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Fault Estimation for Nonlinear Dynamic Systems

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Abstract In this paper, the problems of fault detection and estimation for nonlinear dynamic systems are considered by using fault detection observer and adaptive fault diagnosis observer. Based on Lyapunov stability theory and linear matrix inequality (LMI) techniques, a new sufficient condition in terms of LMIs for the proposed problem is derived. At the same time, we get the adaptive fault estimation algorithm. The LMI condition can be easily solved by MATLAB LMI toolbox. Finally, a flexible joint robotic example is given to illustrate the efficiency of the proposed approach.

Keywords Fault estimation · Fault detection · Fault diagnosis observer · Nonlinear dynamic systems · Linear matrix inequality

1 Introduction

The increasing demand for high performance and reliability has led to more challenging operating conditions for many complex industry systems. Unexpected change of external circumstance, components' normal wear or breakdown may lead to critical or even catastrophic failure of the system. Therefore, to improve system's reliability

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and safety, fault diagnosis and fault-tolerant control have received a great deal of attention in recent years. And a lot of fruitful results can be found in several excellent books [2, 3] and the references therein.

Generally speaking, fault detection and estimation is the first step of the fault diagnosis. Timely detection can avoid the development of more serious faults [4, 6, 9, 10, 17, 22]. In [17], the problem of robust fault detection for discrete-time switched systems with state delays was investigated. And a robust fault detection filter that guaranteed both sensitivity to faults and robustness to disturbances was designed. Using a generalized form of observer-based fault detection filter (FDF) as a residual generator, Ding and Zhong [6] dealt with the problem of robust fault detection for Markovian jump linear systems with polytopic uncertainties, and presented an adaptive sensor fault detection and isolation approach in linear multi-input multi-output systems with unknown system parameters. For nonlinear time-delay systems with unknown inputs, the robust fault detection filter (RFDF) design problems were studied in [1]. On the other hand, fault-tolerant control (FTC) has been used to improve efficiency during the past two decades [5, 8, 15, 18, 19]. Fault estimation can provide the magnitude of the fault, then using the obtained fault information one can design an additive controller to compensate for the fault. Therefore, fault estimation has been studied by many scholars, and the methods they adopted mainly include diagnosis observer, parameter estimation, adaptive fault diagnosis observer, neural network, and so on [7, 11–14, 16, 20, 21]. Zhang and Jiang [20] studied the problem of fault estimation of time-varying delay systems using adaptive fault diagnosis observer. It is well known that most real systems have very strong nonlinearity, so it is significantly practical to study nonlinear systems. Bin et al. [12] investigated process fault accommodation in a class of nonlinear continuous-time systems. There, the derivative of output was not in the adaptive fault estimator and had some conservativeness. Using radial basis function (RBF) neural network, Huang and Kok [11] investigated the problem of fault detection and diagnosis in a class of nonlinear systems with modeling uncertainties. To our knowledge, there are only few conclusions of fault detection and estimation for practical nonlinear systems using adaptive fault diagnose observer.

Motivated by these considerations, the authors of this paper discuss the problems of fault detection and estimation for nonlinear dynamic systems using fault detection observer and adaptive fault diagnosis observer. Based on Lyapunov stability theory combined with linear matrix inequality (LMI) techniques, we get the adaptive fault estimation algorithm. And a new sufficient conditions in terms of LMI, which guarantee the error system stability, are derived. The LMI condition can be easily solved by MATLAB LMI toolbox. Finally, a flexible joint robotic example is given to illustrate the efficiency of the proposed approach.

2 Problem Formulation

Consider the following nonlinear continuous-time system:

$$\begin{cases} \dot{x}(t) = Ax(t) + g(t, x(t)) + Bu(t) + Ef(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input vector and $y(t) \in \Re^q$ is the output vector, $f(t) \in \Re^r$ represent the actuator fault; A, B, C, E are known constant real matrices of appropriate dimensions; column rank and the pair (A, C) are observable; $g(t, x(t))$ is a continuous nonlinear vector function, assumed to be Lipschitz, with a Lipschitz constant l_g, i.e., $\|g(t, x_2(t)) - g(t, x_1(t))\| \le l_g \|x_2(t) - x_1(t)\|$. It is assumed that the derivative of $f(t)$ with respect to time is norm-bounded, i.e. $\|\dot{f}(t)\| \le f_1$, where $f_1 \ge 0$.

Remark 1 It is known that, in the conventional method, the fault $f(t)$ was often seen as a constant one, which has considerable conservativeness. In our paper, we consider time-varying fault rather than the constant fault: $\dot{f}(t) \neq 0$.

