this property to (9), one has

$$\begin{bmatrix} \mathrm{He}(P_i A_i) + \sum_{j=1}^N \pi_{ij} P_j & P_i B_{2i} & C_i^T \\ B_{2i}^T P_i & -\gamma^2 I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix}$$
$$= \begin{bmatrix} \mathrm{He}(P_i A_i) + \mathcal{P}_k^i & P_i B_{2i} & C_i^T \\ B_{2i}^T P_i & -\gamma^2 I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix} + \begin{bmatrix} \pi_{ii} P_i & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \mathcal{P}_{uk}^i & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{bmatrix} < 0. \tag{20}$$

Because $\frac{\sum_{l \in \mathcal{L}_{uk}^i} \pi_{il}}{\lambda_k^i} = 1$, for the case $\pi_{ii} \in \mathcal{I}_k^i$, (20) is transformed as

$$\begin{bmatrix} \mathrm{He}(P_i A_i) + \sum_{j=1}^N \pi_{ij} P_j & P_i B_{2i} & C_i^T \\ B_{2i}^T P_i & -\gamma^2 I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix}$$
$$= \frac{\sum_{l \in \mathcal{L}_{uk}^i} \pi_{il}}{\lambda_k^i} \begin{bmatrix} \mathrm{He}(P_i A_i) + \mathcal{P}_k^i + \pi_{ii} P_i & P_i B_{2i} & C_i^T \\ B_{2i}^T P_i & -\gamma^2 I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix} + \begin{bmatrix} \mathcal{P}_{uk}^i & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{bmatrix}$$
$$= \frac{\sum_{l \in \mathcal{L}_{uk}^i} \pi_{il}}{\lambda_k^i} \begin{bmatrix} \mathrm{He}(P_i A_i) + \mathcal{P}_k^i + \pi_{ii} P_i + \lambda_k^i P_l & P_i B_{2i} & C_i^T \\ B_{2i}^T P_i & -\gamma^2 I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix} < 0. \tag{21}$$

A sufficient condition to make (21) hold is presented below:

$$\begin{bmatrix} \mathrm{He}(P_iA_i)+\bar{\mathcal{P}}_k^i+\bar{\pi}_{ii}P_i+\bar{\lambda}_k^iP_l & P_iB_{2i} & C_i^T \\ B_{2i}^TP_i & -\gamma^2I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix}<0 \quad \left(l\in\mathcal{L}_{uk}^i\right). \tag{22}$$

On the other hand, if $\pi_{ii}\in\mathcal{I}_{uk}^i$, replacing π_{ii} with $(-\sum_{i\in\mathcal{L}_k^i}\pi_{ij}-\sum_{l\in\mathcal{L}_{uk}^i}\pi_{il})$ in (20), one has

$$\begin{bmatrix} \mathrm{He}(P_iA_i)+\mathcal{P}_k^i+\delta_k^iP_i & P_iB_{2i} & C_i^T \\ B_{2i}^TP_i & -\gamma^2I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix}+\sum_{l\in\mathcal{L}_{uk}^i}\begin{bmatrix} (P_l-P_i) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}<0. \tag{23}$$

If the following inequalities are satisfied, then (23) can be guaranteed.

$$\begin{cases} \begin{bmatrix} \mathrm{He}(P_iA_i)+\bar{\mathcal{P}}_k^i+\bar{\delta}_k^iP_i & P_iB_{2i} & C_i^T \\ B_{2i}^TP_i & -\gamma^2I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix}<0, \\ P_l\le P_i \quad (l\in\mathcal{L}_{uk}^i). \end{cases} \tag{24}$$

In the following, our main task is to prove that (22) and (24) can be obtained from (18) and (19), respectively. In fact, there exist equivalence relations among (22) and (18), and (24) and (19), which are shown in the following discussion.

