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Fuzzy Feedback Linearization Control for MIMO Nonlinear System and Its Application to Full-Vehicle Suspension System

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Abstract The paper presents a novel fuzzy feedback linearization control of nonlinear multi-input multi-output (MIMO) systems for the tracking and almost disturbance decoupling (ADD) performances based on the fuzzy logic control (FLC). The main contribution of this study is to construct a controller, under appropriate conditions, such that the resulting closed-loop system is valid for any initial condition and bounded tracking signal with the following characteristics: input-to-state stability with respect to disturbance inputs and almost disturbance decoupling. The feedback linearization control guarantees the almost disturbance decoupling performance and the uniform ultimate bounded stability of the tracking error system. As soon as the tracking errors are driven to touch the global final attractor with the desired radius, the fuzzy logic control immediately is applied via a human expert's knowledge to improve the convergence rate. One example, which cannot be solved by the previous paper on the almost disturbance decoupling problem, is proposed in this paper to exploit the fact that the tracking and the almost disturbance decoupling performances are easily achieved by the proposed approach. In order to demonstrate the applicability, this paper has investigated a full-vehicle suspension system.

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1 Introduction

Two well-known tasks of stabilization and tracking problems are important topics in the field of control. Tracking problems are generally more complicated than stabilization problems for nonlinear control systems. Many approaches for nonlinear systems are introduced, including feedback linearization, variable structure control (sliding mode control), backstepping, regulation control, nonlinear H^{∞} control, the internal model principle and H^{∞} adaptive fuzzy control. Recently, variable structure control has been introduced to deal with nonlinear systems [18]. However, chattering behavior which may create unmodeled high frequency due to the discontinuous switching and imperfect implementation and may even drive systems to instability is inevitable for variable structure control schemes. Backstepping has been a powerful tool for synthesizing the controller for a class of nonlinear systems. However, a disadvantage of the backstepping approach is the explosion of complexity which is caused by the complicated repeated differentiations of some nonlinear functions [32, 42]. An output tracking approach is to utilize the scheme of the output regulation control [14], in which the outputs are assumed to be excited by an exosystem. However, the nonlinear regulation problem requires solving the difficult solution of partial-differential algebraic equations. Another problem of the output regulation control is that the exosystem states need to be switched to describe changes in the output and this will create transient tracking errors [27]. In general, the nonlinear H^{∞} control has to solve the Hamilton–Jacobi equation, which is a difficult nonlinear partial-differential equation [2, 15, 36]. Only for some particular nonlinear systems can we derive a closed-form solution [13]. The control approach based on the internal model principle converts the tracking problem to a nonlinear output regulation problem. This approach depends on solving a first-order partial-differential equation of the center manifold [14]. For some special nonlinear systems and desired trajectories, the asymptotic solutions of this equation via ordinary differential equations have been developed [6, 9]. Recently, H^{∞} adaptive fuzzy control has been proposed to deal with nonlinear systems systematically [4]. The drawback with H^{∞} adaptive fuzzy control is that the complex parameter update law makes this approach impractical. During the past decade, significant progress has been made in the research of control approaches for nonlinear systems based on the feedback linearization theory [7, 12, 18, 26, 30]. Moreover, the feedback linearization approach has been applied successfully to address many real controls. These include the control of electromagnetic suspension systems [16], pendulum systems [5], spacecraft [29], electrohydraulic servosystems [1], car-pole systems [3] and bank-to-turn missile systems [21].

For the past two decades, fuzzy logic control has attracted a great deal of attention [31]. Despite the success, many fundamental issues remain unanswered. Almost disturbance decoupling analysis and systematic design are among the primary issues to be further addressed. The almost disturbance decoupling problem, the design of a controller which attenuated the effect of the disturbance on the output terminal to an arbitrary degree of accuracy, was originally developed for linear and nonlinear control systems in [39] and [24], respectively. Henceforward, the problem has attracted considerable attentions and many significant results have been developed for both linear and nonlinear control systems [23, 28, 38]. The almost disturbance decoupling problem of nonlinear single-input/single-output (SISO) systems was investigated in [24] by state feedback and solved in terms of sufficient conditions for systems with nonlinearities which are not globally Lipschitz and disturbances appearing linearly but possibly multiplying nonlinearities. The resulting state feedback control is constructed following a singular perturbation approach. The sufficient conditions in [24] require that the nonlinearities multiplying the disturbances satisfy structural triangular conditions. The result of [24] indicated that for nonlinear SISO systems the almost disturbance decoupling problem could not be solvable, as the following example showed:

$$\begin{aligned}\dot{x}_1(t) &= \tan^{-1} x_2 + \theta(t), \qquad \left|\theta(t)\right| > \frac{\pi}{2}\\ \dot{x}_2(t) &= u\\ y &= x_1\end{aligned}$$

where u, y denoted the input and output respectively and $\theta(t)$ was the disturbance of the system. On the contrary, this example can be easily solved via the proposed approach in this paper.

Fuzzy logic control has been applied not only to cement kilns and subway trains but also to industrial processes. Its designing procedures are as follows. First, representing the nonlinear system as the famous Takagi–Sugeno fuzzy model offers an alternative to conventional models. The control design is carried out based on an aggregation of linear controllers constructed for each local linear element of the fuzzy model via the parallel distributed compensation scheme [37]. For the stability analysis of fuzzy systems, a lot of studies are reported (see, e.g., [20, 33–35] and the references therein). The stability and controller design of fuzzy systems can be mainly discussed by Tanaka–Sugeno's theorem [33]. However, it's difficult to find the common positive definite matrix P for linear matrix inequality (LMI) problem even if P is a second-order matrix [17]. To overcome the difficulty of finding the common positive definite matrix P for the fuzzy-model approach, we will propose a new method to guarantee that the closed-loop system is stable and the almost disturbance decoupling performance is achieved. The proposed designing structure is as follows. First, based on the feedback linearization approach, a tracking control is proposed to guarantee the almost disturbance decoupling property and the uniform ultimate bounded stability of the tracking error system. As soon as the tracking errors are driven to touch the global final attractor, the conventional fuzzy logic control immediately is applied via a human expert's knowledge to improve the convergence rate. In order to exploit the significant applicability, this paper also has successfully derived the tracking controller with almost disturbance decoupling for a full-vehicle suspension system. Throughout the paper, the notation $\|\cdot\|$ denotes the usual Euclidean norm or the corresponding induced matrix norm.

2 Controller Design

In this paper, we consider the following nonlinear uncertain control system with disturbances:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n) \\ f_2(x_1, x_2, \dots, x_n) \\ \vdots \\ f_n(x_1, x_2, \dots, x_n) \end{bmatrix} + \begin{bmatrix} g_1(x_1, x_2, \dots, x_n) & g_2(x_1, x_2, \dots, x_n) & \cdots & g_m(x_1, x_2, \dots, x_n) \end{bmatrix} \times \begin{bmatrix} u_1(x_1, x_2, \dots, x_n) \\ u_2(x_1, x_2, \dots, x_n) \\ \vdots \\ u_m(x_1, x_2, \dots, x_n) \end{bmatrix} + \sum_{j=1}^{p} q_j^* \theta_{dj} \tag{1a}$$

$$\begin{bmatrix} y_1(x_1, x_2, \dots, x_n) \\ y_2(x_1, x_2, \dots, x_n) \\ \vdots \\ y_m(x_1, x_2, \dots, x_n) \end{bmatrix} = \begin{bmatrix} h_1(x_1, x_2, \dots, x_n) \\ h_2(x_1, x_2, \dots, x_n) \\ \vdots \\ h_m(x_1, x_2, \dots, x_n) \end{bmatrix} \tag{1b}$$

i.e.,

$$\dot{X}(t) = f\big(X(t)\big) + g\big(X(t)\big)u + \sum_{j=1}^{p} q_j^* \theta_{dj}$$

$$y(t) = h\big(X(t)\big)$$

where $X(t) \equiv [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^{\mathrm{T}} \in \Re^n$ is the state vector, $u \equiv [u_1\ u_2\ \cdots\ u_m]^{\mathrm{T}} \in \Re^m$ is the input vector, $y \equiv [y_1\ y_2\ \cdots\ y_m]^{\mathrm{T}} \in \Re^m$ is the output vector and $\theta_d \equiv [\theta_{d1}(t)\ \theta_{d2}(t)\ \cdots\ \theta_{dp}(t)]^{\mathrm{T}}$ is a bounded time-varying disturbances vector. The functions $q_j^* \in \Re^n$, $f \equiv [f_1\ f_2\ \cdots\ f_n]^{\mathrm{T}} \in \Re^n$ and $g \equiv [g_1\ g_2\ \cdots\ g_m] \in \Re^{n\times m}$ are smooth vector fields. The function $h \equiv [h_1\ h_2\ \cdots\ h_m]^{\mathrm{T}} \in \Re^m$ is a smooth vector-valued function. The nominal system is then defined as follows:

$$\dot{X}(t) = f\big(X(t)\big) + g\big(X(t)\big)u \tag{2a}$$

$$y(t) = h\big(X(t)\big) \tag{2b}$$

The nominal system of the form (2) is assumed to have the vector relative degree $\{r_1, r_2, \dots, r_m\}$ [12], i.e., the following conditions are satisfied for all $X \in \Re^n$:

(i)

$$L_{g_j} L_f^k h_i(X) = 0 \tag{3}$$

for all $1 \le i \le m$, $1 \le j \le m$, $k < r_i - 1$, where the operator L is the Lie derivative [12] and $r_1 + r_2 + \cdots + r_m = r$.

(ii) The $m \times m$ matrix

$$A \equiv \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(X) & \cdots & L_{g_m} L_f^{r_1-1} h_1(X) \\ L_{g_1} L_f^{r_2-1} h_2(X) & \cdots & L_{g_m} L_f^{r_2-1} h_2(X) \\ \vdots & & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(X) & \cdots & L_{g_m} L_f^{r_m-1} h_m(X) \end{bmatrix} \tag{4}$$

is nonsingular.

The desired output trajectory y_d^i, $1 \le i \le m$ and its first r_i derivatives are all uniformly bounded and

$$\left\| \left[y_d^i, y_d^{i(1)}, \ldots, y_d^{i(r_i)} \right] \right\| \le B_d^i, \quad 1 \le i \le m \tag{5}$$

where B_d^i is a positive constant.

The objective of the paper is to propose a control that includes a feedback linearization controller and a fuzzy logic controller to achieve the almost disturbance decoupling and tracking performances.

