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A Nonlinear Adaptive Resilient Observer Design for a Class of Lipschitz Systems Using LMI

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Abstract This paper addresses the parameter and state estimation problem in the presence of the observer gain perturbations for Lipschitz systems that are linear in the unknown parameters and nonlinear in the states. A nonlinear adaptive resilient observer is designed, and its stability conditions based on the Lyapunov technique are derived. The gain for this observer is derived systematically using the linear matrix inequality approach. A numerical example and a physical setup are provided to show the effectiveness of the proposed method.

Keywords Nonlinear systems · Resilient observer · Nonlinear observer · Adaptive observer · Robust estimation · Linear matrix inequality

1 Introduction

One of the major difficulties in the design of practical observers for most physical systems are their model uncertainties due to either constant or slow changes of unknown quantities such as unknown physical parameters. Adaptive observers have been used to cope with the lack of knowledge on the system parameters in state estimation problems.

For nonlinear systems with unknown parameters, various adaptive observers have been introduced [19–22]. In [1], the authors reported early results on adaptive observers for nonlinear systems, namely observers estimating the entire state vector using an on-line adaptation for the unknown parameters. The authors in [20–23] focused on a class of nonlinear systems which are transformable by a global parameter-independent state-space diffeomorphism into a system whose dynamics are linear in

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unmeasured states and nonlinear in inputs and measurable outputs. Then they designed an adaptive observer for the new system such that the state and parameter estimates both converge asymptotically under the persistence excitation condition. In these works, nonlinear terms are assumed to be related only to the input and the measured output, and disturbances are neglected.

This design method has been extended in [5] and [26] to cover the slightly more general case of systems where the nonlinear terms depend on the input and the entire state vector (not just measured outputs) with the nonlinearities satisfying Lipschitz conditions. In this work, a systematic algorithm is provided to check the feasibility of an asymptotically stable adaptive observer. An arbitrarily small disturbance may force the parameter estimates to drift toward infinity, while the state estimation error remains small [16, 23]. Several techniques have been introduced to modify the adaptive observer structure to prevent parameter estimation drift. For instance, in [16] and [23], this goal has been achieved by designing robust adaptive observers assuming that the nonlinear terms only depend on the input and the measured outputs.

In [9], a robust adaptive observer for sensorless induction-motor drives was designed based on the linearized dynamic equation and linear matrix inequality (LMI) method. The motor's dynamic equations are formulated in the form of a very special class of nonlinear system which is linear in feed-forward and nonlinear in the feedback. The stability conditions and the observer gain are obtained by solving the corresponding LMIs. In [15] an adaptive observer based fault diagnosis for satellite attitude control systems is used. In [24], LMI technique was used to design an observer for Lipschitz nonlinear systems. The design offers extra degrees of freedom over the classical static gain structure. Another LMI-based observer design for a class of Lipschitz nonlinear dynamical systems can be found in [28]. The differential mean value theorem allows the nonlinear error dynamics to be transformed into a linear parameter varying system. The authors introduced a general Lipschitz-like condition on the Jacobian matrix for differentiable systems. To ensure asymptotic convergence of the state estimation error, sufficient conditions are expressed in terms of LMIs. However, for large values of the Lipschitz constant, the stability conditions may become infeasible.

An observer for which the estimation error diverges by a small perturbation in the observer gain is referred to as fragile or non-resilient [12]. Since the observer gains are usually obtained from offline calculations, in many practical applications the gain may have slow drifts; thus, it is necessary that the observer tolerates some perturbations in its coefficients. The authors in [17] have shown that even vanishingly small perturbations in the control coefficients may destabilize the closed-loop system. Afterwards, more researchers concentrated their attention on this subject. In [6] an overview of the resilient design technique is presented. In [7], synthesis of a resilient regulator for the linear systems is provided. In [27], a robust resilient Kalman filter design for a class of linear systems with norm-bounded multiplicative uncertainties in the filter gain is introduced. In reference [13], an observer is designed using LMI approach to maintain disturbance attenuation performance in the case of randomly varying perturbations in the observer gain. In [14], an LMI solution for nonlinear resilient observer design is presented.

In this paper, we consider a fairly general class of nonlinear systems in which the nonlinearities are assumed to be Lipschitz, containing uncertain piecewise constant parameters in the presence of bounded perturbation on the observer gains. Our objective is to find an LMI-based adaptive observer gain for this class of nonlinear systems that is robust not only against perturbations in the gain matrix but also against perturbations in system parameters at the same time. The proposed observer stabilizes the state estimation error. Moreover, when the persistent excitation condition holds, the parameter estimation vector converges to its true value. Unlike [23], we allow the nonlinear terms in the system to depend on the input and all the states, in general, and we modified the adaptive law to overcome some drawback in parameter estimation. We also show that the proposed design is feasible for much larger values of the Lipschitz constants compared to those of the design in [5].

The rest of the paper is organized as follows: Sect. 2 provides the problem statement. In Sect. 3, the proposed resilient adaptive observer is presented. A numerical example is provided in Sect. 4. A synchronous generator setup is discussed in Sect. 5 as a case study for the method proposed. Finally, the conclusion remarks are given in Sect. 6.