(18) $\Longrightarrow$ (22). Let $R_i=(\gamma^2I-D_{2i}^TD_{2i})$. According to (18), R_i is positive definite. By Schur's complement, one has

$$\begin{bmatrix} \mathrm{He}(-V_i)+V_i^T(T_i^{-1}+\sum_{j\neq i}\bar{\pi}_{ij}Q_j^{-1}+\bar{\lambda}_k^iQ_l^{-1})V_i & (A_iV_i)^T+Q_i \\ A_iV_i+Q_i & \bar{\pi}_{ii}Q_i+T_i-2Q_i \end{bmatrix}$$
$$+\begin{bmatrix} 0 & B_{2i}^T \\ (C_iV_i) & 0 \end{bmatrix}^T\begin{bmatrix} -\gamma^2I & D_{2i}^T \\ D_{2i} & -I \end{bmatrix}^{-1}\begin{bmatrix} 0 & B_{2i}^T \\ (C_iV_i) & 0 \end{bmatrix}<0. \tag{25}$$

According to Lemma 3, (25) is equivalently rewritten as (26),

$$\begin{bmatrix} \Pi_{i11} & * \\ \Pi_{i21} & \Pi_{i22} \end{bmatrix}<0, \tag{26}$$

where

$$\begin{aligned} \Pi_{i11}&=\mathrm{He}(-V_i)+V_i^T\left(T_i^{-1}+\sum_{j\neq i}\bar{\pi}_{ij}Q_j^{-1}+\bar{\lambda}_k^iQ_l^{-1}\right)V_i \\ &\quad+(C_iV_i)^T\left(I+D_{2i}R_i^{-1}D_{2i}^T\right)(C_iV_i), \\ \Pi_{i21}&=\left(A_i+B_{2i}R_i^{-1}D_{2i}^TC_i\right)V_i+Q_i, \\ \Pi_{i22}&=\bar{\pi}_{ii}Q_i+T_i-2Q_i+B_{2i}R_i^{-1}B_{2i}^T. \end{aligned}$$

On the other hand, from (18), we can get the fact that V_i is nonsingular. Let $V_i = W_i^{-1}$, $Q_i = P_i^{-1}$ and pre- and post-multiply (26) by $\begin{bmatrix} W_i & 0 \\ 0 & P_i \end{bmatrix}$. Then one has

$$\begin{bmatrix} \Theta_{i11} & * \\ \Theta_{i21} & \Theta_{i22} \end{bmatrix} < 0, \tag{27}$$

where

$$\Theta_{i11} = \mathrm{He}(-W_i) + T_i^{-1} + \sum_{j \neq i} \bar{\pi}_{ij} P_j + \bar{\lambda}_k^i P_l + C_i^T \big(I + D_{2i} R_i^{-1} D_{2i}^T\big) C_i,$$

$$\Theta_{i21} = P_i \big(A_i + B_{2i} R_i^{-1} D_{2i}^T C_i\big) + W_i,$$

$$\Theta_{i22} = \bar{\pi}_{ii} P_i + P_i T_i P_i - 2P_i + P_i B_{2i} R_i^{-1} B_{2i}^T P_i.$$

Pre- and post-multiplying (27) by $[\, I \quad I \,]$ and its transpose, one gets

$$\begin{aligned} &\mathrm{He}\big(P_i\big(A_i + B_{2i} R_i^{-1} D_{2i}^T C_i\big)\big) + \sum \bar{\pi}_{ij} P_j + \bar{\lambda}_k^i P_l + P_i B_{2i} R_i^{-1} B_{2i}^T P_i \\ &\quad + C_i^T (I + D_{2i} R_i^{-1} D_{2i}^T) C_i + T_i^{-1} + P_i T_i P_i - 2P_i < 0. \end{aligned} \tag{28}$$

Because $T_i^{-1} + P_i T_i P_i - 2P_i = (T_i^{-1} - P_i) T_i (T_i^{-1} - P_i) \geq 0$, it follows that

$$\begin{aligned} &\mathrm{He}\big(P_i\big(A_i + B_{2i} R_i^{-1} D_{2i}^T C_i\big)\big) + \sum \bar{\pi}_{ij} P_j + \bar{\lambda}_k^i P_l \\ &\quad + P_i B_{2i} R_i^{-1} B_{2i}^T P_i + C_i^T \big(I + D_{2i} R_i^{-1} D_{2i}^T\big) C_i < 0 \end{aligned} \tag{29}$$

which can be also rewritten as

$$\begin{aligned} &\mathrm{He}\big(P_i\,(A_i)\big) + \sum \bar{\pi}_{ij} P_j + \bar{\lambda}_k^i P_l \\ &\quad + \begin{bmatrix} P_i B_{2i} & C_i^T \end{bmatrix} \begin{bmatrix} R_i^{-1} & R_i^{-1} D_i^T \\ D_i R_i^{-1} & I + D_{2i} R_i^{-1} D_{2i}^T \end{bmatrix} \begin{bmatrix} (P_i B_{2i})^T \\ C_i \end{bmatrix} < 0. \end{aligned} \tag{30}$$

