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Robust Finite-Time H_∞ Control for Nonlinear Jump Systems via Neural Networks

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Abstract This paper presents a neural network-based robust finite-time H_∞ control design approach for a class of nonlinear Markov jump systems (MJSs). The system under consideration is subject to norm bounded parameter uncertainties and external disturbance. In the proposed framework, the nonlinearities are initially approximated by multilayer feedback neural networks. Subsequently, the neural networks undergo piecewise interpolation to generate a linear differential inclusion model. Then, based on the model, a robust finite-time state-feedback controller is designed such that the nonlinear MJS is finite-time bounded and finite-time stabilizable. The H_∞ control is specified to ensure the elimination of the approximation errors and external disturbances with a desired level. The controller gains can be derived by solving a set of linear matrix inequalities. Finally, simulation results are given to illustrate the effectiveness of the developed theoretic results.

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1 Introduction

In past years, the H_∞ control problem has attracted much attention due to its both practical and theoretical importance. Many results have been reported in the literature on this topic, for example, [5, 10–12, 25] and the references therein. However, much attention has been devoted to the behavior of control dynamics over an infinite-time interval. In many practical applications, the main concern is the behavior of the control dynamics over a fixed finite-time interval (for example, large values of the state are not acceptable in the presence of saturations [3]). In these cases, it is necessary to check the unacceptable values so that the states do not exceed a certain threshold during a fixed finite-time interval by giving some initial conditions. To deal with such problems, Dorato gave the concept of finite-time stability in 1961 [9]. Then, some attempts on finite-time stability were made [2, 27] by using an approach based on Lyapunov theory. Recently, with the aid of linear matrix inequality (LMI) techniques [7], more concepts of finite-time stability have been proposed, such as finite-time control [1, 2], finite-time boundedness [1, 4], finite-time stabilization via feedback control [4], and so on.

On another research front line, a great amount of effort has been devoted to stochastic systems with Markovian jump parameters in the past two decades. This class of systems can be used to model a variety of physical systems which may experience abrupt changes in their structure and parameters caused by phenomena such as component failures, abrupt environmental disturbances, sudden variations of the operating condition, etc. In Markov jump systems (MJSs), the dynamics of the jump modes and continuous states are respectively modeled by finite state Markov chains and differential equations. Since the pioneering work on jumping linear quadratic control in the 1960s [13–15], MJSs have been extensively studied, and a number of achievements related to these systems have been made on control design [8, 20, 21], filtering design [29, 31], and stability analysis [22]. However, most of the results in this field relate to stability and performance criteria defined over an infinite-time interval. In practice, one is not only interested in system stability but also in bounds of system trajectories. A typical example can be found in aircraft control, where it is requested that during the execution of a certain task the state variables should not exceed some threshold under all admissible pilot inputs and in the presences of wind disturbances. What is more, almost all of the systems have inherent nonlinearities, and parameter uncertainties are often encountered in the systems, so their presence must be taken into consideration in realistic controller design.

In recent years, neural network control techniques have been widely used in nonlinear fields due to their universal approximation capability. Details concerning their successful applications can be found in various fields; see, for example, [17, 23, 28, 30, 32] and the references therein. There are, however, some drawbacks in using neural networks (NNs) in any control scheme. Most notably, guaranteeing stability becomes a major problem. Recently, a class of NNs that admit linear difference inclusion (LDI) state-space representation has been proposed in [16, 24] and used in the stability analysis via a Lyapunov function method. Based on the LDI model, some systematic model-based neural network control design techniques have been developed [18].

In this work, with the help of neural approximation, we consider the problem of finite-time H_∞ control for a class of uncertain nonlinear MJSs with norm bounded external disturbance. Firstly, the nonlinearities in the different jump modes are parameterized by NNs. Subsequently, an LDI state-space representation for a class of NNs is established. Then, based on this LDI representation, sufficient conditions are presented to guarantee the closed-loop system finite-time stabilizable (FTS) and eliminate the effect of the approximation errors and external disturbances on the regulated output. These conditions can be reduced to feasibility problems involving (LMIs). Finally, a nonlinear MJS with parameter uncertainties has been used as an example to show the potential of the proposed techniques.

Throughout the paper, the following notation will be used. R^n and $R^{n\times m}$ denote n-dimensional Euclidean space, and the set of all the $n \times m$ real matrices, respectively. A^{T} and A^{-1}denote the matrix transpose and the matrix inverse, $diag\{A \quad B\}$ represents the block diagonal matrix of A and B, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ mean the maximal and minimal eigenvalues of a real matrix A, $\|A\|$ denotes the Euclidean norm of matrix A, $E\{\cdot\}$ stands for the mathematics statistical expectation of the stochastic process or vector, $L_2^n[0 \quad N]$ is the space of n-dimensional square integrable functions vector over $[0 \quad N]$, $P > 0$ stands for a positive definite matrix, I is the unit matrix with appropriate dimensions, 0 is the zero matrix with appropriate dimensions, and "$*$" means the symmetric terms in a symmetric matrix.

2 System Description

We are given a probability space (Ω, F, P) where Ω is the sample space, F is the algebra of events, and P is the probability measure defined on F. Let the random form process $\{r_t, t \geq 0\}$ be the Markov stochastic process taking values on a finite set $M = \{1, 2, \ldots, N\}$ with transition rate matrix $\Pi = \{\pi_{rj}\}, r, j \in M$ and define the following transition probability from mode r at time t to mode j at time $t + \Delta t$ as

$$P_{rj} = P_r\{r_{t+\Delta t} = j | r_t = r\} = \begin{cases} \pi_{rj}\Delta t + o(\Delta t), & r \neq j \\ 1 + \pi_{rr}\Delta t + o(\Delta t), & r = j \end{cases}$$

with transition probability rates $\pi_{rj} \geq 0$ for $r, j \in M$, $r \neq j$ and $\sum_{j=1, j\neq r}^{N} \pi_{rj} = -\pi_{rr}$ where $\Delta t > 0$ and $\lim_{\Delta t \downarrow 0} o(\Delta t)/\Delta t \to 0$.