2.1 Feedback Linearization Controller Design

Under the assumption of well-defined vector relative degree, it has been shown [12] that the mapping

$$\phi : \Re^n \to \Re^n \tag{6}$$

defined as

$$\xi_i \equiv \begin{bmatrix} \xi_1^i \\ \xi_2^i \\ \vdots \\ \xi_{r_i}^i \end{bmatrix} \equiv \begin{bmatrix} \phi_1^i \\ \phi_2^i \\ \vdots \\ \phi_{r_i}^i \end{bmatrix} \equiv \begin{bmatrix} L_f^0 h_i(X) \\ L_f^1 h_i(X) \\ \vdots \\ L_f^{r_i-1} h_i(X) \end{bmatrix}, \quad i = 1, 2, \ldots, m \tag{7}$$

$$\phi_k\big(X(t)\big) \equiv \eta_k(t), \quad k = r+1,\ r+2, \ldots, n \tag{8}$$

satisfying

$$L_{g_j} \phi_k\big(X(t)\big) = 0, \quad k = r+1,\ r+2, \ldots, n,\ 1 \le j \le m \tag{9}$$

is a diffeomorphism onto image, if

(i) The distribution

$$G \equiv \operatorname{span}\{g_1, g_2, \ldots, g_m\} \tag{10}$$

is involutive.

(ii) The vector fields

$$Y_j^k, \quad 1 \le j \le m,\ 1 \le k \le r_j \tag{11}$$

are complete, where

$$Y_j^k \equiv (-1)^{k-1} \mathrm{ad}_{\tilde{f}}^{k-1} \tilde{g}_j, \quad 1 \le j \le m,\ 1 \le k \le r_j \tag{12}$$

$$\tilde{f}(X) \equiv f(X) - g(X) A^{-1}(X) b(X) \tag{13}$$

$$b(X) \equiv \begin{bmatrix} L_f^{r_1} h_1(X) \\ L_f^{r_2} h_2(X) \\ \vdots \\ L_f^{r_m} h_m(X) \end{bmatrix} \tag{14}$$

$$\tilde{g} \equiv \begin{bmatrix} \tilde{g}_1 & \tilde{g}_2 & \cdots & \tilde{g}_m \end{bmatrix} \equiv g(X) A^{-1}(X) \tag{15}$$

$$\mathrm{ad}_f^k g \equiv \left[f,\ \mathrm{ad}_f^{k-1} g \right] \tag{16}$$

$$[f, g] \equiv \frac{\partial g}{\partial X} f(X) - \frac{\partial f}{\partial X} g(X) \tag{17}$$

For the sake of convenience, define the trajectory error to be

$$e_j^i \equiv \xi_j^i - y_d^{i(j-1)}, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, r_i \tag{18}$$

$$e^i \equiv \begin{bmatrix} e_1^i & e_2^i & \cdots & e_{r_i}^i \end{bmatrix}^{\mathrm{T}} \in \Re^{r_i} \tag{19}$$

The trajectory error is multiplied with some adjustable positive constant ε as follows:

$$\overline{e_j^i} \equiv \varepsilon^{j-1} e_j^i, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, r_i \tag{20}$$

$$\overline{e^i} \equiv \begin{bmatrix} \overline{e_1^i} & \overline{e_2^i} & \cdots & \overline{e_{r_i}^i} \end{bmatrix}^{\mathrm{T}} \in \Re^{r_i} \tag{21}$$

$$\overline{e} \equiv \begin{bmatrix} \overline{e^1} \\ \overline{e^2} \\ \vdots \\ \overline{e^m} \end{bmatrix} \in \Re^{r} \tag{22}$$

and let

$$\xi \equiv \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_r \end{bmatrix} \in \Re^{r} \tag{23}$$

$$\eta(t) \equiv \begin{bmatrix} \eta_{r+1}(t) & \eta_{r+2}(t) & \cdots & \eta_n(t) \end{bmatrix}^{\mathrm{T}} \in \Re^{n-r} \tag{24}$$

$$q\big(\xi(t),\eta(t)\big)\equiv\big[L_f\phi_{r+1}(t)\quad L_f\phi_{r+2}(t)\quad\cdots\quad L_f\phi_n(t)\big]^{\mathrm{T}}$$
$$\equiv[q_{r+1}\quad q_{r+2}\quad\cdots\quad q_n]^{\mathrm{T}} \tag{25}$$

Define a phase-variable canonical matrix A_c^i to be

$$A_c^i\equiv\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&&&\vdots\\0&0&0&\cdots&1\\-\alpha_1^i&-\alpha_2^i&-\alpha_3^i&\cdots&-\alpha_{r_i}^i\end{bmatrix}_{r_i\times r_i},\quad 1\le i\le m \tag{26}$$

where $\alpha_1^i,\alpha_2^i,\dots,\alpha_{r_i}^i$ are some chosen parameters such that A_c^i is Hurwitz and define the vector B^i to be

$$B^i\equiv[0\quad 0\quad\cdots\quad 0\quad 1]^{\mathrm{T}}_{r_i\times 1},\quad 1\le i\le m \tag{27}$$

Let P^i be the positive definite solution of the following Lyapunov equation:

$$\big(A_c^i\big)^{\mathrm{T}}P^i+P^iA_c^i=-I,\quad 1\le i\le m \tag{28}$$

$$\lambda_{\max}\big(P^i\big)\equiv\text{the maximum eigenvalue of }P^i,\quad 1\le i\le m \tag{29}$$

$$\lambda_{\min}\big(P^i\big)\equiv\text{the minimum eigenvalue of }P^i,\quad 1\le i\le m \tag{30}$$

$$\lambda_{\max}^*\equiv\min\big\{\lambda_{\max}\big(P^1\big),\lambda_{\max}\big(P^2\big),\dots,\lambda_{\max}\big(P^m\big)\big\} \tag{31}$$

$$\lambda_{\min}^*\equiv\min\big\{\lambda_{\min}\big(P^1\big),\lambda_{\min}\big(P^2\big),\dots,\lambda_{\min}\big(P^m\big)\big\} \tag{32}$$

Assumption 1 For all $t\ge 0$, $\eta\in\Re^{n-r}$ and $\xi\in\Re^r$, there exists a positive constant M such that the following inequality holds:

$$\big\|q_{22}(t,\eta,\bar{e})-q_{22}(t,\eta,0)\big\|\le M\big(\|\bar{e}\|\big) \tag{33}$$

where $q_{22}(t,\eta,\bar{e})\equiv q(\xi,\eta)$.

For the sake of stating precisely the investigated problem, define

$$d_{ij}\equiv L_{g_j}L_f^{r_i-1}h_i(X),\quad 1\le i\le m,\ 1\le j\le m \tag{34}$$

$$c_i\equiv L_f^{r_i}h_i(X),\quad 1\le i\le m \tag{35}$$

and

$$\overline{\overline{e^i}}\equiv\alpha_1^i\overline{e_1^i}+\alpha_2^i\overline{e_2^i}+\cdots+\alpha_{r_i}^i\overline{e_{r_i}^i},\quad 1\le i\le m \tag{36}$$

Definition 1 [18] Consider the system $\dot{x}=f(t,x,\theta)$, where $f:[0,\infty)\times\Re^n\times\Re^n\to\Re^n$ is piecewise continuous in t and locally Lipschitz in x and θ. This system is said to be input-to-state stable if there exists a class KL function β, a class K

function γ and positive constants k_1 and k_2 such that for any initial state $x(t_0)$ with $\|x(t_0)\| < k_1$ and any bounded input $\theta(t)$ with $\sup_{t \ge t_0} \|\theta(t)\| < k_2$, the state exists and satisfies

$$\|x(t)\| \le \beta(\|x(t_0)\|, t - t_0) + \gamma\Big(\sup_{t_0 \le \tau \le t} \|\theta(\tau)\|\Big) \tag{37}$$

for all $t \ge t_0 \ge 0$. Now we formulate the tracking problem with almost disturbance decoupling as follows.

Definition 2 [23] The tracking problem with almost disturbance decoupling is said to be globally solvable by the state feedback controller u for the transformed-error system by a global diffeomorphism (6), if the controller u makes the closed-loop system enjoy the following properties:

(i) It is input-to-state stable with respect to disturbance inputs.
(ii) For any initial value $\bar{x}_{e0} \equiv [\bar{e}(t_0)\ \eta(t_0)]^{\mathrm{T}}$, any $t \ge t_0$ and any $t_0 \ge 0$

$$|y(t) - y_d(t)| \le \beta_{11}(\|x(t_0)\|, t - t_0) + \frac{1}{\sqrt{\beta_{22}}}\beta_{33}\Big(\sup_{t_0 \le \tau \le t} \|\theta(\tau)\|\Big) \tag{38}$$

and

$$\int_{t_0}^{t} [y(\tau) - y_d(\tau)]^2\, d\tau \le \frac{1}{\beta_{44}}\bigg[\beta_{55}(\|\bar{x}_{e0}\|) + \int_{t_0}^{t} \beta_{33}(\|\theta(\tau)\|^2)\, d\tau\bigg] \tag{39}$$

where β_{22}, β_{44} are some positive constants, β_{33}, β_{55} are class K functions and β_{11} is a class KL function.

Theorem 1 *Suppose that there exists a continuously differentiable function* $V : \Re^{n-r} \to \Re^{+}$ *such that the following three inequalities hold for all* $\eta \in \Re^{n-r}$:

$$(a)\quad \omega_1\|\eta\|^2 \le V(\eta) \le \omega_2\|\eta\|^2, \quad \omega_1, \omega_2 > 0 \tag{40a}$$

$$(b)\quad \nabla_t V + (\nabla_\eta V)^{\mathrm{T}} q_{22}(t, \eta, 0) \le -2\alpha_x\|\eta\|^2, \quad \alpha_x > 0 \tag{40b}$$

$$(c)\quad \|\nabla_\eta V\| \le \omega_3\|\eta\|, \quad \omega_3 > 0 \tag{40c}$$

then the tracking problem with almost disturbance decoupling is globally solvable by the controller defined as

$$u_{\text{feedback}} = A^{-1}\{-b + v\} \tag{41}$$

$$b \equiv \big[L_f^{r_1} h_1 \quad L_f^{r_2} h_2 \quad \cdots \quad L_f^{r_m} h_m\big]^{\mathrm{T}} \tag{42}$$

$$v \equiv [v_1 \quad v_2 \quad \cdots \quad v_m]^{\mathrm{T}} \tag{43}$$

$$v_i \equiv y_d^{i(r_i)} - \varepsilon^{-r_i}\alpha_1^i\big[L_f^0 h_i(X) - y_d^i\big] - \varepsilon^{1-r_i}\alpha_2^i\big[L_f^1 h_i(X) - y_d^{i(1)}\big] - \cdots - \varepsilon^{-1}\alpha_{r_i}^i\big[L_f^{r_i-1} h_i(X) - y_d^{i(r_i-1)}\big], \quad 1 \le i \le m \tag{44}$$

and the influence of disturbances on the L_2 norm of the tracking error can be arbitrarily attenuated by increasing the following adjustable parameter $N_2 > 1$:

$$k_{11} \equiv \frac{k}{2\varepsilon} - \frac{1}{36}\frac{k^2\|\phi_\xi^1\|^2\|P^1\|^2}{\varepsilon^2} - \cdots - \frac{1}{36}\frac{k^2\|\phi_\xi^m\|^2\|P^m\|^2}{\varepsilon^2} - 9 \tag{45a}$$