According to Lemma 3 and Schur's complement, (30) is converted as (31),

$$\begin{bmatrix} \mathrm{He}(P_i A_i) + \sum \bar{\pi}_{ij} P_j + \bar{\lambda}_k^i P_l & P_i B_{2i} & C_i^T \\ B_{2i}^T P_i & -\gamma^2 I & D_{2i}^T \\ C_i & D_{2i} & -I \end{bmatrix} < 0 \tag{31}$$

which is just (22).

(22) $\Longrightarrow$ (18). Pre- and post-multiplying (22) by diag$\{\, Q_i,\ I,\ I \,\}$ $(Q_i = P_i^{-1})$ and its transpose, it follows that

$$\begin{bmatrix} \mathrm{He}(A_i Q_i) + \bar{\pi}_{ii} Q_i & * & * & * \\ B_{2i}^T & -\gamma^2 I & * & * \\ C_i Q_i & D_{2i} & -I & * \\ \bar{\mathcal{C}}_k^i & 0 & 0 & \mathcal{D}_k^i \end{bmatrix} < 0, \tag{32}$$

where

$$\bar{\mathcal{C}}_k^i = \left[\, (\sqrt{\bar{\pi}_{i1}} Q_i)^T \;\; \cdots \;\; (\sqrt{\bar{\pi}_{iK_i}} Q_i)^T \,\right]^T,$$
$$\mathcal{D}_k^i = -\operatorname{diag}\left\{ Q_1, \ldots, Q_{K_i} \right\}.$$

Since (32) holds, by the continuity of the LMI, there always exists a set of sufficiently small positive scalars ϵ_i satisfying

$$\begin{bmatrix} \mathrm{He}(A_i Q_i) + \bar{\pi}_{ii} Q_i & * & * & * \\ B_{2i}^T & -\gamma^2 I & D_{2i}^T & * \\ C_i Q_i & D_{2i} & -I & * \\ \mathcal{E}_k^i Q_i & 0 & 0 & \mathcal{D}_k^i \end{bmatrix} + \epsilon_i \begin{bmatrix} A_i \\ 0 \\ C_i \\ \mathcal{E}_k^i \end{bmatrix} Q_i \begin{bmatrix} A_i \\ 0 \\ C_i \\ \mathcal{E}_k^i \end{bmatrix}^T < 0, \tag{33}$$

where

$$\mathcal{E}_k^i = \left[\, \sqrt{\bar{\pi}_{i1}} I \quad \cdots \quad \sqrt{\bar{\pi}_{iK_i}} I \,\right]^T.$$

After performing direct algebraic manipulations, one has

$$\begin{aligned} &\begin{bmatrix} -\epsilon_i^{-1} Q_i + \bar{\pi}_{ii} Q_i & * & * & * \\ B_{2i}^T & -\gamma^2 I & * & * \\ 0 & D_{2i} & -I & * \\ 0 & 0 & 0 & \mathcal{D}_k^i \end{bmatrix} \\ &\quad + \begin{bmatrix} (\epsilon_i A_i + I) Q_i \\ 0 \\ \epsilon_i C_i Q_i \\ \epsilon \mathcal{E}_k^i Q_i \end{bmatrix} (\epsilon_i Q_i)^{-1} \begin{bmatrix} (\epsilon_i A_i + I) Q_i \\ 0 \\ \epsilon_i C_i Q_i \\ \epsilon \mathcal{E}_k^i Q_i \end{bmatrix}^T < 0. \end{aligned} \tag{34}$$