3 Adaptive Diagnostic Observer Design

The following observer is proposed to detect the actuator fault occurred in dynamics:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + g(t, \hat{x}(t)) + Bu(t) - L(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{2}$$

where $\hat{x}(t) \in \Re^n$ is the observer state vector, $\hat{y}(t) \in \Re^q$ is the observer output vector, $L \in \Re^{n\times q}$ is the observer gain matrix, which will be determined later. As the assumption that (A, C) is observable, for gain matrix L, there exists positive definite matrix P such that the following linear matrix inequality hods:

$$(A - LC)^T P + P(A - LC) < 0. \tag{3}$$

Let $\tilde{x}(t) = \hat{x}(t) - x(t)$, $\tilde{y}(t) = \hat{y}(t) - y(t)$, then the error dynamic is described by

$$\begin{cases} \dot{\tilde{x}}(t) = (A - LC)\tilde{x}(t) + G(t, \hat{x}(t), x(t)) - Ef(t) \\ \tilde{y}(t) = C\tilde{x}(t) \end{cases} \tag{4}$$

where $G(t, \hat{x}(t), x(t) = g(t, \hat{x}(t)) - g(t, x(t))$.

If no fault occurs (i.e. $f(t) = 0$), then from (4) it can be seen that $\lim_{t\to\infty} \tilde{y}(t) = 0$. However, if there is fault $f(t)$ and $\lim_{t\to\infty} f(t) \neq 0$, then $\lim_{t\to\infty} \tilde{y}(t) \neq 0$. Therefore the fault detection can be carried out as

$$\begin{cases} \lim_{t\to\infty} \tilde{y}(t) = 0, & \text{no fault occurs} \\ \lim_{t\to\infty} \tilde{y}(t) \neq 0, & \text{fault has occurred} \end{cases} \tag{5}$$

and the observer given by (2) is referred to as the fault detection observer for the system described by (1).

To diagnose the fault after the alarm (5) is generated, consider the following fault diagnostic observer:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + g(t, \hat{x}(t)) + Bu(t) + E\hat{f}(t) - L(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{6}$$

where $\hat{f}(t)$ is the estimation of the actuator fault $f(t)$. Note that if $\tilde{f}(t) = \hat{f}(t) - f(t)$, then the error system is

$$\begin{cases} \dot{\tilde{x}}(t) = (A - LC)\tilde{x}(t) + G(t, \hat{x}(t), x(t)) + E\tilde{f}(t) \\ \tilde{y}(t) = C\tilde{x}(t). \end{cases} \tag{7}$$

Now, based on the above adaptive fault estimator, the following theorem will give the fault estimation algorithm and the conditions that guarantee the stability of error system (7).

Theorem 1 *If there exist symmetric positive definite matrices $P \in \Re^{n\times n}$, $M \in \Re^{r\times r}$, and real matrices $Y \in \Re^{n\times q}$, $F \in \Re^{r\times q}$ such that the following LMIs hold*:

$$E^T P = FC \tag{8}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & P \\ * & \Pi_{22} & -E^T P \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{9}$$

where

$$\Pi_{11} = PA + A^T P - YC - C^T Y^T + \varepsilon l_g^2,$$

$$\Pi_{12} = -A^T PE + C^T Y^T E,$$

$$\Pi_{22} = -2E^T PE + M,$$

then the adaptive fault estimation algorithm

$$\dot{\hat{f}}(t) = -\Gamma F\big(\dot{\tilde{y}}(t) + \tilde{y}(t)\big) \tag{10}$$

can realize $\tilde{x}(t)$ and $\tilde{f}(t)$ uniformly ultimately bounded and the matrix $\Gamma \in \Re^{r\times r}$ is the learning rate with $*$ denoting the symmetric elements in a symmetric matrix.

Proof Choose a positive definite Lyapunov–Krasovskii functional as follows:

$$V = \tilde{x}^T(t)P\tilde{x}(t) + \tilde{f}^T(t)\Gamma^{-1}\tilde{f}(t).$$

Taking the time derivative on $V(t)$ along the trajectories of system (6), we get

$$\begin{aligned} \dot{V}(t) = {} & \tilde{x}^T(t)\big[P(A - LC) + (A - LC)^T P\big]\tilde{x}(t) + 2\tilde{x}^T(t)PG\big(t, \hat{x}(t), x(t)\big) \\ & - 2\tilde{f}^T(t)E^T P(A - LC)\tilde{x}(t) - 2\tilde{f}^T(t)E^T PG\big(t, \hat{x}(t), x(t)\big) \\ & - 2\tilde{f}^T(t)E^T PE\tilde{f}(t) - 2\tilde{f}^T(t)\Gamma^{-1}\dot{f}(t). \end{aligned}$$