2 Problem Statement

Consider an uncertain nonlinear system of the form:

$$\begin{aligned} \dot{x} &= Ax + \phi(x,u) + bf(x,u)\theta \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in \Re^n$, $u \in \Re^q$, $y \in \Re^m$, and $\theta \in \Re^p$ are the state, input, output, and parameter vectors, respectively, $b \in \Re^{n\times m}$, $C \in \Re^{m\times n}$ are constant matrices, and $f : [\Re^n\ \Re^q] \to \Re^{m\times p}$, $\phi : [\Re^n\ \Re^q] \to \Re^n$ are nonlinear functions which are Lipschitz in x with Lipschitz constants γ_1 and γ_2, respectively, i.e.:

$$\left\|\phi(x_1,u) - \phi(x_2,u)\right\| < \gamma_1\|x_1 - x_2\| \tag{2}$$

and

$$\left\|f(x_1,u) - f(x_2,u)\right\| < \gamma_2\|x_1 - x_2\| \tag{3}$$

for all $x_1, x_2 \in R^n$. System (1) is linear in θ and nonlinear in x with Lipschitz nonlinearities. This is a fairly general class, since, in most cases, nonlinearities are bounded in a Lipschitz manner if the states are bounded. We assume that the unknown piecewise constant parameter vector and its distance from nominal parameter vector θ_0 are both bounded in the following sense:

$$\|\theta\| \le \gamma_3 \tag{4}$$

$$\|\theta - \theta_0\| \le M \tag{5}$$

Lemma 1 (Schur complement [3]) *The LMI*:

$$\begin{bmatrix} Q(x) & S(x) \\ S^{\mathrm{T}}(x) & R(x) \end{bmatrix} > 0 \tag{6}$$

where $Q(x) = Q^{\mathrm{T}}(x)$, $R(x) = R^{\mathrm{T}}(x)$, *and* $S(x)$ *affinely depend on* x, *is equivalent to*

$$\begin{cases} R(x) > 0 \\ Q(x) - S(x)R^{-1}(x)S^{\mathrm{T}}(x) > 0 \end{cases} \tag{7}$$

Lemma 2 ([4]) *Let* x, y *be real vectors of the same dimension. Then, for any scalar* $\varepsilon > 0$, *the following inequality holds*:

$$2x^{\mathrm{T}}y \le \varepsilon x^{\mathrm{T}}x + \varepsilon^{-1}y^{\mathrm{T}}y \tag{8}$$

3 Resilient Adaptive Observer Design

Consider a nonlinear adaptive observer of the form [5]

$$\dot{\hat{x}} = A\hat{x} + \phi(\hat{x}, u) + bf(\hat{x}, u)\hat{\theta} + \big[L + \Delta(t)\big](y - C\hat{x}) \tag{9}$$

where $\hat{x}$ and $\hat{\theta}$ are the state and parameter estimates, respectively, L is the observer gain and the resilient term $\Delta(t)$ is an additive perturbation on the gain with known bound $\|\Delta(t)\| \le r$ for all t.

Then, the observer error dynamic equation is obtained as

$$\dot{\tilde{x}} = (A - LC - \Delta C)\tilde{x} + \phi(x, u) - \phi(\hat{x}, u) + bf(x, u)\theta - bf(\hat{x}, u)\hat{\theta} \tag{10}$$

where $\tilde{x} = x - \hat{x}$ is the state estimation error.

The following theorem provides sufficient conditions for the stability of the robust adaptive observer (9).

Theorem 1 *Consider the following parameter adaption law*:

$$\dot{\hat{\theta}} = \Gamma^{-1}\big(f(\hat{x}, u)^{\mathrm{T}}C\tilde{x}\big) - \sigma\Gamma^{-1}(\hat{\theta} - \theta_0) \tag{11}$$

where $\Gamma = \Gamma^{\mathrm{T}} > 0$ *is an arbitrary constant matrix and*:

$$\sigma = \begin{cases} 0 & \text{if } \|\hat{\theta} - \theta_0\| < M \\ \sigma_0\big(\frac{\|\hat{\theta}-\theta_0\|}{M} - 1\big) & \text{if } M \le \|\hat{\theta} - \theta_0\| \le 2M \\ \sigma_0 & \text{if } \|\hat{\theta} - \theta_0\| > 2M \end{cases} \tag{12}$$

with positive constants scalars M *and* σ_0. *If there exist positive real numbers* $\varepsilon_1, \varepsilon_2, \varepsilon_3$ *and matrices* $P = P^{\mathrm{T}} > 0$ *and* S, *such that* $Pb = C^{\mathrm{T}}$ *and*

$$\begin{bmatrix} \Lambda & P & P & P \\ P & -\varepsilon_1 I & 0 & 0 \\ P & 0 & -\varepsilon_2 I & 0 \\ P & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{13}$$