Consider the following nonlinear MJS with parameter uncertainties:

$$\begin{cases} \dot{x}(t) = \big[A(r_t) + \Delta A(r_t)\big]x(t) + \big[B(r_t) + \Delta B(r_t)\big]u(t) + B_d(r_t)w(t) \\ \qquad\quad + f(x(t), r_t) \\ z(t) = C(r_t)x(t) + D(r_t)u(t) + D_d(r_t)w(t) \\ x(t) = x_0, \quad r_t = r_0, \quad t = 0 \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the vector of state variables, $z(t) \in R^l$ is the controlled output, $u(t) \in R^m$ is the controlled input, $w(t) \in L_2^p[0 \quad +\infty)$ is the external disturbances, $f(\cdot)$ is a continuous nonlinear mapping with $f(0) = 0$ but not assumed to be known a priori, x_0 and r_0 respectively represent the initial state and initial mode, and $A(r_t), B(r_t), B_d(r_t), C(r_t), D(r_t)$ and $D_d(r_t)$ are known mode-dependent constant matrices with appropriate dimensions. For notational simplicity, when $r_t = r$, $r \in M$, $A(r_t), \Delta A(r_t, t), B(r_t), \Delta B(r_t, t), B_d(r_t), C(r_t), D(r_t), D_d(r_t)$ and $f(x(t), r_t)$ are respectively denoted as A_r, ΔA_r, B_r, ΔB_r, B_{dr}, C_r, D_r, D_{dr} and $f_r(x(t))$. ΔA_r and ΔB_r are the time-varying but norm bounded uncertainties satisfying

$$[\,\Delta A_r \quad \Delta B_r\,] = S_r F_r(t)[\,H_{1r} \quad H_{2r}\,] \tag{2}$$

where S_r, H_{1r} and H_{2r} are known mode-dependent matrices with appropriate dimensions and $F_r(t)$ is the time-varying unknown matrix function with Lebesgue norm measurable elements satisfying $F_r^{\mathrm{T}}(t) F_r(t) \le I$.

Remark 1 The parameter uncertainty structure in (2) is an extension of the admissible condition. The matrix S_r is chosen as full row rank matrix. The motivation for us to consider uncertainties derives from the fact that it is always impossible to obtain the exact mathematical model of a practical dynamics due to the complexity of the process, environmental noises, time-varying parameters, and many difficulties in measuring various and uncertain dynamics, etc. Hence, the uncertainties ΔA_r and ΔB_r in (1)–(2) reflect the inaccuracy in the mathematical modeling of jump dynamical systems. Note that the unknown mode-dependent matrix $F_r(t)$ in (2) can also be allowed to be state dependent, i.e., $F_r(t) = F_r(t, x(t))$, as long as $F_r(t, x(t)) \le 1$ is satisfied.

Assumption 1 *For any given positive numberd*, *the external disturbance $w(t)$ is time varying and satisfies*

$$\int_0^N w^{\mathrm{T}}(t) w(t) dt \le d, \quad d \ge 0 \tag{3}$$

For each mode r, nonlinear function $f_r(x(t))$ is to be parameterized by NNs. Such a parameterization makes sense because any continuous nonlinear function can be approximated arbitrarily well on a compact interval by NNs. Without loss of generality, let the single hidden layer perceptron $N_r(x(t), W_{r1}, W_{r2})$ be suitably trained to approximate the nonlinear term $f_r(x(t))$, which is described in matrix-vector notation as

$$N_r\big(x(t), W_{r1}, W_{r2}\big) = \psi_{r2}\big[W_{r2}\psi_{r1}\big[W_{r1}x(t)\big]\big] \tag{4}$$

where $W_{ri} \in \mathbf{R}^{n_h \times n}$, $i = 1, 2$ denote the connecting weight matrices of neurons, and $\psi_{ri}(\cdot)$ denotes the activation function vector of the NNs, which is defined as

$$\psi_{ri}[\zeta_r] = \big[\varphi_{r1}(\varsigma_{r1}), \varphi_{r2}(\varsigma_{r2}), \ldots, \varphi_{rn_i}(\varsigma_{rn_i})\big]^{\mathrm{T}}$$

in which we let

$$\varphi_{rj}(\varsigma_{rj}) = \lambda_{rj}\left(\frac{1-e^{-\varsigma_{rj}/q_{rj}}}{1+e^{-\varsigma_{rj}/q_{rj}}}\right), \quad q_{rj}, \lambda_{rj} > 0, \quad j = 1, 2, \ldots, n_i$$

The maximum and minimum derivatives of activation function φ_{rj} are defined as follows:

$$s_{rj}(k, \varphi_{rj}) = \begin{cases} \min_{\zeta_{rj}} \frac{\partial \varphi_{rj}(\zeta_{rj})}{\partial \zeta_{rj}}, & k = 0 \\ \max_{\zeta_{rj}} \frac{\partial \varphi_{rj}(\zeta_{rj})}{\partial \zeta_{rj}}, & k = 1 \end{cases} \tag{5}$$

The activation function φ_{rj} can be rewritten in the following min-max form:

$$\varphi_{rj} = h_{rj}(0)s_{rj}(0, \varphi_{rj}) + h_{rj}(1)s_{rj}(1, \varphi_{rj})$$

where $h_{rj}(k)$, $k = 0, 1$ is a set of positive real numbers associated with φ_{rj} satisfying $h_{rj}(k) > 0$ and $h_{rj}(0) + h_{rj}(1) = 1$.

