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SHORT PAPER

Stochastic Stabilization of Markovian Jump Systems with Partial Unknown Transition Probabilities and Actuator Saturation

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Abstract The stochastic stabilization problem of Markovian jump systems subject to both partial unknown transition probabilities and actuator saturation is considered in this paper. Different from the previous results where complete knowledge on the transition probabilities is available, a new controller synthesis scheme is proposed as well as an estimate of the domain of attraction in mean square sense. A sufficient condition is first established to guarantee the stochastic stability of the closed-loop system. An optimization problem with LMI constraints is then formulated to determine the largest contractively invariant set in mean square sense. Finally, a numerical example is provided to show the effectiveness of our method.

Keywords Markovian jump systems · Partial unknown transition probabilities · Actuator saturation · Domain of attraction in mean square sense

1 Introduction

Markov jump systems (MJS) is a class of systems that has been an attractive subject of research during the last decades; to mention a few references, we refer to [3, 4]. This class of system is very appropriate to model plants whose structure is subject

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to random abrupt changes due to, for instance, random component failures, abrupt environment disturbance, changes of the operating point of a linearized model of a nonlinear system, etc. Many fundamental results for deterministic systems have been extended to stochastic systems [2, 6, 10, 13, 14].

There is a common assumption made in most of the existing literature, that is, a complete knowledge of the transition probabilities has to be available precisely a priori. However, it is now recognized that only partial transition probabilities can be obtained in some practical situations. This leads to another interesting topic concerning MJS: robust stability, robust stabilization and robust performance for Markovian jump systems with uncertain switching probabilities. Xiong et al. studied the robust controller design problem for systems involving parameter uncertainties in both system matrices and mode transition probability matrix [15]. Recently, an interesting result was proposed in [16], where the stability and stabilization problems are addressed for MJS with partly unknown transition probabilities.

On the other hand, nearly all practical control problems involve plants whose actuators or sensors are limited by inherent physical constraints. It is well known that such nonlinearities can produce significant performance degradation and possible instability for the closed-loop systems. Because of its practical and theoretical importance, stability analysis and synthesis for control systems with actuator or sensor saturation has been studied by many researchers for a long time, for example [1, 7, 9, 11, 17–19]. However, as far as we know, there is little research on MJS subject to actuator saturation. Boukas considered the output feedback control for uncertain time-delay systems with saturating actuators [5]. In [8], the quadratic optimal control problem of a discrete-time MJS with constraints on the state and control variables was presented. Liu et al. [12] studied the control design problem for MJS subject to actuator saturation based on the work of [9].

Since both actuator saturation and uncertainty in transition probabilities are often encountered in practice, it is of great importance to study the robust stochastic stabilization for MJS subject to both undesirable phenomena. To the best of our knowledge, there is no relevant document concerning all the factors mentioned above which motivates our study.

In this paper, we are concerned with designing state feedback controller for MJS in the presence of both partial unknown transition probabilities and saturating actuators. A sufficient condition is first established to guarantee the stochastic stability of the closed-loop system. An optimization problem with LMI constraints is then formulated to determine the largest contractively invariant set in mean square sense. Finally, a numerical example is provided to show the effectiveness of our method.

Notation: In this paper, the superscript "T" stands for matrix transposition, R^n denotes the n dimensional Euclidean space, $R^{n\times m}$ is the set of all $n \times m$ real matrices and N^+ represents the sets of positive integers, respectively. For notation $(\varXi, \varUpsilon, \varTheta)$, $\varXi$ represents the sample space, $\varUpsilon$ is the σ-algebra of subsets of the sample space and $\varTheta$ is the probability measure on $\varUpsilon$. The notation $P > 0$ (respectively, $P \geq 0$) means that P is symmetric and positive definite (respectively, positive semi-definite). $\mathrm{diag}\{M_1, M_2, \ldots, M_N\}$ denotes a block diagonal matrix with diagonal blocks being the matrix $M_1, M_2, \ldots, M_N$. $G + G^T$ is denoted as $G + (\cdot)^T$ for simplicity. In symmetric block matrices, we use $\star$ as an ellipsis for terms that are induced by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2 Problem Statement and Preliminaries

