Circuits Syst Signal Process (2014) 33:1189–1214
DOI 10.1007/s00034-013-9689-x

Reduced-Order H_∞ Filters for Uncertain 2-D Continuous Systems, Via LMIs and Polynomial Matrices

Chakir El-Kasri · Abdelaziz Hmamed · Fernando Tadeo

Received: 19 April 2013 / Revised: 26 September 2013 / Published online: 22 October 2013

Abstract This paper deals with designing H_∞ filters of reduced order for two dimensional (2-D) continuous systems described by Roesser models, with uncertain state space matrices. These filters are characterized in terms of linear matrix inequalities (LMI), to minimize a bound on the H_∞ noise attenuation, by using homogeneous polynomially parameter-dependent matrices of arbitrary degree. The methodology is also particularized for full order and zero order (static) filters, where more simple LMI conditions are derived. Numerical examples are presented to illustrate the proposed methodology.

Keywords 2-D Continuous systems · Uncertain systems · H_∞ filtering · Linear matrix inequality (LMI)

1 Introduction

H_∞ filtering, first presented in [13], has the main aim to minimize the H_∞ norm of the error of a filtering system, in order to ensure that the L_2-induced gain from the noise signals to the estimation error will be less than a prescribed level. In contrast with Kalman filtering, H_∞ filtering does not require the exact knowledge of the

C. El-Kasri · A. Hmamed
Department of Physics, Faculty of Sciences Dhar El Mehraz, LESSI, BP 1796, Fes-Atlas, Morocco

C. El-Kasri
e-mail: chakir_elkasri@yahoo.fr

A. Hmamed
e-mail: hmamed_abdelaziz@yahoo.fr

F. Tadeo (✉)
Department of Systems Engineering and Automatic Control, University of Valladolid, 47005 Valladolid, Spain
e-mail: fernando@autom.uva.es

Birkhäuser

noise signals, which renders this approach appropriate in some practical applications. A great number of results on the H_∞ filtering have been proposed in the literature, in both the deterministic and stochastic contexts: see, for example, [27, 29, 33], and references therein. When uncertainties appear in a system model, the robust H_∞ filtering has been also investigated: see, for example, [6, 23, 36].

Currently, there is an increased interest in the design of reduced-order H_∞ filters, as presented in [4, 16, 30, 35], since reduced-order filters are easier to implement than full-order ones: this is an important issue when fast data processing is needed.

Note that the results discussed so far were obtained for one-dimensional (1-D) systems. However, many practical systems are better modeled as two-dimensional (2-D) systems, such as those in image data processing and transmission, thermal processes, gas absorption, water stream heating, etc. [26]. The study of 2-D systems is of both practical and theoretical importance [20, 25, 38]. Therefore, in recent years, much attention has been devoted to the analysis and synthesis problems for 2-D systems: controlability [21, 22]; stability [17, 18]; the stability and stabilization in the presence of delays [2, 15, 19, 28]; 2-D dynamic output feedback control [37], model approximation [8], etc. For the specific problem of 2-D H_∞ filtering, several results have already been obtained: for example, for Roesser models [7]; for Fornasini–Marchesini second model [31, 34]; for 2-D systems with delays [7, 10–12, 34], etc.

Interested in the design of reduced-order H_∞ filters and in order to obtain less conservative results, we present a new approach, the structured polynomially parameter-dependent method, for designing robust H_∞ filters for uncertain 2-D continuous systems described by the Roesser model. Given a stable system with parameter uncertainties residing in polytope vertices, the focus is on designing a robust filter such that the filtering error system is robustly asymptotically stable and minimizing the H_∞ norm of the filtering error system for the entire uncertainty domain. It should be pointed out that not only the full-order filters are established, but also the reduced-order filters are designed. Furthermore, when the reduced-order model is restricted to be of zeroth-order, the dimension constraint is removed and a simpler condition expressed by LMIs is obtained.

In this paper, the reduced-order H_∞ filtering problem for uncertain 2-D continuous systems with new structure of the key slack variable matrix is treated. The class of 2-D systems under consideration corresponds to continuous 2-D systems described by a Roesser state space model subject to polytopic uncertainties in both the state and output matrices. A sufficient condition for the solvability of the robust H_∞ filtering problem is derived in terms of a set of LMIs, based on homogeneous polynomial dependence on the uncertain parameters of arbitrary degree. The more the degree increases, the less conservative filter designs can be obtained. It is shown that the H_∞ filter result includes the quadratic framework, and the linearly parameter-dependent framework as special cases for zeroth degree and first degree, respectively. Two examples will illustrate the feasibility of the proposed methodology.

Notation Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix of appropriate dimension. The superscript T represents the transpose of a matrix; diag$\{\dots\}$, denotes a block-diagonal matrix; the Euclidean vector norm is denoted by $\|\cdot\|$. and the symmetric term in a symmetric matrix is denoted by $*$, e.g., $\left[\begin{smallmatrix} X & Y \\ * & Z \end{smallmatrix}\right] = \left[\begin{smallmatrix} X & Y \\ Y^T & Z \end{smallmatrix}\right]$. Finally, the ℓ_2 norm of a 2-D signal $w(t_1, t_2)$ is given by $\|w(t_1, t_2)\| = \sqrt{\int_0^\infty \int_0^\infty w(t_1, t_2)^T w(t_1, t_2)\, dt_1\, dt_2}$, where $w(t_1, t_2)$ is said to be in the space $\ell_2\{[0, \infty], [0, \infty]\}$ or ℓ_2 if $\|w(t_1, t_2)\| < \infty$.

2 Problem Formulation

Consider an uncertain 2-D continuous system described by the following Roesser state-space model:

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = A_\alpha \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix} + B_\alpha \omega(t_1, t_2), \tag{1}$$

$$y(t_1, t_2) = C_{1_\alpha} \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix} + D_{1_\alpha} \omega(t_1, t_2), \tag{2}$$

$$z(t_1, t_2) = C_\alpha \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix} + D_\alpha \omega(t_1, t_2), \tag{3}$$

where $x^h(t_1, t_2) \in \Re^{n_h}$ and $x^v(t_1, t_2) \in \Re^{n_v}$ are the horizontal and vertical states, respectively; $y(t_1, t_2) \in \Re^p$ is the measured output; $z(t_1, t_2) \in \Re^r$ is the signal to be estimated, and $w(t_1, t_2) \in \Re^m$ is the exogenous input with bounded energy (i.e., $w(t_1, t_2) \in \ell_2$). The system matrices are assumed to belong to a known polyhedral domain Γ described by N vertices, that is,

$$\mathcal{P}_\alpha \triangleq [A_\alpha, B_\alpha, C_{1_\alpha}, D_{1_\alpha}, C_\alpha, D_\alpha] \in \Gamma, \tag{4}$$

where

$$\Gamma \triangleq \left\{ \mathcal{P}(\alpha) \mid \mathcal{P}(\alpha) = \sum_{m=1}^{N} \alpha_m \mathcal{P}_m : \sum_{i=1}^{N} \alpha_m = 1, \alpha_m \geq 0 \right\},$$

with $\mathcal{P}_m \triangleq \{A_m, B_m, C_{1_m}, D_{1_m}, C_m, D_m\}$ denoting the mth vertex of the polyhedral domain Γ. It is assumed that the parameter α is unknown (not measured online) and does not depend explicitly on the time variable (t_1, t_2).

The boundary conditions are defined by

$$x^h(0, t_2) = g(t_2), \qquad x^v(t_1, 0) = f(t_1) \quad \forall (t_1, t_2) \geq 0.$$

Inspired by [5], we make the following assumption:

Assumption 1 The boundary conditions satisfy

$$\left\| x^h(0, t_2) \right\| < \infty, \qquad \lim_{t_2 \to \infty} \left\| x^h(0, t_2) \right\| = 0,$$

$$\left\| x^v(t_1, 0) \right\| < \infty, \qquad \lim_{t_1 \to \infty} \left\| x^v(t_1, 0) \right\| = 0.$$

Similar to [5], we give the following definition:

Definition 1 The 2-D continuous system (1)–(3) with Assumption 1 is said to be asymptotically stable if

$$\lim_{(t_1+t_2)\to\infty} \left\| x^h(t_1,t_2) \right\| = 0; \qquad \lim_{(t_1+t_2)\to\infty} \left\| x^v(t_1,t_2) \right\| = 0.$$

Now, we want to find a 2-D continuous linear time-invariant filter, with input $y(t_1,t_2)$ and output $z_f(t_1,t_2)$, which is an estimation of $z(t_1,t_2)$. Here, we consider the following state space description for this filter:

$$\begin{bmatrix} \frac{\partial x_f^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x_f^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = A_f \begin{bmatrix} x_f^h(t_1,t_2) \\ x_f^v(t_1,t_2) \end{bmatrix} + B_f y(t_1,t_2), \tag{5}$$

$$\begin{aligned} z_f(t_1,t_2) &= C_f \begin{bmatrix} x_f^h(t_1,t_2) \\ x_f^v(t_1,t_2) \end{bmatrix} + D_f y(t_1,t_2), \\ x_f^h(0,t_2) &= 0, \qquad x_f^v(t_1,0) = 0, \quad \forall t_1, t_2, \end{aligned} \tag{6}$$

where $x_f^h(t_1,t_2) \in \Re^{n_{h_f}}$ is the vector of the reduced-order filter horizontal states with $1 \le n_{h_f} < n_f$, and $x_f^v(t_1,t_2) \in \Re^{n_{v_f}}$ is the vector of vertical states, with $1 \le n_{v_f} < n_f$ (for full-order filter, we have $n_{h_f} = n_f$ and $n_{v_f} = n_f$); A_f, B_f, and C_f are constant matrices to be determined, partitioned as follows:

$$A_f \triangleq \begin{bmatrix} A_f^{11} & A_f^{12} \\ A_f^{21} & A_f^{22} \end{bmatrix}, \qquad B_f \triangleq \begin{bmatrix} B_f^1 \\ B_f^2 \end{bmatrix}, \qquad C_f \triangleq \begin{bmatrix} C_f^1 & C_f^2 \end{bmatrix}. \tag{7}$$

Denote

$$\begin{aligned} \tilde{x}^h(t_1,t_2) &= \begin{bmatrix} x^h(t_1,t_2)^T & x_f^h(t_1,t_2)^T \end{bmatrix}^T, \\ \tilde{x}^v(t_1,t_2) &= \begin{bmatrix} x^v t_1,t_2)^T & x_f^v(t_1,t_2)^T \end{bmatrix}^T, \\ \tilde{z}(t_1,t_2) &= z(t_1,t_2) - z_f(t_1,t_2). \end{aligned} \tag{8}$$

Augmenting system (1)–(3) to include the states of filter (5)–(6), we obtain the following filtering error system:

$$\begin{bmatrix} \frac{\partial \tilde{x}^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial \tilde{x}^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = \tilde{A}_\alpha \begin{bmatrix} \tilde{x}^h(t_1,t_2) \\ \tilde{x}^v(t_1,t_2) \end{bmatrix} + \tilde{B}_\alpha w(t_1,t_2), \tag{9}$$

$$\tilde{z}(t_1,t_2) = \tilde{C}_\alpha \begin{bmatrix} \tilde{x}^h(t_1,t_2) \\ \tilde{x}^v(t_1,t_2) \end{bmatrix} + \tilde{D}_\alpha w(t_1,t_2), \tag{10}$$

where

$$\tilde{A}_\alpha = \Upsilon \hat{A}_\alpha \Upsilon^T, \qquad \tilde{B}_\alpha = \Upsilon \hat{B}_\alpha, \qquad \tilde{C}_\alpha = \hat{C}_\alpha \Upsilon^T, \qquad \tilde{D}_\alpha = \hat{D}_\alpha,$$

$$\Upsilon_1 = \begin{bmatrix} I_{n_h} & 0_{n_h \times n_h} \\ 0_{n_{h_f} \times n_h} & 0_{n_{h_f} \times n_v} \\ 0_{n_v \times n_h} & I_{n_v} \\ 0_{n_{v_f} \times n_h} & 0_{n_{v_f} \times n_v} \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} 0_{n_h \times n_{h_f}} & 0_{n_h \times n_{v_f}} \\ I_{n_{h_f}} & 0_{n_{h_f} \times n_{v_f}} \\ 0_{n_v \times n_{h_f}} & 0_{n_v \times n_{v_f}} \\ 0_{n_{v_f} \times n_{h_f}} & I_{n_{v_f}} \end{bmatrix}, \qquad \Upsilon = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \end{bmatrix},$$

$$\hat{A}_\alpha = \begin{bmatrix} A_\alpha & 0 \\ B_f C_{1_\alpha} & A_f \end{bmatrix}, \qquad \hat{B}_\alpha = \begin{bmatrix} B_\alpha \\ B_f D_{1_\alpha} \end{bmatrix},$$

$$\hat{C}_\alpha = \begin{bmatrix} C_\alpha - D_f C_{1_\alpha} & -C_f \end{bmatrix}, \qquad \hat{D}_\alpha = D_\alpha - D_f D_{1_\alpha}.$$

The matrix transfer function of the error system (9)–(10) is then given by

$$\tilde{G}(s_1, s_2) = \tilde{C}_\alpha \big[I(s_1, s_2) - \tilde{A}_\alpha \big]^{-1} \tilde{B}_\alpha + \tilde{D}_\alpha, \tag{11}$$

and the H_∞ norm of the system is, by definition,

$$\|\tilde{G}\|_\infty = \sup_{w_1, w_2 \in R} \sigma_{\max}\big[\tilde{G}(jw_1, jw_2)\big], \tag{12}$$

where $\sigma(\cdot)$ denotes the maximum singular value.

Remark 1 By using the 2-D Parseval's theorem [25], it is not difficult to show that, under zero boundary conditions and with asymptotic stability of (9)–(10), the condition $\|\tilde{G}\|_\infty < \gamma$ is equivalent to

$$\sup_{0 \neq w(t_1, t_2) \in \ell_2} \frac{\|\tilde{z}(t_1, t_2)\|}{\|w(t_1, t_2)\|} \leq \gamma. \tag{13}$$

Our aim in is to design reduced-order H_∞ filters of the form (5)–(6) such that:

1. The filter error system (9)–(10) is asymptotically stable when $w(t_1, t_2) = 0$.
2. The filter error system (9)–(10) fulfills a prescribed level γ of the H_∞ norm; i.e., under the zero boundary condition, $\|\tilde{z}(t_1, t_2)\| < \gamma \|w(t_1, t_2)\|$ is satisfied for any $w(t_1, t_2) \in \ell_2$.