According to the approximation theorem, for a given accuracy $\rho_r > 0$, there exist constant weight matrices W_{ri}^* defined as

$$\left(W_{r1}^*, W_{r2}^*\right) = \underset{(W_{r1}^*, W_{r2}^*)}{\arg\min} \left\{ \max_{x(t)\in D} \left\| f_r\left(x(t)\right) - N_r\left(x(t), W_{r1}, W_{r2}\right) \right\| \right\}$$

where D is a compact set $D \in R^m$, such that

$$\max_{x(t)\in D} \left\| f_r\left(x(t)\right) - N_r\left(x(t), W_{r1}^*, W_{r2}^*\right) \right\| \le \rho_r \left\| x(t) \right\| \tag{6}$$

For each mode r, denote a set of n_i-dimensional index vectors of the ith layer ($i = 1, 2$) as

$$\gamma_{n_i} = \gamma_{n_i}(\sigma_r) = \left\{ \sigma_r \in R^{n_i} \mid \sigma_{rj} \in \{0, 1\},\ j = 1, \ldots, n_i \right\}$$

where σ_r is used as a binary indicator. Obviously, the ith layer with n_i neurons has 2^{n_i} combinations of binary indicator with $k = 0, 1$, and the elements of index vectors for two-layer NNs have $2^{n_2} \times 2^{n_1}$ combinations in the set $\Theta = \gamma_{n_2} \oplus \gamma_{n_1}$.

By using (5) and adopting the compact representation [16], the NNs (4) can be expressed as follows:

$$\begin{aligned} N\left(x(t), W_{r1}^*, W_{r2}^*\right) &= \psi_{r2}\left[W_{r2}^* \begin{bmatrix} \sum_{k=0}^{1} h_{r11}(k)s_{r11}(k, \varphi_{r11}) \times (W_{r1}^* x)_{r1} \\ \vdots \\ \sum_{k=0}^{1} h_{r1n_1}(k)s_{r1n_1}(k, \varphi_{r1n_1}) \times (W_{r1}^* x)_{rn_1} \end{bmatrix} \right] \\ &= \sum_{\sigma_r \in \Theta} \mu_{\sigma_r} A_{\sigma_r}\left(\sigma_r, \psi_r, W_r^*\right) x(t) \end{aligned}$$

where

$$
\begin{aligned}
&A_{\sigma_r} = diag\big[s_{r2j}(\sigma_{r2j}, \varphi_{r2j})\big] W_{r2}^* \, diag\big[s_{r1j}(\sigma_{r1j}, \varphi_{r1j})\big] W_{r1}^* \\
&\sum_{\sigma \in \gamma_{n_2} \oplus \gamma_{n_1}} \mu_\sigma = \sum_{\substack{k_{r2n_2}=0 \\ \vdots \\ k_{r21}=0}}^{1} \sum_{\substack{k_{r2n_1}=0 \\ \vdots \\ k_{r11}=0}}^{1} h_{r2n_2}(k_{r2n_2}) \cdots h_{r21}(k_{r21}) h_{r1n_1}(k_{r1n_1}) \cdots \\
&\qquad\qquad \times h_{r11}(k_{r11}) = 1 \\
&h_{rij}(\sigma_{rij}) \ge 0, \quad \sigma_{rij} = 0, 1 \\
&h_{rij}(0) + h_{rij}(1) = 1, \quad i = 1, 2,\ j = n_1, n_2
\end{aligned}
\tag{7}
$$

Thus, by means of NNs, the nonlinear MJS (1) is transformed into a group of LDIs with error bound, in which the different inclusion is powered by a stochastic Markovian process, i.e.,

$$
\begin{cases}
\dot{x}(t) = \Big[\sum_{\sigma_r \in \Theta} \mu_{\sigma_r} A_{\sigma_r} + A_r + \Delta A_r\Big] x(t) + [B_r + \Delta B_r] u(t) \\
\qquad\quad + B_{dr} w(t) + \Delta f_r\big(x(t)\big) \\
z(t) = C_r x(t) + D_r u(t) + D_{dr} w(t) \\
x(t) = x_0, \quad r_t = r_0, \quad t = 0
\end{cases}
\tag{8}
$$

where

$$
\Delta f_r\big(x(t)\big) = \max_{x(t) \in D} \big\| f_r\big(x(t)\big) - N_r\big(x(t), W_{r1}^*, W_{r2}^*\big)\big\| \le \rho_r \big\|x(t)\big\|
\tag{9}
$$

denotes the approximation errors of the NNs.

Remark 2 It should be noted that, in our study, the detailed structure and quantitative size of error dynamics $\Delta f_r(x(t))$ is not needed, and only the norm bounded condition is required. This condition can be easily met in practical applications. Also, the bound of the norm may vary according to different nonlinearities in different modes.

Based on the LDI model (8), we construct the following state-feedback controller for system (1):

$$
u(t) = K_r x(t)
\tag{10}
$$

Then the resulting closed-loop system can be obtained:

$$
\begin{cases}
\dot{x}(t) = \big(\bar{A}_r + \Delta \bar{A}_r\big) x(t) + B_{dr} w(t) + \Delta f_r\big(x(t)\big) \\
z(t) = \bar{C}_r x(t) + D_{dr} w(t) \\
x(t) = x_0, \quad r_t = r_0, \quad t = 0
\end{cases}
\tag{11}
$$

where

$$
\bar{A}_r = \sum_{\sigma_r \in \Theta} \mu_{\sigma_r} A_{\sigma_r} + A_r + B_r K_r, \qquad \Delta \bar{A}_r = \Delta A_r + \Delta B_r K_r, \qquad \bar{C}_r = C_r + D_r K_r
$$

The finite-time H_∞ control problem to be addressed in this paper can be formulated as designing a state-feedback controller in the form of (10) such that

(1) The closed-loop system (11) is finite-time bounded (FTB).
(2) Under zero initial condition, the controller output $z(t)$ satisfies

$$E\left\{\int_0^t z^{\mathrm{T}}(t)z(t)\,dt\right\} \le \eta^2 \int_0^t w^{\mathrm{T}}(t)w(t)\,dt \tag{12}$$

for any nonzero $w(t)$ satisfied boundary condition (3).

3 Main Results

The general idea of finite-time control can be formalized through the following definitions over a finite-time interval for some given initial conditions.