Let us consider the following dynamic system defined in a probability space (Ξ, Υ, Θ):

$$\dot{x}(t) = A(\gamma(t))x(t) + B(\gamma(t))\sigma(u(t)) \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input. The continuous-time, discrete-state homogeneous Markov jumping process $\{\gamma(t), t \geq 0\}$ takes discrete values in a given finite set $S = \{1, 2, \ldots, s\}$ and has the following mode transition rates:

$$P\{\gamma(t+\Delta) = k|\gamma(t) = i\} = \begin{cases} \lambda_{ik}\Delta + o(\Delta) & i \neq k \\ 1 + \lambda_{ii}\Delta + o(\Delta) & i = k \end{cases}$$

where $\Delta > 0$, $\lambda_{ik} \geq 0$ for $i \neq k$ and $\lambda_{ii} = -\sum_{k=1,k\neq i}^{s} \lambda_{ik}$ for each mode i, $o(\Delta)/\Delta \to 0$ as $\Delta \to 0$. For $\gamma(t) = i \in S$, the system matrices of the ith mode are denoted by A_i, B_i. On the other hand, the transition rates of the Markov jumping process are considered to be partially known. That is, some elements in the transition rate matrix are not accessible. For example, for system (1) with four operation modes, the transition rate matrix may be of the form

$$\Lambda = \begin{bmatrix} \lambda_{11} & ? & ? & \lambda_{14} \\ ? & \lambda_{22} & \lambda_{23} & ? \\ ? & \lambda_{32} & ? & \lambda_{34} \\ \lambda_{41} & ? & \lambda_{43} & ? \end{bmatrix}$$

where "?" represents the inaccessible elements. Note that this kind of more general expression of transition rate was first proposed in [16]. For the simplicity of presentation, let us introduce the following notations. For all $i \in S$, let

$$S = S_\kappa^i + S_{u\kappa}^i$$

with

$$\begin{aligned} S_\kappa^i &\triangleq \{k : \lambda_{ik} \text{ is known}\} \\ S_{u\kappa}^i &\triangleq \{k : \lambda_{ik} \text{ is unknown}\} \end{aligned} \tag{2}$$

In addition, if $S_\kappa^i \neq \emptyset$, then S_κ^i can be also rewritten as

$$S_\kappa^i = \left(\kappa_1^i, \ldots, \kappa_m^i\right), \quad \forall\, 1 \leq m \leq s \tag{3}$$

where $\kappa_m^i \in N^+$ represents the mth known element with the index κ_m^i in the ith row of matrix Λ. Moreover, we denote $\lambda_\kappa^i \triangleq \sum_{k\in S_\kappa^i} \lambda_{ik}$ in the following discussion.

The function $\sigma(\cdot) : R^m \to R^m$ is the standard saturation function, i.e.,

$$\sigma(u) = [\,\sigma(u_1)\;\; \sigma(u_2)\;\; \cdots\;\; \sigma(u_m)\,]^T$$

where

$$\sigma(u_i) = \operatorname{sign}(u_i)\min\{1, |u_i|\}$$

Remark 1 In probability theory, a continuous-time Markov process is a stochastic process $\{\gamma(t), t \geq 0\}$ that satisfied the Markov property and takes values from a given set. The process is characterized by "transition rates" λ_{ij} between states i and j. The transition rates λ_{ij} are typically given as the ijth elements of the transition rate matrix Λ (also known as an intensity matrix). Since the transition rate matrix contains rates, the rate of departing from one state to arrive at another should be positive. On the other hand, the rates for a given state should sum to zero, yielding the diagonal elements to be

$$\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$$

As a result, the rate that the system remains in a state should be negative.