Remark 2 In the reduced-order case, we consider three particular scenarios: First, ($n_{h_f} \neq 0$, $n_{v_f} = 0$); then, ($n_{h_f} = 0$, $n_{v_f} \neq 0$), and finally the zeroth-order filter: ($n_{h_f} = 0, n_{v_f} = 0$).

Case 1: $n_{h_f} \neq 0, n_{v_f} = 0$.

In this case, the reduced-order H_∞ filter in (5)–(6) is given by

$$\frac{\partial x_f^h(t_1, t_2)}{\partial t_2} = A_f^{11} x_f^h(t_1, t_2) + B_f^1 y(t_1, t_2), \tag{14}$$

$$z_f(t_1, t_2) = C_f^1 x_f^h(t_1, t_2) + D_f y(t_1, t_2). \tag{15}$$

Augmenting system (1)–(3) to include the states of the filter (14)–(15) and using (8), we obtain the following filtering error system:

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial \tilde{x}^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = \tilde{A}_\alpha \begin{bmatrix} x^h(t_1,t_2) \\ \tilde{x}^v(t_1,t_2) \end{bmatrix} + \tilde{B}_\alpha w(t_1,t_2), \tag{16}$$

$$\tilde{z}(t_1,t_2) = \tilde{C}_\alpha \begin{bmatrix} x^h(t_1,t_2) \\ \tilde{x}^v(t_1,t_2) \end{bmatrix} + \tilde{D}_\alpha w(t_1,t_2), \tag{17}$$

where

$$\tilde{A}_\alpha = \Upsilon \hat{A}_\alpha \Upsilon^T, \qquad \tilde{B}_\alpha = \Upsilon \hat{B}_\alpha, \qquad \tilde{C}_\alpha = \hat{C}_\alpha \Upsilon^T, \qquad \tilde{D}_\alpha = \hat{D}_\alpha,$$

$$\Upsilon_1 = \begin{bmatrix} I_{n_h} & 0_{n_h \times n_h} \\ 0_{n_{h_f} \times n_h} & 0_{n_{h_f} \times n_v} \\ 0_{n_v \times n_h} & I_{n_v} \end{bmatrix}, \qquad \Upsilon_2 = \begin{bmatrix} 0_{n_h \times n_{h_f}} \\ I_{n_{h_f}} \\ 0_{n_v \times n_{h_f}} \end{bmatrix}, \qquad \Upsilon = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \end{bmatrix},$$

$$\hat{A}_\alpha = \begin{bmatrix} A_\alpha & 0 \\ B_f^1 C_{1_\alpha} & A_f^{11} \end{bmatrix}, \qquad \hat{B}_\alpha = \begin{bmatrix} B_\alpha \\ B_f^1 D_{1_\alpha} \end{bmatrix},$$

$$\hat{C}_\alpha = \begin{bmatrix} C_\alpha - D_f C_{1_\alpha} & -C_f^1 \end{bmatrix}, \qquad \hat{D}_\alpha = D_\alpha - D_f D_{1_\alpha}.$$

Case 2: $n_{h_f} = 0$, $n_{v_f} \neq 0$.

The reduced-order H_∞ filter in (5)–(6) is now

$$\frac{\partial x_f^v(t_1,t_2)}{\partial t_2} = A_f^{22} x_f^v(t_1,t_2) + B_f^2 y(t_1,t_2), \tag{18}$$

$$z_f(t_1,t_2) = C_f^2 x_f^v(t_1,t_2) + D_f y(t_1,t_2). \tag{19}$$

Augmenting system (1)–(3) to include the states of the filter (18)–(19) and using (8), we obtain the following filtering error system:

$$\begin{bmatrix} \frac{\partial \tilde{x}^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = \tilde{A}_\alpha \begin{bmatrix} \tilde{x}^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + \tilde{B}_\alpha w(t_1,t_2), \tag{20}$$

$$\tilde{z}(t_1,t_2) = \tilde{C}_\alpha \begin{bmatrix} \tilde{x}^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} + \tilde{D}_\alpha w(t_1,t_2), \tag{21}$$

where

$$\tilde{A}_\alpha = \Upsilon \hat{A}_\alpha \Upsilon^T, \qquad \tilde{B}_\alpha = \Upsilon \hat{B}_\alpha, \qquad \tilde{C}_\alpha = \hat{C}_\alpha \Upsilon^T, \qquad \tilde{D}_\alpha = \hat{D}_\alpha,$$

$$\Upsilon_1 = \begin{bmatrix} I_{n_h} & 0_{n_h \times n_h} \\ 0_{n_v \times n_h} & I_{n_v} \\ 0_{n_{v_f} \times n_h} & 0_{n_{v_f} \times n_v} \end{bmatrix}, \qquad \Upsilon_2 = \begin{bmatrix} 0_{n_h \times n_{v_f}} \\ 0_{n_v \times n_{v_f}} \\ I_{n_{v_f}} \end{bmatrix}, \qquad \Upsilon = \begin{bmatrix} \Upsilon_1 & \Upsilon_2 \end{bmatrix},$$

$$\hat{A}_\alpha = \begin{bmatrix} A_\alpha & 0 \\ B_f^2 C_{1_\alpha} & A_f^{22} \end{bmatrix}, \qquad \hat{B}_\alpha = \begin{bmatrix} B_\alpha \\ B_f^2 D_{1_\alpha} \end{bmatrix},$$

$$\hat{C}_\alpha = \begin{bmatrix} C_\alpha - D_f C_{1_\alpha} & -C_f^2 \end{bmatrix}, \qquad \hat{D}_\alpha = D_\alpha - D_f D_{1_\alpha}.$$

Case 3: $n_{h_f} = 0$, $n_{v_f} = 0$.

The reduced-order H_∞ filter in (5)–(6) is now the following static filter:

$$z_f(t_1, t_2) = D_f y(t_1, t_2). \tag{22}$$

Connecting this filter (22) to system (1)–(3), we obtain the following filtering error system:

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = A_\alpha \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix} + B_\alpha w(t_1, t_2), \tag{23}$$

$$\tilde{z}(t_1, t_2) = \tilde{C}_\alpha \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix} + \tilde{D}_\alpha w(t_1, t_2), \tag{24}$$

with

$$\tilde{C}_\alpha = C_\alpha - D_f C_{1_\alpha}, \qquad \tilde{D}_\alpha = D_\alpha - D_f D_{1_\alpha}.$$

3 Preliminaries

This section is devoted to some preliminary results used later.

Consider now the following 2-D continuous system:

$$\begin{bmatrix} \frac{\partial x^h(t_1,t_2)}{\partial t_1} \\ \frac{\partial x^v(t_1,t_2)}{\partial t_2} \end{bmatrix} = A_\alpha \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix} = \begin{bmatrix} A_{11_\alpha} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x^h(t_1, t_2) \\ x^v(t_1, t_2) \end{bmatrix}. \tag{25}$$

To test the asymptotic stability of (25), the following condition, based on properties of the characteristic polynomial, could be used:

$$\mathcal{C}(s_1, s_2) \neq 0, \quad \forall (s_1, s_2), \qquad \mathrm{Re}(s_1) \geq 0, \qquad \mathrm{Re}(s_2) \geq 0, \tag{26}$$

where

$$\mathcal{C}(s_1, s_2) = \det \begin{bmatrix} s_1 I_{n_h} - A_{11} & -A_{12} \\ -A_{21} & s_2 I_{n_v} - A_{22} \end{bmatrix}.$$

However, this condition is difficult to use to design filters, so an alternative is used here, based on testing stability using Lyapunov matrices. This methodology makes possible to derive a condition in terms of Linear Matrices Inequalities (LMIs).

Theorem 1 [14] *The 2-D system (25) is asymptotically stable if there exists a matrix* $P = \left[\begin{smallmatrix} P_h & 0 \\ 0 & P_v \end{smallmatrix}\right] > 0$ *(block diagonal positive definite) such that*

$$A^T P + PA < 0. \tag{27}$$

In this case, a Lyapunov function of system (25) is defined as

$$V(t_1, t_2) \triangleq V_1(t_1, t_2) + V_2(t_1, t_2), \tag{28}$$

where

$$V_1(t_1, t_2) \triangleq x^{hT}(t_1, t_2) P_h x^h(t_1, t_2),$$
$$V_2(t_1, t_2) \triangleq x^{vT}(t_1, t_2) P_v x^v(t_1, t_2).$$

Definition 2 [19] The unidirectional derivative of $V(t_1, t_2)$ in (28) is defined to be

$$\dot{V}_u(t_1, t_2) \triangleq \frac{\partial V_1(t_1, t_2)}{\partial t_1} + \frac{\partial V_2(t_1, t_2)}{\partial t_2}. \tag{29}$$

Note that this unidirectional derivative can be seen as a particular case of the derivative of the function $V(t_1, t_2)$ in one direction, independently of the other direction.

Lemma 1 [19] *The 2-D system (25) is asymptotically stable if its unidirectional derivative (29) is negative definite.*

Proof We now give an alternative proof based on Definition 2. From (29) and Lemma 1 we have that

$$\frac{\partial V_1(t_1, t_2)}{\partial t_1} + \frac{\partial V_2(t_1, t_2)}{\partial t_2} < 0,$$

which implies

$$\begin{aligned} &V_1(t_1 + \Delta t_1, t_2) < V_1(t_1, t_2) \quad \text{with } \left\| x^h(t_1, t_2) \right\| > 0 \quad \text{or} \\ &V_2(t_1, t_2 + \Delta t_2) < V_2(t_1, t_2) \quad \text{with } \left\| x^v(t_1, t_2) \right\| > 0. \end{aligned} \tag{30}$$

Let $t_1 \to \infty$ with t_2 finite: substituting them into (30), we get $V_1(\infty, t_2) < V_1(\infty, t_2)$ if $\|x^h(\infty, t_2)\| > 0$ or, equivalently,

$$V_2(\infty, t_2 + \Delta t_2) < V_2(\infty, t_2) < V_2(\infty, 0). \tag{31}$$

Since both $V_1(\infty, t_2) < V_1(\infty, t_2)$ and $V_2(\infty, t_2) < 0$ are false if $\|x^h(\infty, t_2)\| > 0$, it follows that (31) is false. Thus, $\|x^h(\infty, t_2)\| = 0$. Similarly, we can get that $\|x^v(t_1, \infty)\| > 0$, which completes the proof. □

By using a parameter-dependent Lyapunov function $P(\alpha)$ we can obtain the following result.

Lemma 2 [38] *Given $\gamma > 0$, the estimation error system* (9)–(10) *is asymptotically stable with* $\|\tilde{G}\|_\infty < \gamma$ *if there exists a block diagonal positive-definite matrix* $P_\alpha = \mathrm{diag}(P_{h_\alpha}, P_{v_\alpha}) > 0$ *satisfying*

$$\begin{bmatrix} \tilde{A}_\alpha^T P_\alpha^T + P_\alpha \tilde{A}_\alpha & \star & \star \\ \tilde{B}_\alpha^T P_\alpha & -\gamma^2 I & \star \\ \tilde{C}_\alpha & \tilde{D}_\alpha & -I \end{bmatrix} < 0. \tag{32}$$

Lemma 3 *Let* $\xi \in \Re^n$, $Q \in \Re^{n\times n}$, *and* $\mathcal{B} \in \Re^{m\times n}$ *with* rank $\mathcal{B} < n$ *and* $\mathcal{B}^\perp$ *such that* $\mathcal{B}\mathcal{B}^\perp = 0$. *Then, the following conditions are equivalent*:

1. $\xi^T Q \xi < 0\ \forall \xi \neq 0 : \mathcal{B}\xi = 0$.
2. $\mathcal{B}^{\perp T} Q \mathcal{B}^\perp < 0$.
3. $\exists \mu \in \Re : Q - \mu \mathcal{B}^T \mathcal{B} < 0$.
4. $\exists \chi \in \Re^{n\times m} : Q + \chi \mathcal{B} + \mathcal{B}^T \chi^T < 0$.

4 Main Results

In this section, an LMI approach will be developed to solve the robust H_∞ filtering problem formulated in the previous section. First, we propose the following results derived from those in [32] and [38].