Definition 1 [9] For a given constant $N > 0$, uncertain nonlinear MJS (1) (setting $u(t) = 0$, $w(t) \equiv 0$) is said to be finite-time stable with respect to $(c_1 \quad c_2 \quad N \quad R_r)$, where $c_1 < c_2$, $R_r > 0$, if

$$E\left\{x_0^{\mathrm{T}} R_r x_0\right\} \le c_1 \quad \Rightarrow \quad E\left\{x^{\mathrm{T}}(t) R_r x(t)\right\} < c_2, \quad \forall t \in [0 \quad N] \tag{13}$$

Definition 2 (FTB) [3] For a given constant $N > 0$, uncertain nonlinear MJS (1) (setting $u(t) = 0$) is said to be FTB with respect to $(c_1 \quad c_2 \quad N \quad R_r \quad d)$, where $c_1 < c_2$, $R_r > 0$, if condition (13) holds.

Definition 3 (FTS) [4] Given a time constant $N > 0$, positive scalars c_1 and c_2, with $c_1 < c_2$, and mode-dependent positive definite matrix $R_r > 0$, uncertain nonlinear MJS (1) is said to be finite-time stabilizable (FTS) with respect to $(c_1 \quad c_2 \quad N \quad R_r \quad d)$, if there exists a state-feedback controller in the form of (10), such that system (1) is finite-time stable.

Remark 3 It should be pointed out that there is a great difference between Lyapunov stability and finite-time stability. The concept of Lyapunov stability (or Lyapunov almost asymptotic stability) is largely known to the control community, but an MJS is finite-time stable if, once we fix a finite-time interval, its state does not exceed some bonds during this time interval. Moreover, an MJS which is finite-time stable may not be Lyapunov stable; conversely, a Lyapunov stable MJS could be not finite-time stable if its state exceeds the prescribed bounds during the transients.

Remark 4 In fact, finite-time stability can be recovered as a particular case of finite-time boundedness by setting $w(t) = 0$. In the presence of external inputs, finite-time stability leads to the concept of finite-time boundedness. That is, a system is FTB if, given a bound initial condition and a characterization of the set of admissible inputs, the system states remain below the prescribed limit for all inputs in the bound set. Finite-time stability and finite-time boundedness are open-loop concepts. But the finite-time control problem concerns the design of a finite-time controller which guarantees the FTB and FTS of a closed-loop system via state feedback.

Definition 4 [19] In the Euclidean space $\{R^n \times M \times R_+\}$, we introduce the stochastic Lyapunov–Krasovskii function of uncertain nonlinear MJS (1) as $V(x(t), r_t = r, t > 0)$, the weak infinitesimal operator satisfying

$$\begin{aligned}
&\Gamma V\big(x(t), r\big)\\
&\quad = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\big[E\big\{V\big(x(t+\Delta t), r_{t+\Delta t}, t+\Delta t\big) \mid x(t)=x, r_t = r\big\} - V\big(x(t), r, t\big)\big]\\
&\quad = \frac{\partial}{\partial t} V\big(x(t), r, t\big) + \frac{\partial}{\partial x} V\big(x(t), r, t\big)\dot{x}(t, r) + \sum_{j=1}^{\mathrm{N}} \pi_{rj} V\big(x(t), j, t\big) \qquad (14)
\end{aligned}$$

Lemma 1 [26] *Let T, M, F and N be real matrices of appropriate dimension with $F^{\mathrm{T}}F \le I$. Then, for a positive scalar $\alpha > 0$, we have*

$$T + MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} \le T + \alpha MM^{\mathrm{T}} + \alpha^{-1}N^{\mathrm{T}}N \qquad (15)$$

Lemma 2 *For a given time constant $N > 0$, the uncertain nonlinear MJS (1) is FTS via a state-feedback controller with respect to $(c_1 \quad c_2 \quad N \quad R_r \quad d)$, if there exist positive constant $\alpha > 0$, mode-dependent symmetric positive definite matrix $P_r \in R^{n\times n}$, $r \in M$, symmetric positive definite matrix $Q \in R^{p\times p}$, and positive real number ε_{r1} for all $\sigma_r \in \Theta$ such that*

$$\begin{bmatrix} \Lambda_r & P_r B_{dr} \\ B_{dr}^T P_r & -\alpha Q \end{bmatrix} < 0 \qquad (16)$$

$$\frac{c_1 \lambda_{\max}(\tilde{P}_r) + d\lambda_{\max}(Q)(1 - e^{-\alpha t})}{\lambda_{\min}(\tilde{P}_r)} < e^{-\alpha t} c_2 \qquad (17)$$

where

$$\Lambda_r = \big(\bar{A}_r + \Delta\bar{A}_r\big)^{\mathrm{T}} P_r + P_r\big(\bar{A}_r + \Delta\bar{A}_r\big) + \sum_{j=1}^{N} \pi_{rj} P_j + \varepsilon_{r1} P_r P_r + \varepsilon_{r1}^{-1}\rho^2 I - \alpha P_r$$

$$\tilde{P}_r = R_r^{-1/2} P_r R_r^{-1/2}$$

$\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximal and minimal eigenvalues of the augment, respectively.

Proof For the closed-loop neural system (11) of nonlinear MJS (1), choose a stochastic Lyapunov function candidate as $V(x(t), r_t = r) = V(x, r) = x^{\mathrm{T}} P_r x$, where P_r is a mode-dependent positive definite symmetric matrix for each r. Along the trajectories of system (11), the corresponding time derivative of $V(x(t), r)$ is given by

$$\begin{aligned}
\Gamma V(x(t), r) = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\big[&E\big\{V\big(x(t+\Delta t), r_{t+\Delta t}, t+\Delta t\big) \mid x(t)=x, r_t = r\big\}\\
&- V\big(x(t), r, t\big)\big]
\end{aligned}$$

$$= x^{\mathrm{T}}\left(\left(\bar{A}_r + \Delta\bar{A}_r\right)^{\mathrm{T}} P_r + P_r\left(\bar{A}_r + \Delta\bar{A}_r\right) + \sum_{j=1}^{N} \pi_{rj} P_j\right)x$$
$$+ 2x^{\mathrm{T}} P_r B_{dr} w + 2x^{\mathrm{T}} P_r \Delta f_r(x) \tag{18}$$