Our aim is to design mode-dependent state feedback controller

$$u(t) = F(\gamma(t))x(t) \tag{4}$$

where $F_i \in R^{m \times n}$ ($\forall\, \gamma(t) = i \in S$) is the controller gain to be determined, such that the closed-loop system is stochastically stable. Moreover, an estimate of the domain of attraction in mean square sense will also be provided.

The following definition will be adopted in the rest of this paper.

Definition 1 [12] A set $\$ \subset R^n$ is called the domain of attraction in mean square sense of system (1), if for any initial mode $\gamma_0 \in S$ and initial state $x_0 \in \$$, the solution $x(t, x_0, \gamma_0)$ of (1) satisfies

$$\lim_{T_f \to \infty} E\left\{ \int_0^{T_f} x^T(t, x_0, \gamma_0) x(t, x_0, \gamma_0)\, dt \mid x_0, \gamma_0 \right\} \leq x_0^T \Upsilon x_0$$

for some matrix $\Upsilon > 0$.

As pointed out in [12], if the initial state x_0 is located in the domain of attraction in mean square sense, then the closed-loop system will be stochastically stable at the origin.

Let the jth row of the matrix F_i be f_{ij}. We define the following symmetric polyhedron:

$$\mathbf{L}(F_i) = \left\{ x \in R^n : -1 \leq f_{ij} x \leq 1,\ j = 1, 2, \ldots, m \right\}$$

For any matrix $P_i > 0$, one can define the ellipsoid

$$\Omega(P_i) = \left\{ x \in R^n : x^T P_i x \leq 1 \right\}$$

Let $\mathcal{D}$ be the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. It is obvious that there are 2^m elements in $\mathcal{D}$. In the case of $m = 2$,

$$\mathcal{D} = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

Denote each element of $\mathcal{D}$ as D_j, $j = 1, \ldots, 2^m$, and let $D_j^- = I - D_j$. Note that D_j^- is also an element of $\mathcal{D}$ if $D_j \in \mathcal{D}$.

In order to obtain our main result in this paper, the following lemma is needed.

Lemma 1 [9] *For given matrices* $F, H \in R^{m\times n}$, *if* $x \in \mathbf{L}(H)$, *we have*

$$\sigma(Fx) \in Co\{D_j Fx + D_j^- Hx\} \quad j = 1, \ldots, 2^m$$

Hence, $\sigma(Fx)$ *can be expressed as*

$$\sigma(Fx) = \sum_{j=1}^{2^m} \eta_j (D_j F + D_j^- H)x \tag{5}$$

where $\sum_{j=1}^{2^m} \eta_j = 1$ *and* $\eta_j \geq 0$.

3 Main Results

In this section, we design state feedback controller of the form (4) that will stochastically stabilize system (1) with partial unknown transition rates (2).

Theorem 1 *Given system* (1) *with partly unknown transition probabilities* (2), *if there exist matrices* H_i, *and symmetric positive definite matrices* P_i *with appropriate dimensions such that for all* $i = 1, 2, \ldots, s$, $j = 1, 2, \ldots, 2^m$

$$(1 + \lambda_\kappa^i)\{[A_i + B_i(D_j F_i + D_j^- H_i)]^T P_i + (\cdot)^T\} + \sum_{k\in S_\kappa^i} \lambda_{ik} P_k < 0 \tag{6}$$

$$[A_i + B_i(D_j F_i + D_j^- H_i)]^T P_i + (\cdot)^T + P_k \geq 0, \quad \forall k \in S_{u\kappa}^i,\ k = i \tag{7}$$

$$[A_i + B_i(D_j F_i + D_j^- H_i)]^T P_i + (\cdot)^T + P_k \leq 0, \quad \forall k \in S_{u\kappa}^i,\ k \neq i \tag{8}$$

and

$$\Omega(P_i) \subset \mathbf{L}(H_i) \tag{9}$$

then the set of intersection of ellipsoids $\Omega(P_i)$, *i.e.*, $\bigcap_{i=1}^s \Omega(P_i)$ *is contained in the domain of attraction in mean square sense of the closed-loop system under the controller* (4).