Theorem 2 *Given $\gamma > 0$, the filter error system* (9)–(10) *is asymptotically stable with* $\|\tilde{G}\|_\infty < \gamma$ *if there exist* $P = \mathrm{diag}(P_h, P_v) > 0$ *with* $P_h \in R^{n_h+n_{h_f}}$ *and* $P_v \in R^{n_v+n_{v_f}}$ *and matrices* $E_\alpha \in R^{(n+n_f)\times(n+n_f)}$, $F_\alpha \in R^{p\times(n+n_f)}$, $K_\alpha \in R^{(n+n_f)\times(n+n_f)}$, *and* $Q_\alpha \in R^{r\times(n+n_f)}$ *satisfying*

$$\begin{bmatrix} K\tilde{A}_\alpha + \tilde{A}_\alpha^T K^T & \star & \star & \star \\ P_\alpha + E_\alpha \tilde{A}_\alpha - K_\alpha^T & -E_\alpha - E_\alpha^T & \star & \star \\ \tilde{B}_\alpha^T K_\alpha^T + Q_\alpha \tilde{A}_\alpha & \tilde{B}_\alpha^T E_\alpha^T - Q_\alpha & Q_\alpha \tilde{B}_\alpha + \tilde{B}_\alpha^T Q_\alpha^T - \gamma^2 I & \star \\ F_\alpha \tilde{A}_\alpha + \tilde{C}_\alpha & -F_\alpha & F_\alpha \tilde{B}_\alpha + \tilde{D}_\alpha & -I \end{bmatrix} < 0. \tag{33}$$

Proof The equivalence is obtained by considering

$$\chi = \begin{bmatrix} K_\alpha \\ E_\alpha \\ Q_\alpha \\ F_\alpha \end{bmatrix}, \qquad \mathcal{B}^T = \begin{bmatrix} \tilde{A}_\alpha^T \\ -I_{n+n_f} \\ \tilde{B}_\alpha^T \\ 0_{p\times(n+n_f)} \end{bmatrix},$$

$$\mathcal{Q} = \begin{bmatrix} 0_{(n+n_f)\times(n+n_f)} & \star & \star & \star \\ P_\alpha & 0_{(n+n_f)\times(n+n_f)} & \star & \star \\ 0_{r\times(n+n_f)} & 0_{r\times(n+n_f)} & -\gamma^2 I_r & \star \\ \tilde{C} & 0_{p\times(n+n_f)} & \tilde{D} & -I_p \end{bmatrix},$$

under condition (4) of Lemma 3, with

$$\mathcal{B}^{\perp T} = \begin{bmatrix} I_{n+n_f} & \tilde{A}_\alpha^T & 0_{2n\times r} & 0_{(n+n_f)\times p} \\ 0_{r\times(n+n_f)} & \tilde{B}_\alpha^T & I_r & 0_{r\times p} \\ 0_{p\times(n+n_f)} & 0_{p\times(n+n_f)} & 0_{p\times r} & I_p \end{bmatrix},$$

which, using condition (2) of Lemma 3, gives (32). □

The additional variable matrices F_α and Q_α provide additional degrees of freedom for the solution of the robust H_∞ filtering problems presented below. Note that when $F_\alpha = 0$ and $Q_\alpha = 0$, the LMI (33) reduces to LMI (34). From Theorem 2 we have the following corollary.

Corollary 1 *Given* $\gamma > 0$, *the filter error system* (9)–(10) *is asymptotically stable with* $\|\tilde{G}\|_\infty < \gamma$ *if there exist* $P = \mathrm{diag}(P_h, P_v) > 0$ *with* $P_h \in R^{n_h + n_{h_f}}$ *and* $P_v \in R^{n_v + n_{v_f}}$ *and matrices* $E_\alpha \in R^{(n+n_f)\times(n+n_f)}$ *and* $K_\alpha \in R^{(n+n_f)\times(n+n_f)}$ *satisfying*

$$\begin{bmatrix} K\tilde{A}_\alpha + \tilde{A}_\alpha^T K^T & \star & \star & \star \\ P_\alpha + E_\alpha \tilde{A}_\alpha - K_\alpha^T & -E_\alpha - E_\alpha^T & \star & \star \\ \tilde{B}_\alpha^T K_\alpha^T & \tilde{B}_\alpha^T E_\alpha^T & -\gamma^2 I & \star \\ \tilde{C}_\alpha & 0 & \tilde{D}_\alpha & -I \end{bmatrix} < 0. \tag{34}$$

Proof The proof can be easily extended from that for 1-D systems in [9]. □

Remark 3 E_α, F_α, K_α, and Q_α act as slack variables to provide extra degrees of freedom in the solution space of the robust H_∞ filtering problem. Thanks to these matrices, we obtain an LMI in which the Lyapunov matrix P_α is not involved in any product with the system matrices. This enables us to derive a robust H_∞ filtering condition that is less conservative than previous results due to the extra degrees of freedom (see the numerical example at the end of the paper).

In the sequel, based on Theorem 2, we will first design full-order parameter-independent H_∞ filters of the form (5)–(6). The results are then extended to reduced-order filters, providing the main results of the paper.

4.1 Full-Order H_∞ filter design

The following result provides sufficient conditions for the existence of a full-order H_∞ filter ($n_{h_f} = h_n, n_{v_f} = n_v$) for system (9)–(10) satisfying (13).

Theorem 3 *Consider system* (1)–(3) *and let* $\gamma > 0$ *be a given constant. Then the estimation error system* (9)–(10) *is asymptotically stable with* $\|\tilde{G}\|_\infty < \gamma$ *if there exist* $\bar{P}_\alpha \triangleq \mathrm{diag}\{\bar{P}_{h\alpha}, \bar{P}_{v\alpha}\} > 0$ *and matrices* $N_\alpha \triangleq \mathrm{diag}\{N_{h\alpha}, N_{v\alpha}\}$, $T_\alpha \triangleq \mathrm{diag}\{T_{h\alpha}, T_{v\alpha}\}$, $E_{1\alpha} \triangleq \mathrm{diag}\{E_{1_{h\alpha}}, E_{1_{v\alpha}}\}$, $K_{1\alpha} \triangleq \mathrm{diag}\{K_{1_{h\alpha}}, K_{1_{v\alpha}}\}$, F_{1_α}, G_{1_α}, Q_{1_α}, $X \triangleq \mathrm{diag}\{X_h, X_v\}$, S_a, S_b, S_c, *and* S_d *such that*

$$\begin{bmatrix} M_{11_\alpha} + M_{11_\alpha}^T & \star & \star & \star \\ M_{21_\alpha} & M_{22_\alpha} + M_{22_\alpha}^T & \star & \star \\ M_{31_\alpha} & M_{32_\alpha} & M_{33_\alpha} & \star \\ (F_{1_\alpha} A_\alpha + C_\alpha - S_d C_{1_\alpha})\Upsilon_1^T - S_c \Upsilon_2^T & -F_{1_\alpha}\Upsilon_1^T & M_{43_\alpha} & -I \end{bmatrix} < 0, \tag{35}$$

where

$$M_{11_\alpha} = \Upsilon_1 (K_{1_\alpha} A_\alpha + S_b C_{1_\alpha})\Upsilon_1^T + (\Upsilon_1 + \Upsilon_2) S_a \Upsilon_2^T + \Upsilon_2 (N_\alpha A_\alpha + S_b C_{1_\alpha})\Upsilon_1^T,$$

$$
\begin{aligned}
M_{21_\alpha} &= \bar{P}_\alpha + \Upsilon_1\big(E_{1_\alpha}A_\alpha + \lambda_1 S_b C_{1_\alpha} - K_{1_\alpha}^T\big)\Upsilon_1^T + \Upsilon_1\big(\lambda_1 S_a - N_\alpha^T\big)\Upsilon_2^T \\
&\quad + \Upsilon_2\big(T_\alpha A_\alpha + \lambda_1 S_b C_{1_\alpha} - X^T\big)\Upsilon_1^T + \Upsilon_2\big(\lambda_1 S_a - X^T\big)\Upsilon_2^T, \\
M_{22_\alpha} &= -\Upsilon_1 E_{1_\alpha}\Upsilon_1^T - \Upsilon_2 T_\alpha \Upsilon_1^T - \lambda_1 \Upsilon_1 X \Upsilon_2^T - \lambda_2 \Upsilon_2 X \Upsilon_1^T, \\
M_{31_\alpha} &= \big(B_\alpha^T K_{1_\alpha}^T + D_{1_\alpha}^T S_b^T + Q_{1_\alpha} A_\alpha\big)\Upsilon_1^T + \big(B_\alpha^T N_\alpha^T + D_{1_\alpha}^T S_b^T\big)\Upsilon_2^T, \\
M_{32_\alpha} &= \big(B_\alpha^T E_{1_\alpha}^T + \lambda_1 D_{1_\alpha}^T S_b^T - Q_{1_\alpha}\big)\Upsilon_1^T + \big(B_\alpha^T T_\alpha^T + \lambda_2 D_{1_\alpha}^T S_b^T\big)\Upsilon_2^T, \\
M_{33_\alpha} &= Q_{1_\alpha} B_\alpha + B_\alpha^T Q_{1_\alpha}^T - \gamma^2 I, \qquad M_{43_\alpha} = F_{1_\alpha} B_\alpha + D_\alpha - S_d D_{1_\alpha}.
\end{aligned}
$$

In this case, the desired 2-D *continuous filter in the form of* (5)–(6) *can be selected with the following parameters*:

$$
\begin{bmatrix} A_f & B_f \\ C_f & D_f \end{bmatrix} = \begin{bmatrix} X^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} S_a & S_b \\ S_c & S_d \end{bmatrix}. \tag{36}
$$

Proof Let P_α, E_α, F_α, K_α, and Q_α have the following structures:

$$
\begin{aligned}
P_\alpha &= \operatorname{diag}\left\{ \begin{bmatrix} P_{1h_\alpha} & P_{2h_\alpha} \\ P_{2h_\alpha}^T & P_{3h_\alpha} \end{bmatrix}, \begin{bmatrix} P_{1v_\alpha} & P_{2v_\alpha} \\ P_{2v_\alpha}^T & P_{3v_\alpha} \end{bmatrix} \right\}, \qquad Q_\alpha = \begin{bmatrix} Q_{1h} & 0 & Q_{1v} & 0 \end{bmatrix}, \\
E_\alpha &= \operatorname{diag}\left\{ \begin{bmatrix} E_{1h_\alpha} & \lambda_1 K_{4h} \\ E_{2h_\alpha} & \lambda_2 K_{3h} \end{bmatrix}, \begin{bmatrix} E_{1v_\alpha} & \lambda_1 K_{4v} \\ E_{2v_\alpha} & \lambda_2 K_{3v} \end{bmatrix} \right\}, \qquad F_\alpha = \begin{bmatrix} F_{1h} & 0 & F_{1v} & 0 \end{bmatrix}, \\
K_\alpha &= \operatorname{diag}\left\{ \begin{bmatrix} K_{1h_\alpha} & K_{4h} \\ K_{2h_\alpha} & K_{3h} \end{bmatrix}, \begin{bmatrix} K_{1v_\alpha} & K_{4v} \\ K_{2v_\alpha} & K_{3v} \end{bmatrix} \right\}.
\end{aligned}
$$

Without loss of generality, we suppose that K_{3h}, K_{4h}, K_{3v}, and K_{4v} are nonsingular. Introducing the transformation matrix

$$
\Phi = \operatorname{diag}\big\{ I_h, K_{4h}K_{3h}^{-1}, I_v, K_{4v}K_{3v}^{-1} \big\}
$$

and pre- and post-multiplying (33) by $\operatorname{diag}\{\Phi, \Phi, I, I\}$, we get

$$
\begin{bmatrix}
\Phi(K\tilde{A}_\alpha + \tilde{A}_\alpha^T K^T)\Phi^T & \star & \star & \star \\
\Phi(P_\alpha + E_\alpha \tilde{A}_\alpha - K_\alpha^T)\Phi^T & -\Phi(E_\alpha + E_\alpha^T)\Phi^T & \star & \star \\
\tilde{B}_\alpha^T K_\alpha^T \Phi^T + Q_\alpha \tilde{A}_\alpha \Phi^T & \tilde{B}_\alpha^T E_\alpha^T \Phi^T - Q_\alpha \Phi^T & Q_\alpha \tilde{B}_\alpha + \tilde{B}_\alpha^T Q_\alpha^T - \gamma^2 I & \star \\
F_\alpha \tilde{A}_\alpha \Phi^T + \tilde{C}_\alpha \Phi^T & -F_\alpha \Phi^T & F_\alpha \tilde{B}_\alpha + \tilde{D}_\alpha & -I
\end{bmatrix}
< 0. \tag{37}
$$

Defining

$$
\begin{aligned}
\bar{P}_\alpha &= \Phi P_\alpha \Phi^T = \operatorname{diag}\left\{ \begin{bmatrix} \bar{P}_{1h_\alpha} & \bar{P}_{2h_\alpha} \\ \bar{P}_{2h_\alpha}^T & \bar{P}_{1h_\alpha} \end{bmatrix}, \begin{bmatrix} \bar{P}_{1v_\alpha} & \bar{P}_{2v_\alpha} \\ \bar{P}_{2v_\alpha}^T & \bar{P}_{1v_\alpha} \end{bmatrix} \right\}, \\
X &= \operatorname{diag}\big\{ K_{4h}K_{3h}^{-1}K_{4h}, K_{4v}K_{3v}^{-1}K_{4v} \big\}, \\
N_\alpha &= \operatorname{diag}\big\{ K_{4h}K_{3h}^{-1}K_{2h_\alpha}, K_{4v}K_{3v}^{-1}K_{2v_\alpha} \big\},
\end{aligned}
$$

$$T_\alpha = \mathrm{diag}\{K_{4h}K_{3h}^{-1}E_{2h_\alpha}, K_{4v}K_{3v}^{-1}E_{2v_\alpha}\},$$
$$K_{1\alpha} = \mathrm{diag}\{K_{1h_\alpha}, K_{1v_\alpha}\}, \qquad K_{4\alpha} = \mathrm{diag}\{K_{4h_\alpha}, K_{4v_\alpha}\},$$
$$S_a = \begin{bmatrix} S_{a_{1h}} & S_{a_{1v}} \\ S_{a_{2h}} & S_{a_{2v}} \end{bmatrix} = \begin{bmatrix} K_{4h}A_{f_{1h}}K_{3h}^{-1}K_{4h}^T & K_{4h}A_{f_{1v}}K_{3v}^{-1}K_{4v}^T \\ K_{4v}A_{f_{2h}}K_{3h}^{-1}K_{4h}^T & K_{4v}A_{f_{2v}}K_{3v}^{-1}K_{4v}^T \end{bmatrix},$$
$$S_b = \begin{bmatrix} S_{b_h} \\ S_{b_v} \end{bmatrix} = \begin{bmatrix} K_{4h} & 0 \\ 0 & K_{4v} \end{bmatrix} \begin{bmatrix} B_{f_h} \\ B_{f_v} \end{bmatrix}, \qquad S_d = D_f,$$
$$S_c = \begin{bmatrix} S_{c_h} & S_{c_v} \end{bmatrix} = \begin{bmatrix} C_{f_h} & C_{f_v} \end{bmatrix} \begin{bmatrix} K_{3h}^{-1}K_{4h} & 0 \\ 0 & K_{3v}^{-1}K_{4v} \end{bmatrix},$$
$$\begin{bmatrix} S_a & S_b \\ S_c & S_d \end{bmatrix} = \begin{bmatrix} K_{4h} & 0 & 0 \\ 0 & K_{4v} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} A_{f_{1h}} & A_{f_{1v}} & B_{f_h} \\ A_{f_{2h}} & A_{f_{2v}} & B_{f_v} \\ C_{f_h} & C_{f_v} & D_f \end{bmatrix} \begin{bmatrix} K_{3h}^{-1}K_{4h}^T & 0 & 0 \\ 0 & K_{3v}^{-1}K_{4v}^T & 0 \\ 0 & 0 & I \end{bmatrix} \tag{38}$$

(see the proof of (38)), we know that the transfer function of the filter in (5)–(6) from $y(t_1,t_2)$ to $z_f(t_1,t_2)$ is $G_{z_f y}(s_1,s_2) = C_f[\mathrm{diag}\{s_1 I_{nh}, s_2 I_{nv}\} - A_f]^{-1}B_f + D_f$. Substituting (38) into this transfer function and considering $X_h = K_{4h}K_{3h}^{-1}K_{4h}^T$ and $X_v = K_{4v}K_{3v}^{-1}K_{4v}^T$, we get

$$\mathrm{T}_{z_f y}(s_1,s_2) = S_c[\mathrm{diag}\{s_1 I_{nh}, s_2 I_{nv}\} - X^{-1}S_a]^{-1}X^{-1}S_b + S_d.$$

Therefore, the filter can be given by (36), and the proof is completed. □

Remark 4 Observe that, for given λ_1 and λ_2, (35) is convex and can be solved using standard LMI tools. Finding optimal values of λ_1 and $\lambda_2$can be completed, for example, by using the Matlab command Fminsearch.