According to Lemma 1 and condition (9), $2x^{\mathrm{T}} P_r \Delta f_r(x)$ can be presented in the following form:

$$2x^{\mathrm{T}} P_r \Delta f_r(x) \le x^{\mathrm{T}}\left(\varepsilon_{r1} P_r P_r + \varepsilon_{r1}^{-1} \rho^2 I\right)x$$

so (18) can be rewritten as

$$\Gamma V\left(x(t), r\right) \le x^{\mathrm{T}}\left(\bar{A}_r + \Delta\bar{A}_r\right)^{\mathrm{T}} P_r + P_r\left(\bar{A}_r + \Delta\bar{A}_r\right)$$
$$+ \sum_{j=1}^{N} \pi_{rj} P_j + \varepsilon_{r1} P_r P_r + \varepsilon_{r1}^{-1} \rho^2 I\big)x + 2x^{\mathrm{T}} P_r B_{dr} w \tag{19}$$

From (16) and (19), the following condition is satisfied:

$$\Gamma V\left(x(t), r\right) < \alpha V\left(x(t), r\right) + \alpha w^{\mathrm{T}}(t) Q w(t) \tag{20}$$

Multiplying (20) by $e^{-\alpha t}$ yields

$$\Gamma\left[e^{-\alpha t} V\left(x(t), r\right)\right] < \alpha e^{-\alpha t} w^{\mathrm{T}}(t) Q w(t) \tag{21}$$

Integrating (21) from 0 to t, it follows that

$$e^{-\alpha t} V\left(x(t), r\right) - V(x_0, r_0) < \alpha \int_0^t e^{-\alpha\tau} w^{\mathrm{T}}(\tau) Q w(\tau)\, d\tau$$

Then, the above inequality is equivalent to

$$V\left(x(t), r\right) < e^{\alpha t} V(x_0, r_0) + \alpha e^{\alpha t} \int_0^t e^{-\alpha\tau} w^{\mathrm{T}}(\tau) Q w(\tau)\, d\tau$$
$$< e^{\alpha t} V(x_0, r_0) + \alpha d \lambda_{\max}(Q) e^{\alpha t} \int_0^t e^{-\alpha\tau}\, d\tau$$
$$= e^{\alpha t}\left[V(x_0, r_0) + \alpha d \lambda_{\max}(Q) \frac{1 - e^{-\alpha t}}{\alpha}\right] \tag{22}$$

Noting that $\tilde{P}_r = R_r^{-1/2} P_r R_r^{-1/2}$, from (22) it follows that

$$V\left(x(t), r\right) < e^{\alpha t}\left[c_1 \lambda_{\max}\left(\tilde{P}_r\right) + d \lambda_{\max}(Q)\left(1 - e^{-\alpha t}\right)\right] \tag{23}$$

On the other hand, the following condition holds:

$$V\left(x(\tau), r\right) = x(t)^{\mathrm{T}} P_r x(t) \ge \lambda_{\min}\left(\tilde{P}_r\right) x(t)^{\mathrm{T}} R_r x(t) \tag{24}$$

Putting together (23) and (24), we have

$$E\{x_k^{\mathrm{T}} R_r x_k\} < \frac{e^{\alpha t}[c_1\lambda_{\max}(\tilde{P}_r) + d\lambda_{\max}(Q)(1-e^{-\alpha t})]}{\lambda_{\min}(\tilde{P}_r)}$$

Condition (17) implies that for $\forall t \in [0 \quad N]$, $E\{x_k^{\mathrm{T}} R_r x_k\} < c_2$. Then uncertain nonlinear MJS (1) is said to be FTS with respect to $(c_1 \quad c_2 \quad N \quad R_r \quad d)$. This completes the proof. □

Lemma 3 *For a given time constant $N > 0$, the uncertain nonlinear MJS (1) is FTS via a state-feedback controller with respect to $(c_1 \quad c_2 \quad N \quad R_r \quad d)$ if there exist positive constants $\alpha > 0$ and $\eta > 0$, mode-dependent symmetric positive definite matrix $P_r \in R^{n\times n}$, $r \in M$, and positive real number ε_{r1} for all $\sigma_r \in \Theta$ such that*

$$\begin{bmatrix} \Lambda_r & P_r B_{dr} \\ B_{dr}^{\mathrm{T}} P_r & -\eta^2 I \end{bmatrix} < 0 \tag{25}$$

$$c_1\lambda_{\max}(\tilde{P}_r) + \frac{\eta^2 d}{\alpha}(1-e^{-\alpha t}) < e^{-\alpha t} c_2 \lambda_{\min}(\tilde{P}_r) \tag{26}$$

Proof Consider the similar Lyapunov–Krasovskii function $V(x(t), r) = x^{\mathrm{T}} P_r x$. Along the trajectories of system (11), and recalling condition (25), we have

$$\Gamma V(x(t), r) < \alpha V(x(t), r) + \eta^2 w^{\mathrm{T}}(t) w(t)$$

Then following a similar proof to that of Lemma 2, inequalities (25) and (26) can be obtained. This completes the proof. □

Theorem 1 *For a given time constant $N > 0$, the uncertain nonlinear MJS (1) is FTS via a state-feedback controller with respect to $(c_1 \quad c_2 \quad N \quad R_r \quad d)$ and satisfies the cost function inequality (12) for all admissible $w(t)$ with the constraint condition (3) if there exist positive constant $\alpha > 0$ and $\eta > 0$, mode-dependent symmetric positive definite matrix $P_r \in R^{n\times n}$, $r \in M$, and positive real number ε_{r1} for all $\sigma_r \in \Theta$ such that*

$$\begin{bmatrix} \Lambda_r + \bar{C}_r^{\mathrm{T}} \bar{C}_r & P_r B_{dr} + \bar{C}_r^{\mathrm{T}} D_{dr} \\ B_{dr}^{\mathrm{T}} P_r + D_{dr}^{\mathrm{T}} \bar{C}_r & -\eta^2 I + D_{dr}^{\mathrm{T}} D_{dr} \end{bmatrix} < 0 \tag{27}$$