$$k_{22} \equiv 2\alpha_x - \frac{\omega_3^2 M^2}{36} - \frac{1}{36}\omega_3^2\|\phi_\eta\|^2 \tag{45b}$$

$$N_2 \equiv \min\{k_{11}, k_{22}\} \tag{45c}$$

$$N_1 \equiv \frac{m+1}{(1/9)}\left(\sup_{t_0\le\tau\le t}\left\|\theta_d(\tau)\right\|\right)^2 \tag{45d}$$

$$\phi_\xi^i(\varepsilon) \equiv \begin{bmatrix} \varepsilon\frac{\partial}{\partial X}h_i q_1^* & \cdots & \varepsilon\frac{\partial}{\partial X}h_i q_p^* \\ \vdots & & \vdots \\ \varepsilon^{r_i}\frac{\partial}{\partial X}L_f^{r_i-1}h_i q_1^* & \cdots & \varepsilon^{r_i}\frac{\partial}{\partial X}L_f^{r_i-1}h_i q_p^* \end{bmatrix}, \quad 1 \le i \le m \tag{45e}$$

$$\phi_\eta(\varepsilon) \equiv \begin{bmatrix} \frac{\partial}{\partial X}\phi_{r+1}q_1^* & \cdots & \frac{\partial}{\partial X}\phi_{r+1}q_p^* \\ \vdots & & \vdots \\ \frac{\partial}{\partial X}\phi_n q_1^* & \cdots & \frac{\partial}{\partial X}\phi_n q_p^* \end{bmatrix} \tag{45f}$$

where $k(\varepsilon): \Re^+ \to \Re^+$ is any continuous function satisfying

$$\lim_{\varepsilon\to 0} k(\varepsilon) = 0 \quad \textit{and} \quad \lim_{\varepsilon\to 0}\frac{\varepsilon}{k(\varepsilon)} = 0 \tag{45g}$$

Moreover, the output tracking error of system (1) is exponentially attracted into a sphere $B_{\underline{r}}$, $\underline{r} = \sqrt{\frac{N_1}{N_2}}$, with an exponential rate of convergence

$$\frac{1}{2}\left(\frac{N_2}{\Delta_{\max}} - \frac{N_1}{\Delta_{\max}\underline{r}^2}\right) \equiv \frac{1}{2}\alpha^* \tag{45h}$$

where

$$\Delta_{\max} \equiv \max\left\{\omega_2, \frac{k}{2}\lambda_{\max}^*\right\} \tag{45i}$$

Proof Applying the co-ordinate transformation (6) yields

$$\begin{aligned} \dot{\xi}_1^1 &= \frac{\partial\phi_1^1}{\partial X}\frac{dX}{dt} = \frac{\partial h_1}{\partial X}\left[f + g\cdot u + \sum_{j=1}^p q_j^*\theta_{dj}\right] \\ &= \frac{\partial h_1}{\partial X}f + \frac{\partial h_1}{\partial X}g_1u_1 + \cdots + \frac{\partial h_1}{\partial X}g_m u_m + \sum_{j=1}^p \frac{\partial h_1}{\partial X}q_j^*\theta_{dj} \\ &= \frac{\partial h_1}{\partial X}f + \sum_{j=1}^p \frac{\partial h_1}{\partial X}q_j^*\theta_{dj} = \xi_2^1 + \sum_{j=1}^p \frac{\partial h_1}{\partial X}q_j^*\theta_{dj} \end{aligned} \tag{46}$$

$$\vdots$$

$$\dot{\xi}^1_{r_1-1} = \frac{\partial \phi^1_{r_1-1}}{\partial X}\frac{dX}{dt} = \frac{\partial L_f^{r_1-2}h_1}{\partial X}\left[f + g\cdot u + \sum_{j=1}^{p} q_j^*\theta_{dj}\right]$$

$$= \frac{\partial L_f^{r_1-2}h_1}{\partial X}f + \frac{\partial L_f^{r_1-2}h_1}{\partial X}g_1u_1 + \cdots + \frac{\partial L_f^{r_1-2}h_1}{\partial X}g_mu_m + \sum_{j=1}^{p}\frac{\partial L_f^{r_1-2}h_1}{\partial X}q_j^*\theta_{dj}$$

$$= \frac{\partial L_f^{r_1-2}h_1}{\partial X}f + \sum_{j=1}^{p}\frac{\partial L_f^{r_1-2}h_1}{\partial X}q_j^*\theta_{dj} = L_f^{r_1-1}h_1 + \sum_{j=1}^{p}\frac{\partial L_f^{r_1-2}h_1}{\partial X}q_j^*\theta_{dj} \tag{47}$$

$$\dot{\xi}^1_{r_1} = \frac{\partial \phi^1_{r_1}}{\partial X}\frac{dX}{dt} = \frac{\partial L_f^{r_1-1}h_1}{\partial X}\left[f + g\cdot u + \sum_{j=1}^{p} q_j^*\theta_{dj}\right]$$

$$= \frac{\partial L_f^{r_1-1}h_1}{\partial X}f + \frac{\partial L_f^{r_1-1}h_1}{\partial X}g_1u_1 + \cdots + \frac{\partial L_f^{r_1-1}h_1}{\partial X}g_mu_m + \sum_{j=1}^{p}\frac{\partial L_f^{r_1-1}h_1}{\partial X}q_j^*\theta_{dj}$$

$$= L_f^{r_1}h_1 + L_{g_1}L_f^{r_1-1}h_1u_1 + \cdots + L_{g_m}L_f^{r_1-1}h_1u_m + \sum_{j=1}^{p}\frac{\partial L_f^{r_1-1}h_1}{\partial X}q_j^*\theta_{dj}$$

$$= c_1 + d_{11}u_1 + \cdots + d_{1m}u_m + \sum_{j=1}^{p}\frac{\partial L_f^{r_1-1}h_1}{\partial X}q_j^*\theta_{dj} \tag{48}$$

$$\vdots$$

$$\dot{\xi}^m_1 = \frac{\partial \phi^m_1}{\partial X}\frac{dX}{dt} = \frac{\partial h_m}{\partial X}\left[f + g\cdot u + \sum_{j=1}^{p} q_j^*\theta_{dj}\right]$$

$$= \frac{\partial h_m}{\partial X}f + \frac{\partial h_m}{\partial X}g_1u_1 + \cdots + \frac{\partial h_m}{\partial X}g_mu_m + \sum_{j=1}^{p}\frac{\partial h_m}{\partial X}q_j^*\theta_{dj}$$

$$= L_f^1h_m + \sum_{j=1}^{p}\frac{\partial h_m}{\partial X}q_j^*\theta_{dj} = \xi_2^m + \sum_{j=1}^{p}\frac{\partial h_m}{\partial X}q_j^*\theta_{dj} \tag{49}$$

$$\vdots$$

$$\begin{aligned}
\dot{\xi}^m_{r_m-1} &= \frac{\partial \phi^m_{r_m-1}}{\partial X}\frac{dX}{dt} = \frac{\partial L_f^{r_m-2}h_m}{\partial X}\left[f + g\cdot u + \sum_{j=1}^{p} q_j^* \theta_{dj}\right] \\
&= \frac{\partial L_f^{r_m-2}h_m}{\partial X} f + \frac{\partial L_f^{r_m-2}h_m}{\partial X} g_1 u_1 + \cdots + \frac{\partial L_f^{r_m-2}h_m}{\partial X} g_m u_m \\
&\quad + \sum_{j=1}^{p} \frac{\partial L_f^{r_m-2}h_m}{\partial X} q_j^* \theta_{dj} \\
&= L_f^{r_m-1}h_m + \sum_{j=1}^{p} \frac{\partial L_f^{r_m-2}h_m}{\partial X} q_j^* \theta_{dj} = \xi^m_{r_m} + \sum_{j=1}^{p} \frac{\partial L_f^{r_m-2}h_m}{\partial X} q_j^* \theta_{dj}
\end{aligned} \tag{50}$$

$$\begin{aligned}
\dot{\xi}^m_{r_m} &= \frac{\partial \phi^m_{r_m}}{\partial X}\frac{dX}{dt} = \frac{\partial L_f^{r_m-1}h_m}{\partial X}\left[f + g\cdot u + \sum_{j=1}^{p} q_j^* \theta_{dj}\right] \\
&= \frac{\partial L_f^{r_m-1}h_m}{\partial X} f + \frac{\partial L_f^{r_m-1}h_m}{\partial X} g_1 u_1 + \cdots + \frac{\partial L_f^{r_m-1}h_m}{\partial X} g_m u_m \\
&\quad + \sum_{j=1}^{p} \frac{\partial L_f^{r_m-1}h_m}{\partial X} q_j^* \theta_{dj} \\
&= L_f^{r_m}h_m + L_{g_1}L_f^{r_m-1}h_m u_1 + \cdots + L_{g_m}L_f^{r_m-1}h_m u_m + \sum_{j=1}^{p} \frac{\partial L_f^{r_m-1}h_m}{\partial X} q_j^* \theta_{dj} \\
&= c_m + d_{m1}u_1 + \cdots + d_{mm}u_m + \sum_{j=1}^{p} \frac{\partial L_f^{r_m-1}h_m}{\partial X} q_j^* \theta_{dj}
\end{aligned} \tag{51}$$

$$\begin{aligned}
\dot{\eta}_k(t) &= \frac{\partial \phi_k}{\partial X}\frac{dX}{dt} = \frac{\partial \phi_k}{\partial X}\left[f + g\cdot u + \sum_{j=1}^{p} q_j^* \theta_{dj}\right] \\
&= \frac{\partial \phi_k}{\partial X} f + \frac{\partial \phi_k}{\partial X} g_1 u_1 + \cdots + \frac{\partial \phi_k}{\partial X} g_m u_m + \sum_{j=1}^{p} \frac{\partial \phi_k}{\partial X} q_j^* \theta_{dj} \\
&= L_f \phi_k + \sum_{j=1}^{p} \frac{\partial \phi_k}{\partial X} q_j^* \theta_{dj} \\
&= q_k + \sum_{j=1}^{p} \frac{\partial \phi_k}{\partial X} q_j^* \theta_{dj}, \quad k = r+1,\ r+2, \ldots, n
\end{aligned} \tag{52}$$