Similar to Theorem 3, by Corollary 1 we have the following:

Corollary 2 *Consider system* (1)–(3) *and let* $\gamma > 0$ *be a given constant. Then the estimation error system* (9)–(10) *is asymptotically stable with* $\|\tilde{G}\|_\infty < \gamma$ *if there exist* $\bar{P}_\alpha \triangleq \mathrm{diag}\{\bar{P}_{h\alpha}, \bar{P}_{v\alpha}\} > 0$ *and matrices* $N_\alpha \triangleq \mathrm{diag}\{N_{h\alpha}, N_{v\alpha}\}$, $T_\alpha \triangleq \mathrm{diag}\{T_{h\alpha}, T_{v\alpha}\}$, $E_{1\alpha} \triangleq \mathrm{diag}\{E_{1_{h\alpha}}, E_{1_{v\alpha}}\}$, $K_{1\alpha} \triangleq \mathrm{diag}\{K_{1_{h\alpha}}, K_{1_{v\alpha}}\}$, G_{1_α}, $X \triangleq \mathrm{diag}\{X_h, X_v\}$, S_a, S_b, S_c, *and* S_d *such that*

$$\begin{bmatrix} M_{11_\alpha} + M_{11_\alpha}^T & \star & \star & \star \\ M_{21_\alpha} & M_{22_\alpha} + M_{22_\alpha}^T & \star & \star \\ M_{31_\alpha} & M_{32_\alpha} & -\gamma^2 I & \star \\ (C_\alpha - S_d C_{1_\alpha})\Upsilon_1^T - S_c\Upsilon_2^T & -F_{1_\alpha}\Upsilon_1^T & D_\alpha - S_d D_{1_\alpha} & -I \end{bmatrix} < 0, \tag{39}$$

where

$$M_{11_\alpha} = \Upsilon_1(K_{1_\alpha}A_\alpha + S_b C_{1_\alpha})\Upsilon_1^T + (\Upsilon_1 + \Upsilon_2)S_a\Upsilon_2^T + \Upsilon_2(N_\alpha A_\alpha + S_b C_{1_\alpha})\Upsilon_1^T,$$
$$\begin{aligned} M_{21_\alpha} = {} & \bar{P}_\alpha + \Upsilon_1(E_{1_\alpha}A_\alpha + \lambda_1 S_b C_{1_\alpha} - K_{1_\alpha}^T)\Upsilon_1^T + \Upsilon_1(\lambda_1 S_a - N_\alpha^T)\Upsilon_2^T \\ & + \Upsilon_2(T_\alpha A_\alpha + \lambda_1 S_b C_{1_\alpha} - X^T)\Upsilon_1^T + \Upsilon_2(\lambda_1 S_a - X^T)\Upsilon_2^T, \end{aligned}$$

$$M_{22_\alpha} = -\Upsilon_1 E_{1_\alpha} \Upsilon_1^T - \Upsilon_2 T_\alpha \Upsilon_1^T - \lambda_1 \Upsilon_1 X \Upsilon_2^T - \lambda_2 \Upsilon_2 X \Upsilon_1^T,$$

$$M_{31_\alpha} = \left(B_\alpha^T K_{1_\alpha}^T + D_{1_\alpha}^T S_b^T\right)\Upsilon_1^T + \left(B_\alpha^T N_\alpha^T + D_{1_\alpha}^T S_b^T\right)\Upsilon_2^T,$$

$$M_{32_\alpha} = \left(B_\alpha^T E_{1_\alpha}^T + \lambda_1 D_{1_\alpha}^T S_b^T\right)\Upsilon_1^T + \left(B_\alpha^T T_\alpha^T + \lambda_2 D_{1_\alpha}^T S_b^T\right)\Upsilon_2^T.$$

4.2 Reduced-Order H_∞ filter design

In this subsection, we provide a solution of the H_∞ reduced-order filtering problem in terms of LMIs.

First, it must be pointed out that for the reduced-order $1 \le n_{h_f} < n_h$, $1 \le n_{vf} < n_v$, the LMI (35) is no longer applicable because the matrices K_{4h} and K_{4h} are rectangular, of dimensions $n_{hf} \times n_h$ and $n_{vf} \times n_v$, respectively. We get rid of this difficulty by proposing a special structure for the matrices:

$$V_h = \begin{bmatrix} I_{n_{h_f} \times n_{h_f}} \\ 0_{n_h - n_{h_f} \times n_{h_f}} \end{bmatrix}, \qquad V_v = \begin{bmatrix} I_{n_{v_f} \times n_{v_f}} \\ 0_{n_v - n_{v_f} \times n_{v_f}} \end{bmatrix}.$$

Then, replacing matrices K_{4h}, K_{4v} by $V_h K_{4h}$ and $V_v K_{4v}$, respectively, makes possible to derive the corresponding result, as it is now presented:

Theorem 4 *Define* $V_h = \left[\begin{smallmatrix} I_{n_{h_f} \times n_{h_f}} \\ 0_{n_h - n_{h_f} \times n_{h_f}} \end{smallmatrix}\right]$, $V_v = \left[\begin{smallmatrix} I_{n_{v_f} \times n_{v_f}} \\ 0_{n_v - n_{v_f} \times n_{v_f}} \end{smallmatrix}\right]$, *and* $V \triangleq \operatorname{diag}\{V_h, V_v\}$. *Consider system* (1)–(3), *and let* $\gamma > 0$ *be a given constant. Then, there exists a reduced-order* H_∞ *filter in the form of* (5)–(6) *such that the estimation error system* (9)–(10) *is asymptotically stable with* $\|\tilde{G}\|_\infty < \gamma$ *if there exist* $\bar{P}_\alpha \triangleq \operatorname{diag}\{\bar{P}_{h\alpha}, \bar{P}_{v\alpha}\} > 0$ *and matrices* $N_\alpha \triangleq \operatorname{diag}\{N_{h\alpha}, N_{v\alpha}\}$, $T_\alpha \triangleq \operatorname{diag}\{T_{h\alpha}, T_{v\alpha}\}$, $E_{1\alpha} \triangleq \operatorname{diag}\{E_{1_{h\alpha}}, E_{1_{v\alpha}}\}$, $K_{1\alpha} \triangleq \operatorname{diag}\{K_{1_{h\alpha}}, K_{1_{v\alpha}}\}$, F_{1_α}, G_{1_α}, Q_{1_α}, $X \triangleq \operatorname{diag}\{X_h, X_v\}$, S_a, S_b, S_c, *and* S_d *such that*

$$\begin{bmatrix} M_{11_\alpha} + M_{11_\alpha}^T & \star & \star & \star \\ M_{21_\alpha} & M_{22_\alpha} + M_{22_\alpha}^T & \star & \star \\ M_{31_\alpha} & M_{32_\alpha} & M_{33_\alpha} & \star \\ (F_{1_\alpha} A_\alpha + C_\alpha - S_d C_{1_\alpha})\Upsilon_1^T - S_c \Upsilon_2^T & -F_{1_\alpha}\Upsilon_1^T & M_{43_\alpha} & -I \end{bmatrix} < 0, \qquad (40)$$

where

$$M_{11_\alpha} = \Upsilon_1 (K_{1_\alpha} A_\alpha + V S_b C_{1_\alpha})\Upsilon_1^T + (\Upsilon_1 V + \Upsilon_2) S_a \Upsilon_2^T + \Upsilon_2 (N_\alpha A_\alpha + S_b C_{1_\alpha})\Upsilon_1^T,$$

$$M_{21_\alpha} = \bar{P}_\alpha + \Upsilon_1\left(E_{1_\alpha} A_\alpha + \lambda_1 S_b C_{1_\alpha} - K_{1_\alpha}^T\right)\Upsilon_1^T + \Upsilon_1\left(\lambda_1 V S_a - N_\alpha^T\right)\Upsilon_2^T$$
$$+ \Upsilon_2\left(T_\alpha A_\alpha + \lambda_1 S_b C_{1_\alpha} - X^T V^T\right)\Upsilon_1^T + \Upsilon_2\left(\lambda_1 S_a - X^T\right)\Upsilon_2^T,$$

$$M_{22_\alpha} = -\Upsilon_1 E_{1_\alpha} \Upsilon_1^T - \Upsilon_2 T_\alpha \Upsilon_1^T - \lambda_1 \Upsilon_1 V X \Upsilon_2^T - \lambda_2 \Upsilon_2 X \Upsilon_1^T,$$

$$M_{31_\alpha} = \left(B_\alpha^T K_{1_\alpha}^T + D_{1_\alpha}^T S_b^T V^T + Q_{1_\alpha} A_\alpha\right)\Upsilon_1^T + \left(B_\alpha^T N_\alpha^T + D_{1_\alpha}^T S_b^T\right)\Upsilon_2^T,$$

$$M_{32_\alpha} = \left(B_\alpha^T E_{1_\alpha}^T + \lambda_1 D_{1_\alpha}^T S_b^T V^T - Q_{1_\alpha}\right)\Upsilon_1^T + \left(B_\alpha^T T_\alpha^T + \lambda_2 D_{1_\alpha}^T S_b^T\right)\Upsilon_2^T,$$

$$M_{33_\alpha} = Q_{1_\alpha} B_\alpha + B_\alpha^T Q_{1_\alpha}^T - \gamma^2 I, \qquad M_{43_\alpha} = F_{1_\alpha} B_\alpha + D_\alpha - S_d D_{1_\alpha}.$$

In this case, the 2-D filter in the form of (5)–(6) *is given by*

$$\begin{bmatrix} A_f & B_f \\ C_f & D_f \end{bmatrix} = \begin{bmatrix} X^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} S_a & S_b \\ S_c & S_d \end{bmatrix}. \tag{41}$$

Proof The proof is parallel to that of Theorem 3. We obtain (40) when the matrices P_α, E_α, F_α, K_α, and Q_α have the following structures:

$$P_\alpha = \operatorname{diag}\left\{ \begin{bmatrix} P_{1h_\alpha} & P_{2h_\alpha} \\ P_{2h_\alpha}^T & P_{3h_\alpha} \end{bmatrix}, \begin{bmatrix} P_{1v_\alpha} & P_{2v_\alpha} \\ P_{2v_\alpha}^T & P_{3v_\alpha} \end{bmatrix} \right\}, \qquad Q_\alpha = \begin{bmatrix} Q_{1h} & 0 & Q_{1v} & 0 \end{bmatrix},$$

$$K_\alpha = \operatorname{diag}\left\{ \begin{bmatrix} K_{1h_\alpha} & V_h K_{4h} \\ K_{2h_\alpha} & K_{3h} \end{bmatrix}, \begin{bmatrix} K_{1v_\alpha} & V_v K_{4v} \\ K_{2v_\alpha} & K_{3v} \end{bmatrix} \right\}, \qquad F_\alpha = \begin{bmatrix} F_{1h} & 0 & F_{1v} & 0 \end{bmatrix},$$

$$E_\alpha = \operatorname{diag}\left\{ \begin{bmatrix} E_{1h_\alpha} & \lambda_1 V_h K_{4h} \\ E_{2h_\alpha} & \lambda_2 K_{3h} \end{bmatrix}, \begin{bmatrix} E_{1v_\alpha} & \lambda_1 V_v K_{4v} \\ E_{2v_\alpha} & \lambda_2 K_{3v} \end{bmatrix} \right\}.$$

□

Remark 5 In the filter model (5)–(6), when $n_{h_f} = n_h$ and $n_{v_f} = n_v$, then $V = I_{2n}$, so it is a full-order filter; therefore, Theorems 3 and 4 are equivalent for this specific case. The reduced-order filter is then studied when $(1 \le n_{h_f} < n_h, 1 \le n_{v_f} < n_v)$, as when $(n_{h_f} = 0$ or $n_{v_f} = 0)$, we directly get the following corollaries from Theorem 4.

Case 1: $n_{h_f} \ne 0$, $n_{v_f} = 0$.