Proof Considering Lemma 3 and the closed-loop system (11), we introduce the following condition by defining the similar Lyapunov–Krasovskii function $V(x(t), r) = x^{\mathrm{T}} P_r x$:

$$\Gamma V(x(t), r) < \alpha V(x(t), r) + \eta^2 w^{\mathrm{T}}(t) w(t) - z^{\mathrm{T}}(t) z(t)$$

Obviously, this condition can be guaranteed by inequality (27). On the other hand, multiplying the above inequality by $e^{-\alpha t}$, it follows that

$$\Gamma\left[e^{-\alpha t}V\left(x(t), r\right)\right] < e^{-\alpha t}\left[\eta^2 w^{\mathrm{T}}(t)w(t) - z^{\mathrm{T}}(t)z(t)\right]$$

In zero initial condition, by integrating the above inequality from 0 to N, we can get

$$e^{-\alpha t}V\left(x(t), r\right) < \int_0^t e^{-\alpha\tau}\left[\eta^2 w^{\mathrm{T}}(t)w(t) - z^{\mathrm{T}}(t)z(t)\right]d\tau$$

Thus, the following condition holds:

$$\int_0^t e^{-\alpha\tau} z^{\mathrm{T}}(t)z(t)\,d\tau < \eta^2\int_0^t e^{-\alpha\tau} w^{\mathrm{T}}(t)w(t)\,d\tau$$

Note that $t \in [0 \quad N]$ then yields

$$\int_0^N z^{\mathrm{T}}(t)z(t)\,d\tau < e^{-\alpha N}\eta^2\int_0^N w^{\mathrm{T}}(t)w(t)\,d\tau$$

Therefore, condition (12) can be guaranteed by letting $\bar{\eta} = \sqrt{e^{-\alpha N}}\eta$. This completes the proof. □

Theorem 2 *For given time constants $N > 0, \alpha > 0$ and $\eta^2 > 0$, the uncertain nonlinear MJS (1) is FTS with respect to $(c_1 \quad c_2 \quad N \quad R_r \quad d)$, via a state-feedback controller $K_r = Y_r X_r^{-1}$, and satisfies the inequality (12) for all admissible $w(t)$ with the constraint condition (3), if there exist mode-dependent symmetric positive definite matrix $X_r \in R^{n\times n}$, mode-dependent matrix $Y_r \in R^{m\times n}$, positive real number $\varepsilon_{r1}, \varepsilon_{r2}$ and λ for all $\sigma_r \in \Theta$ such that*

$$\begin{bmatrix} M_r & B_{dr} & X_r C_r^{\mathrm{T}} + Y_r^{\mathrm{T}} D_r^{\mathrm{T}} & X_r H_{1r}^{\mathrm{T}} + Y_r^{\mathrm{T}} H_{2r}^{\mathrm{T}} & \rho_r X_r & N_r \\ B_{dr}^{\mathrm{T}} & -\eta^2 I & D_{dr}^{\mathrm{T}} & 0 & 0 & 0 \\ C_r X_r + D_r Y_r & D_{dr} & -I & 0 & 0 & 0 \\ H_{1r} X_r + H_{2r} Y_r & 0 & 0 & -\varepsilon_{r2} I & 0 & 0 \\ \rho_r X_r & 0 & 0 & 0 & -\varepsilon_{r1} I & 0 \\ N_r^{\mathrm{T}} & 0 & 0 & 0 & 0 & -L_r \end{bmatrix} < 0 \tag{28}$$

$$\lambda R_r^{-1} < X_r < R_r^{-1} \tag{29}$$

$$\begin{bmatrix} -e^{-\alpha t}c_2 + \frac{\eta^2 d}{\alpha}(1 - e^{-\alpha t}) & \sqrt{c_1} \\ \sqrt{c_1} & -\lambda \end{bmatrix} < 0 \tag{30}$$

where

$$\begin{aligned} M_r = {} & X_r(A_{\sigma_r} + A_r)^{\mathrm{T}} + (A_{\sigma_r} + A_r)X_r + Y_r^{\mathrm{T}} B_r^{\mathrm{T}} + B_r Y_r + \pi_{rr} X_r \\ & + \varepsilon_{r1} I + \varepsilon_{r2} S_r S_r^{\mathrm{T}} - \alpha X_r \end{aligned}$$

$$N_r = \left[\sqrt{\pi_{r1}} X_r \quad \cdots \quad \sqrt{\pi_{r(r-1)}} X_r \quad \sqrt{\pi_{r(r+1)}} X_r \quad \cdots \quad \sqrt{\pi_{rN}} X_r \right]$$
$$L_r = diag\left\{ X_1, \quad \ldots, \quad X_{r-1}, \quad X_{r+1}, \quad \ldots, \quad X_N \right\}$$

Proof Note that inequality (27) is equivalent to the following condition:

$$Z_r = \begin{bmatrix} \Lambda_r & P_r B_{dr} & \bar{C}_r^{\mathrm{T}} \\ B_{dr}^{\mathrm{T}} P_r & -\eta^2 I & D_{dr}^{\mathrm{T}} \\ \bar{C}_r & D_{dr} & -I \end{bmatrix} < 0$$

In order to deal with the uncertainties described as the form in (2), we use the following approach:

$$Z_r = \Xi_r + \Delta \Xi_r < 0$$

where

$$\Xi_r = \begin{bmatrix} \Omega_r & P_r B_{dr} & C_r^{\mathrm{T}} + K_r^{\mathrm{T}} D_r^{\mathrm{T}} \\ B_{dr}^{\mathrm{T}} P_r & -\eta^2 I & D_{dr}^{\mathrm{T}} \\ C_r + D_r K_r & D_{dr} & -I \end{bmatrix} < 0$$

$$\Delta \Xi_r = \begin{bmatrix} (\Delta A_r + \Delta B_r K_r)^{\mathrm{T}} P_r + P_r (\Delta A_r + \Delta B_r K_r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Omega_r = \left(A_r' + B_r K_r\right)^{\mathrm{T}} P_r + P_r \left(A_r' + B_r K_r\right) + \sum_{j=1}^{N} \pi_{rj} P_j + \varepsilon_{r1} P_r P_r + \varepsilon_{r1}^{-1} \rho^2 I - \alpha P_r$$