Proof Consider system (1) with the controller (4), and let the mode $\gamma(t) = i \in S$ and $P_i > 0$. Let us choose a stochastic Lyapunov function candidate $V(x(t), \gamma(t) = i) = x^T(t)P_i x(t)$. The weak infinitesimal operator ∇ of the stochastic process $\{x(t), \gamma(t), t\}$ is

$$\begin{aligned}
&\nabla V(x(t), \gamma(t) = i) \\
&\quad = x^T(t)\left\{\sum_{k=1}^{s} \lambda_{ik} P_k\right\} x(t) + 2[A_i x(t) + B_i\sigma(F_i x(t))]^T P_i x(t)
\end{aligned}$$

From (9), it follows that if $x(t) \in \bigcap_{i=1}^{s} \Omega(P_i)$, then $x(t) \in \mathbf{L}(H_i)$. Using Lemma 1, we have

$$\begin{aligned}
&\nabla V(x(t), \gamma(t) = i) \\
&\quad = 2x^T(t)\left[A_i + B_i \sum_{j=1}^{2^m} \eta_j (D_j F_i + D_j^- H_i)\right]^T P_i x(t) + x^T(t)\left\{\sum_{k=1}^{s} \lambda_{ik} P_k\right\} x(t) \\
&\quad = \sum_{j=1}^{2^m} \eta_j \, x^T(t) \Gamma_{ij} x(t)
\end{aligned}$$

with

$$\Gamma_{ij} = \left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T + \sum_{k=1}^{s} \lambda_{ik} P_k$$

By the fact that $\sum_{k \in S} \lambda_{ik} = 0$, we can rewrite Γ_{ij} as

$$\begin{aligned}
\Gamma_{ij} \triangleq{}& \left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T + \sum_{k=1}^{s} \lambda_{ik} P_k \\
&+ \sum_{k \in S} \lambda_{ik} \left\{\left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T\right\}
\end{aligned}$$

To decompose the transition rate matrix into the known and the unknown parts, one has

$$\begin{aligned}
\Gamma_{ij} ={}& \left(1 + \sum_{k \in S_\kappa^i} \lambda_{ik}\right)\left\{\left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T\right\} + \sum_{k \in S_\kappa^i} \lambda_{ik} P_k \\
&+ \sum_{k \in S_{u\kappa}^i} \lambda_{ik} \left\{\left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T\right\} + \sum_{k \in S_{u\kappa}^i} \lambda_{ik} P_k \\
={}& (1 + \lambda_\kappa^i)\left\{\left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T\right\} + \sum_{k \in S_\kappa^i} \lambda_{ik} P_k \\
&+ \sum_{k \in S_{u\kappa}^i} \lambda_{ik} \left\{\left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T + P_k\right\}
\end{aligned}$$

Note that $\forall k \in S_{u\kappa}^i$ and if $i \in S_\kappa^i$, we can get $\Gamma_{ij} < 0$ by conditions (6), (8) and the fact $\lambda_{ik} \geq 0\, (\forall i, k \in S, i \neq k)$. On the other hand, in the case of $\forall k \in S_{u\kappa}^i$ and if $i \in S_{u\kappa}^i$, one can obtain

$$\begin{aligned}
\Gamma_{ij} ={}& (1 + \lambda_\kappa^i)\left\{\left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T\right\} + \sum_{k \in S_\kappa^i} \lambda_{ik} P_k \\
&+ \sum_{k \in S_{u\kappa}^i, \kappa \neq i} \lambda_{ik} \left(\left\{\left[A_i + B_i(D_j F_i + D_j^- H_i)\right]^T P_i + (\cdot)^T\right\} + P_k\right)
\end{aligned}$$

$$+\lambda_{ii}\big\{\big[A_i+B_i(D_jF_i+D_j^-H_i)\big]^T P_i+(\cdot)^T+P_k\big\}$$