Corollary 3 *Define* $V_h = \begin{bmatrix} I_{n_{h_f} \times n_{h_f}} \\ 0_{n_h - n_{h_f} \times n_{h_f}} \end{bmatrix}$. *Consider system* (1)–(3) *and let* $\gamma > 0$ *be a given constant. Then, there exists a reduced-order* H_∞ *filter in the form of* (18)–(19) *such that the estimation error system* (20)–(21) *is asymptotically stable with* $\|\tilde{G}\|_\infty < \gamma$ *if there exist* $\bar{P}_\alpha \triangleq \operatorname{diag}\{\bar{P}_{h\alpha}, \bar{P}_{v\alpha}\} > 0$ *and matrices* $N_\alpha \triangleq \operatorname{diag}\{N_{h\alpha}, N_{v\alpha}\}$, $T_\alpha \triangleq \operatorname{diag}\{T_{h\alpha}, T_{v\alpha}\}$, $E_{1\alpha} \triangleq \operatorname{diag}\{E_{1_{h\alpha}}, E_{1_{v\alpha}}\}$, $K_{1\alpha}$, F_{1_α}, G_{1_α}, Q_{1_α}, $X \triangleq \operatorname{diag}\{X_h, X_v\}$, S_a, S_b, S_c, *and* S_d *such that*

$$\begin{bmatrix} M_{11_\alpha} + M_{11_\alpha}^T & \star & \star & \star \\ M_{21_\alpha} & M_{22_\alpha} + M_{22_\alpha}^T & \star & \star \\ M_{31_\alpha} & M_{32_\alpha} & M_{33_\alpha} & \star \\ (F_{1_\alpha} A_\alpha + C_\alpha - S_d C_{1_\alpha})\Upsilon_1^T - S_c \Upsilon_2^T & -F_{1_\alpha}\Upsilon_1^T & M_{43_\alpha} & -I \end{bmatrix} < 0, \tag{42}$$

where

$$\begin{aligned}
M_{11_\alpha} &= \Upsilon_1 (K_{1_\alpha} A_\alpha + V_h S_b C_{1_\alpha}) \Upsilon_1^T + (\Upsilon_1 V_h + \Upsilon_2) S_a \Upsilon_2^T + \Upsilon_2 (N_\alpha A_\alpha + S_b C_{1_\alpha}) \Upsilon_1^T, \\
M_{21_\alpha} &= \bar{P}_\alpha + \Upsilon_1 \big(E_{1_\alpha} A_\alpha + \lambda_1 S_b C_{1_\alpha} - K_{1_\alpha}^T\big) \Upsilon_1^T + \Upsilon_1 \big(\lambda_1 V_h S_a - N_\alpha^T\big) \Upsilon_2^T \\
&\quad + \Upsilon_2 \big(T_\alpha A_\alpha + \lambda_1 S_b C_{1_\alpha} - X^T V_h^T\big) \Upsilon_1^T + \Upsilon_2 \big(\lambda_1 S_a - X^T\big) \Upsilon_2^T, \\
M_{22_\alpha} &= -\Upsilon_1 E_{1_\alpha} \Upsilon_1^T - \Upsilon_2 T_\alpha \Upsilon_1^T - \lambda_1 \Upsilon_1 V_h X \Upsilon_2^T - \lambda_2 \Upsilon_2 X \Upsilon_1^T, \\
M_{31_\alpha} &= \big(B_\alpha^T K_{1_\alpha}^T + D_{1_\alpha}^T S_b^T V_h^T + Q_{1_\alpha} A_\alpha\big) \Upsilon_1^T + \big(B_\alpha^T N_\alpha^T + D_{1_\alpha}^T S_b^T\big) \Upsilon_2^T, \\
M_{32_\alpha} &= \big(B_\alpha^T E_{1_\alpha}^T + \lambda_1 D_{1_\alpha}^T S_b^T V_h^T - Q_{1_\alpha}\big) \Upsilon_1^T + \big(B_\alpha^T T_\alpha^T + \lambda_2 D_{1_\alpha}^T S_b^T\big) \Upsilon_2^T, \\
M_{33_\alpha} &= Q_{1_\alpha} B_\alpha + B_\alpha^T Q_{1_\alpha}^T - \gamma^2 I, \qquad M_{43_\alpha} = F_{1_\alpha} B_\alpha + D_\alpha - S_d D_{1_\alpha}.
\end{aligned}$$

Proof Let matrices P_α, E_α, F_α, K_α, and P_α take the following structures:

$$P_\alpha=\begin{bmatrix}P_{1h_\alpha} & P_{2h_\alpha} & 0\\ P_{2h_\alpha}^T & P_{3h_\alpha} & 0\\ 0 & 0 & P_{v_\alpha}\end{bmatrix},\qquad K_\alpha=\begin{bmatrix}K_{1h_\alpha} & K_{2h_\alpha} & 0\\ K_{2h_\alpha}^T & K_{3h_\alpha} & 0\\ 0 & 0 & K_{1v_\alpha}\end{bmatrix},$$

$$E_\alpha=\begin{bmatrix}E_{1h_\alpha} & \lambda_1 K_{4h} & 0\\ E_{2h_\alpha}^T & \lambda_2 K_{3h} & 0\\ 0 & 0 & E_{1v_\alpha}\end{bmatrix},\qquad F_\alpha=\begin{bmatrix}F_{1h} & 0 & F_{1v}\end{bmatrix},$$

$$Q_\alpha=\begin{bmatrix}Q_{1h} & 0 & Q_{1v}\end{bmatrix}.$$

Without loss of generality, K_{3h} and K_{4h} are nonsingular. Introduce now the transformation matrix

$$\Phi=\mathrm{diag}\{I_h, K_{4h}K_{3h}^{-1}, I_v\}$$

and define

$$\bar{P}_\alpha=\Phi P(\alpha)\Phi^T=\begin{bmatrix}\bar{P}_{1h_\alpha} & \bar{P}_{2h_\alpha} & 0\\ \bar{P}_{2h_\alpha}^T & \bar{P}_{3h_\alpha} & 0\\ 0 & 0 & \bar{P}_{v_\alpha}\end{bmatrix},\qquad X=K_{4h}^T K_{3h}^{-1}K_{4h},$$

$$N(\alpha)=K_{4h}^T K_{3h}^{-1}K_{2h}(\alpha),\qquad T(\alpha)=K_{4h}^T K_{3h}^{-1}E_{2h}(\alpha),\qquad K_1(\alpha)=K_{1h}(\alpha),$$

$$S_a=K_{4h}^T A_f^{11}K_{3h}^{-1}K_{4h}^T,\qquad S_b=K_{4h}^T B_f^1,\qquad S_c=C_f^1 K_{3h}^{-1}K_{4h},\qquad S_d=D_f,$$

$$\begin{bmatrix}S_a & S_b\\ S_c & S_d\end{bmatrix}=\begin{bmatrix}K_{4h} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}A_f^{11} & B_f^1\\ C_f^1 & D_f\end{bmatrix}\begin{bmatrix}K_{3h}^{-1}K_{4h}^T & 0\\ 0 & I\end{bmatrix}.$$

Following proof of Theorem 3, it is possible to obtain the LMI of (42), and

$$\begin{bmatrix}A_f^{11} & B_f^1\\ C_f^1 & D_f\end{bmatrix}=\begin{bmatrix}X^{-1} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}S_a & S_b\\ S_c & S_d\end{bmatrix},\tag{43}$$

which completes the proof. □

Similarly to Corollary 3, we can get the following result to design the reduced-order common filter in the form of (14)–(15).

Case 2: $n_{h_f}\neq 0$, $n_{v_f}=0$.

Corollary 4 *Define* $V_v=\begin{bmatrix}I_{n_{v_f}\times n_{v_f}}\\ 0_{n_v-n_{v_f}\times n_{v_f}}\end{bmatrix}$. *Consider system* (1)–(3) *and let* $\gamma>0$ *be a given constant. Then, there exists a reduced-order* H_∞ *filter in the form of* (14)–(15) *such that the estimation error system* (16)–(17) *is asymptotically stable with* $\|\tilde{G}\|_\infty<\gamma$ *if there exist positive definite matrices* $\bar{P}_\alpha\triangleq\mathrm{diag}\{\bar{P}_{h\alpha},\bar{P}_{v\alpha}\}>0$ *and matrices* $N_\alpha\triangleq$

diag$\{N_{h\alpha}, N_{v\alpha}\}$, $T_\alpha \triangleq$ diag$\{T_{h\alpha}, T_{v\alpha}\}$, $E_{1\alpha} \triangleq$ diag$\{E_{1_{h\alpha}}, E_{1_{v\alpha}}\}$, $K_{1\alpha}$, F_{1_α}, G_{1_α}, Q_{1_α}, $X \triangleq$ diag$\{X_h, X_v\}$, S_a, S_b, S_c, *and* S_d *such that*

$$\begin{bmatrix} M_{11_\alpha} + M_{11_\alpha}^T & \star & \star & \star \\ M_{21_\alpha} & M_{22_\alpha} + M_{22_\alpha}^T & \star & \star \\ M_{31_\alpha} & M_{32_\alpha} & M_{33_\alpha} & \star \\ (F_{1_\alpha} A_\alpha + C_\alpha - S_d C_{1_\alpha})\Upsilon_1^T - S_c \Upsilon_2^T & -F_{1_\alpha}\Upsilon_1^T & M_{43_\alpha} & -I \end{bmatrix} < 0, \quad (44)$$

where

$$M_{11_\alpha} = \Upsilon_1(K_{1_\alpha} A_\alpha + V_v S_b C_{1_\alpha})\Upsilon_1^T + (\Upsilon_1 + \Upsilon_2) V_v S_a \Upsilon_2^T + \Upsilon_2(N_\alpha A_\alpha + S_b C_{1_\alpha})\Upsilon_1^T,$$
$$M_{21_\alpha} = \bar{P}_\alpha + \Upsilon_1\big(E_{1_\alpha} A_\alpha + \lambda_1 S_b C_{1_\alpha} - K_{1_\alpha}^T\big)\Upsilon_1^T + \Upsilon_1\big(\lambda_1 V_v S_a - N_\alpha^T\big)\Upsilon_2^T$$
$$+ \Upsilon_2\big(T_\alpha A_\alpha + \lambda_1 S_b C_{1_\alpha} - X^T V_v^T\big)\Upsilon_1^T + \Upsilon_2\big(\lambda_1 S_a - X^T\big)\Upsilon_2^T,$$
$$M_{22_\alpha} = -\Upsilon_1 E_{1_\alpha}\Upsilon_1^T - \Upsilon_2 T_\alpha \Upsilon_1^T - \lambda_1 \Upsilon_1 V_v X \Upsilon_2^T - \lambda_2 \Upsilon_2 X \Upsilon_1^T,$$
$$M_{31_\alpha} = \big(B_\alpha^T K_{1_\alpha}^T + D_{1_\alpha}^T S_b^T V_v^T + Q_{1_\alpha} A_\alpha\big)\Upsilon_1^T + \big(B_\alpha^T N_\alpha^T + D_{1_\alpha}^T S_b^T\big)\Upsilon_2^T,$$
$$M_{32_\alpha} = \big(B_\alpha^T E_{1_\alpha}^T + \lambda_1 D_{1_\alpha}^T S_b^T V_v^T - Q_{1_\alpha}\big)\Upsilon_1^T + \big(B_\alpha^T T_\alpha^T + \lambda_2 D_{1_\alpha}^T S_b^T\big)\Upsilon_2^T,$$
$$M_{33_\alpha} = Q_{1_\alpha} B_\alpha + B_\alpha^T Q_{1_\alpha}^T - \gamma^2 I, \qquad M_{43_\alpha} = F_{1_\alpha} B_\alpha + D_\alpha - S_d D_{1_\alpha}.$$

Proof Let matrices P_α, E_α, F_α, K_α, and Q_α have the following structures:

$$P_\alpha = \begin{bmatrix} \bar{P}_{h_\alpha} & & 0 \\ 0 & \bar{P}_{1v_\alpha} & \bar{P}_{2v_\alpha} \\ 0 & \bar{P}_{2v_\alpha}^T & \bar{P}_{3v_\alpha} \end{bmatrix}, \qquad K_\alpha = \begin{bmatrix} K_{1h_\alpha} & 0 & 0 \\ 0 & K_{1v_\alpha} & K_{4v} \\ 0 & K_{2v_\alpha} & K_{3v} \end{bmatrix},$$
$$E_\alpha = \begin{bmatrix} E_{1h_\alpha} & 0 & 0 \\ 0 & E_{1v_\alpha} & \lambda_1 K_{4v} \\ 0 & E_{2v_\alpha} & \lambda_2 K_{3v} \end{bmatrix}, \qquad F_\alpha = \begin{bmatrix} F_{1h} & F_{1v} & 0 \end{bmatrix},$$
$$Q_\alpha = \begin{bmatrix} Q_{1h} & Q_{1v} & 0 \end{bmatrix}.$$

Without loss of generality, we again assume that K_{3v} and K_{4v} are nonsingular. We define the transformation matrix

$$\Phi = \text{diag}\{I_h, I_v, K_{4v} K_{3v}^{-1}\}$$

and

$$\bar{P}_\alpha = \Phi P(\alpha)\Phi^T = \begin{bmatrix} \bar{P}_{h_\alpha} & 0 & 0 \\ 0 & \bar{P}_{1v_\alpha} & \bar{P}_{2v_\alpha} \\ 0 & \bar{P}_{2v_\alpha}^T & \bar{P}_{3v_\alpha} \end{bmatrix}, \qquad X = K_{4v}^T K_{3v}^{-1} K_{4v},$$
$$N(\alpha) = K_{4v}^T K_{3v}^{-1} K_{2v}(\alpha), \qquad T(\alpha) = K_{4v}^T K_{3v}^{-1} E_{2v}(\alpha), \qquad K_1(\alpha) = K_{1v}(\alpha),$$
$$S_a = K_{4v}^T A_f^{22} K_{3v}^{-1} K_{4v}^T, \qquad S_b = K_{4v}^T B_f^2, \qquad S_c = C_f^2 K_{3v}^{-1} K_{4v}, \qquad S_d = D_f,$$

$$\begin{bmatrix} S_a & S_b \\ S_c & S_d \end{bmatrix} = \begin{bmatrix} K_{4v} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_f^{22} & B_f^2 \\ C_f^2 & D_f \end{bmatrix} \begin{bmatrix} K_{3v}^{-1} K_{4v}^T & 0 \\ 0 & I \end{bmatrix}.$$

Similarly to the proof of Theorem 3, the LMI (44) is obtained with

$$\begin{bmatrix} A_f^{22} & B_f^2 \\ C_f^2 & D_f \end{bmatrix} = \begin{bmatrix} X^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} S_a & S_b \\ S_c & S_d \end{bmatrix}, \tag{45}$$

completing the proof. □

Case 3: $n_{h_f} = 0$, $n_{v_f} = 0$.