Since

$$c_i\big(\xi(t), \eta(t)\big) \equiv L_f^{r_i} h_i\big(X(t)\big), \quad 1 \le i \le m \tag{53}$$

$$d_{ij} \equiv L_{g_j} L_f^{r_i-1} h_i(X), \quad 1 \le i \le m,\ 1 \le j \le m \tag{54}$$

$$q_k\big(\xi(t), \eta(t)\big) = L_f \phi_k(X), \quad k = r+1,\ r+2, \dots, n \tag{55}$$

the dynamic equations of system (1) in the new co-ordinates are as follows:

$$\dot{\xi}_i^1(t) = \xi_{i+1}^1(t) + \sum_{j=1}^{p} \frac{\partial}{\partial X} L_f^{i-1} h_1 q_j^* \theta_{dj}, \quad i = 1, 2, \dots, r_1 - 1 \tag{56}$$

$$\begin{aligned} \dot{\xi}_{r_1}^1(t) &= c_1\big(\xi(t), \eta(t)\big) + d_{11}\big(\xi(t), \eta(t)\big) u_1 + \cdots + d_{1m}\big(\xi(t), \eta(t)\big) u_m \\ &\quad + \sum_{j=1}^{p} \frac{\partial}{\partial X} L_f^{r_1-1} h_1 q_j^* \theta_{dj} \end{aligned} \tag{57}$$

$$\vdots$$

$$\dot{\xi}_i^m(t) = \xi_{i+1}^m(t) + \sum_{j=1}^{p} \frac{\partial}{\partial X} L_f^{i-1} h_m q_j^* \theta_{dj}, \quad i = 1, 2, \dots, r_m - 1 \tag{58}$$

$$\begin{aligned} \dot{\xi}_{r_m}^m(t) &= c_m\big(\xi(t), \eta(t)\big) + d_{m1}\big(\xi(t), \eta(t)\big) u_1 + \cdots + d_{mm}\big(\xi(t), \eta(t)\big) u_m \\ &\quad + \sum_{j=1}^{p} \frac{\partial}{\partial X} L_f^{r_m-1} h_m q_j^* \theta_{dj} \end{aligned} \tag{59}$$

$$\dot{\eta}_k(t) = q_k\big(\xi(t), \eta(t)\big) + \sum_{j=1}^{p} \frac{\partial}{\partial X} \phi_k(X) q_j^* \theta_{dj}, \quad k = r+1, \dots, n \tag{60}$$

$$y_i(t) = \xi_1^i(t), \quad 1 \le i \le m \tag{61}$$

According to (18), (44), (53), and (54), the tracking controller can be rewritten as

$$u = A^{-1}[-b + v] \tag{62}$$

Substituting (62) into (57) and (59) yields the dynamic equations of system (1) as follows:

$$\begin{aligned} \begin{bmatrix} \dot{\xi}_1^i(t) \\ \dot{\xi}_2^i(t) \\ \vdots \\ \dot{\xi}_{r_i-1}^i(t) \\ \dot{\xi}_{r_i}^i(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ & \vdots & & & & \vdots \\ 0 & 0 & 0 & & \cdots & 1 \\ 0 & 0 & 0 & & \cdots & 0 \end{bmatrix} \begin{bmatrix} \xi_1^i(t) \\ \xi_2^i(t) \\ \vdots \\ \xi_{r_i-1}^i(t) \\ \xi_{r_i}^i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} v_i \\ &\quad + \begin{bmatrix} \sum_{j=1}^{p} \frac{\partial}{\partial X} h_i q_j^* \theta_{dj} \\ \sum_{j=1}^{p} \frac{\partial}{\partial X} L_f^1 h_i q_j^* \theta_{dj} \\ \vdots \\ \sum_{j=1}^{p} \frac{\partial}{\partial X} L_f^{r_i-1} h_i q_j^* \theta_{dj} \end{bmatrix} \end{aligned} \tag{63}$$

$$\begin{bmatrix} \dot{\eta}_{r+1}(t) \\ \dot{\eta}_{r+2}(t) \\ \vdots \\ \dot{\eta}_{n-1}(t) \\ \dot{\eta}_n(t) \end{bmatrix} = \begin{bmatrix} q_{r+1}(t) \\ q_{r+2}(t) \\ \vdots \\ q_{n-1}(t) \\ q_n(t) \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^{p} \frac{\partial}{\partial X}\phi_{r+1} q_j^* \theta_{dj} \\ \sum_{j=1}^{p} \frac{\partial}{\partial X}\phi_{r+2} q_j^* \theta_{dj} \\ \vdots \\ \sum_{j=1}^{p} \frac{\partial}{\partial X}\phi_{n-1} q_j^* \theta_{dj} \\ \sum_{j=1}^{p} \frac{\partial}{\partial X}\phi_{n} q_j^* \theta_{dj} \end{bmatrix} \tag{64}$$

$$y_i = [1 \quad 0 \quad \cdots \quad 0 \quad 0]_{1\times r_i} \begin{bmatrix} \xi_1^i(t) \\ \xi_2^i(t) \\ \vdots \\ \xi_{r_i-1}^i(t) \\ \xi_{r_i}^i(t) \end{bmatrix}_{r_i\times 1} = \xi_1^i(t), \quad 1 \le i \le m \tag{65}$$

When we combine (18), (20), (21), (26), and (44), it is easily verified that (63)–(65) can be transformed into the following forms:

$$\dot{\eta}(t) = q\big(\xi(t), \eta(t)\big) + \phi_\eta \theta_d \equiv q_{22}\big(t, \eta(t), \bar{e}\big) + \phi_\eta \theta_d \tag{66a}$$

$$\varepsilon \dot{\overline{e^i}}(t) = A_c^i \overline{e^i} + \phi_\xi^i \theta_d, \quad 1 \le i \le m \tag{66b}$$

$$y_i(t) = \xi_1^i(t), \quad 1 \le i \le m \tag{67}$$

We consider $L(\bar{e}, \eta)$, defined by

$$L(\bar{e}, \eta) \equiv V(\eta) + k(\varepsilon)W(\bar{e}) \equiv V(\eta) + k(\varepsilon)\big(W^1\big(\overline{e^1}\big) + \cdots + W^m\big(\overline{e^m}\big)\big) \tag{68}$$

and

$$W(\bar{e}) \equiv W^1\big(\overline{e^1}\big) + \cdots + W^m\big(\overline{e^m}\big) \tag{69}$$

to be a composite Lyapunov function of the subsystems (66a) and (66b) [19, 22], where $W(\overline{e^i})$ satisfies

$$W^i\big(\overline{e^i}\big) \equiv \frac{1}{2}\overline{e^i}^{\mathrm{T}} P^i \overline{e^i} \tag{70}$$

In view of (18), (33), and (40), the derivative of L along the trajectories of (66a) and (66b) is given by

$$\begin{aligned} \dot{L} &= \big[\nabla_t V + (\nabla_\eta V)^{\mathrm{T}}\dot{\eta}\big] + \frac{k}{2}\big[\big(\dot{\overline{e^1}}\big)^{\mathrm{T}} P^1 \overline{e^1} + \big(\overline{e^1} e^1\big)^{\mathrm{T}} P^1 \big(\dot{\overline{e^1}}\big) + \cdots \\ &\quad + \big(\dot{\overline{e^m}}\big)^{\mathrm{T}} P^m \overline{e^m} + \big(\overline{e^m}\big)^{\mathrm{T}} P^m \big(\dot{\overline{e^m}}\big)\big] \\ &= \big[\nabla_t V + (\nabla_\eta V)^{\mathrm{T}}\dot{\eta}\big] + \frac{k}{2}\bigg[\bigg(\frac{1}{\varepsilon}A_c^1\overline{e^1} + \frac{1}{\varepsilon}\phi_\xi^1\theta_d\bigg)^{\mathrm{T}} P^1 \overline{e^1} \\ &\quad + \big(\overline{e^1}\big)^{\mathrm{T}} P^1 \bigg(\frac{1}{\varepsilon}A_c^1\overline{e^1} + \frac{1}{\varepsilon}\phi_\xi^1\theta_d\bigg) + \cdots \end{aligned}$$

$$
\begin{aligned}
&+\left(\frac{1}{\varepsilon}A_c^m\overline{e^m}+\frac{1}{\varepsilon}\phi_\xi^m\theta_d\right)^{\mathrm T}P^m\overline{e^m}+\left(\overline{e^m}\right)^{\mathrm T}P^m\left(\frac{1}{\varepsilon}A_c^m\overline{e^m}+\frac{1}{\varepsilon}\phi_\xi^m\theta_d\right)\Bigg]\\
&=\left[\nabla_t V+(\nabla_\eta V)^{\mathrm T}\big(q_{22}\big(t,\eta(t),\overline{e}\big)+\phi_\eta\theta_d\big)\right]+\Bigg\{\frac{k}{2\varepsilon}\left(\overline{e^1}\right)^{\mathrm T}\left[\left(A_c^1\right)^{\mathrm T}P^1+P^1\left(A_c^1\right)\right]\overline{e^1}\\
&\quad+\cdots+\frac{k}{2\varepsilon}\left(\overline{e^m}\right)^{\mathrm T}\left[\left(A_c^m\right)^{\mathrm T}P^m+P^m\left(A_c^m\right)\right]\overline{e^m}\\
&\quad+\frac{k}{\varepsilon}\left[(\theta_d)^{\mathrm T}\left(\phi_\xi^1\right)^{\mathrm T}P^1\overline{e^1}+\cdots+(\theta_d)^{\mathrm T}\left(\phi_\xi^m\right)^{\mathrm T}P^m\overline{e^m}\right]\Bigg\}\\
&\le\left[\nabla_t V+(\nabla_\eta V)^{\mathrm T}q_{22}\big(t,\eta(t),\overline{e}\big)+(\nabla_\eta V)^{\mathrm T}\phi_\eta\theta_d\right]\\
&\quad-\frac{k}{2\varepsilon}\left[\left(\overline{e^1}\right)^{\mathrm T}\overline{e^1}+\cdots+\left(\overline{e^m}\right)^{\mathrm T}\overline{e^m}\right]\\
&\quad+\frac{k}{\varepsilon}\left[\|\theta_d\|\left\|\phi_\xi^1\right\|\left\|P^1\right\|\left\|\overline{e^1}\right\|+\cdots+\|\theta_d\|\left\|\phi_\xi^m\right\|\left\|P^m\right\|\left\|\overline{e^m}\right\|\right]\\
&\le\left[\nabla_t V+(\nabla_\eta V)^{\mathrm T}q_{22}\big(t,\eta(t),0\big)\right]+(\nabla_\eta V)^{\mathrm T}\left[q_{22}\big(t,\eta(t),\overline{e}\big)-q_{22}\big(t,\eta(t),0\big)\right]\\
&\quad+\|\nabla_\eta V\|\|\phi_\eta\|\|\theta_d\|-\frac{k}{2\varepsilon}\left[\left\|\overline{e^1}\right\|^2+\left\|\overline{e^2}\right\|^2+\cdots+\left\|\overline{e^m}\right\|^2\right]\\
&\quad+\frac{1}{36}\frac{k^2}{\varepsilon^2}\left\|\phi_\xi^1\right\|^2\left\|P^1\right\|^2\left\|\overline{e^1}\right\|^2+9\|\theta_d\|^2+\cdots\\
&\quad+\frac{1}{36}\frac{k^2}{\varepsilon^2}\left\|\phi_\xi^m\right\|^2\left\|P^m\right\|^2\left\|\overline{e^m}\right\|^2+9\|\theta_d\|^2\\
&\le\left[\nabla_t V+(\nabla_\eta V)^{\mathrm T}q_{22}\big(t,\eta(t),0\big)\right]+\|\nabla_\eta V\|\left\|q_{22}\big(t,\eta(t),\overline{e}\big)-q_{22}\big(t,\eta(t),0\big)\right\|\\
&\quad+\|\nabla_\eta V\|\|\phi_\eta\|\|\theta_d\|-\frac{k}{2\varepsilon}\left[\|\bar e\|^2\right]+\frac{1}{36}\frac{k^2}{\varepsilon^2}\left\|\phi_\xi^1\right\|^2\left\|P^1\right\|^2\left\|\overline{e^1}\right\|^2+9\|\theta_d\|^2\\
&\quad+\cdots+\frac{1}{36}\frac{k^2}{\varepsilon^2}\left\|\phi_\xi^m\right\|^2\left\|P^m\right\|^2\left\|\overline{e^m}\right\|^2+9\|\theta_d\|^2\\
&\le-2\alpha_x\|\eta\|^2+\omega_3\|\eta\|M\|\bar e\|+\omega_3\|\eta\|\|\phi_\eta\|\|\theta_d\|\\
&\quad-\frac{k}{2\varepsilon}\|\bar e\|^2+\frac{1}{36}\frac{k^2}{\varepsilon^2}\left\|\phi_\xi^1\right\|^2\left\|P^1\right\|^2\left\|\overline{e^1}\right\|^2+9\|\theta_d\|^2+\cdots\\
&\quad+\frac{1}{36}\frac{k^2}{\varepsilon^2}\left\|\phi_\xi^m\right\|^2\left\|P^m\right\|^2\left\|\overline{e^m}\right\|^2+9\|\theta_d\|^2\\
&\le-2\alpha_x\|\eta\|^2+\frac{1}{36}\omega_3^2M^2\|\eta\|^2+9\|\bar e\|^2+\frac{1}{36}\omega_3^2\|\phi_\eta\|^2\|\eta\|^2+9\|\theta_d\|^2\\
&\quad-\frac{k}{2\varepsilon}\|\bar e\|^2+\frac{1}{36}\frac{k^2}{\varepsilon^2}\left\|\phi_\xi^1\right\|^2\left\|P^1\right\|^2\|\bar e\|^2+9\|\theta_d\|^2+\cdots\\
&\quad+\frac{1}{36}\frac{k^2}{\varepsilon^2}\left\|\phi_\xi^m\right\|^2\left\|P^m\right\|^2\|\bar e\|^2+9\|\theta_d\|^2
\end{aligned}
$$