Via Schur's complement, (34) is rewritten as (35),

$$\begin{bmatrix} -\epsilon_i Q_i & * & * & * & * \\ (\epsilon_i A_i + I) Q_i & -\epsilon_i^{-1} Q_i + \bar{\pi}_{ii} Q_i & * & * & * \\ 0 & B_{2i}^T & -\gamma^2 I & * & * \\ \epsilon_i C_i Q_i & 0 & D_{2i} & -I & * \\ \epsilon \mathcal{E}_k^i Q_i & 0 & 0 & 0 & \mathcal{D}_k^i \end{bmatrix} < 0 \tag{35}$$

which can be also rewritten as (36),

$$\begin{bmatrix} -2\epsilon_i Q_i & * & * & * & * & * \\ \epsilon_i A_i Q_i + Q_i & -\epsilon_i^{-1} Q_i + \bar{\pi}_{ii} Q_i & * & * & * & * \\ 0 & B_{2i}^T & -\gamma^2 I & * & * & * \\ \epsilon_i C_i Q_i & 0 & D_{2i} & -I & * & * \\ \epsilon_i Q_i & 0 & 0 & 0 & -\epsilon_i Q_i & * \\ \epsilon_i \mathcal{E}_k^i Q_i & 0 & 0 & 0 & 0 & \mathcal{D}_k^i \end{bmatrix} < 0. \tag{36}$$

Let $V_i = \epsilon_i Q_i$; then we have

$$\begin{bmatrix} \mathrm{He}(-V_i) & * & * & * & * & * \\ A_i V_i + Q_i & -\epsilon_i^{-1} Q_i + \bar{\pi}_{ii} Q_i & * & * & * & * \\ 0 & B_{2i}^T & -\gamma^2 I & * & * & * \\ C_i V_i & 0 & D_{2i} & -I & * & * \\ V_i & 0 & 0 & 0 & -\epsilon_i Q_i & * \\ \mathcal{C}_k^i & 0 & 0 & 0 & 0 & \mathcal{D}_k^i \end{bmatrix} < 0, \tag{37}$$

where

$$\mathcal{C}_k^i = \left[(\sqrt{\bar{\pi}_{i1}} V_i)^T \quad \cdots \quad (\sqrt{\bar{\pi}_{iK_i}} V_i)^T \right]^T.$$

On the other hand, by using $\epsilon_i - 2 \geq -\epsilon_i^{-1}$ in (37), (37) can be guaranteed from the following equality:

$$\begin{bmatrix} \mathrm{He}(-V_i) & * & * & * & * & * \\ A_i V_i + Q_i & (\epsilon_i - 2) Q_i + \bar{\pi}_{ii} Q_i & * & * & * & * \\ 0 & B_{2i}^T & -\gamma^2 I & * & * & * \\ C_i V_i & 0 & D_{2i} & -I & * & * \\ V_i & 0 & 0 & 0 & -\epsilon_i Q_i & * \\ \mathcal{C}_k^i & 0 & 0 & 0 & 0 & \mathcal{D}_k^i \end{bmatrix} < 0. \tag{38}$$

Let $T_i = \epsilon_i Q_i$. Then (18) is obtained.

Along a similar line to (24), we can get that (19) is equivalent to (24). Therefore, if the conditions given in (18) and (19) hold, the autonomous system (1) is SS and has the prescribed H_∞ performance index γ. □

Remark 1 In Theorem 1, two sets of slack variables are introduced and the parameter-dependent Lyapunov function approach is enabled by a matrix transformation technique.

Remark 2 In the above proof, we make full use of property (2). Namely, if $\pi_{ii} \in \mathcal{I}_k^i$, we hold it. Otherwise, it is replaced by $(-\sum_{j \in \mathcal{I}_k^i} \pi_{ij} - \sum_{l \in \mathcal{I}_{uk}^i} \pi_{il})$. Due to this equivalence transformation, the proposed method in this paper is less conservative than Lemma 2, which will be shown in Theorem 3.

Remark 3 Without considering the performance factor, the results given in Theorem 1 can be directly reduced to the state feedback control result in [27].

Based on the conditions given in Theorem 1, the following theorem presents an H_∞ state feedback controller design method for continuous MJLSs with incomplete TPs in the framework of LMIs.