Corollary 5 *Given $\gamma > 0$, There exists a zero-order H_∞ filter in the form of (22) such that the estimation error system (23)–(24) is asymptotically stable with $\|\tilde{G}\|_\infty < \gamma$ if there exist a positive-definite matrix $P = \mathrm{diag}(P_h, P_v) > 0$ with $P_h \in R^{n_h}$ and $P_v \in R^{n_v}$ and matrices $E_\alpha \in R^{n\times n}$, $F_\alpha \in R^{p\times n}$, $K_\alpha \in R^{n\times n}$, and $Q_\alpha \in R^{r\times n}$ satisfying*

$$\begin{bmatrix} KA_\alpha + A_\alpha^T K^T & \star & \star & \star \\ P_\alpha + E_\alpha A_\alpha - K_\alpha^T & -E_\alpha - E_\alpha^T & \star & \star \\ B_\alpha^T K_\alpha^T + Q_\alpha A_\alpha & B_\alpha^T E_\alpha^T - Q_\alpha & Q_\alpha B_\alpha + B_\alpha^T Q_\alpha^T - \gamma^2 I & \star \\ F_\alpha A_\alpha + C_\alpha - D_f C_{1\alpha} & -F_\alpha & F_\alpha B_\alpha + D_\alpha - D_f D_{1\alpha} & -I \end{bmatrix} < 0. \tag{46}$$

4.3 Homogeneous Polynomial Solutions

Before presenting the formulation of Theorem 4 using homogeneous polynomially parameter-dependent matrices, some definitions and preliminaries are needed to represent and handle products and sums of homogeneous polynomials. First, we define the homogeneous polynomially parameter-dependent matrices of degree $\mathfrak{g}$ by

$$\bar{P}_{\alpha(\mathfrak{g})} = \sum_{j=1}^{J(\mathfrak{g})} \alpha_1^{k_1} \alpha_2^{k_2} \dots \alpha_N^{k_N} \bar{P}_{\mathfrak{K}_j(\mathfrak{g})}, \quad k_1 k_2 \dots k_N = \mathfrak{K}_j(\mathfrak{g}). \tag{47}$$

Similarly, matrices N_α, T_α, $E_{1\alpha}$, $K_{1\alpha}$, $F_{1\alpha}$, $G_{1\alpha}$, and $Q_{1\alpha}$ take the same form.

The notations above are as follows: $\alpha_1^{k_1} \alpha_2^{k_2} \dots \alpha_N^{k_N}$, $\alpha \in \Omega$, $k_i \in \mathbb{N}$, $i = 1, \dots, N$ are monomials; $\mathfrak{K}_j(\mathfrak{g})$ is the jth N-tuples of $\mathfrak{K}(\mathfrak{g})$, lexically ordered, $j = 1, \dots, \mathfrak{J}(\mathfrak{g})$, and $\mathfrak{K}(\mathfrak{g})$ is the set of N-tuples obtained as all possible combinations of $k_1 k_2 \dots k_N$ that fulfill $k_1 + k_2 + \dots + k_N = \mathfrak{g}$. Since the number of vertices in the polytope $\mathcal{P}$ is N, the number of elements in $\mathfrak{K}(\mathfrak{g})$ is then given by $\mathfrak{J}(\mathfrak{g}) = (N + \mathfrak{g} - 1)!/(\mathfrak{g}!(N-1)!)$.

For each $i = 1, \dots, N$, we define the N-tuples $\mathfrak{K}_j^i(\mathfrak{g})$ that are equal to $\mathfrak{K}_j(\mathfrak{g})$ but with $k_i > 0$ replaced by $k_i - 1$. Note that the N-tuples $\mathfrak{K}_j^i(\mathfrak{g})$ are defined only in the cases where the corresponding k_i are positive. Note also that, when applied to the elements of $\mathfrak{K}(\mathfrak{g} + 1)$, the N-tuples $\mathfrak{K}_j^i(\mathfrak{g} + 1)$ define subscripts $k_1 k_2 \dots k_N$ of matrices $\bar{P}_{k_1 k_2 \dots k_N}$, $T_{k_1 k_2 \dots k_N}$, $N_{k_1 k_2 \dots k_N}$, $E_{1_{k_1 k_2 \dots k_N}}$, $F_{1_{k_1 k_2 \dots k_N}}$, $G_{1_{k_1 k_2 \dots k_N}}$, $K_{1_{k_1 k_2 \dots k_N}}$, and $Q_{1_{k_1 k_2 \dots k_N}}$, associated to homogeneous polynomial parameter-dependent matrices of degree $\mathfrak{g}$.

Finally, we define the scalar constant coefficients $\beta_j^i(j+1) = \mathfrak{g}!/(k_1!k_2!\dots k_N!)$, with $k_1k_2\dots k_N \in \mathfrak{K}_j^i(\mathfrak{g}+1)$.

To facilitate the presentation of our main results, denote $\beta_j^i(j+1)$ by $\mathfrak{h}$; using this notation, we now present the main result in this section.

Theorem 5 *Define* $V_h = \begin{bmatrix} I_{n_{h_f}\times n_{h_f}} \\ 0_{n_h-n_{h_f}\times n_{h_f}} \end{bmatrix}$, $V_v = \begin{bmatrix} I_{n_{v_f}\times n_{v_f}} & 0_{n_v-n_{v_f}\times n_{v_f}} \end{bmatrix}$, *and* $V \triangleq \operatorname{diag}\{V_h, V_v\}$. *Suppose that there exist symmetric parameter-dependent positive definite matrices* $\bar{P}_{\mathfrak{K}_\mathfrak{j}(\mathfrak{g})} > 0$ *and matrices* $T_{\mathfrak{K}_\mathfrak{j}(\mathfrak{g})}$, $N_{\mathfrak{K}_\mathfrak{j}(\mathfrak{g})}$, $E1_{\mathfrak{K}_\mathfrak{j}(\mathfrak{g})}$, $F1_{\mathfrak{K}_\mathfrak{j}(\mathfrak{g})}$, $G1_{\mathfrak{K}_\mathfrak{j}(\mathfrak{g})}$, $K1_{\mathfrak{K}_\mathfrak{j}(\mathfrak{g})}$, *and* $Q1_{\mathfrak{K}_\mathfrak{j}(\mathfrak{g})}$, $\mathfrak{K}_\mathfrak{j}(\mathfrak{g}) \in \mathfrak{K}(\mathfrak{g})$, $j = 1, \dots, \mathfrak{J}(\mathfrak{g})$, *such that the following LMIs hold for all* $\mathfrak{K}_\mathfrak{l}(\mathfrak{g}+\mathfrak{1}) \in \mathfrak{K}(\mathfrak{g}+\mathfrak{l})$, $l = 1, \dots, \mathfrak{J}(\mathfrak{g}+\mathfrak{l})$:

$$\Psi_\alpha = \sum_{i\in l_l(\mathfrak{g}+\mathfrak{l})} \begin{bmatrix} M_{11}+M_{11}^T & \star & \star & \star \\ M_{21} & M_{22} & \star & \star \\ M_{31} & M_{32} & M_{33} & \star \\ M_{41} & -F1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}\Upsilon_1^T & M_{43} & -\mathfrak{h}I \end{bmatrix} < 0, \tag{48}$$

where

$$\begin{aligned}
M_{11} &= \Upsilon_1\big(K1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}A_i + \mathfrak{h}VS_bC_{1_i}\big)\Upsilon_1^T + \Upsilon_1\mathfrak{h}VS_a\Upsilon_2^T \\
&\quad + \Upsilon_2\big(N_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}A_i + \mathfrak{h}S_bC_{1_i}\big)\Upsilon_1^T + \Upsilon_2\mathfrak{h}S_a\Upsilon_2^T, \\
M_{21} &= \bar{P}_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})} + \Upsilon_1\big(E1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}A_i + \lambda_1\mathfrak{h}S_bC_{1_i} - K1^T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}\big)\Upsilon_1^T \\
&\quad + \Upsilon_2\big(\lambda_1\mathfrak{h}S_a - \mathfrak{h}X^T\big)\Upsilon_2^T + \Upsilon_2\big(T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}A_i + \lambda_1\mathfrak{h}S_bC_{1_i} - \mathfrak{h}X^TV^T\big)\Upsilon_1^T \\
&\quad + \Upsilon_1\big(\lambda_1\mathfrak{h}VS_a - N^T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}\big)\Upsilon_2^T, \\
M_{22} &= -\Upsilon_1E1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}\Upsilon_1^T - \Upsilon_2T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}\Upsilon_1^T - \lambda_1\Upsilon_1\mathfrak{h}VX\Upsilon_2^T - \lambda_2\Upsilon_2\mathfrak{h}X\Upsilon_1^T, \\
M_{31} &= \big(B_i^TK1^T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})} + \mathfrak{h}D_{1_i}^TS_b^TV^T + Q1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}A_i\big)\Upsilon_1^T \\
&\quad + \big(B_i^TN^T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})} + \mathfrak{h}D_{1_i}^TS_b^T\big)\Upsilon_2^T, \\
M_{32} &= \big(B_i^TE1^T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})} + \lambda_1\mathfrak{h}D_{1_i}^TS_b^TV^T - Q1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}\big)\Upsilon_1^T \\
&\quad + \big(B_i^TT^T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})} + \lambda_2\mathfrak{h}D_{1_i}^TS_b^T\big)\Upsilon_2^T, \\
M_{33} &= Q1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}B_i + B_i^TQ1^T_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})} - \mathfrak{h}\gamma^2I, \\
M_{43} &= F1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}B_i + \mathfrak{h}D_i - \mathfrak{h}S_dD_{1_i}, \\
M_{41} &= \big(F1_{\mathfrak{K}_\mathfrak{l}^\mathfrak{i}(\mathfrak{g}+\mathfrak{1})}A_i + \mathfrak{h}C_i - \mathfrak{h}S_dC_{1_i}\big)\Upsilon_1^T - \mathfrak{h}S_c\Upsilon_2^T.
\end{aligned}$$

Then the homogeneous polynomially parameter-dependent matrices given by (47) *ensure* (40) *for all* $\alpha \in \Omega$; *moreover, if the LMI* (48) *is fulfilled for a given degree* $\mathfrak{g}$, *then the LMIs corresponding to any degree* $\mathfrak{g} > \hat{\mathfrak{g}}$ *are also satisfied.*

Proof Note that (40) for $(A(\alpha), B(\alpha), C_1(\alpha), D_1(\alpha), C(\alpha), D(\alpha)) \in \mathcal{P}$ and P_α, T_α, N_α, $K_{1\alpha}$, $E_{1\alpha}$, $F_{1\alpha}$, $G_{1\alpha}$, $Q_{1\alpha}$ given by (48) are homogeneous polynomial matrix equations of degree $\mathfrak{g}+1$ that can be written as

$$\sum_{l=1}^{J(g+1)} \alpha_1^{k_1}\alpha_2^{k_2}\ldots\alpha_N^{k_N}\{\Psi_\alpha\} < 0, \quad k_1k_2\ldots k_N = \mathfrak{K}_l(\mathfrak{g}+1). \tag{49}$$

Condition (48) imposed for all $l = 1, \ldots, \mathfrak{J}(\mathfrak{g}+1)$ ensures condition in (40) for all $\alpha \in \Omega$, and thus the first part is proved.

Suppose that the LMIs of (48) are fulfilled for a certain degree $\hat{\mathfrak{g}}$, that is, there exist $\mathfrak{J}(\hat{\mathfrak{g}})$ matrices $\bar{P}_{\mathfrak{K}_j(\hat{g})}$, $T_{\mathfrak{K}_j(\hat{g})}$, $N_{\mathfrak{K}_j(\hat{g})}$, $K_{1_{\mathfrak{K}_j(\hat{g})}}$, $E_{1_{\mathfrak{K}_j(\hat{g})}}$, $F_{1_{\mathfrak{K}_j(\hat{g})}}$, and $Q_{1_{\mathfrak{K}_j(\hat{g})}}$, $j = 1, \ldots, \mathfrak{J}(\hat{\mathfrak{g}})$, such that $\bar{P}_{\hat{\mathfrak{g}}\alpha}$, $T_{\hat{\mathfrak{g}}\alpha}$, $N_{\hat{\mathfrak{g}}\alpha}$, $K_{1_{\hat{\mathfrak{g}}\alpha}}$, $E_{1_{\hat{\mathfrak{g}}\alpha}}$, $F_{1_{\hat{\mathfrak{g}}\alpha}}$, and $Q_{1_{\hat{\mathfrak{g}}\alpha}}$, are homogeneous polynomially parameter-dependent matrices ensuring condition (40). Then, the terms of the polynomial matrices $\bar{P}_{\alpha(\hat{\mathfrak{g}}+1)} = (\alpha_1 + \cdots + \alpha_N)\bar{P}_{\alpha(\hat{\mathfrak{g}})}$, $T_{\alpha(\hat{\mathfrak{g}}+1)} = (\alpha_1 + \cdots + \alpha_N)T_{\alpha(\hat{\mathfrak{g}})}$, $N_{\alpha(\hat{\mathfrak{g}}+1)} = (\alpha_1 + \cdots + \alpha_N)N_{\alpha(\hat{\mathfrak{g}})}$, $E_{1\alpha(\hat{\mathfrak{g}}+1)} = (\alpha_1 + \cdots + \alpha_N)E_{1\alpha(\hat{\mathfrak{g}})}$, $K_{1\alpha(\hat{\mathfrak{g}}+1)} = (\alpha_1 + \cdots + \alpha_N)K_{1\alpha(\hat{\mathfrak{g}})}$, $F_{1\alpha(\hat{\mathfrak{g}}+1)} = (\alpha_1 + \cdots + \alpha_N)F_{1\alpha(\hat{\mathfrak{g}})}$, and $Q_{1\alpha(\hat{\mathfrak{g}}+1)} = (\alpha_1 + \cdots + \alpha_N)Q_{1\alpha(\hat{\mathfrak{g}})}$ satisfy the LMIs of Theorem 4 corresponding to the degree $\hat{\mathfrak{g}}+1$, which can be obtained in this case by a linear combination of the LMIs of Theorem 4 for $\hat{\mathfrak{g}}$. □

It must be pointed out that when $n_{h_f} = 0$ or $n_{v_f} = 0$, by Theorem 5, we have the following corollary.