From the definition of transition rate matrix, it follows that $\lambda_{ii}=-\sum_{k=1,k\neq i}^{s}\lambda_{ik}<0$. Therefore, from (6)–(8), we can also get $\Gamma_{ij}<0$. Therefore, one has

$$\nabla V(x(t),i)\le -\min_{i\in S}\left\{\lambda_{\min}\left(-\sum_{j=1}^{2^m}\eta_j\Gamma_{ij}\right)\right\}x^T(t)x(t)$$

By Dynkin's formula, we know

$$\begin{aligned}
&E\,[\,V(x(t),i)\,]-V(x_0,\gamma_0)\\
&\quad=E\left[\int_0^t \nabla V\big(x(\theta),\gamma(\theta)\big)\,d\theta\right]\\
&\quad\le -\min_{i\in S}\left\{\lambda_{\min}\left(-\sum_{j=1}^{2^m}\eta_j\Gamma_{ij}\right)\right\}\times E\left[\int_0^t x^T(\theta)x(\theta)\,d\theta \mid (x_0,\gamma_0)\right]
\end{aligned}$$

which, in turn, implies

$$\begin{aligned}
&\min_{i\in S}\left\{\lambda_{\min}\left(-\sum_{j=1}^{2^m}\eta_j\Gamma_{ij}\right)\right\}\times E\left[\int_0^{Tf} x^T(t)x(t)\,dt \mid (x_0,\gamma_0)\right]\\
&\quad\le V(x_0,\gamma_0)-E\,[V(x(t),i)]\\
&\quad\le V(x_0,\gamma_0)
\end{aligned}$$

This leads to

$$E\left[\int_0^{T_f} x^T(t)x(t)\,dt \mid (x_0,\gamma_0)\right]\le \frac{V(x_0,\gamma_0)}{\min_{i\in S}\{\lambda_{\min}(-\sum_{j=1}^{2^m}\eta_j\Gamma_{ij})\}}$$

Thus

$$\lim_{T_f\to\infty} E\left[\int_0^{T_f} x^T(t)x(t)\,dt \mid (x_0,\gamma_0)\right]\le x_0^T\frac{\lambda_{\max}(P_i)I}{\min_{i\in S}\{\lambda_{\min}(-\sum_{j=1}^{2^m}\eta_j\Gamma_{ij})\}}x_0$$

Consequently,

$$\lim_{T_f\to\infty} E\left[\int_0^{T_f} x^T(t)x(t)\,dt \mid (x_0,\gamma_0)\right]\le x_0^T\Upsilon x_0$$

where

$$\Upsilon=\lambda_{\max}(P_i)I\Big/\min_{i\in S}\left\{\lambda_{\min}\left(-\sum_{j=1}^{2^m}\eta_j\,\Gamma_{ij}\right)\right\}$$

This implies that the set $\bigcap_{i=1}^{s}\Omega(P_i)$ is contained in the domain of attraction in mean square sense of the closed-loop system. This completes the proof of Theorem 1. □

Remark 2 It is noted that for some applications the complete access for the jumping mode is not available. In such a case, it is of great interest to design a mode-independent state feedback control. To solve this problem, we just need to add the constraints $F_i = F$, $H_i = H$, $P_i = P$ $(i = 1, 2, \dots, s)$ in Theorem 1.

Theorem 1 just presents a sufficient condition to ensure the stochastic stability of the closed-loop system. In what follows, we will design a stabilizing feedback controller (4) for system (1). More specifically, we will convert the condition in Theorem 1 into LMI, which can be solved efficiently by interior-point method.