Theorem 2 *For the considered system* (1) *with incomplete TPs, where* γ *is a prescribed positive scalar, if there exist* $P_i > 0$, V_i, T_i, *and* L_i *satisfying the following*:

for $\pi_{ii} \in \mathcal{I}_k^i$

$$\begin{bmatrix} \mathrm{He}(-V_i) & * & * & * & * & * & * \\ A_iV_i + B_{1i}L_i + Q_i & \bar{\pi}_{ii}Q_i + T_i - 2Q_i & * & * & * & * & * \\ 0 & B_{2i}^T & -\gamma^2 I & * & * & * & * \\ C_iV_i + D_{1i}L_i & 0 & D_{2i} & -I & * & * & * \\ V_i & 0 & 0 & 0 & -T_i & * & * \\ \mathcal{C}_{K_1}V_i & 0 & 0 & 0 & 0 & -\mathcal{D}_{K_1} & * \\ \sqrt{\bar{\lambda}_k^i}V_i & 0 & 0 & 0 & 0 & 0 & -Q_l \end{bmatrix} < 0, \tag{39}$$

for $\pi_{ii} \in \mathcal{I}_{uk}^i$

$$\begin{cases} \begin{bmatrix} \mathrm{He}(-V_i) & * & * & * & * & * \\ A_iV_i + B_{1i}L_i + Q_i & \bar{\delta}_iQ_i + T_i - 2Q_i & * & * & * & * \\ 0 & B_{2i}^T & -\gamma^2 I & * & * & * \\ C_iV_i + D_{1i}L_i & 0 & D_{2i} & -I & * & * \\ V_i & 0 & 0 & 0 & -T_i & * \\ \mathcal{C}_{K_1}V_i & 0 & 0 & 0 & 0 & -\mathcal{D}_{K_1} \end{bmatrix} < 0 \\ Q_i \le Q_l \quad (l \in \mathcal{L}_{uk}^i) \end{cases} \tag{40}$$

then the considered system is stochastically stabilizable and has the prescribed H_∞ *performance index* γ *via the controller* (6). *Moreover, the controller gain matrices are given as*

$$K_i = L_iV_i^{-1}. \tag{41}$$

Proof Let $L_i = K_iV_i$ and take the closed-loop system matrices to Theorem 1. □

Remark 4 The controller design method given in Theorem 2 can be directly extended to deal with the robust controller design problem for system matrices subject to polytopic uncertainties.

Remark 5 In Theorem 2, by setting $\sigma = \gamma^2$ and minimizing σ subject to (39) and (40), the optimal H_∞ performance index σ^* as well as the corresponding controller gains can be obtained.

The following Theorem 3 is given to show that the method proposed in Theorem 2 is less conservative than that of Lemma 2.

Theorem 3 *For the TPs to be known or unknown considered in* [42], *if the conditions given in Lemma* 2 *hold, then the conditions given in Theorem* 2 *hold.*

Proof For $i \in \mathcal{I}_k^i$, multiplying λ_k^i by (12) of Lemma 2 after using Schur's complement, one has

$$\lambda_k^i \Omega_i + \lambda_k^i X_i X_l^{-1} X_i \geq 0 \quad \Longrightarrow \quad \lambda_k^i X_i X_l^{-1} X_i \geq (-\lambda_k^i \Omega_i). \tag{42}$$

Taking (42) into (13), we have

$$\begin{bmatrix} \Omega_i + \pi_{ii} X_i + \lambda_k^i X_i X_l^{-1} X_i & B_{2i} & (C_{1i} X_i)^T & \mathcal{S}_k^i \\ * & -\gamma^2 I & D_{2i}^T & 0 \\ * & * & -I & 0 \\ * & * & * & -\mathcal{X}_k^i \end{bmatrix} \leq 0. \tag{43}$$

Using Schur's complement once again, the above inequality is equivalently converted to

$$\begin{bmatrix} \mathrm{He}(A_i X_i) + \pi_{ii} X_i & B_{2i} & (C_{1i} X_i)^T & \mathcal{S}_k^i & \sqrt{\lambda_k^i} X_i \\ * & -\gamma^2 I & D_{2i}^T & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\mathcal{X}_k^i & 0 \\ * & * & * & * & -X_l \end{bmatrix} \leq 0. \tag{44}$$

The equivalence between (44) and (39) can be obtained by using the same method as in Theorem 1. For $i \in \mathcal{I}_{uk}^i$, by utilizing the same method as above, the second part can be proved. □

4 Numerical Examples

In the following, two numerical examples are provided to illustrate the effectiveness of the proposed method. The eigenvalues of each mode matrix in Example 1, which is borrowed from [42], are in the open left plane. Unlike Example 1, the eigenvalues of each mode matrix in Example 2 are unstable.