where

$$\Lambda = A^{\mathrm{T}}P - C^{\mathrm{T}}S + PA - S^{\mathrm{T}}C + \big(\varepsilon_1\gamma_1^2 + \varepsilon_2\gamma_2^2\gamma_3^2\,\|b\|^2\big)I + r^2\varepsilon_3 C^{\mathrm{T}}C$$

with γ_1, γ_2, and γ_3 defined in (2), (3), and (4), respectively, then the observer gain $L = P^{-1}S^{\mathrm{T}}$ stabilizes the state estimation error dynamics in (10) while the parameter estimation error remains bounded. Moreover, if the following persistency excitation condition holds $\forall t_0, \exists \xi, \delta > 0$ such that:

$$\int_{t_0}^{t_0+\delta} bf\big(x(\tau), u(\tau)\big) f^{\mathrm{T}}\big(x(\tau), u(\tau)\big) b^{\mathrm{T}}\, d\tau > \xi I \tag{14}$$

then, the parameter estimate vector converges to its true value for all disturbances satisfying $\|\Delta(t)\| \le r$.

Proof Consider the following Lyapunov function candidate for error dynamic (10):

$$V = \tilde{x}^{\mathrm{T}} P \tilde{x} + \tilde{\theta}^{\mathrm{T}} \Gamma \tilde{\theta} \tag{15}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter estimation error. Taking the derivative of (15) and using (10), results in

$$\begin{aligned} \dot{V} = {} & \tilde{x}^{\mathrm{T}}\big[(A - LC - \Delta C)^{\mathrm{T}} P + P(A - LC - \Delta C)\big]\tilde{x} + 2\big[\phi(x,u) - \phi(\hat{x},u)\big]^{\mathrm{T}} P\tilde{x} \\ & + 2\big[bf(x,u)\theta - bf(\hat{x},u)\hat{\theta}\big]^{\mathrm{T}} P\tilde{x} + 2\tilde{\theta}^{\mathrm{T}}\Gamma\dot{\tilde{\theta}} \end{aligned} \tag{16}$$

Using Lemma 2 and inequality (2) on the second term, and substituting $\hat{\theta} = \theta - \tilde{\theta}$ in the third term of (16) result in

$$\begin{aligned} \dot{V} \le {} & \tilde{x}^{\mathrm{T}}\big[(A - LC)^{\mathrm{T}} P + P(A - LC)\big]\tilde{x} - 2\tilde{x}^{\mathrm{T}} C^{\mathrm{T}} \Delta^{\mathrm{T}} P\tilde{x} \\ & + \tilde{x}^{\mathrm{T}}\big(\varepsilon_1\gamma_1^2 + \varepsilon_1^{-1} PP\big)\tilde{x} + 2\big[bf(x,u)\theta - bf(\hat{x},u)\theta\big]^{\mathrm{T}} P\tilde{x} \\ & + 2\big[bf(\hat{x},u)\tilde{\theta}\big]^{\mathrm{T}} P\tilde{x} + 2\tilde{\theta}^{\mathrm{T}}\Gamma\dot{\tilde{\theta}} \end{aligned} \tag{17}$$

Again, applying Lemma 2 to the second and the fourth term of inequality (17) with ε_3 and ε_2, respectively, and using (3) and (4) and $\|\Delta(t)\| \le r$ it follows that

$$\begin{aligned} \dot{V} \le {} & \tilde{x}^{\mathrm{T}}\big[\Omega + \varepsilon_1^{-1} PP + \varepsilon_2^{-1} PP + \varepsilon_3^{-1} PP\big]\tilde{x} + \varepsilon_3 r^2 \tilde{x}^{\mathrm{T}} C^{\mathrm{T}} C\tilde{x} \\ & + 2\big[bf(\hat{x},u)\tilde{\theta}\big]^{\mathrm{T}} P\tilde{x} + 2\tilde{\theta}^{\mathrm{T}}\Gamma\dot{\tilde{\theta}} \end{aligned} \tag{18}$$

where

$$\Omega = (A - LC)^{\mathrm{T}} P + P(A - LC) + \varepsilon_1\gamma_1^2 I + \varepsilon_2\gamma_2^2\gamma_3^2\|b\|^2 I.$$

Since θ is piecewise constant, thus, we assume $\dot{\theta} = 0$ and thus $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$. Using this fact, substituting (11) in (18), and using $b^{\mathrm{T}} P = C$ yield

$$\dot{V} \le \tilde{x}^{\mathrm{T}}\big[\Omega + \big(\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_3^{-1}\big) PP + \varepsilon_3 r^2 C^{\mathrm{T}} C\big]\tilde{x} + 2\sigma\tilde{\theta}^{\mathrm{T}}\big(\hat{\theta} - \theta_0\big) \tag{19}$$

Then, using (5) it follows that

$$\begin{aligned}\sigma\tilde{\theta}^{\mathrm{T}}(\hat{\theta}-\theta_0) &= \sigma\left[\theta-\hat{\theta}\right]^{\mathrm{T}}(\hat{\theta}-\theta_0)\\ &= \sigma(\theta-\theta_0)^{\mathrm{T}}(\hat{\theta}-\theta_0)-\sigma(\hat{\theta}-\theta_0)^{\mathrm{T}}(\hat{\theta}-\theta_0)\\ &\le \sigma M\|\hat{\theta}-\theta_0\|-\sigma\|\hat{\theta}-\theta_0\|^2\\ &\le N\end{aligned} \tag{20}$$

where:

$$N=\sigma\|\hat{\theta}-\theta_0\|\left(M-\|\hat{\theta}-\theta_0\|\right) \tag{21}$$