$$= -\|\eta\|^2\left[2\alpha_x - \frac{1}{36}\omega_3^2 M^2 - \frac{1}{36}\omega_3^2\|\phi_\eta\|^2\right]$$
$$- \|\bar{e}\|^2\left[\frac{k}{2\varepsilon} - \frac{1}{36}\frac{k^2}{\varepsilon^2}\|\phi_\xi^1\|^2\|P^1\|^2 - \cdots - \frac{1}{36}\frac{k^2}{\varepsilon^2}\|\phi_\xi^m\|^2\|P^m\|^2 - 9\right]$$
$$+ \frac{m+1}{(1/9)}\|\theta_d\|^2 \tag{71}$$

i.e.

$$\dot{L} \le -k_{11}\|\bar{e}\|^2 - k_{22}\|\eta\|^2 + \frac{m+1}{(1/9)}\|\theta_d\|^2 \tag{72}$$

From (45c), we obtain

$$\dot{L} \le -N_2\big(\|\bar{e}\|^2 + \|\eta\|^2\big) + \frac{m+1}{(1/9)}\|\theta_d\|^2 \tag{73}$$

Define

$$\bar{e} \equiv \begin{bmatrix} \overline{e^1} \\ \overline{e^2} \\ \vdots \\ \overline{e^m} \end{bmatrix} \equiv \begin{bmatrix} \overline{e_1^1} \\ \overline{e_{\text{rem}}^1} \end{bmatrix}, \quad \overline{e_{\text{rem}}^1} \in \Re^{r-1} \tag{74}$$

Hence

$$\dot{L} \le -N_2\Big(\|\eta\|^2 + \big\|\overline{e_1^1}\big\|^2 + \big\|\overline{e_{\text{rem}}^1}\big\|^2\Big) + \frac{m+1}{(1/9)}\|\theta_d\|^2 \tag{75}$$

Utilizing (75) easily yields

$$\int_{t_0}^{t}\big(y_1(\tau) - y_d^1(\tau)\big)^2 d\tau \le \frac{L(t_0)}{N_2} + \frac{m+1}{(1/9)N_2}\int_{t_0}^{t}\big\|\theta_d(\tau)\big\|^2 d\tau \tag{76}$$

Similarly, it is easy to prove that

$$\int_{t_0}^{t}\big(y_i(\tau) - y_d^i(\tau)\big)^2 d\tau \le \frac{L(t_0)}{N_2} + \frac{m+1}{(1/9)N_2}\int_{t_0}^{t}\big\|\theta_d(\tau)\big\|^2 d\tau, \quad 2 \le i \le m \tag{77}$$

so that statement (39) is satisfied. From (73), we get

$$\dot{L} \le -N_2\big(\|y_{\text{total}}\|^2\big) + \frac{m+1}{(1/9)}\big\|\theta_d(\tau)\big\|^2 \tag{78a}$$

where

$$\|y_{\text{total}}\|^2 \equiv \|\bar{e}\|^2 + \|\eta\|^2. \tag{78b}$$

By virtue of ([18], Theorem 5.2), (78a) implies the input-to-state stability for the closed-loop system. Furthermore, it is easy to see that

$$\Delta_{\min}\big(\|\bar{e}\|^2 + \|\eta\|^2\big) \le L \le \Delta_{\max}\big(\|\bar{e}\|^2 + \|\eta\|^2\big) \tag{79}$$

i.e.

$$\Delta_{\min}\left(\|y_{\text{total}}\|^2\right) \le L \le \Delta_{\max}\left(\|y_{\text{total}}\|^2\right) \tag{80}$$

where $\Delta_{\min} \equiv \min\{\omega_1, \frac{k}{2}\lambda^*_{\min}\}$ and $\Delta_{\max} \equiv \max\{\omega_2, \frac{k}{2}\lambda^*_{\max}\}$. From (73) and (80), we get

$$\dot{L} \le -\frac{N_2}{\Delta_{\max}} L + \frac{m+1}{(1/9)}\Big(\sup_{t_0 \le \tau \le t} \|\theta_d(\tau)\|\Big)^2 \tag{81}$$

Hence,

$$\begin{aligned} \left|y_i(t) - y_d^i(t)\right| \le {} & \sqrt{\frac{2L(t_0)}{k\lambda^*_{\min}}} e^{-\frac{N_2}{2\Delta_{\max}}(t-t_0)} \\ & + \sqrt{\frac{\Delta_{\max}(m+1)}{(1/18)k\lambda^*_{\min}N_2}}\Big(\sup_{t_0 \le \tau \le t} \|\theta_d(\tau)\|\Big), \quad 2 \le i \le m \end{aligned} \tag{82}$$

which implies

$$\left|y_1(t) - y_d^1(t)\right| \le \sqrt{\frac{2L(t_0)}{k\lambda^*_{\min}}} e^{-\frac{N_2}{2\Delta_{\max}}(t-t_0)} + \sqrt{\frac{\Delta_{\max}(m+1)}{(1/18)k\lambda^*_{\min}N_2}}\Big(\sup_{t_0 \le \tau \le t} \|\theta_d(\tau)\|\Big) \tag{83}$$

Similarly, it is easy to prove that

$$\begin{aligned} \left|y_i(t) - y_d^i(t)\right| \le {} & \sqrt{\frac{2L(t_0)}{k\lambda^*_{\min}}} e^{-\frac{N_2}{2\Delta_{\max}}(t-t_0)} \\ & + \sqrt{\frac{\Delta_{\max}(m+1)}{(1/18)\,k\lambda^*_{\min}N_2}}\Big(\sup_{t_0 \le \tau \le t} \|\theta_d(\tau)\|\Big), \quad 2 \le i \le m \end{aligned} \tag{84}$$

Thus statement (38) is proved and then the tracking problem with almost disturbance decoupling is globally solved. Finally, we will prove that the sphere $B_{\underline{r}}$ is a global attractor for the output tracking error of system (1). From (78a) and (45d), we get

$$\dot{L} \le -N_2\left(\|y_{\text{total}}\|^2\right) + N_1 \tag{85}$$

For $\|y_{\text{total}}\| > \underline{r}$, we have $\dot{L} < 0$. Hence any closed ball defined by

$$B_{\underline{r}} \equiv \left\{\begin{bmatrix}\bar{e}\\ \eta\end{bmatrix} : \|\bar{e}\|^2 + \|\eta\|^2 \le \underline{r}\right\} \tag{86}$$

is a global final attractor for the tracking error system of the nonlinear control systems (1). Furthermore, for $y \notin B_{\overline{r}}$, we have

$$\begin{aligned} \frac{\dot{L}}{L} &\le \frac{-N_2\|y_{\text{total}}\|^2 + N_1}{L} \le \frac{-N_2\|y_{\text{total}}\|^2 + N_1}{\Delta_{\max}\|y_{\text{total}}\|^2} \le \frac{-N_2}{\Delta_{\max}} + \frac{N_1}{\Delta_{\max}\|y_{\text{total}}\|^2} \\ &\le \frac{-N_2}{\Delta_{\max}} + \frac{N_1}{\Delta_{\max}\underline{r}^2} \equiv -\alpha^* \end{aligned} \tag{87}$$

i.e.,

$$\dot{L} \le -\alpha^* L$$

According to the comparison theorem [25], we get

$$L(y_{\text{total}}(t)) \le L(y_{\text{total}}(t_0)) \exp[-\alpha^*(t - t_0)]$$

Therefore,

$$\begin{aligned} \Delta_{\min} \|y_{\text{total}}\|^2 &\le L(y_{\text{total}}(t)) \le L(y_{\text{total}}(t_0)) \exp[-\alpha^*(t - t_0)] \\ &\le \Delta_{\max} \|y_{\text{total}}(t_0)\|^2 \exp[-\alpha^*(t - t_0)] \end{aligned} \tag{88}$$

Consequently, we get

$$\|y_{\text{total}}\| \le \sqrt{\frac{\Delta_{\max}}{\Delta_{\min}}} \|y_{\text{total}}(t_0)\| \exp\left[-\frac{1}{2}\alpha^*(t - t_0)\right]$$

i.e., the convergence rate toward the sphere $B_{\underline{r}}$ is equal to $\alpha^*/2$. This completes our proof. □

If the sum $r_1 + r_2 + \cdots + r_m$ is equal to the system dimension n, then Theorem 1 will be reduced to the following simplified version with cancelling Assumption 1 and (40).