$$A_r' = \sum_{\sigma_r \in \theta} \mu_{\sigma_r} A_{\sigma_r} + A_r$$

According to Lemma 1, $\Delta \Xi_r$ can be presented in the following form:

$$\Delta \Xi_r = \begin{bmatrix} P_r S_r \\ 0 \\ 0 \end{bmatrix} F_r \left[H_{1r} + H_{2r} K_r \quad 0 \quad 0 \right] + \begin{bmatrix} H_{1r}^{\mathrm{T}} + K_r^{\mathrm{T}} H_{2r}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} F_r^{\mathrm{T}} \left[S_r^{\mathrm{T}} P_r \quad 0 \quad 0 \right]$$
$$< \varepsilon_{r2} \begin{bmatrix} P_r S_r \\ 0 \\ 0 \end{bmatrix} \left[S_r^{\mathrm{T}} P_r \quad 0 \quad 0 \right] + \varepsilon_{r2}^{-1} \begin{bmatrix} H_{1r}^{\mathrm{T}} + K_r^{\mathrm{T}} H_{2r}^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix} \left[H_{1r} + H_{2r} K_r \quad 0 \quad 0 \right]$$

Then

$$Z_r = \begin{bmatrix} \Omega_r + \varepsilon_{r2} P_r S_r S_r^{\mathrm{T}} P_r & P_r B_{dr} & C_r^{\mathrm{T}} + K_r^{\mathrm{T}} D_r^{\mathrm{T}} & H_{1r}^{\mathrm{T}} + K_r^{\mathrm{T}} H_{2r}^{\mathrm{T}} \\ B_{dr}^{\mathrm{T}} P_r & -\eta^2 I & D_{dr}^{\mathrm{T}} & 0 \\ C_r + D_r K_r & D_{dr} & -I & 0 \\ H_{1r} + H_{2r} K_r & 0 & 0 & -\varepsilon_{r2} I \end{bmatrix} < 0$$

Pre- and post-multiplying the inequality $Z_r < 0$ by block diagonal matrix $diag\{ P_r^{-1} \quad I \quad I \quad I \}$, letting $X_r = P_r^{-1}$, $Y_r = K_r X_r$ and applying the Schur complement formula, we obtain inequality (28) using $\sum_{\sigma_r \in \Theta} \mu_{\sigma_r} = 1$.

Define $\tilde{X}_r = \tilde{P}_r^{-1} = R_r^{1/2} X_r R_r^{1/2}$, and consider

$$\lambda_{\max}(\tilde{X}_r) = \frac{1}{\lambda_{\min}(\tilde{P}_r)}$$

From condition (26) it follows that

$$\frac{c_1}{\lambda_{\min}(\tilde{X}_r)} + \frac{\eta^2 d}{\alpha}\left(1 - e^{-\alpha t}\right) < \frac{e^{-\alpha t} c_2}{\lambda_{\max}(\tilde{X}_r)} \tag{31}$$

Define

$$\lambda_{\max}(\tilde{X}_r) < 1, \quad \lambda = \lambda_{\min}(\tilde{X}_r) \tag{32}$$

The above definitions imply condition (29). Putting (31) and (32) together, condition (30) can be obtained. This completes the proof. □

Remark 5 Note that Theorem 2 has presented the sufficient condition of designing the finite-time stabilized controller for nonlinear MJSs, and the coupled LMIs (28)–(30) are with respect to $X_r, Y_r, \varepsilon_{r1}, \varepsilon_{r2}, c_1, c_2, \lambda, d, N, \alpha$ and η^2. Therefore, for given scalars c_1, c_2, N, α and d, we can take η^2 as an optimized variable, i.e., to obtain an optimal finite-time stabilized controller, the attenuation level η^2 can be reduced to the minimum possible level such that LMIs (28)–(30) are satisfied. The optimization problem can be described as follows:

$$\begin{aligned} &\min_{X_r, Y_r, \varepsilon_{r1}, \varepsilon_{r2}, \lambda} \beta \\ &\text{s.t.} \quad \text{LMI (28)–(30)} \quad \text{with } \beta = \eta^2 \end{aligned} \tag{33}$$

Remark 6 Let $\alpha = 0$, $u(t) = 0$, $w(t) = 0$, and $f(x) = 0$. We can get

$$(A_r + \Delta A_r)^{\mathrm{T}} P_r + P_r (A_r + \Delta A_r) + \sum_{j=1}^{N} \pi_{rj} P_j < 0$$

which can guarantee the Lyapunov stochastic stability (or almost asymptotic stability) of uncertain MJSs (see [6]). By using MATLAB LMI Toolbox, it is straightforward to check the feasibility of Theorem 2 and Remark 5.

Remark 7 The class of neural network-based LDI representations is similar to Takagi–Sugeno's fuzzy model [11], which is obtained by interpolating several linearized systems at different operating points through fuzzy certainty functions. Therefore, the results presented above can be applied to their fuzzy models.

4 Numerical Examples

Consider the nonlinear MJS (1) with parameters given by
Mode 1:

$$A_1 = \begin{bmatrix} -1 & -3 \\ 0 & -5 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}^{\mathrm{T}}, \qquad B_{d1} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \qquad H_{11} = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.3 \end{bmatrix}$$

$$f_1(x) = \begin{bmatrix} 0 \\ \sin(x_1(t)) \end{bmatrix}, \qquad H_{21} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \qquad D_1 = D_{d1} = 0.1$$

Mode 2:

$$A_2 = \begin{bmatrix} 0 & -2 \\ 0 & -3 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 1 \\ 1.8 \end{bmatrix}^{\mathrm{T}}, \qquad B_{d2} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad H_{12} = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.3 \end{bmatrix}$$

$$f_2(x) = \begin{bmatrix} 0 \\ \sin(0.1x_1(t)) \end{bmatrix}, \qquad H_{22} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \qquad D_2 = D_{d2} = 0.2$$

Mode 3:

$$A_3 = \begin{bmatrix} -1 & -3 \\ 0 & -4 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} 0 \\ 1.7 \end{bmatrix}, \qquad C_3 = \begin{bmatrix} 1.2 \\ 1.5 \end{bmatrix}^{\mathrm{T}}, \qquad B_{d3} = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \qquad H_{13} = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.5 \end{bmatrix}$$

$$f_3(x) = \begin{bmatrix} 0 \\ \sin(0.5x_1(t)) \end{bmatrix}, \qquad H_{23} = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix}, \qquad D_3 = D_{d3} = 0.4$$

The transition rate matrix is defined by

$$\Pi = \begin{bmatrix} -3 & 1.8 & 1.2 \\ 0.3 & -2 & 1.7 \\ 0.3 & 0.7 & -1 \end{bmatrix}$$

and the corresponding evolution of the jump mode is shown in Fig. 1.