If condition (12) holds, the derived upper bound N in (20) is always is non-positive, because of the following. For $\|\hat{\theta}-\theta_0\|<M$, since $\sigma=0$, we have $N=0$. For $M\le\|\hat{\theta}-\theta_0\|\le 2M$, we have $N=-\frac{\sigma_0}{M}\|\hat{\theta}-\theta_0\|(M-\|\hat{\theta}-\theta_0\|)^2\le 0$. For $\|\hat{\theta}-\theta_0\|>2M$, we have $N\le-\sigma_0 M\|\hat{\theta}-\theta_0\|\le 0$.

Therefore, it follows that

$$2\sigma\tilde{\theta}^{\mathrm{T}}(\hat{\theta}-\theta_0)\le 0 \tag{22}$$

Substituting the above inequality in (19), the sufficient condition for $\dot{V}<0$ is

$$\left[\Omega+\left(\varepsilon_1^{-1}+\varepsilon_2^{-1}+\varepsilon_3^{-1}\right)PP+r^2\varepsilon_3C^{\mathrm{T}}C\right]<0 \tag{23}$$

To convert the above inequality to LMI, using Schur complement, we rewrite it as

$$\begin{bmatrix}\Omega+r^2\varepsilon_3C^{\mathrm{T}}C & P & P & P\\ P & -\varepsilon_1 I & 0 & 0\\ P & 0 & -\varepsilon_2 I & 0\\ P & 0 & 0 & -\varepsilon_3 I\end{bmatrix}<0 \tag{24}$$

Thus LMI (13) is obtained where$S=L^{\mathrm{T}}P$ and $\Lambda=\Omega+r^2\varepsilon_3C^{\mathrm{T}}C$.

The inequality (23) follows:

$$\left[\Omega+\left(\varepsilon_1^{-1}+\varepsilon_2^{-1}+\varepsilon_3^{-1}\right)PP+r^2\varepsilon_3C^{\mathrm{T}}C\right]\le-\alpha I \tag{25}$$

Substituting (25) and (22) into (19), yields:

$$\dot{V}\le-\alpha\tilde{x}^{\mathrm{T}}\tilde{x} \tag{26}$$

Integrating both sides of inequality (26) from $t=0$ to $t=t_f$ it follows that

$$V(t_f)\le V(0)-\alpha\int_0^{t_f}\tilde{x}^{\mathrm{T}}(\tau)\tilde{x}(\tau)\,d\tau \tag{27}$$

Since $V(x,\theta)\ge 0$ and is non-increasing, $V\in L_\infty$. Consequently, from the definition (15) it follows that $\tilde{x}\in L_\infty$ and $\tilde{\theta}\in L_\infty$. Moreover, (27) implies that $\tilde{x}\in L_2$. Moreover, since both $\phi(x,u)$ and $f(x,u)$ are Lipschitz, (10) yields $\dot{\tilde{x}}\in L_\infty$. With $\tilde{x}\in L_\infty, \tilde{x}\in L_2$ and $\dot{\tilde{x}}\in L_\infty$, and using Barbalat's lemma [17] it follows that

$\lim_{t\to\infty} \tilde{x}(t) = 0$, and consequently, it can also be concluded that $\lim_{t\to\infty} \dot{\tilde{x}}(t) = 0$. Therefore, considering (10), we have

$$\lim_{t\to\infty} \left(bf(x,u)\theta - bf\left(\hat{x},u\right)\hat{\theta}\right) = 0 \tag{28}$$

Since $\lim_{t\to\infty} \hat{x} = x$, (28) reduces to

$$\lim_{t\to\infty} \left(bf(x,u)\left(\theta - \hat{\theta}\right)\right) = 0 \tag{29}$$

Thus if the persistency excitation condition (14) holds, we can say that the parameter estimates converge to their true values ($\hat{\theta} \to \theta$) for all gain perturbations satisfying $\|\Delta(t)\| \le r$. □

Remark 1 The second term in (11) is added as leakage modification which modifies the adaptive law so that the time derivative of the Lyapunov function remains negative in the parameter estimate space when these parameters exceed certain bounds [11]. As we have shown, if the persistency excitation condition (14) holds, this bound shrinks to a point, which is the true parameter vector.