Theorem 2 *The tracking problem with almost disturbance decoupling is globally solvable by the controller defined by* (41) *and the influence of disturbances on the* L_2 *norm of the tracking error can be arbitrarily attenuated by increasing the following adjustable parameter* $k_{11} > 1$:

$$k_{11} \equiv \frac{k}{2\varepsilon} - \frac{1}{36}\frac{k^2\|\phi_\xi^1\|^2\|P^1\|^2}{\varepsilon^2} - \cdots - \frac{1}{36}\frac{k^2\|\phi_\xi^m\|^2\|P^m\|^2}{\varepsilon^2} \tag{89}$$

$$N_1 \equiv \frac{m}{(1/9)}\Big(\sup_{t_0 \le \tau \le t} \|\theta_d(\tau)\|\Big)^2 \tag{90}$$

$$\phi_\xi^i(\varepsilon) \equiv \begin{bmatrix} \varepsilon \frac{\partial}{\partial X} h_i q_1^* & \cdots & \varepsilon \frac{\partial}{\partial X} h_i q_p^* \\ \vdots & & \vdots \\ \varepsilon^{r_i} \frac{\partial}{\partial X} L_f^{r_i - 1} h_i q_1^* & \cdots & \varepsilon^{r_i} \frac{\partial}{\partial X} L_f^{r_i - 1} h_i q_p^* \end{bmatrix}, \quad 1 \le i \le m \tag{91}$$

2.2 Fuzzy Controller Design

Based on using the feedback linearization control as a guarantee of uniform ultimate bounded stability, the multiple-input/single-output fuzzy control design can be technically applied via expert's knowledge to improve the convergence rate of tracking error. The block diagram of the fuzzy control is shown in Fig. 1. In general, the tracking error $e(t)$ and its time derivative $\dot{e}(t)$ are utilized as the input fuzzy variables of

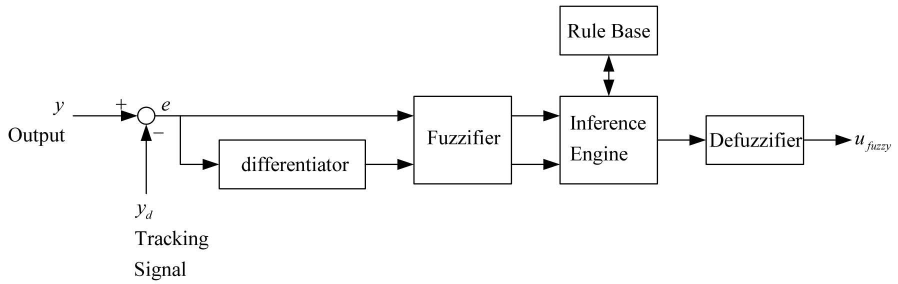


Fig. 1 Fuzzy logic controller

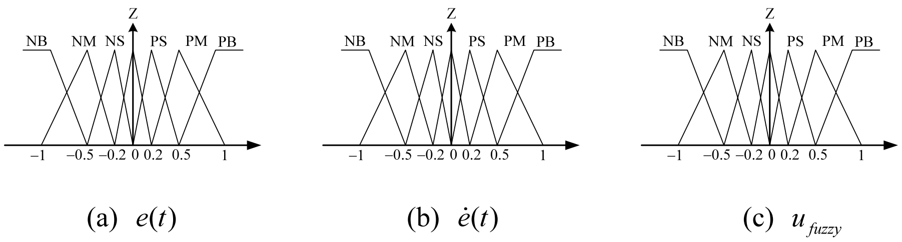


Fig. 2 Membership functions for (**a**) $e(t)$, (**b**) $\dot{e}(t)$ and (**c**) u_{fuzzy}

Table 1 Fuzzy control rule base

u_{fuzzy}		$e(t)$						
		NB	NM	NS	ZE	PS	PM	PB
$\dot{e}(t)$	NB	PB	PB	PB	PB	PM	PS	ZE
	NM	PB	PB	PB	PM	PS	ZE	NS
	NS	PB	PB	PM	PS	ZE	NS	NM
	ZE	PB	PM	PS	ZE	NS	NM	NB
	PS	PM	PS	ZE	NS	NM	NB	NB
	PM	PS	ZE	NS	NM	NB	NB	NB
	PB	ZE	NS	NM	NB	NB	NB	NB

the IF-THEN control rules and the output is the control variable u_{fuzzy}. To make it easier to compute, the membership functions of the linguistic terms for $e(t)$, $\dot{e}(t)$ and u_{fuzzy} are all chosen to be the triangular shape functions. We define seven linguistic terms: PB (Positive big), PM (Positive medium), PS (Positive small), ZE (Zero), NS (Negative small), NM (Negative medium) and NB (Negative big), for each fuzzy variable as shown in Fig. 2.

The fuzzy control rule table for u_{fuzzy} is shown in Table 1. The rule base is heuristically built by the standard MacVicar-Whelan rule base [40] for usual servo control systems. The Mamdani method is used for fuzzy inference. The defuzzification of the output set membership value is obtained by the centroid method. Therefore, we can combine the designs of feedback linearization control and fuzzy control to construct the overall controller as follows:

$$u_{\text{fe+fu}} \equiv A^{-1}\{-b+v\}u_s(t) + \begin{bmatrix} u_{\text{fuzzy1}}u_s(t-t_1) \\ u_{\text{fuzzy2}}u_s(t-t_2) \\ \vdots \\ u_{\text{fuzzy}m}u_s(t-t_m) \end{bmatrix}, \quad 1 \le i \le m \tag{92}$$

where $u_s(t)$ denotes the unit step function and t_i, $1 \le i \le m$ are the time variables that the tracking error of states touch the global final attractor $B_{\underline{r}}$.

According to the previous theorems and discussions, an efficient algorithm for deriving the almost disturbance decoupling control is proposed as follows:

(1) Calculate the vector relative degree $r_1, r_2, \ldots, r_m$ of the given control system.
(2) Choose the diffeomorphism ϕ such that Assumption 1 is satisfied.
(3) Adjust some parameters $\alpha_1^i, \alpha_2^i, \ldots, \alpha_{r_i}^i$ such that the matrices A_c^i are Hurwitz and calculate the positive definite matrices P^i of the Lyapunov equation (28) by some software package, such as MATLAB.
(4) Based on the famous Lyapunov approach, design a Lyapunov function to solve the conditions (40a)–(40c). If the relative degree $r_1 + r_2 + \cdots + r_m$ is equal to the system dimension n, then this step should be omitted and we immediately go to the next step.
(5) Appropriately tune the parameters k, ε such that $N_2 > 1$ and go to the next step. Otherwise, we go to step 3 and repeat the overall designing procedures.
(6) According to (41), the desired feedback linearization controller u_{feedback} can be constructed such that the uniform ultimate bounded stability is guaranteed. That is, the system dynamics enter a neighborhood of zero state and remain within it thereafter.
(7) Finally, the conventional fuzzy control u_{fuzzy} is immediately applied to improve the convergence rate of the tracking error dynamics.

3 Illustrative Example

A full-car model active suspension system is introduced in this section. The system entails four suspension mechanisms and four control units as shown in Fig. 3 [8, 10, 11, 41], and is represented as a nonlinear seven-degree-of-freedom (7-DOF). All suspensions consist of a spring, a damper and an actuator to generate a pushing force between the body and the axle. The sprung mass is assumed to be a rigid body and has freedom of motion in the vertical (heave), pitch and roll directions, while the unsprung masses are free to bounce vertically with respect to the sprung mass. The suspension between the sprung mass and the unsprung masses are modeled as linear viscous dampers and spring elements. The tire is assumed to contact the surface of the road when the automobile is travelling; and is regarded as a spring with a mass. From Fig. 3, the displacements of the sprung mass are given in the following:

- *Front-left wheel*

$$z_{s1} = z + a\sin\theta + d\sin\varphi \tag{93}$$

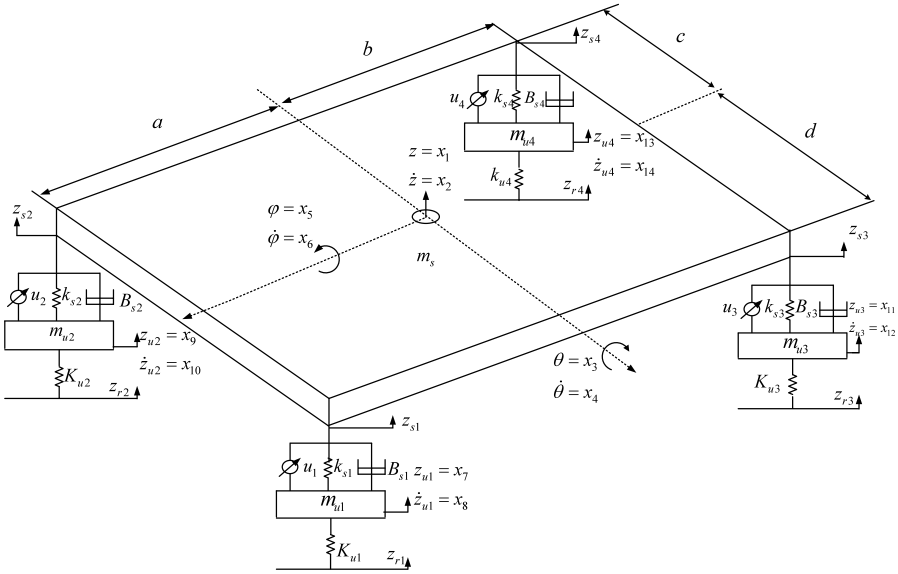


Fig. 3 Full-vehicle suspension system

- *Front-right wheel*

$$z_{s2} = z + a\sin\theta - c\sin\varphi \tag{94}$$

- *Rear-left wheel*

$$z_{s3} = z - b\sin\theta + d\sin\varphi \tag{95}$$

- *Rear-right wheel*

$$z_{s4} = z - b\sin\theta - c\sin\varphi \tag{96}$$

Here z_{s1} is the front-left body displacement, z_{s2} is the front-right body displacement, z_{s3} is the rear-left body displacement, z_{s4} is the rear-right body displacement, a and b are the distances from front wheels and the rear wheels to the center of mass of the sprung mass, c and d are the distances from the right wheels and the left wheels to the center of mass of the sprung mass, and z, θ, φ are the vertical displacement, the pitch angular displacement and the roll angular displacement of the center of mass of the sprung mass. The equivalent forces in each portion of half-cars are given in the following:

- *Front half-car*

$$\begin{aligned} F_1 = {} & k_{s2}(z_{s2} - z_{u2}) + B_{s2}(\dot{z}_{s2} - \dot{z}_{u2}) + k_{s1}(z_{s1} - z_{u1}) \\ & + B_{s1}(\dot{z}_{s1} - \dot{z}_{u1}) - u_1 - u_2 \end{aligned} \tag{97}$$

- *Rear half-car*

$$F_2 = -k_{s4}(z_{s4} - z_{u4}) - B_{s4}(\dot{z}_{s4} - \dot{z}_{u4}) - k_{s3}(z_{s3} - z_{u3}) \\ - B_{s3}(\dot{z}_{s3} - \dot{z}_{u3}) + u_3 + u_4 \tag{98}$$

- *Right half-car*

$$F_3 = -k_{s2}(z_{s2} - z_{u2}) - B_{s2}(\dot{z}_{s2} - \dot{z}_{u2}) - k_{s4}(z_{s4} - z_{u4}) \\ - B_{s4}(\dot{z}_{s4} - \dot{z}_{u4}) - u_2 - u_4 \tag{99}$$

- *Left half-car*

$$F_4 = k_{s1}(z_{s1} - z_{u1}) + B_{s1}(\dot{z}_{s1} - \dot{z}_{u1}) + k_{s3}(z_{s3} - z_{u3}) \\ - B_{s3}(\dot{z}_{s3} - \dot{z}_{u3}) + u_1 + u_3 \tag{100}$$

where u_1, u_2, u_3 and u_4 are the front-left, front-right, rear-left and rear-right force inputs. B_{s1}, B_{s2}, B_{s3} and B_{s4} are the front-left, front-right, rear-left and rear-right damping coefficients k_{s1}, k_{s2}, k_{s3} and k_{s4} are the front-left, front-right, rear-left and rear-right spring coefficients and z_{u1}, z_{u2}, z_{u3} and z_{u4} are the front-left, front-right, rear-left and rear-right wheel displacements.