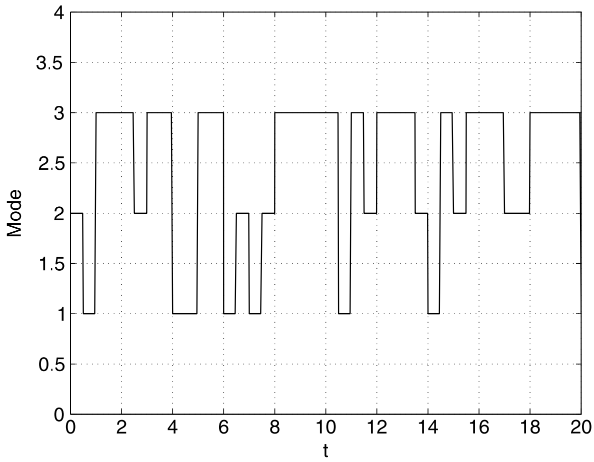

Fig. 1 Evolution of mode versus time

It is to be noted that the jumping $\gamma(r)$ appears in the MJS (1) as cyclic frequency of nonlinear function. Thus, the same three single hidden layer neural networks with 2 hidden neurons were chosen to approximate the nonlinear functions $f_r(x)$ for each mode. All parameters of activation functions associated with the hidden layer were chosen to be $q_{rj} = 0.5$ and $\lambda_{rj} = 1$. For these activation functions, we have $s_{rj}(0, \varphi_{rj}) = 0$ and $s_{rj}(1, \varphi_{rj}) = 1$, and three LDI are obtained as

$$A_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} 0 & 0 \\ 0.5152 & 0 \end{bmatrix},$$
$$A_{13} = \begin{bmatrix} 0 & 0 \\ -0.2257 & 0 \end{bmatrix}, \qquad A_{14} = \begin{bmatrix} 0 & 0 \\ 0.7409 & 0 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_{22} = \begin{bmatrix} 0 & 0 \\ 1.2480 & 0 \end{bmatrix},$$
$$A_{23} = \begin{bmatrix} 0 & 0 \\ 4.4304 & 0 \end{bmatrix}, \qquad A_{24} = \begin{bmatrix} 0 & 0 \\ -3.1824 & 0 \end{bmatrix},$$
$$A_{31} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_{32} = \begin{bmatrix} 0 & 0 \\ 1.3656 & 0 \end{bmatrix},$$
$$A_{33} = \begin{bmatrix} 0 & 0 \\ 2.6068 & 0 \end{bmatrix}, \qquad A_{34} = \begin{bmatrix} 0 & 0 \\ -2.6065 & 0 \end{bmatrix}$$

The upper bounds of the approximation error are $\rho_1 = 0.15$, $\rho_2 = 0.23$ and $\rho_3 = 0.85$ respectively.

Introducing the initial value for $c_1 = 0.4$, $c_2 = 4$, $N = 3$, $R_r = 2I_2$, $d = 4$ and $\alpha = 0.5$, and applying Theorem 2, the mode-dependent gains of the feedback control law are obtained:

$$K_1 = [-2.6912 \quad -3.9730]$$
$$K_2 = [-35.3945 \quad -18.6009]$$
$$K_3 = [-24.2483 \quad -22.2155]$$

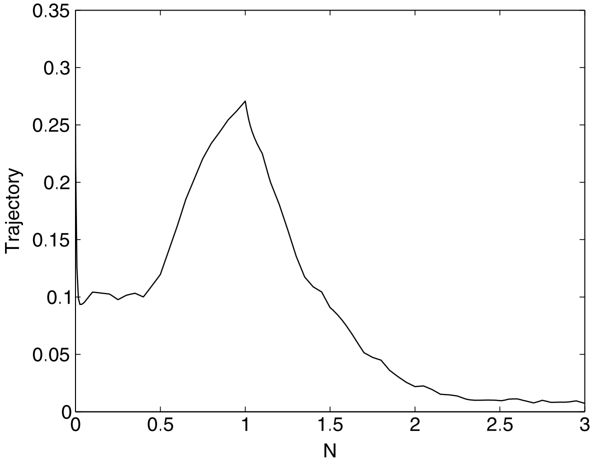

Fig. 2 Trajectory of the system energy in finite-time interval

To demonstrate the effectiveness of the design method, assuming the initial condition is $x_0 = [0.1 \quad 0.2]^{\mathrm{T}}$, the trajectory of the system energy under the state-feedback control law in a finite-time interval is shown in Fig. 2. It can be seen that the system stays within a certain bound over a finite-time horizon by the designed controller.

Remark 8 It should be pointed out that, in the simulation example, as long as the choice of initial condition is satisfied with $\|x_0^{\mathrm{T}} R_r x_0\| \leq c_1$, the system is robustly finite-time stabilizable, i.e., system trajectories stay within a given bound.

5 Conclusion

In this paper, the problem of robust finite-time H_∞ control for a class of uncertain nonlinear MJSs has been investigated. The uncertain parameters are assumed to be unknown, but norm bounded. By means of LDI state-space representation, a general design methodology for neural network-based control systems is extended such that the uncertain nonlinear MJS is finite-time stabilizable (FTS) and satisfies a given H_∞ performance index. The main results are presented in the form of linear matrix inequalities. A simulation example is given to demonstrate the effectiveness and the potential of the proposed techniques.

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