Example 1 Consider continuous MJLSs (1) with four operation modes and the following data:

$$A_1 = \begin{bmatrix} -0.25 & -0.25 \\ 0.5 & -0.5 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.05 & -0.17 \\ 0.5 & -0.1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.6 & -0.05 \\ 0.5 & -0.6 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} -0.3 & -0.12 \\ 0.5 & -0.15 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}, \qquad B_{12} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \qquad B_{13} = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$B_{14} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \qquad B_{21} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}^T, \qquad B_{22} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^T,$$

$$B_{23} = \begin{bmatrix} -0.2 & -0.1 \end{bmatrix}^T, \qquad B_{24} = \begin{bmatrix} 0.12 & 0.9 \end{bmatrix}^T,$$

$$C_{11} = \begin{bmatrix} 0.5 & 1 \end{bmatrix}, \quad C_{12} = \begin{bmatrix} -0.2 & 0.8 \end{bmatrix}, \quad C_{13} = \begin{bmatrix} 0.1 & -1 \end{bmatrix},$$
$$C_{14} = \begin{bmatrix} 0.3 & -0.9 \end{bmatrix}, \quad C_{21} = \begin{bmatrix} 0.5 & 1 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix},$$
$$C_{23} = \begin{bmatrix} 0.5 & -1 \end{bmatrix}, \quad C_{24} = \begin{bmatrix} -0.5 & 0 \end{bmatrix},$$
$$D_{11} = -0.5, \quad D_{12} = -0.4, \quad D_{13} = -0.2, \quad D_{14} = -0.2,$$
$$D_{21} = 0.5, \quad D_{22} = -0.4, \quad D_{23} = 0.2, \quad D_{24} = -0.62.$$

The TP matrix is as follows:

$$\begin{bmatrix} -1.3 & 0.2 & ? & ? \\ ? & ? & 0.3 & 0.3 \\ 0.6 & ? & -1.5 & ? \\ 0.4 & ? & ? & ? \end{bmatrix}, \tag{45}$$

where ? denotes the completely unknown TPs.

Solving the conditions given in Theorem 1 and Lemma 2, respectively, the corresponding optimal H_∞ indices σ^* are given in Table 1.

Employing the method proposed in [42] to generate a possible modes evolution (Fig. 1), for given initial state $x_0 = [-1.2 \quad 0.6]^T$, the state response curves of the closed-loop systems for different methods are shown in Fig. 2.

From the obtained optimal performance indices and the above state curves, it can be seen that the method proposed in this paper is more effective than the existing result [42].

In the following, an open-loop unstable system example further verifies the effectiveness of the proposed method.

Table 1 Comparison of H_∞ performance indices for different methods (Example 1)