For a symmetric positive definite matrix M, it is easy to show that

$$\begin{aligned} -2\tilde{f}^T(t)\Gamma^{-1}\dot{f}(t) & \le \tilde{f}^T(t)M\tilde{f}(t) + \dot{f}^T(t)\Gamma^{-1}M^{-1}\Gamma^{-1}\dot{f}(t) \\ & \le \tilde{f}^T(t)M\tilde{f}(t) + f_1^2\lambda_{\max}\big(\Gamma^{-1}M^{-1}\Gamma^{-1}\big). \end{aligned}$$

As the nonlinear term $g(t,x(t))$ satisfies the Lipschitz condition, for a scalar $\varepsilon > 0$ we have

$$\varepsilon l_g^2 \tilde{x}^T(t)\tilde{x}(t) - \varepsilon G^T\big(t,\hat{x}(t),x(t)\big)G\big(t,\hat{x}(t),x(t)\big) \geq 0.$$

Then we can further obtain that

$$\begin{aligned}\dot{V}(t) = {}& \tilde{x}^T(t)\big[P(A-LC)+(A-LC)^T P\big]\tilde{x}(t) + 2\tilde{x}^T(t)PG\big(t,\hat{x}(t),x(t)\big)\\ &- 2\tilde{f}^T(t)E^T P(A-LC)\tilde{x}(t) - 2\tilde{f}^T(t)E^T PG\big(t,\hat{x}(t),x(t)\big)\\ &- 2\tilde{f}^T(t)E^T PE\tilde{f}(t) + \tilde{f}^T(t)M\tilde{f}(t) + f_1^2\lambda_{\max}\big(\Gamma^{-1}M^{-1}\Gamma^{-1}\big)\\ &+ \varepsilon l_g^2\tilde{x}^T(t)\tilde{x}(t) - \varepsilon G^T\big(t,\hat{x}(t),x(t)\big)G\big(t,\hat{x}(t),x(t)\big)\\ = {}& \xi^T(t)\Pi\xi(t)+\delta,\end{aligned}$$

where

$$\delta = f_1^2\lambda_{\max}\big(\Gamma^{-1}M^{-1}\Gamma^{-1}\big).$$

So, if (9) holds, $\dot{V}(t) < -\varepsilon\|\xi(t)\|^2 + \delta$, where ε is the minimum eigenvalue of $-\Pi$. It follows that $V(t) < 0$ for $\varepsilon\|\xi(t)\|^2 > \delta$, which means that the $\xi(t)$ converges to a set according to Lyapunov stability theory. Therefore, estimation errors of the fault and the state are uniformly bounded. □

Remark 2 It is well known that most real systems have very strong nonlinearity, therefore, conclusions obtained in our paper are more general than in [20]. We will consider the fault estimation and accommodation for nonlinear system with time-varying delay later.

Remark 3 Compared with [12], the fault estimation algorithm in our paper contains the derivative of output error. It can make the estimation more accurate.

Remark 4 Here, we discuss the method of how to solve the conditions in Theorem 1. Using LMI toolbox, it is easy to solve the inequality (9), but for (8) there are some difficulties. Practically, it is difficult to solve (8) and (9) simultaneously, which is not stated in [20]. In our paper, equation constraint (8) in Theorem 1 can be transformed into solving the optimization problem: Minimize β such that the following linear matrix inequality holds:

$$\begin{bmatrix} \beta I & E^T P - FC \\ PE - C^T F^T & \beta I \end{bmatrix} > 0. \tag{11}$$

If fault $f(t)$ is a constant, i.e. $\dot{f}(t) = 0$, then we have $\dot{\tilde{f}}(t) = \dot{\hat{f}}(t)$. From this point, we can get the following corollary.

Corollary 1 *If there exist symmetric positive definite matrices $P \in \Re^{n\times n}$, and real matrices $Y \in \Re^{n\times q}$, $F \in \Re^{r\times q}$ such that the following LMIs hold*:

$$E^T P = FC \tag{12}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & P \\ * & -2E^T PE & -E^T P \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{13}$$

where

$$\Pi_{11} = PA + A^T P - YC - C^T Y^T + \varepsilon l_g^2,$$

$$\Pi_{12} = -A^T PE + C^T Y^T E,$$

then the adaptive fault estimation algorithm

$$\dot{\hat{f}}(t) = -\Gamma F\big(\dot{\tilde{y}}(t) + \tilde{y}(t)\big) \tag{14}$$

can realize $\tilde{x}(t)$ and $\tilde{f}(t)$ uniformly ultimately bounded and the matrix $\Gamma \in \Re^{r\times r}$ is the learning rate with $*$ denoting the symmetric elements in a symmetric matrix.