By applying Newton's second law and using the static equilibrium position as the origin for the displacement of the center of gravity and the angular displacement of the vehicle body, the motion equations for the suspension system can be formulated. The equations of motion for heave are written as follows:

$$\begin{aligned}\ddot{z} = \frac{1}{m_s}\big\{&-(k_{s1}+k_{s2}+k_{s3}+k_{s4})z - (B_{s1}+B_{s2}+B_{s3}+B_{s4})\dot{z} \\ &+ \big[-a(k_{s1}+k_{s2}) + b(k_{s3}+k_{s4})\big]\sin\theta + k_{s1}z_{u1} + B_{s1}\dot{z}_{u1} \\ &+ \big[-a(B_{s1}+B_{s2}) + b(B_{s3}+B_{s4})\big]\dot{\theta}\cos\theta + k_{s2}z_{u2} + B_{s2}\dot{z}_{u2} \\ &+ \big[c(k_{s2}+k_{s4}) - d(k_{s1}+k_{s3})\big]\sin\varphi + k_{s3}z_{u3} + B_{s3}\dot{z}_{u3} \\ &+ \big[c(B_{s2}+B_{s4}) - d(B_{s1}+B_{s3})\big]\dot{\varphi}\cos\varphi + k_{s4}z_{u4} + B_{s4}\dot{z}_{u4} \\ &- u_1 - u_2 - u_3 - u_4\big\}\end{aligned} \tag{101}$$

For the pitch motion of the sprung mass:

$$\begin{aligned}I_\theta\ddot{\theta} &= aF_1\cos\theta - bF_2\cos\theta \\ &= a\cos\theta\big\{k_{s1}(z_{s1} - z_{u1}) + B_{s1}(\dot{z}_{s1} - \dot{z}_{u1}) \\ &\quad + k_{s2}(z_{s2} - z_{u2}) + B_{s2}(\dot{z}_{s2} - \dot{z}_{u2}) - u_1 - u_2\big\} \\ &\quad - b\cos\theta\big\{k_{s3}(z_{s3} - z_{u3}) + B_{s3}(\dot{z}_{s3} - \dot{z}_{u3}) \\ &\quad + k_{s4}(z_{s4} - z_{u4}) + B_{s4}(\dot{z}_{s4} - \dot{z}_{u4}) - u_3 - u_4\big\}\end{aligned} \tag{102}$$

For the roll motion of the sprung mass:

$$\begin{aligned} I_\varphi \ddot{\varphi} &= -cF_3 \cos\varphi + dF_4 \cos\varphi \\ &= -c\cos\varphi \big\{k_{s2}(z_{s2} - z_{u2}) + B_{s2}(\dot{z}_{s2} - \dot{z}_{u2}) \\ &\quad + k_{s4}(z_{s4} - z_{u4}) + B_{s4}(\dot{z}_{s4} - \dot{z}_{u4}) - u_2 - u_4\big\} \\ &\quad + d\cos\varphi \big\{k_{s1}(z_{s1} - z_{u1}) + B_{s1}(\dot{z}_{s1} - \dot{z}_{u1}) \\ &\quad + k_{s3}(z_{s3} - z_{u3}) + B_{s3}(\dot{z}_{s3} - \dot{z}_{u3}) - u_1 - u_3\big\} \end{aligned} \tag{103}$$

where m_s is the mass of the car body, and I_θ and I_φ are the centroidal moments of inertia for pitch and roll.

Applying Newton's second law once again on the front-left, front-right, rear-left and rear-right wheels' unsprung masses, for each side of the wheel motion (vertical direction) can also be formulated as follows:

- *Front-left wheel*

$$\begin{aligned} m_{u1}\ddot{z}_{u1} &= K_{u1}(z_{r1} - z_{u1}) + k_{s1}(z_{s1} - z_{u1}) \\ &\quad + B_{s1}(\dot{z}_{s1} - \dot{z}_{u1}) + u_1 \end{aligned} \tag{104}$$

- *Front-right wheel*

$$\begin{aligned} m_{u2}\ddot{z}_{u2} &= K_{u2}(z_{r2} - z_{u2}) + k_{s2}(z_{s2} - z_{u2}) \\ &\quad + B_{s2}(\dot{z}_{s2} - \dot{z}_{u2}) + u_2 \end{aligned} \tag{105}$$

- *Rear-left wheel*

$$\begin{aligned} m_{u3}\ddot{z}_{u3} &= K_{u3}(z_{r3} - z_{u3}) + k_{s3}(z_{s3} - z_{u3}) \\ &\quad + B_{s3}(\dot{z}_{s3} - \dot{z}_{u3}) + u_3 \end{aligned} \tag{106}$$

- *Rear-right wheel*

$$\begin{aligned} m_{u4}\ddot{z}_{u4} &= K_{u4}(z_{r4} - z_{u4}) + k_{s4}(z_{s4} - z_{u4}) \\ &\quad + B_{s4}(\dot{z}_{s4} - \dot{z}_{u4}) + u_4 \end{aligned} \tag{107}$$

where m_{u1}, m_{u2}, m_{u3} and m_{u4} are the unsprung masses of the front-left, front-right, rear-left and rear-right wheels. K_{u1}, K_{u2}, K_{u3} and K_{u4} are the front-left, front-right, rear-left and rear-right tire spring coefficients, and z_{r1}, z_{r2}, z_{r3} and z_{r4} are the front-left, front-right, rear-left and rear-right terrain height disturbances.

From equations (93)–(96) and (101)–(107), the system state variables of the full-car active suspension model are assigned as follows:

$x_1 = z$, ride height (heave)

$x_2 = \dot{z}$, payload velocity

$x_3 = \theta,$ pitch angle

$x_4 = \dot{\theta},$ pitch velocity

$x_5 = \varphi,$ roll angle

$x_6 = \dot{\varphi},$ roll velocity

$x_7 = z_{u1},$ front-left wheel unsprung mass height

$x_8 = \dot{z}_{u1},$ front-left wheel unsprung mass velocity

$x_9 = z_{u2},$ front-right wheel unsprung mass height

$x_{10} = \dot{z}_{u2},$ front-right wheel unsprung mass velocity

$x_{11} = z_{u3},$ rear-left wheel unsprung mass height

$x_{12} = \dot{z}_{u3},$ rear-left wheel unsprung mass velocity

$x_{13} = z_{u4},$ rear-right wheel unsprung mass height

$x_{14} = \dot{z}_{u4},$ rear-right wheel unsprung mass velocity

Therefore, (93)–(96) and (101)–(107) can be rewritten in the following form of state space model;

$$\dot{x}_1 = x_2 \tag{108a}$$

$$\begin{aligned}\dot{x}_2 = \frac{1}{m_s}\{&-(k_{s1}+k_{s2}+k_{s3}+k_{s4})x_1 - (B_{s1}+B_{s2}+B_{s3}+B_{s4})x_2 \\ &+ \left[-a(k_{s1}+k_{s2})+b(k_{s3}+k_{s4})\right]\sin x_3 + k_{s1}x_7 + B_{s1}x_8 - u_1 \\ &+ \left[-a(B_{s1}+B_{s2})+b(B_{s3}+B_{s4})\right]x_4\cos x_3 + k_{s2}x_9 + B_{s2}x_{10} - u_2 \\ &+ \left[c(k_{s2}+k_{s4})-d(k_{s1}+k_{s3})\right]\sin x_5 + k_{s3}x_{11} + B_{s3}x_{12} - u_3 \\ &+ \left[c(B_{s2}+B_{s4})-d(B_{s1}+B_{s3})\right]x_6\cos x_5 + k_{s4}x_{13} + B_{s4}x_{14} - u_4\}\end{aligned} \tag{108b}$$

$$\dot{x}_3 = x_4 \tag{108c}$$

$$\begin{aligned}\dot{x}_4 = \frac{\cos x_3}{I_\varphi}\{&\left[-a(k_{s1}+k_{s2})+b(k_{s3}+k_{s4})\right]x_1 - a(u_1+u_3) \\ &- \left[a^2(k_{s1}+k_{s2})+b^2(k_{s3}+k_{s4})\right]\sin x_3 + ak_{s1}x_7 + aB_{s1}x_8 \\ &- \left[a^2(B_{s1}+B_{s2})+b^2(B_{s3}+B_{s4})\right]x_4\cos x_3 + ak_{s2}x_9 + aB_{s2}x_{10} \\ &+ \left[c(ak_{s2}-bk_{s4})-d(ak_{s1}-bk_{s3})\right]\sin x_5 - bk_{s3}x_{11} - bB_{s3}x_{12} \\ &+ \left[c(aB_{s2}-bB_{s4})-d(aB_{s1}-bB_{s3})\right]x_6\cos x_5 - bk_{s4}x_{13} - bB_{s4}x_{14} \\ &+ \left[-a(B_{s1}+B_{s2})+b(B_{s3}+B_{s4})\right]x_2 + b(u_2+u_4)\}\end{aligned} \tag{108d}$$