Case 1: $n_{h_f} \neq 0, n_{v_f} = 0$.

Corollary 6 *Define* $V_h = \begin{bmatrix} I_{n_{h_f}\times n_{h_f}} \\ 0_{n_h - n_{h_f}\times n_{h_f}} \end{bmatrix}$. *Suppose that there exist symmetric parameter-dependent positive definite matrices* $\bar{P}_{\mathfrak{K}_j(\mathfrak{g})} > 0$ *and matrices* $T_{\mathfrak{K}_j(\mathfrak{g})}$, $N_{\mathfrak{K}_j(\mathfrak{g})}$, $E_{1_{\mathfrak{K}_j(\mathfrak{g})}}$, $F_{1_{\mathfrak{K}_j(\mathfrak{g})}}$, $G_{1_{\mathfrak{K}_j(\mathfrak{g})}}$, $K_{1_{\mathfrak{K}_j(\mathfrak{g})}}$, *and* $Q_{1_{\mathfrak{K}_j(\mathfrak{g})}}$, $\mathfrak{K}_j(\mathfrak{g}) \in \mathfrak{K}(\mathfrak{g})$, $j = 1, \ldots, \mathfrak{J}(\mathfrak{g})$, *such that the following LMIs hold for all* $\mathfrak{K}_l(\mathfrak{g}+1) \in \mathfrak{K}(\mathfrak{g}+\mathfrak{l})$, $l = 1, \ldots, \mathfrak{J}(\mathfrak{g}+\mathfrak{l})$:

$$\Psi_\alpha = \sum_{i\in l_l(\mathfrak{g}+\mathfrak{l})} \begin{bmatrix} M_{11}+M_{11}^T & \star & \star & \star \\ M_{21} & M_{22} & \star & \star \\ M_{31} & M_{32} & M_{33} & \star \\ M_{41} & -F_{1_{\mathfrak{K}_l^i(\mathfrak{g}+1)}}\Upsilon_1^T & M_{43} & -\mathfrak{h}I \end{bmatrix} < 0, \tag{50}$$

where

$$M_{11} = \Upsilon_1(K_{1_{\mathfrak{K}_l^i(\mathfrak{g}+1)}}A_i + \mathfrak{h}V_hS_bC_{1_i})\Upsilon_1^T + \Upsilon_1\mathfrak{h}V_hS_a\Upsilon_2^T$$
$$+ \Upsilon_2(N_{\mathfrak{K}_l^i(\mathfrak{g}+1)}A_i + \mathfrak{h}S_bC_{1_i})\Upsilon_1^T + \Upsilon_2\mathfrak{h}S_a\Upsilon_2^T,$$

$$M_{21} = \bar{P}_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} + \Upsilon_1\big(E_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} A_i + \lambda_1 \mathfrak{h} S_b C_{1_i} - K^T_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}}\big)\Upsilon_1^T$$

$$+ \Upsilon_2\big(\lambda_1 \mathfrak{h} S_a - \mathfrak{h} X^T\big)\Upsilon_2^T + \Upsilon_2\big(T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} A_i + \lambda_1 \mathfrak{h} S_b C_{1_i} - \mathfrak{h} X^T V_h^T\big)\Upsilon_1^T$$

$$+ \Upsilon_1\big(\lambda_1 \mathfrak{h} V S_a - N^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}\big)\Upsilon_2^T,$$

$$M_{22} = -\Upsilon_1 E_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} \Upsilon_1^T - \Upsilon_2 T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} \Upsilon_1^T - \lambda_1 \Upsilon_1 \mathfrak{h} V_h X \Upsilon_2^T - \lambda_2 \Upsilon_2 \mathfrak{h} X \Upsilon_1^T,$$

$$M_{31} = \big(B_i^T K^T_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} + \mathfrak{h} D_{1_i}^T S_b^T V_h^T + Q_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} A_i\big)\Upsilon_1^T$$

$$+ \big(B_i^T N^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} + \mathfrak{h} D_{1_i}^T S_b^T\big)\Upsilon_2^T,$$

$$M_{32} = \big(B_i^T E^T_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} + \lambda_1 \mathfrak{h} D_{1_i}^T S_b^T V^T - Q_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}}\big)\Upsilon_1^T$$

$$+ \big(B_i^T T^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} + \lambda_2 \mathfrak{h} D_{1_i}^T S_b^T\big)\Upsilon_2^T,$$

$$M_{33} = Q_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} B_i + B_i^T Q^T_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} - \mathfrak{h}\gamma^2 I,$$

$$M_{43} = F_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} B_i + \mathfrak{h} D_i - \mathfrak{h} S_d D_{1_i},$$

$$M_{41} = (F_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} A_i + \mathfrak{h} C_i - \mathfrak{h} S_d C_{1_i})\Upsilon_1^T - \mathfrak{h} S_c \Upsilon_2^T.$$

Then the homogeneous polynomially parameter-dependent matrices given by (47) *ensure* (42) *for all* $\alpha \in \Omega$. *Moreover, if the LMI of* (50) *is fulfilled for a given degree* $\mathfrak{g}$, *then the LMIs corresponding to any degree* $\mathfrak{g} > \hat{\mathfrak{g}}$ *are also satisfied.*

Similar to Corollary 6 ($n_{h_f} = 0, n_{v_f} \neq 0$), we have the following corollary.

Case 2: $n_{h_f} = 0, n_{v_f} \neq 0$.

Corollary 7 *Define* $V_v = \begin{bmatrix} I_{n_{v_f} \times n_{v_f}} \\ 0_{n_v - n_{v_f} \times n_{v_f}} \end{bmatrix}$ *Suppose that there exist symmetric parameter-dependent positive definite matrices* $\bar{P}_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})} > 0$ *and matrices* $T_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}$, $N_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}$, $E_{1_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}}$, $F_{1_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}}$, $G_{1_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}}$, $K_{1_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}}$, *and* $Q_{1_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}}$, $\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g}) \in \mathfrak{K}(\mathfrak{g})$, $j = 1, \ldots, \mathfrak{J}(\mathfrak{g})$, *such that the following LMIs hold for all* $\mathfrak{K}_{\mathfrak{l}}(\mathfrak{g}+\mathfrak{1}) \in \mathfrak{K}(\mathfrak{g}+\mathfrak{l})$, $l = 1, \ldots, \mathfrak{J}(\mathfrak{g}+\mathfrak{l})$:

$$\Psi_\alpha = \sum_{i \in l_l(\mathfrak{g}+\mathfrak{l})} \begin{bmatrix} M_{11} + M_{11}^T & \star & \star & \star \\ M_{21} & M_{22} & \star & \star \\ M_{31} & M_{32} & M_{33} & \star \\ M_{41} & -F_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} \Upsilon_1^T & M_{43} & -\mathfrak{h} I \end{bmatrix} < 0, \tag{51}$$

where

$$M_{11} = \Upsilon_1 (K_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} A_i + \mathfrak{h} V_v S_b C_{1_i})\Upsilon_1^T + \Upsilon_1 \mathfrak{h} V_v S_a \Upsilon_2^T$$

$$+ \Upsilon_2 (N_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} A_i + \mathfrak{h} S_b C_{1_i})\Upsilon_1^T + \Upsilon_2 \mathfrak{h} S_a \Upsilon_2^T,$$

$$M_{21} = \bar{P}_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} + \Upsilon_1\big(E_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} A_i + \lambda_1 \mathfrak{h} S_b C_{1_i} - K^T_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}}\big)\Upsilon_1^T$$

$$+ \Upsilon_2\big(\lambda_1 \mathfrak{h} S_a - \mathfrak{h} X^T\big)\Upsilon_2^T + \Upsilon_2\big(T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} A_i + \lambda_1 \mathfrak{h} S_b C_{1_i} - \mathfrak{h} X^T V_v^T\big)\Upsilon_1^T$$

$$+ \Upsilon_1\big(\lambda_1 \mathfrak{h} V S_a - N^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}\big)\Upsilon_2^T,$$

$$M_{22} = -\Upsilon_1 E_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} \Upsilon_1^T - \Upsilon_2 T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} \Upsilon_1^T - \lambda_1 \Upsilon_1 \mathfrak{h} V_v X \Upsilon_2^T - \lambda_2 \Upsilon_2 \mathfrak{h} X \Upsilon_1^T,$$

$$M_{31} = \big(B_i^T K^T_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} + \mathfrak{h} D_{1_i}^T S_b^T V_v^T + Q_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} A_i\big)\Upsilon_1^T$$

$$+ \big(B_i^T N^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} + \mathfrak{h} D_{1_i}^T S_b^T\big)\Upsilon_2^T,$$

$$M_{32} = \big(B_i^T E^T_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} + \lambda_1 \mathfrak{h} D_{1_i}^T S_b^T V^T - Q_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}}\big)\Upsilon_1^T$$

$$+ \big(B_i^T T^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} + \lambda_2 \mathfrak{h} D_{1_i}^T S_b^T\big)\Upsilon_2^T,$$

$$M_{33} = Q_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} B_i + B_i^T Q^T_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} - \mathfrak{h}\gamma^2 I,$$

$$M_{43} = F_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} B_i + \mathfrak{h} D_i - \mathfrak{h} S_d D_{1_i},$$

$$M_{41} = (F_{1_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}} A_i + \mathfrak{h} C_i - \mathfrak{h} S_d C_{1_i})\Upsilon_1^T - \mathfrak{h} S_c \Upsilon_2^T.$$

Then the homogeneous polynomially parameter-dependent matrices given by (47) *ensure* (44) *for all* $\alpha \in \Omega$. *Moreover, if the LMI of* (51) *is fulfilled for a given degree* $\mathfrak{g}$, *then the LMIs corresponding to any degree* $\mathfrak{g} > \hat{\mathfrak{g}}$ *are also satisfied.*

Case 3: $n_{h_f} = 0, n_{v_f} = 0$.

Corollary 8 *Suppose that there exist symmetric parameter-dependent matrices* $P_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})} > 0$, $E_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}$, $F_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}$, $K_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}$, *and* $Q_{\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g})}$ $\mathfrak{K}_{\mathfrak{j}}(\mathfrak{g}) \in \mathfrak{K}(\mathfrak{g})$, $j = 1, \ldots, \mathfrak{J}(\mathfrak{g})$, *such that the following LMIs hold for all* $\mathfrak{K}_{\mathfrak{l}}(\mathfrak{g}+\mathfrak{1}) \in \mathfrak{K}(\mathfrak{g}+\mathfrak{l})$, $l = 1, \ldots, \mathfrak{J}(\mathfrak{g}+\mathfrak{l})$:

$$\begin{bmatrix} K_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} A_i + A_i^T K^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} & \star & \star & \star \\ P_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} + E_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} A_i - K^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} & M_{22} & \star & \star \\ B_i^T K^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} + Q_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} A_i & M_{32} & M_{33} & \star \\ F_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} A_i + \mathfrak{h}(C_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} - D_f C_{1i}) & -F_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} & M_{43} & -\mathfrak{h} I \end{bmatrix} < 0, \quad (52)$$

where

$$M_{22} = -E_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} - E^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}, \qquad M_{33} = Q_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} B_i + B_i^T Q^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} - \mathfrak{h}\gamma^2 I,$$

$$M_{32} = B_i^T E^T_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} - Q_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})}, \qquad M_{43} = F_{\mathfrak{K}_{\mathfrak{l}}^{\mathfrak{i}}(\mathfrak{g}+\mathfrak{1})} B_i + \mathfrak{h}(D_i - D_f D_{1i}).$$

Then the homogeneous polynomially parameter-dependent matrices given by (47) *ensure* (46) *for all* $\alpha \in \Omega$. *Moreover, if* (52) *is fulfilled for a given degree* $\mathfrak{g}$, *then the LMIs corresponding to any degree* $\mathfrak{g} > \hat{\mathfrak{g}}$ *are also satisfied.*

Remark 6 Theorem 5 presents a sufficient condition for the solvability of the reduced-order H_∞ filtering problem. A reduced-order H_∞ filter can be selected by solving the following convex optimization problem:

$$\min \delta \quad \text{subject to (48)} \quad \text{with } \delta = \gamma^2. \tag{53}$$

5 Numerical Examples

Example 1 The system under consideration corresponds to the uncertain 2-D continuous system (1)–(3) with matrices given by

$$A_1 = \begin{bmatrix} -0.468 & 0.845 \\ 0.20 & -0.423 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.825 & 0.427 \\ 0.299 & -0.346 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.744 & 0 \\ 0.52 & -0.545 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} -1.33 & -1.14 \\ 0.322 & -0.309 \end{bmatrix},$$
$$B = \begin{bmatrix} -0.4545 \\ 0.9090 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 100 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0 & 100 \end{bmatrix},$$
$$D_1 = 1, \qquad D = 0.$$

By solving the convex optimization problem in (53), when the parameters $\lambda_1 = 0.8851$ and $\lambda_2 = 1.0568$ are searched, according to Theorem 5, Corollaries 6, 7, and 8, the following filter matrices were obtained:

Case 1: $n_h = 1, n_v = 1, n_{h_f} = 1, n_{v_f} = 1, \gamma = 0.8272.$

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{cc|c} -1.8256 & -45.5051 & -0.4819 \\ 0.0565 & -65.2281 & -0.6504 \\ \hline -0.0040 & -49.9005 & 0.5001 \end{array}\right].$$

Case 2: $n_h = 1, n_v = 1, n_{h_f} = 0, n_{v_f} = 1, \gamma = 0.9984.$

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{c|c} -176.4641 & -1.7499 \\ \hline -6.4619e-005 & 0.9983 \end{array}\right].$$

Case 3: $n_h = 1, n_v = 1, n_{h_f} = 1, n_{v_f} = 0, \gamma = 1.0030.$

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{c|c} -1.8816 & -0.0328 \\ \hline -0.0051 & 0.9987 \end{array}\right].$$

Case 4: $n_h = 1, n_v = 1, n_{h_f} = 0, n_{v_f} = 0, \gamma = 1.0041.$

$$D_f = 0.9987.$$

For comparison, Theorem 3 with $\lambda_1 = -0.0031$ and $\lambda_1 = 0.0057$ provides a guaranted H_∞ cost 0.8272, while [38] yields 0.8936.