4 Simulation Results

Consider a one-link manipulator, whose revolution joint is actuated by a dc motor. The joint elasticity is modeled by a linear torsional spring [12]. The states are the angular positions and velocities of the motor and of the link $x^T = (x_1, x_2, x_3, x_4) = (\theta_m, \omega_m, \theta_1, \omega_1)$. The state-pace model is

$$\begin{cases} \dot{\theta}_m = \omega_m \\ \dot{\omega}_m = \frac{k}{J_m}(\theta_1 - \theta_m) - \frac{b}{J_m}\omega_m + \frac{K_\tau}{J_m}u \\ \dot{\theta}_m = \omega_1 \\ \dot{\omega}_1 = \frac{k}{J_1}(\theta_1 - \theta_m) - \frac{mgh}{J_1}\sin(\theta_1) \end{cases} \tag{15}$$

with J_m and J_1 the inertia of the motor and the link. The system dynamics is nonlinear of the form (1). The numerical values of the parameters given in [12] are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, \qquad g\big(t, x(t)\big) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333 \sin x_3 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \qquad E = \begin{bmatrix} 0 \\ 12.5 \\ 0 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

By solving (8) and (9), one obtains that

$$P = \begin{bmatrix} 26.4820 & -0.8851 & 4.8286 & -15.3387 \\ -0.8851 & 0.0577 & -0.2463 & 1.1474 \\ 4.8286 & -0.2463 & 3.1286 & -4.0254 \\ -15.3387 & 1.1474 & -4.0254 & 38.0104 \end{bmatrix},$$

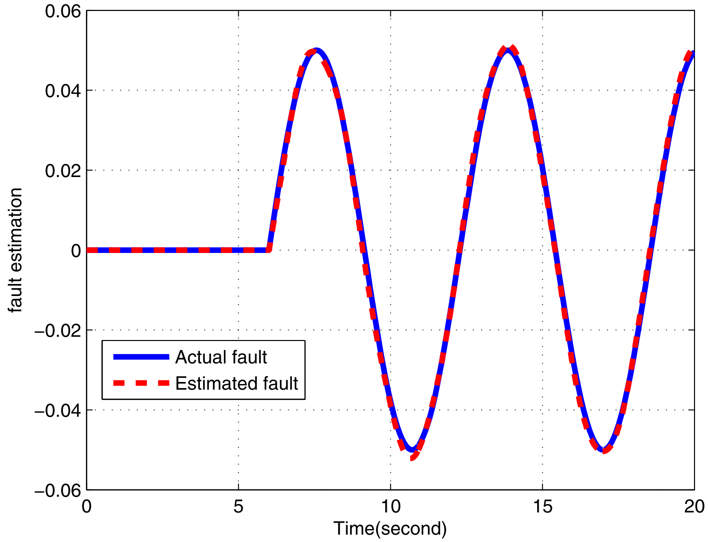

Fig. 1 The fault and its estimation

$$Y = \begin{bmatrix} 131.9468 & 78.7166 \\ -2.0634 & -0.8728 \\ -5.4386 & 15.8781 \\ 93.9841 & -51.1876 \end{bmatrix}, \qquad F = [-11.0643\ 0.7215],$$

$$L = \begin{bmatrix} 7.4263 & 5.2611 \\ -153.3504 & 149.9675 \\ -14.2173 & 4.5578 \\ 8.5929 & -3.2680 \end{bmatrix}.$$

Consider the following fault:

$$f(t) = \begin{cases} 0, & t < 6\ \text{s} \\ 0.05 \sin(t-6), & t \geq 6\ \text{s}. \end{cases}$$

By taking $\Gamma = 25$ and the sampling period $T = 0.01$ s, the simulation result can be obtained as follows with the help of the Matlab Simulink. Figure 1 shows that the proposed fault estimation observer and algorithm have a good performance to estimate the fault $f(t)$. It follows from Figs. 2–4 that the error states, system states and observer states respective responses $\tilde{x}(t)$, $x(t)$ and $\hat{x}(t)$ are obviously stable. From Figs. 3 and 4, the system (1) can be reflected by the designed observer very well.