	Theorem 2	Lemma 2 ([42])
γ	1.3456	1.9923

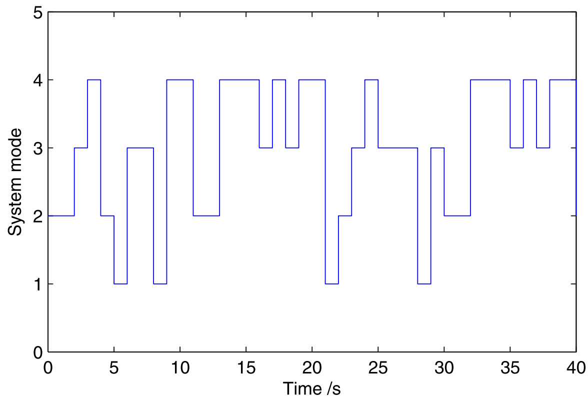

Fig. 1 A possible modes evolution

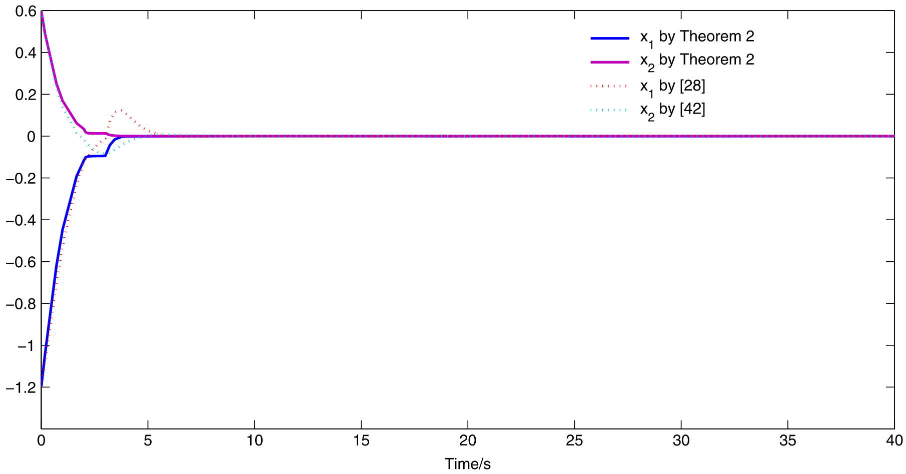

Fig. 2 State response curves for different methods

Example 2 Consider continuous MJLSs (1) with four operation modes and the following data:

$$A_1 = \begin{bmatrix} -0.15 & -0.05 \\ 0.1 & 0.2 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.05 & -0.17 \\ 0.5 & 0.1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.6 & -0.05 \\ 0.5 & 0.6 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 0.3 & -0.12 \\ 0.5 & -0.15 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}, \qquad B_{12} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix},$$
$$B_{13} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \qquad B_{14} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}.$$

The other parts of the system matrices are the same as in Example 1, and the TP matrix is assumed to be as follows:

$$\begin{bmatrix} -1.3 & 0.2 & ? & ? \\ ? & ? & 0.3 & 0.3 \\ \alpha_1 & ? & \alpha_2 & ? \\ 0.4 & ? & ? & ? \end{bmatrix},$$

where α_1 and α_2 are unknown but satisfy $0.5 \le \alpha_1 \le 0.8$ and $-1.9 \le \alpha_1 \le -1.2$ and ? denotes the completely unknown TPs.

Using the same procedure as in Example 1, the obtained H_∞ performance indices are given in Table 2.

This table further shows the effectiveness of the proposed method. Furthermore, by solving Theorem 2, the corresponding state feedback controller gains are obtained,

Table 2 Comparison of H_∞ performance indices for different methods (Example 2)

	Theorem 2	Lemma 2 [42]
γ	2.5822	unfeasible

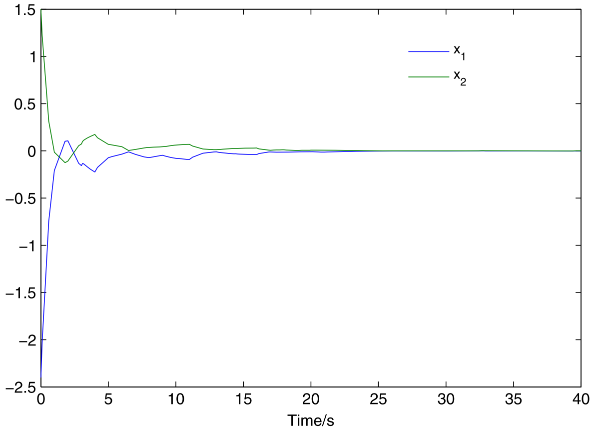

Fig. 3 State response curves

as given below

$$K_1 = \begin{bmatrix} 14.8700 & 21.8006 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 18.5128 & 15.7338 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 46.2946 & 81.3308 \end{bmatrix}, \qquad K_4 = \begin{bmatrix} -57.4605 & -74.1942 \end{bmatrix}.$$

Taking the system mode given in Example 1 (Fig. 1), the state response curves of the closed-loop system (7) are shown in Fig. 3 under the given initial state $x_0 = [-2.4\ 1.5]^T$ and energy-bounded noise $w(t) = 0.5e^{-0.2t}$.

5 Conclusions

The H_∞ state feedback controller design problem for continuous MJLSs with incomplete TPs has been investigated in this paper. Combining the continuous TPs property and a matrix transformation technique, an LMI-based H_∞ controller synthesis method has been proposed. Numerical examples have been given to illustrate the effectiveness of the proposed method.

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