Here we assume that the initial state is a guaranteed region of the state space X_0 with $X_0 \in Co\{x_0^1, \dots, x_0^w\}$, where $x_0^1, \dots, x_0^w$ are some given points in R^n. In order to find the largest estimate of the domain of attraction which includes X_0 in mean square sense, we present the following optimization problem:

$$\begin{aligned} &\sup_{P_i>0, F_i, H_i} \quad \alpha \\ &\text{s.t.} \quad \text{(a)} \quad \alpha x_0^l \in \bigcap_{i=1}^{s} \Omega(P_i) \quad l = 1, 2, \dots, w \\ &\qquad\ \ \text{(b)} \quad \text{Inequalities (6)–(8)} \\ &\qquad\ \ \text{(c)} \quad |h_{ij} x(t)| \le 1, \quad \forall\, x(t) \in \bigcap_{i=1}^{s} \Omega(P_i) \end{aligned} \tag{10}$$

where h_{ij} denote the jth row of H_i. Note that (c) holds if and only if (9) holds. If the computed maximal α is greater than 1, then the initial state X_0 is located in the domain of attraction in mean square sense. Now, we introduce the following transformation:

$$\beta = \alpha^{-2}, \quad X_i = P_i^{-1}, \quad Y_i = F_i X_i, \quad Z_i = H_i X_i. \tag{11}$$

It is observed that condition (a) is equivalent to $\alpha^2 (x_0^l)^T P_i x_0^l \le 1$, which can be further expressed as

$$\begin{bmatrix} -\beta & \star \\ x_0^l & -X_i \end{bmatrix} \le 0 \quad l = 1, 2, \dots, w, \quad i = 1, 2, \dots, s. \tag{12}$$

The expression in (6)–(8) is nonlinear in the design parameter P_i for every $i \in S$. To cast it into an LMI, let us pre- and post-multiply (6) by X_i. Using Schur complement, we get the following conditions by considering two different cases:

$$\begin{bmatrix} (1 + \lambda_\kappa^i)\{A_i X_i + B_i D_j Y_i + B_i D_j^- Z_i + (\cdot)^T\} + \lambda_{ii} X_i & \aleph_\kappa^i \\ \star & -\chi_\kappa^i \end{bmatrix} < 0$$
$$\forall\ i \in S_\kappa^i \tag{13}$$

$$\begin{bmatrix} (1 + \lambda_\kappa^i)\{A_i X_i + B_i D_j Y_i + B_i D_j^- Z_i + (\cdot)^T\} & \aleph_\kappa^i \\ \star & -\chi_\kappa^i \end{bmatrix} < 0$$
$$\forall\ i \notin S_\kappa^i \tag{14}$$

where

$$\aleph_\kappa^i \triangleq \left(\sqrt{\lambda_{i\kappa_1^i}}X_i, \ldots, \sqrt{\lambda_{i\kappa_m^i}}X_i\right)$$

$$\chi_\kappa^i \triangleq \operatorname{diag}\{X_{\kappa_1^i}, \ldots, X_{\kappa_m^i}\}$$

Using Schur complement and performing a similar procedure, we can also convert (7), (8) into their equivalent form

$$A_iX_i + B_iD_jY_i + B_iD_j^- Z_i + (\cdot)^T + X_k \geq 0, \quad \forall k \in S_{u\kappa}^i,\ k = i \tag{15}$$

$$\begin{bmatrix} A_iX_i + B_iD_jY_i + B_iD_j^- Z_i + (\cdot)^T & X_i \\ \star & -X_k \end{bmatrix} \leq 0 \quad \forall k \in S_{u\kappa}^i,\ k \neq i \tag{16}$$

On the other hand, by using the similar technique in [9], (c) is equivalent to

$$\begin{bmatrix} -X_i & \star \\ z_{iq} & -1 \end{bmatrix} \leq 0, \quad i = 1, 2, \ldots, s; \quad q = 1, 2, \ldots, m \tag{17}$$

where z_{iq} is the qth row of Z_i. In summary, the optimization problem (10) can transformed as

$$\begin{aligned} &\inf_{X_i>0, Y_i, Z_i} \quad \beta \\ &\text{s.t.} \quad \text{Inequalities (12)–(17)} \end{aligned} \tag{18}$$

If $\beta_{\min} < 1$ (which in turn implies that $\alpha_{\max} > 1$), then the initial state is located in the domain of attraction in mean square sense and the designed controller can be obtained by $u(t) = F_ix(t)$ with the gain $F_i = Y_iX_i^{-1}$.