Remark 2 From the theorem, since (27) is non-increasing, we have

$$\tilde{x}^{\mathrm{T}} P \tilde{x} + \tilde{\theta}^{\mathrm{T}} \Gamma \tilde{\theta} \le V(0) \tag{30}$$

This implies that

$$\begin{aligned} \tilde{\theta}^{\mathrm{T}} \Gamma \tilde{\theta} &\le V(0) \\ &\le \tilde{x}^{\mathrm{T}}(0) P \tilde{x}(0) + \tilde{\theta}^{\mathrm{T}}(0) \Gamma \tilde{\theta}(0) \end{aligned} \tag{31}$$

Using the Rayleigh–Ritz inequality in (31) gives

$$\|\tilde{\theta}\| \le \sqrt{\frac{\lambda_{\max}(P)\|\tilde{x}(0)\|^2 + \lambda_{\max}(\Gamma)\|\tilde{\theta}(0)\|^2}{\lambda_{\min}(\Gamma)}} \tag{32}$$

where $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ denote the minimum and maximum singular values of its argument, respectively. By increasing $\lambda_{\min}(\Gamma)$, dependency of the parameter estimation error bound to the initial state estimation decreases. However, increasing Γ slows down the convergence of parameter estimate vector (11). Therefore, the trade off in selecting Γ should be considered in the design. For the case that the persistent excitation condition is not met, (32) gives the resulting worst case bound on the parameter estimation error.

Remark 3 Assume that Lipschitz constants in (2) and (3) are defined in the local regions such that $\|x_1\| \le \varepsilon$ and $\|x_2\| \le \varepsilon$ are always satisfied; then we have

$$\begin{aligned} &\dot{V} < 0, \qquad V = \tilde{x}^{\mathrm{T}} P \tilde{x} \\ &V > 0, \qquad \dot{V} < 0 \quad \Rightarrow \quad V(t) < V(0) \end{aligned} \tag{33}$$

thus

$$\lambda_{\min}(P)\|\tilde{x}\|^2 \le V(t) < V(0) \tag{34}$$

This implies that

$$\|\tilde{x}\| \le \sqrt{\frac{V(0)}{\lambda_{\min}(P)}} \le \|\tilde{x}(0)\|\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \tag{35}$$

We know $\|\hat{x}\| \le \|\tilde{x}\| + \|x\|$, thus in order to guarantee the $\|\hat{x}\| \le \varepsilon$, x should satisfy $\|\tilde{x}\| + \|x\| \le \varepsilon$, and thus

$$\|x\| \le \varepsilon - \|\tilde{x}(0)\|\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \tag{36}$$

Inequality (36) shows the region of stability of the observer.

Remark 4 The equality constraint $b^{\mathrm{T}}P = C$ is restrictive. However, the LMI toolbox and Yalmip package among others can efficiently solve the combination of equality and inequality constraints. Alternatively one can find a set of matrices $\{P_i\}$ which form a basis for P such that $b^{\mathrm{T}}PC^{\perp} = 0$. This set of P_i can then be used to check if they make LMI (13) feasible.

4 Numerical Example

Consider the following nonlinear system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\sin(x_2) + 4u(t) \end{bmatrix} + 2\theta\begin{bmatrix} -\cos(x_2) + \sin(0.5t) \\ 0 \end{bmatrix}$$
$$y = [1\ 0]x$$

with a unit step function input as $u(t)$, and unknown parameter $\theta = 3$ for $0 \le t < 20$, with abrupt change to $\theta = 5$ for $t \ge 20$. Moreover, consider that at time $t = 20$ and $t = 25$ to the observer gain are added values of -0.8 and -3.2, respectively, as an additive perturbation. The design parameters are chosen as $\gamma_1^2 = 0.3$, $\gamma_2^2 = 0.2$, $\gamma_3^2 = 5$, $\Gamma = 0.1$, $M = 5$, $\theta_0 = 2.5$, $\sigma_0 = 0.1$. Moreover, $\|\Delta(t)\| \le 3.5$ is considered as an uncertainty bound in the design. Using YALMIP toolbox as parser [19] and LMI Control Toolbox in MATLAB as solver [8], the solution is derived as $S = [6.97\ 2.33]$ and $P = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 0.73 \end{smallmatrix}\right]$, and $\varepsilon_1 = 3.0$, $\varepsilon_2 = 3.1$, $\varepsilon_3 = 2.98$. Hence, the observer gain is obtained as $L = [6.97\ 3.18]^{\mathrm{T}}$. For comparison purposes, we also implement the design method in [5] for the above system.

As is shown in Fig. 1, the gain obtained from the proposed resilient observer design causes the estimator to accurately track the system states while the method in [5] yields an unstable state estimation due to gain perturbation. Figure 2 shows that the parameter estimate in the proposed method also converges to its true value despite the abrupt changes of the real parameter. As we can see from the figures, when the strong