Remark 5 Compared Fig. 1 in [12] with that of our paper we can find that the method proposed in our paper has better performance and less conservativeness. By using the proposed algorithm, we can estimate the fault more accurately.

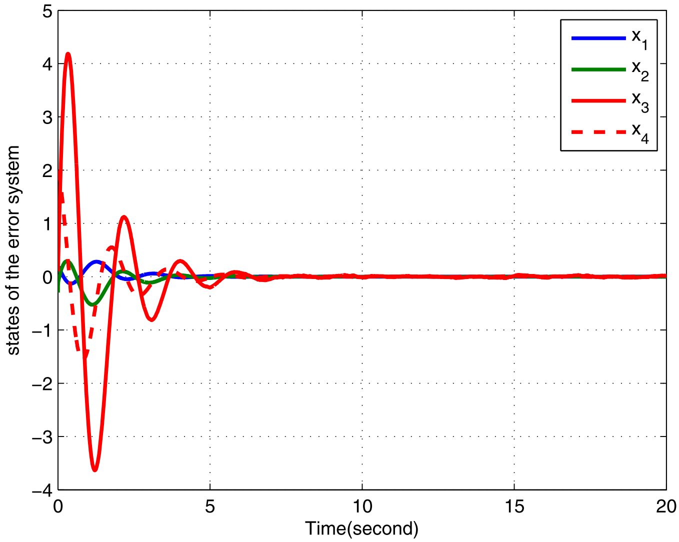

Fig. 2 The states of error system

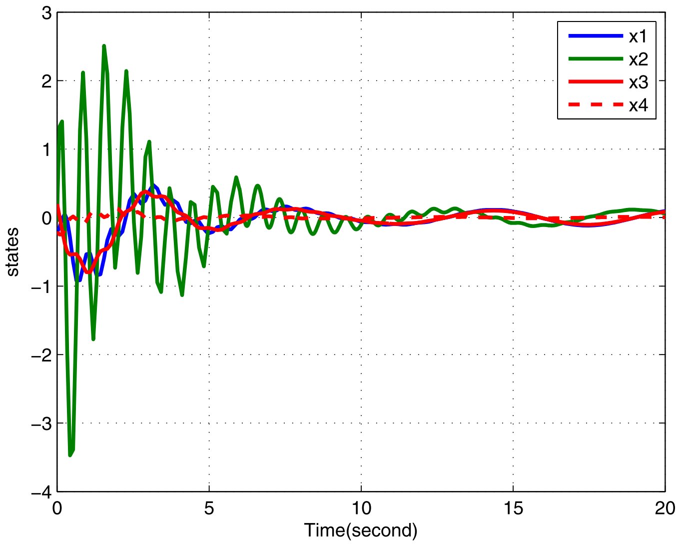

Fig. 3 The system states

5 Conclusions

This paper mainly discusses the fault detection and estimation for nonlinear dynamic systems. By constructing fault detection observer and fault estimation observer, the fault information is obtained. Based on the Lyapunov theorem, we give the adaptive fault estimation algorithm and a less conservative criteria, which guarantee the error

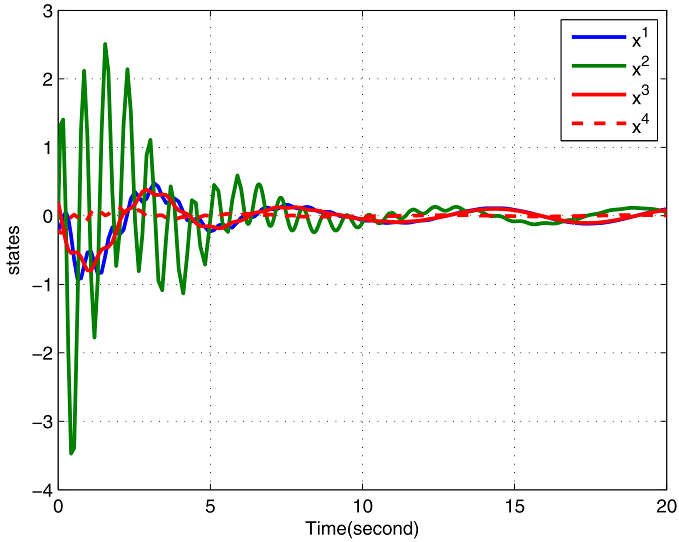

Fig. 4 The observer states

system stability. Consequently, a flexible joint robotic example is given to illustrate the efficiency of the proposed approach. Since most of industrial systems are uncertain and nonlinear, extension of the proposed method to robust fault diagnosis for uncertain nonlinear systems is another interesting issue.

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