Theorem 2 *Given system* (1) *with partly unknown transition rates* (2), *if there exist a scalar* $\beta > 0$, *matrices* Y_i, Z_i, *and* $X_i > 0$ *with appropriate dimensions such that for all* $i = 1, 2, \ldots, s$ *there exists an optimal solution for the optimization problem* (18), *then we can design mode-dependent state feedback controller* (4) *with the maximal domain of attraction* $\bigcap_{i=1}^s \Omega(X_i^{-1})$ *in mean square sense. Furthermore, the feedback gains are computed by*

$$F_i = Y_iX_i^{-1}, \quad i = 1, 2, \ldots, s \tag{19}$$

4 Example

In this section, a numerical example is given to demonstrate the effectiveness of our method presented in this paper.

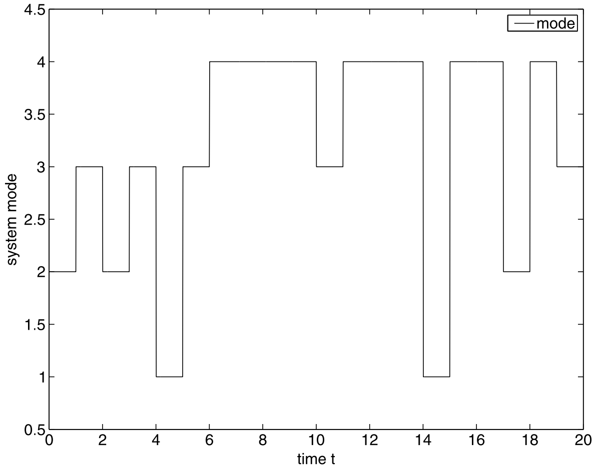

Fig. 1 Mode evolution

Consider a system described by (1) with the following parameters: $S=\{1,2,3,4\}$, $r_0=2$,

$$A_1=\begin{bmatrix}-0.75 & -0.75\\ 1.50 & -1.50\end{bmatrix},\qquad A_2=\begin{bmatrix}-0.15 & 0.49\\ 1.50 & -2.10\end{bmatrix}$$

$$A_3=\begin{bmatrix}-0.30 & -0.15\\ 1.50 & -1.80\end{bmatrix},\qquad A_4=\begin{bmatrix}-0.90 & -0.34\\ 1.50 & -1.65\end{bmatrix}$$

$$B_1=\begin{bmatrix}5\\ -1\end{bmatrix},\qquad B_2=\begin{bmatrix}2\\ -1\end{bmatrix},\qquad B_3=\begin{bmatrix}1\\ -1\end{bmatrix},\qquad B_4=\begin{bmatrix}3\\ -1\end{bmatrix}$$

$$\Lambda=\begin{bmatrix}-1.3 & 0.2 & ? & ?\\ ? & ? & 0.3 & 0.3\\ 0.6 & ? & -1.5 & ?\\ 0.4 & ? & ? & ?\end{bmatrix},\qquad x_0=\begin{bmatrix}-1\\ 0.5\end{bmatrix}$$

To design the mode-dependent state feedback controller of the form (4) such that the closed-loop system with both partial unknown transition probabilities and actuator saturation is stochastically stable, we solve the optimization problem (18) to get $\beta=0.5758$, and the stabilizing gain matrices are computed as

$$F_1=[-0.2065\quad 0.1216],\qquad F_2=[0.3941\quad -0.4641]$$

$$F_3=[-1.0565\quad 0.8280]\times 10^4,\qquad F_4=[0.4823\quad -0.3291]$$

Giving a possible system mode evolution as in Fig. 1, the state trajectories of the closed-loop system and the corresponding control input are shown in Fig. 2 and Fig. 3 (see the solid curve). Obviously, the system is stochastically stable in the presence of partial unknown transition probabilities and actuator saturation. For comparison, we