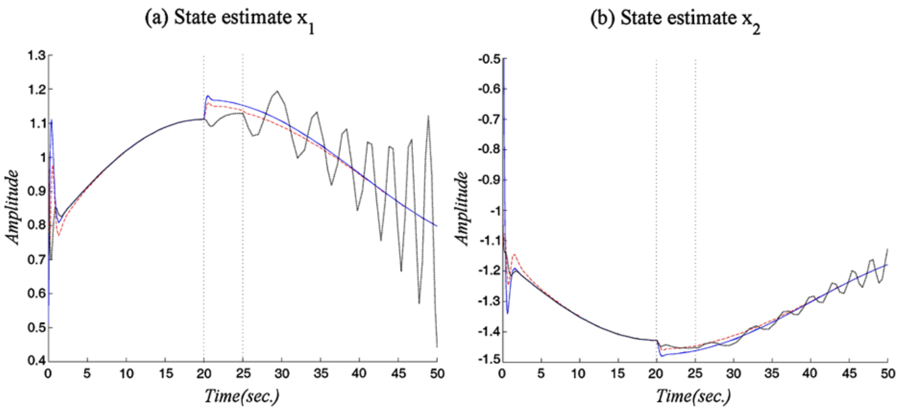


Fig. 1 Actual states (*solid*), state estimates of the proposed method (*dashed*), state estimates of [5] (*dotted*)

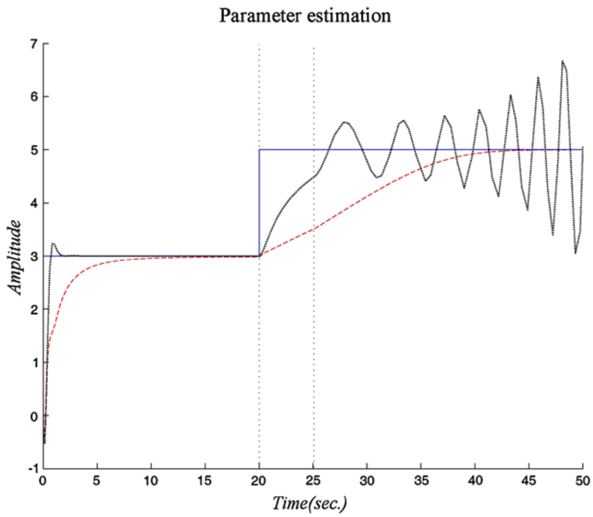


Fig. 2 Parameter estimate using the proposed adaptive observer (*dashed*) vs. the method in [5] (*dotted*)

gain perturbation at $t = 25$ s occurs, the proposed design remains robust, while the conventional adaptive observer [5] become unstable. As is expected, the gain perturbation at $t = 25$ s does not have much effect on the estimation in the proposed method because in the observer dynamics (9), the observer gain is multiplied by output error, and since the estimation error in the proposed method converges to zero the effect of gain perturbation is omitted. Moreover, Fig. 3 shows that the feasibility region of the LMI for the proposed method is much larger than that of the method in [5] for the changes of the Lipschitz constants γ_1 and γ_2.

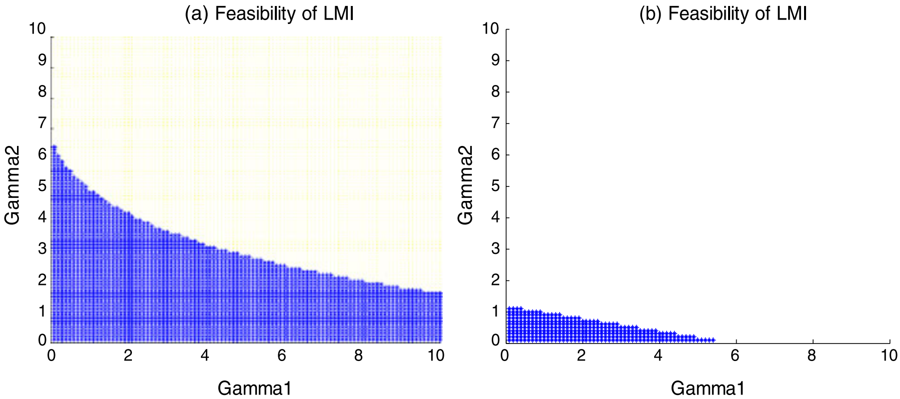


Fig. 3 LMI Feasibility space of Lipschitz constants: **a** the proposed method, **b** method in reference [5]

5 Synchronous Generator Setup

We use a model of a single machine connected to an infinite bus through a reactive transmission lines to the rest of the network, which is represented by an infinite bus. According to reference [2], the mechanical equation and the electrical equation of the synchronous machine can be expressed as follows:

$$M_g\ddot{\delta} + D\dot{\delta} + P_g = P_m \tag{37}$$

$$T'_{do}\dot{E}'_q + \frac{X_d}{X'_d}E'_q = -\left(\frac{X'_d - X_d}{X'_d}\right)V\cos(\delta) + E_{fd} \tag{38}$$

$$P_g = \frac{1}{X'_d}E'_q V\sin(\delta_m) + \frac{1}{2}\left(\frac{1}{X_q} - \frac{1}{X'_q}\right)V^2\sin(2\delta_m) \tag{39}$$

$$E_{fd} = \frac{\omega_0 M_f}{\sqrt{2}r_f}v_f \tag{40}$$

where the parameters are defined in Table 1. The state-space model of the system can be written as [2]

$$\begin{aligned}
\dot{\delta} &= \omega - \omega_0\\
\dot{\omega} &= \frac{\omega_0}{2H_g}P_m - \frac{\omega_0}{2H_g}\left(\frac{V}{x'_d}\right)\sin(\delta)E'_q - \frac{\omega_0}{2H_g}V^2\left(\frac{1}{x_q} - \frac{1}{x'_d}\right)\cos(\delta)\sin(\delta)\\
&\quad - \frac{D}{2H_g}(\omega - \omega_0)\\
\dot{E}'_q &= -\left(\frac{x_d}{T'_{do}x'_d}\right)E'_q + \left(\frac{x_d - x'_d}{T'_{do}x'_d}\right)V\cos(\delta) + \frac{1}{T'_{do}}E_f
\end{aligned} \tag{41}$$