Example 2 Consider an uncertain 2-D continuous system (1)–(3) with the following system matrices [3]:

$$A_1 = \left[\begin{array}{cc|cc} -1.1 & -0.6 & 0.1 & 0.9 \\ 0.2 & -0.2 & -0.5 & -0.2 \\ \hline -0.4 & 0.2 & -1.2 & 0.4 \\ -0.4 & 0.9 & 0.2 & -0.2 \end{array}\right],$$

$$A_2 = \left[\begin{array}{cc|cc} -0.7 & -0.4 & -0.4 & 0.8 \\ -0.5 & -1.5 & 0.8 & 0.7 \\ \hline -0.8 & -0.4 & -0.9 & 0.0 \\ -0.7 & -0.6 & 0.6 & -0.1 \end{array}\right],$$

$$A_3 = \left[\begin{array}{cc|cc} -1.0 & -0.9 & -0.1 & 0.4 \\ -0.6 & -0.8 & -0.7 & -0.8 \\ \hline 0.7 & 0.5 & -1.0 & 0.5 \\ -0.5 & 0.2 & 0.3 & -0.8 \end{array}\right],$$

$$B = \left[\begin{array}{c} 0.2 \\ -0.5 \\ \hline -0.8 \\ 0.3 \end{array}\right], \qquad D_1 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.8 & -0.9 & 0.2 & -0.1 \\ 0.5 & -0.3 & 0 & 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.6 & 0.1 & -0.8 & 0.5 \end{bmatrix}, \qquad D = 0.5.$$

H_∞ upper bounds for the error dynamics have been computed by means of the conditions of Theorem 5 for $\hat{\mathfrak{g}} = 0, \ldots, 3$: $\lambda_1 = 0.2972$, $\lambda_2 = 0.2940$ are searched, with the results and the numbers K of scalar variables and L of LMI rows shown in Table 1.

In the full-order case, with $\mathfrak{g} = 1$ (linearly parameter-dependent approach) and $\lambda_1 = 0.2972$, $\lambda_2 = 0.2940$, Theorem 5 provides a guaranteed H_∞ cost of 0.6157, while the method provided by Corollary 1 in [38] is infeasible, and Corollary 1 yields 0.6202. It is clear that the conditions from Theorem 5 provide the best results. The H_∞ performance value achieved with parameter searching and the corresponding filter for different orders are the following:

Case 1: $n_h = 2, n_v = 2, n_{h_f} = 2, n_{v_f} = 2, \gamma = 0.6157$.

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{cccc|cc} -14.0853 & 15.1244 & -2.2679 & 0.3804 & -15.1165 & -3.4041 \\ 8.7768 & -15.5475 & 1.6635 & -2.6867 & 15.9193 & -4.1822 \\ -3.3899 & 4.6270 & -1.4256 & 1.3822 & -5.2773 & 1.8189 \\ -0.7912 & -1.7643 & 0.2711 & -3.6742 & 4.6220 & -6.4909 \\ \hline 0.1471 & 0.5206 & 0.0042 & -0.0821 & -1.1207 & 1.6979 \end{array}\right].$$

Table 1 Guaranteed H_∞ filtering performance for different orders

g	Full order $n_{h_f}=2, n_{v_f}=2$			Reduced order $n_{h_f}=1, n_{v_f}=1$			Zero order $n_{h_f}=0, n_{v_f}=0$		
	γ	K	L	γ	K	L	γ	K	L
0	–	–	–	–	–	–	–	–	–
1	0.6157	217	73	0.6653	150	73	0.8866	93	73
2	0.5811	397	125	0.6376	285	125	0.7348	183	125
3	0.5785	637	191	0.6341	465	191	0.7345	303	191

K is the number of scalar variables, and L is the number of LMI rows in the optimization

Case 2: $n_h=2, n_v=2, n_{h_f}=2, n_{v_f}=1, \gamma=0.6353$.

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{ccc|cc} -6.5807 & 5.1808 & -1.5236 & -5.3909 & -2.8771 \\ 7.2218 & -10.3792 & 1.9444 & 9.5894 & 0.5859 \\ -1.0667 & 1.1847 & -0.8232 & -0.8678 & -0.2120 \\ \hline 0.2938 & 0.3485 & -0.0359 & -1.0238 & 1.8624 \end{array}\right].$$

Case 3: $n_h=2, n_v=2, n_{h_f}=1, n_{v_f}=2, \gamma=0.6604$.

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{ccc|cc} -2.8058 & 0.2969 & -0.9032 & 0.1439 & -3.8048 \\ -3.1560 & -7.4976 & 1.0784 & -1.6726 & -0.0991 \\ -2.4413 & -1.2240 & -3.4190 & 2.2719 & -6.0941 \\ \hline 0.3555 & 0.0396 & -0.1203 & -0.6365 & 1.6357 \end{array}\right].$$

Case 4: $n_h=2, n_v=2, n_{h_f}=1, n_{v_f}=1, \gamma=0.6653$.

$$\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{cc|cc} -2.7151 & -0.0000 & -0.5525 & -2.3005 \\ -5.5532 & -18.6370 & -0.0732 & -6.4629 \\ \hline 0.4161 & -0.0000 & -0.7106 & 1.8585 \end{array}\right].$$

Case 5: $n_h=2, n_v=2, n_{h_f}=0, n_{v_f}=0, \gamma=0.8866$.

$$D_f = \begin{bmatrix} -1.0378 & 1.7119 \end{bmatrix}.$$

From the comparison it can be seen that the proposed result is less conservative than those given in Corollary 1 and [38].

6 Conclusion

A solution to the reduced-order H_∞ filtering problem has been provided for uncertain 2D systems to solve the H_∞ filter problem of the 2D continuous systems in the Roesser state space model, with uncertain matrices belonging to a given polytope. The proposed methodology, based on using polynomially parameter-dependent matrices and slack variables, provides less conservative results than those in the literature by using extra degrees of freedom in the solution space. Some numerical examples have been provided to demonstrate the feasibility and effectiveness of the proposed methodology.

It must be pointed out that the proposed approach could be extended to other related problems, such as Marchesini–Fornasini models, or even multidimensional systems of more than two dimensions (see [1] and [24]).

Acknowledgements This work is funded by AECI A/030426/10, AP/034911/11 and MiCInn DPI2010-21589-c05-05.

References

1. P. Agathoklis, The Lyapunov equation for n-dimensional discrete systems. IEEE Trans. Circuits Syst. **CAS-35**, 448–451 (1988)
2. M. Benhayoun, A. Benzaouia, F. Mesquine, F. Tadeo, Stabilization of 2D continuous systems with multi-delays and saturated control, in *Proceedings of the 18th Mediterranean Conference on Control and Automation, MED'10*, (2010), pp. 993–999
3. P.A. Bliman, R.C.L.F. Oliveira, V.F. Montagner, P.L.D. Peres, Existence of homogeneous polynomial solutions for parameter-dependent linear matrix inequalities with parameters in the simplex, in *Proceedings of the 45th IEEE Conference on Decision & Control*, San Diego, CA, USA (2006), pp. 13–15
4. H.C. Choi, D. Chwa, S.K. Hong, LMI approach to robust reduced-order H_∞ filter design for polytopic uncertain systems. Int. J. Control. Autom. Syst. **7**, 487–494 (2009)
5. J.R. Cui, G.D. Hu, Q. Zhu, Stability and robust stability of 2D discrete stochastic systems. Discrete Dyn. Nat. Soc. **2011**, 545361 (2011), 11 pp.
6. C.E. De Sousa, L. Xie, Y. Wang, H_∞ filtering for a class of uncertain nonlinear systems. Syst. Control Lett. **20**, 419–426 (1993)
7. C. Du, L. Xie, *H_∞ Control and Filtering of Two-Dimensional Systems* (Springer, Heidelberg, 2002)
8. C. Du, L. Xie, Y.C. Soh, H_∞ Reduced-order approximation of 2-D digital filters. IEEE Trans. Circuits Syst. I, Fundam. Theory Appl. **48**(6), 688–698 (2001)
9. Z. Duan, J. Zhang, C. Zhang, E. Mosca, Robust H_2 and H_∞ filtering for uncertain linear systems. Automatica **42**, 1919–1926 (2006)
10. C. El-Kasri, A. Hmamed, T. Alvarez, F. Tadeo, Uncertain 2D continuous systems with state delay: filter design using an H_∞ polynomial approach. Int. J. Comput. Appl. **44**(18), 13–21 (2012)
11. C. El-Kasri, A. Hmamed, E.H. Tissir, F. Tadeo, Robust H_∞ filtering for uncertain two-dimensional continuous systems with time-varying delays. Submitted to Multidimens. Syst. Signal. Process. (2012)
12. C. El-Kasri, A. Hmamed, E.H. Tissir, F. Tadeo, Robust H_∞ filtering of 2D Roesser discrete systems: a polynomial approach. Math. Probl. Eng. **2012**, 1–15 (2012)
13. A. Elsayed, M.J. Grimble, An approach to H_∞ design of optimal digital filters. IMA J. Math. Control Inf. **6**, 233–251 (1989)
14. K. Galkowski, LMI based stability analysis for 2D continuous systems. Int. Conf. Electron. Circuits Syst. **3**, 923–926 (2002)
15. M. Ghamgui, N. Yeganefar, O. Bachelier, D. Mehdi, Stability and stabilization of 2D continuous time varying delay systems. Int. J. Sci. Tech. Autom. Control Comput. Eng. **6**(1), 1734–1745 (2012)
16. K.M. Grigoriadis, J.T. Watson, Reduced-order H_∞ and L_2–L_∞ filtering via linear matrix inequalities. IEEE Trans. Aerosp. Electron. Syst. **33**, 1326–1338 (1997)
17. T. Hinamoto, 2-D Lyapunov equation and filter design based on Fornasini–Marchesini second model. IEEE Trans. Circuits Syst. I, Fundam. Theory Appl. **40**(2), 102–110 (1993)
18. A. Hmamed, M. Alfidi, A. Benzaouia, F. Tadeo, LMI conditions for robust stability of 2-D linear discrete-time systems. Math. Probl. Eng. **2008**, 356124 (2008), 11 pp.
19. A. Hmamed, F. Mesquine, F. Tadeo, M. Benhayoun, A. Benzaouia, Stabilization of 2-D saturated systems by state feedback control. Multidimens. Syst. Signal Process. **21**(3), 277–292 (2010)
20. A. Hmamed, C. El-Kasri, E.H. Tissir, T. Alvarez, F. Tadeo, Robust H_∞ filtering for uncertain 2-D continuous systems with delays. Int. J. Innov. Comput. Inf. Control **9**(5), 2167–2183 (2013)

21. J. Klamka, Controllability of nonlinear 2-D systems. Nonlinear Anal. Theory Methods Appl. **30**(5), 2963–2968 (1997)
22. J. Klamka, Controllability of infinite-dimensional 2-D linear systems. Adv. Syst. Sci. Appl. **1**(1), 537–543 (1997)
23. M.J. Lacerda, R.C.L.F. Oliveira, P.D. Peres, Robust H_2 and H_∞ filter design for uncertain linear systems via LMIs and polynomial matrices. Signal Process. **91**, 1115–1122 (2011)
24. Z. Lin, Feedback stabilizability of MIMO n-D linear systems. Multidimens. Syst. Signal Process. **9**, 149–172 (1998)
25. W.S. Lu, A. Antonio, *Two-Dimensional Digital Filters* (Marcel Dekker, New York, 1992)
26. N.E. Mastorakis, M. Swamy, A new method for computing the stability margin of two dimensional continuous systems. IEEE Trans. Circuits Syst. I, Fundam. Theory Appl. **49**(6), 869–872 (2002)
27. P.G. Park, T. Keileth, H_∞ via convex optimization. Int. J. Control **66**, 15–22 (1997)
28. W. Paszke, E. Rogers, K. Galkowski, H_2/H_∞ output information-based disturbance attenuation for differential linear repetitive processes. Int. J. Robust Nonlinear Control **21**(17), 1981–1993 (2011)
29. U. Shaked, H_∞ minimum error state estimation of linear stationary processes. IEEE Trans. Autom. Control **35**, 554–558 (1990)
30. H.D. Tuan, P. Apkarian, T.Q. Nguyen, Robust and reduced-order filtering: new LMI based characterizations and methods. IEEE Trans. Signal Process. **49**, 2975–2984 (2001)
31. H.D. Tuan, P. Apkarian, T.Q. Nguyen, T. Narikiys, Robust mixed H_2/H_∞ filtering of 2-D systems. IEEE Trans. Signal Process. **50**, 1759–1771 (2002)
32. A.G. Wu, H. Dong, G. Duan, Improved Robust H_∞ estimation for uncertain continuous-time systems. J. Syst. Sci. Complex. **20**, 362–369 (2007)
33. L. Wu, J. Lam, W. Paszke, K. Galkowski, E. Rogers, Robust H_∞ filtering for uncertain differential linear repetitive processes. Int. J. Adapt. Control Signal Process. **22**, 243–265 (2008)
34. L. Xie, C. Du, C. Zhang, Y.C. Soh, H_2/H_∞ deconvolution filtering of 2-D digital systems. IEEE Trans. Signal Process. **50**, 2319–2332 (2003)
35. S. Xu, J. Lam, Reduced-order H_∞ filtering for singular systems. Syst. Control Lett. **56**, 48–57 (2007)
36. S. Xu, P. Van Dooren, Robust H_∞ filtering for a class of nonlinear systems with state delay and parameter uncertainty. Int. J. Control **75**, 766–774 (2002)
37. L. Xu, O. Saito, K. Abe, Output feedback stabilizability and stabilization algorithms for 2-D systems. Multidimens. Syst. Signal Process. **5**(1), 41–60 (1994)
38. S. Xu, J. Lam, Y. Zou, Z. Lin, W. Paszke, Robust H_∞ filtering for uncertain 2-D continuous systems. IEEE Trans. Signal Process. **53**, 1731–1738 (2005)