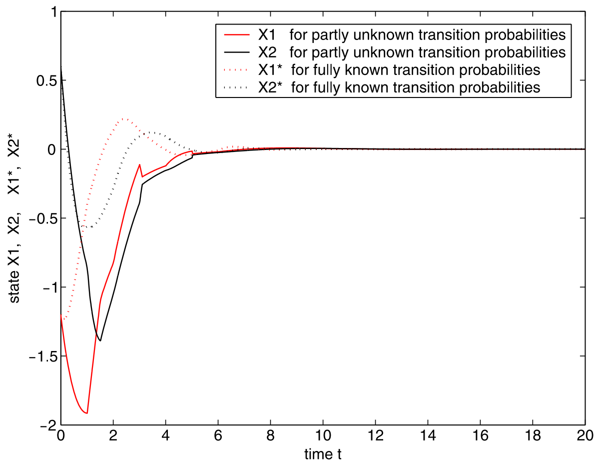

Fig. 2 The state trajectories of the closed-loop system

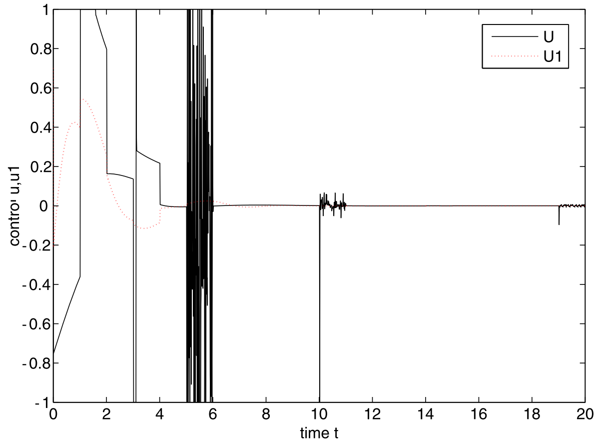

Fig. 3 The control input

also consider the fully known transition matrix case with

$$\Lambda^* = \begin{bmatrix} -1.3 & 0.2 & 0.2 & 0.9 \\ 0.3 & -0.9 & 0.3 & 0.3 \\ 0.6 & 0.3 & -1.5 & 0.6 \\ 0.4 & 0.1 & 0.9 & -1.4 \end{bmatrix}$$

The corresponding feedback gains are computed as

$$F_1^* = [0.0528 \quad -0.1408], \qquad F_2^* = [-0.1286 \quad -0.6112]$$

$$F_3^* = [-0.0645 \quad -0.9204], \qquad F_4^* = [-0.0889 \quad -0.9256]$$

Furthermore, the state trajectories and the control input with fully known transition probability matrix Λ^* are also plotted in Fig. 2 and Fig. 3 (see the dotted curve). As we can see, the convergence speed of fully known transition probability matrix to the origin is faster than that of partial known transition probability matrix, which is reasonable.

Remark 3 It is noted that there exist some computational complexity met in practice for our results. One is that we need to obtain the complete access for the jumping mode $\gamma(t)$ during the operating process, which may not be possible for some applications. Another one is that the complexity will grow exponentially with the increase of the dimension of input m since there are 2^m elements in $\mathcal{D}$.

5 Conclusion

In this paper, we have presented an approach designing of mode-dependent state feedback control law for Markov jumping linear system in the presence of partial unknown transition rate probabilities and saturating actuators. Such a problem has been converted into an optimization problem with LMI constraints. Simulations are given to illustrate the usefulness of our result. Possible future research directions include real-life application of the proposed models and the related stability results, and further extensions of the present result to networked control systems.

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