The equilibrium points of the above system are solutions of

Table 1 Parameter definitions for the synchronous machine

Parameter	Definition
δ	Generator rotor angle referred to the infinite bus power angle
ω	Rotor angular speed
M_g	Per unit inertia constant
E'_q	Transient voltage of armature
D	Per unit damping constant
T'_{do}	Open circuit transient time constant
P_m	Constant mechanical power supplied by the turbine
x_d	Direct axis reactance
x_q	Quadrature axis reactance
x'_d	Transient direct axis reactance
x_l	Line reactance
$X_d = x_d + x_l$	Augmented reactance
X'_d	Transient augmented reactance
X_q	Quadrature axis augmented reactance
P_g	Generated power
E_{fd}	Equivalent EMF in the excitation coil
v_f	Field excitation voltage
V	Infinite bus voltage
M_f	Mutual inductance between stator and rotor windings
r_f	Field resistance

$$\begin{aligned}
&\omega^* - \omega_0 = 0\\
&m_1 - m_2 \sin(\delta^*) E'^*_q - m_3 \cos(\delta^*) \sin(\delta^*) - m_4(\omega^* - \omega_0) = 0\\
&-m_5 E'^*_q + m_6 \cos(\delta^*) + m_7 E^*_{fd} = 0
\end{aligned} \tag{42}$$

where the parameters m_i depend on the machine type, the transmission-line parameters, the rotor inertia and the infinite bus voltage, which are constant values depending on the operating point. These constants are defined as follows:

$$\begin{aligned}
&m_1 = \frac{P_m}{M_g}, \qquad m_2 = \frac{V}{M'_d}, \qquad m_3 = \frac{V^2}{M_g}\left(\frac{1}{X_q} - \frac{1}{X'_d}\right),\\
&m_4 = \frac{D}{M_g}, \qquad m_5 = \frac{X_d}{T'_{do}X'_d}, \qquad m_6 = \left(\frac{X_d - X'_d}{T'_{do}X'_d}\right), \qquad m_7 = \frac{1}{T'_{do}}.
\end{aligned} \tag{43}$$

For a given constant field voltage $E_{fd} = E^*_{fd}$, the generator has two equilibrium points: one stable and one unstable point. In the following analysis and design, the stable equilibrium point, which we denote by $[\delta^*\ \omega^*\ E'^*_q]^{\mathrm{T}}$ is considered. The system equations in terms of the set point error variables $\tilde{\delta} = \delta - \delta^*$, $\tilde{\omega} = \omega - \omega^*$, $\tilde{E}'_q = E'_q - E'^*_q$ and $u = E'_{fd} - E^*_{fd}$ are written as follows:

$$
\begin{aligned}
&\dot{\tilde{\delta}} = \tilde{\omega} \\
&\dot{\tilde{\omega}} = m_1 - m_2 \sin\bigl(\tilde{\delta} + \delta^*\bigr)\bigl(\tilde{E}'_q + E'^*_q\bigr) - \frac{m_3}{2} \sin\bigl(2\bigl(\tilde{\delta} + \delta^*\bigr)\bigr) - m_4\tilde{\omega} \\
&\dot{\tilde{E}}'_q = -m_5\bigl(\tilde{E}'_q + E'^*_q\bigr) + m_6 \cos\bigl(\tilde{\delta} + \delta^*\bigr) + m_7\bigl(u + E^*_{fd}\bigr)
\end{aligned}
\tag{44}
$$

We assume that synchronous generator's electrical parameters such as resistances, reactances, and time constants either can be obtained from the manufacturer datasheet or can be measured by the standard test procedures such as SSFR tests [10, 25] and only the mechanical power (P_m) supplied by the turbine is an unknown parameter. Therefore m_1 has been taken as an uncertain parameter and the other parameters m_i $(i = 2, \ldots, 7)$ are considered as fixed known parameters. Moreover, ω and armature's voltage are assumed as available measurements and the generator's power angle is bounded. Considering the states and parameters: $x = [\tilde{\delta}, \tilde{\omega}, E'_q]^{\mathrm{T}}$, $\theta = m_1$, we can write (44) in the form of (9) as

$$
\begin{aligned}
\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & -m_4 & 0 \\ 0 & 0 & -m_5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\
&\quad + \begin{pmatrix} 0 \\ -m_2 \sin(x_1 + \delta^*)(x_3 + E'^*_q) - \frac{m_3}{2} \sin 2(x_1 + \delta^*) \\ -m_5 E'^*_q + m_6 \cos(x_1 + \delta^*) + m_7(u + E^*_{fd}) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \theta \\
y &= (0\ 1\ 1) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}
\end{aligned}
\tag{45}
$$

Now, the proposed adaptive observer can be used for joint estimation of states and parameters in the synchronous machine given in (45).

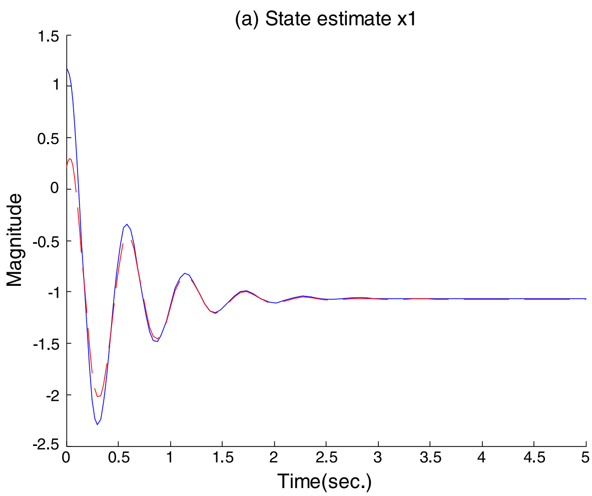


Fig. 4 Synchronous generator's power angle estimation

Fig. 5 Synchronous generator's rotor speed estimation

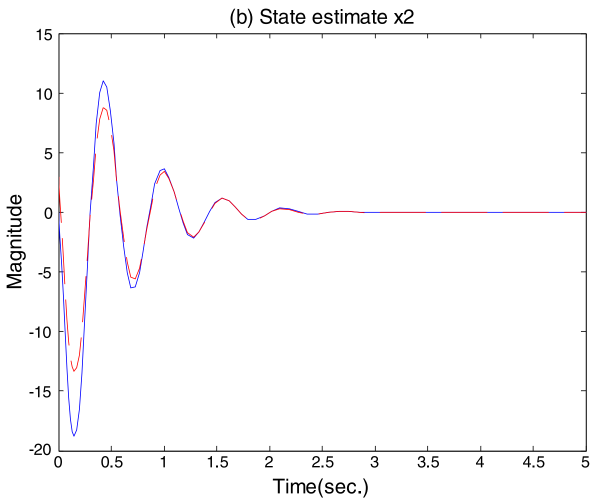
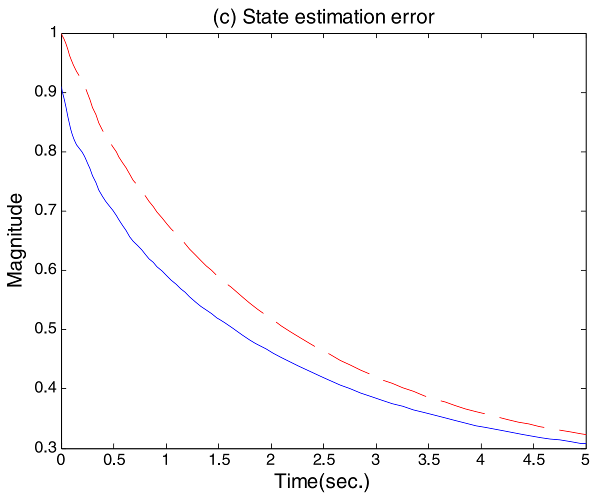


Fig. 6 Synchronous generator's transient voltage estimation



The synchronous generator parameter values (per unit) for this simulation are considered to be as follows:

$$X'_d = 0.5, \qquad X_d = 1.1, \qquad H = 8, \qquad T'_{do} = 5.5, \qquad X_l = 0.415,$$
$$E_f = 1.3, \qquad P_m = 1, \qquad D = 0.1, \qquad \omega_s = 377$$

The design parameters $\Gamma = 0.02$ and $\sigma_0 = 0.1$ are arbitrary fixed constants, and as our analysis shows, they may affect the convergence rates of the signals in the adaptive loops. The parameters $\theta_0 = 2$ and $\gamma_3 = 7$ are a priori knowledge of the system, and $M = 6$ is chosen such that $M > \|\theta_0\|$. Design parameters $\gamma_1 = 2$, $\gamma_2 = 0.2$ are Lipschitz constants and we assumed that our accuracy in implementation is such that we can guarantee $\|\Delta_1\| \leq 2$ and $\|\Delta_2\| \leq 2$. Using the proposed method, the observer gain is obtained as $L = [5.12\ 15.2\ 0.2]$.

Figures 4, 5, and 6 show the states estimation for synchronous generator in the presence of unknown mechanical power by using proposed resilient adaptive observer.

6 Conclusion

In this paper, we offered a systematic algorithm for designing an adaptive resilient observer for a class of nonlinear systems containing uncertain time-varying parameters in the presence of a bounded perturbation on the observer's gain. The resulting LMIs can systematically obtain the robust adaptive observer gains, which ensures that state estimates are under a certain bound, however, convergence of all the parameters depends on the persistency of the excitation.

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