$$\dot{x}_5 = x_6 \tag{108e}$$

$$\dot{x}_6 = \frac{\cos x_5}{I_\varphi}\big\{\big[c(k_{s2}+k_{s4}) - d(k_{s1}+k_{s3})\big]x_1 - c(u_2+u_4) + \big[c(ak_{s2}-bk_{s4}) - d(ak_{s1}-bk_{s3})\big]\sin x_3 + dk_{s1}x_7 + dB_{s1}x_8 + \big[c(aB_{s2}-bB_{s4}) - d(aB_{s1}-bB_{s3})\big]x_4\cos x_3 - cK_{s2}x_9 - cB_{s2}x_{10} - \big[c^2(k_{s2}+k_{s4}) + d^2(k_{s1}+k_{s3})\big]\sin x_5 + dk_{s3}x_{11} + dB_{s3}x_{12} - \big[c^2(B_{s2}+B_{s4}) + d^2(B_{s1}+B_{s3})\big]x_6\cos x_5 - ck_{s4}x_{13} - cB_{s4}x_{14} + \big[c(B_{s2}+B_{s4}) - d(B_{s1}+B_{s3})\big]x_2 + d(u_1+u_3)\big\} \tag{108f}$$

$$\dot{x}_7 = x_8 \tag{108g}$$

$$\dot{x}_8 = \frac{1}{m_{u1}}\big\{k_{s1}x_1 + B_{s1}x_2 + ak_{s1}\sin x_3 + aB_{s1}x_4\cos x_3 + dk_{s1}\sin x_5 + K_{u1}z_{r1} + dB_{s1}x_6\cos x_5 - (K_{u1}+k_{s1})x_7 - B_{s1}x_8 + u_1\big\} \tag{108h}$$

$$\dot{x}_9 = x_{10} \tag{108i}$$

$$\dot{x}_{10} = \frac{1}{m_{u2}}\big\{k_{s2}x_1 + B_{s2}x_2 + ak_{s2}\sin x_3 + aB_{s2}x_4\cos x_3 - ck_{s2}\sin x_5 + K_{u2}z_{r2} - cB_{s2}x_6\cos x_5 - (K_{u2}+k_{s2})x_9 - B_{s2}x_{10} + u_2\big\} \tag{108j}$$

$$\dot{x}_{11} = x_{12} \tag{108k}$$

$$\dot{x}_{12} = \frac{1}{m_{u3}}\big\{k_{s3}x_1 + B_{s3}x_2 - bk_{s3}\sin x_3 - bB_{s3}x_4\cos x_3 + dk_{s3}\sin x_5 + K_{u3}z_{r3} + dB_{s3}x_6\cos x_5 - (K_{u3}+k_{s3})x_{11} - B_{s3}x_{12} + u_3\big\} \tag{108l}$$

$$\dot{x}_{13} = x_{14} \tag{108m}$$

$$\dot{x}_{14} = \frac{1}{m_{u4}}\big\{k_{s4}x_1 + B_{s4}x_2 - bk_{s4}\sin x_3 - bB_{s4}x_4\cos x_3 - ck_{s4}\sin x_5 + K_{u4}z_{r4} - cB_{s4}x_6\cos x_5 - (K_{u4}+k_{s4})x_{13} - B_{s4}x_{14} + u_4\big\} \tag{108n}$$

$$y_1 = x_1 + x_2 := h_1 \tag{109a}$$

$$y_2 = x_3 + x_4 := h_2 \tag{109b}$$

$$y_3 = x_{11} + x_{12} := h_3 \tag{109c}$$

$$y_4 = x_{13} + x_{14} := h_4 \tag{109d}$$

In our simulations, the parameters of the full-car suspension model selected for this study are listed in Table 2. Hence the mathematical model can be rewritten as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -97.33x_1 - 2.8x_2 + 20.8\sin x_3 + 0.63x_4\cos x_3 - 48.67\sin x_5 - 1.4x_6\cos x_5 + 23.33x_7 + 0.67x_8 + 23.33x_9 + 0.67x_{10} + 25.33x_{11} + 0.73x_{12} + 25.33x_{13} + 0.73x_{14} - 0.00067(u_1+u_2+u_3+u_4)$$

Table 2 System parameter values used in the full-car suspension model

Parameter	Value	Parameter	Value	Parameter	Value
m_s	1500 kg	b	1.7 m	$k_{s2}=k_{s4}$	3800 N/m
$m_{ui,i=1,\dots,4}$	59 kg	c	1 m	$B_{s1}=B_{s3}$	1000 N/m/s
I_θ	2160 kg/m^2	d	2 m	$B_{s2}=B_{s4}$	1100 N/m/s
I_ϕ	460 kg/m^2	$K_{ui,i=1,\dots,4}$	190000 N/m	$z_{ri,i=1,\dots,4}$	$0.1\sin(4\pi t)$
a	1.4 m	$k_{s1}=k_{s3}$	3500 N/m		

$$
\begin{aligned}
\dot{x}_3 &= x_4\\
\dot{x}_4 &= 14.44x_1\cos x_3 + 0.44x_2\cos x_3 - 165.2\sin x_3\cos x_3 - 4.76x_4\cos^2 x_3\\
&\quad + 7.22\sin x_5\cos x_3 + 0.22x_6\cos x_5\cos x_3 + 22.69x_7\cos x_3 + 0.65x_8\cos x_3\\
&\quad + 22.69x_9\cos x_3 + 0.65x_{10}\cos x_3 - 29.91x_{11}\cos x_3 - 0.87x_{12}\cos x_3\\
&\quad - 29.91x_{13}\cos x_3 - 0.87x_{14}\cos x_3 - 6.48\times 10^{-4}(u_1+u_3)\cos x_3\\
&\quad + 7.87\times 10^{-4}(u_2+u_4)\cos x_3\\
\dot{x}_5 &= x_6\\
\dot{x}_6 &= -158.70x_1\cos x_5 - 4.57x_2\cos x_5 + 33.91\sin x_3\cos x_5 + 1.02x_4\cos x_3\cos x_5\\
&\quad - 793.48\sin x_5\cos x_5 - 22.83x_6\cos^2 x_5 + 152.17x_7\cos x_5 + 4.35x_8\cos x_5\\
&\quad - 76.09x_9\cos x_5 - 2.17x_{10}\cos x_5 + 165.22x_{11}\cos x_5 + 4.78x_{12}\cos x_5\\
&\quad - 82.61x_{13}\cos x_5 - 2.39x_{14}\cos x_5 - 2.17\times 10^{-3}(u_2+u_4)\cos x_5\\
&\quad + 4.35\times 10^{-3}(u_1+u_3)\cos x_5\\
\dot{x}_7 &= x_8\\
\dot{x}_8 &= 593.22x_1 + 16.95x_2 + 830.50\sin x_3 + 23.73x_4\cos x_3 + 1186.44\sin x_5\\
&\quad + 33.90x_6\cos x_5 - 3813.56x_7 - 16.95x_8 + 3220.34z_{r1} + 0.01695u_1\\
\dot{x}_9 &= x_{10}\\
\dot{x}_{10} &= 593.22x_1 + 16.95x_2 + 830.50\sin x_3 + 23.73x_4\cos x_3 - 593.22\sin x_5\\
&\quad - 16.95x_6\cos x_5 - 3813.56x_9 - 16.95x_{10} + 3220.34z_{r2} + 0.01695u_2\\
\dot{x}_{11} &= x_{12}\\
\dot{x}_{12} &= 644.07x_1 + 18.64x_2 - 1094.91\sin x_3 - 31.70x_4\cos x_3 + 1288.14\sin x_5\\
&\quad + 37.29x_6\cos x_5 - 3864.41x_{11} - 18.64x_{12} + 3220.34z_{r3} + 0.01695u_3\\
\dot{x}_{13} &= x_{14}\\
\dot{x}_{14} &= 644.07x_1 + 18.64x_2 - 1094.91\sin x_3 - 31.70x_4\cos x_3 - 644.07\sin x_5\\
&\quad - 18.64x_6\cos x_5 - 3864.41x_{13} - 18.64x_{14} + 3220.34z_{r4} + 0.01695u_4
\end{aligned}
\tag{110a}
$$

$$\begin{aligned} y_1 &= x_1 + x_2 := h_1 \\ y_2 &= x_3 + x_4 := h_2 \\ y_3 &= x_{11} + x_{12} := h_3 \\ y_4 &= x_{13} + x_{14} := h_4 \end{aligned} \tag{110b}$$

Now we will show how to explicitly construct a controller that tracks the desired signals $y_d^1 = y_d^2 = y_d^3 = y_d^4 = 0$ and attenuates the disturbance's effect on the output terminal to an arbitrary degree of accuracy. Let's arbitrarily choose $\alpha_1^1 = \alpha_1^2 = \alpha_1^3 = \alpha_1^4 = 0.06$, $A_c^1 = A_c^2 = A_c^3 = A_c^4 = -0.06$, $P^1 = P^2 = P^3 = P^4 = 25/3$ and $\lambda_{\min}^* = \lambda_{\max}^* = 25/3$. From (41), we obtain the desired tracking controllers

$$\begin{aligned} u_1 = {} & 818.564\big[(x_2 - 97.33x_1 - 2.8x_2 + 20.8\sin x_3 + 0.63x_4\cos x_3 - 48.67\sin x_5 \\ & - 1.4x_6\cos x_5 + 23.33x_7 + 0.67x_8 + 23.33x_9 + 0.67x_{10} + 25.33x_{11} \\ & + 0.73x_{12} + 25.33x_{13} + 0.73x_{14}) + 0.06\varepsilon^{-1}(x_1 + x_2)\big] \\ & + \frac{696.874}{\cos x_3}\big[\big(x_4 + 14.44x_1\cos x_3 + 0.44x_2\cos x_3 - 165.2\sin x_3\cos x_3 \\ & - 4.76x_4\cos^2 x_3 + 7.22\sin x_5\cos x_3 + 0.22x_6\cos x_5\cos x_3 + 22.69x_7\cos x_3 \\ & + 0.65x_8\cos x_3 + 22.69x_9\cos x_3 + 0.65x_{10}\cos x_3 - 29.91x_{11}\cos x_3 \\ & - 0.87x_{12}\cos x_3 - 29.91x_{13}\cos x_3 - 0.87x_{14}\cos x_3\big) + 0.06\varepsilon^{-1}(x_3 + x_4)\big] \\ & + 59.069\big[(x_{12} + 644.07x_1 + 18.64x_2 - 1094.91\sin x_3 - 31.70x_4\cos x_3 \\ & + 1288.14\sin x_5 + 37.29x_6\cos x_5 - 3864.41x_{11} - 18.64x_{12}) \\ & + 0.06\varepsilon^{-1}(x_{11} + x_{12})\big] \end{aligned} \tag{111}$$

$$\begin{aligned} u_2 = {} & 673.989\big[(x_2 - 97.33x_1 - 2.8x_2 + 20.8\sin x_3 + 0.63x_4\cos x_3 - 48.67\sin x_5 \\ & - 1.4x_6\cos x_5 + 23.33x_7 + 0.67x_8 + 23.33x_9 + 0.67x_{10} + 25.33x_{11} \\ & + 0.73x_{12} + 25.33x_{13} + 0.73x_{14}) + 0.06\varepsilon^{-1}(x_1 + x_2)\big] \\ & - \frac{696.874}{\cos x_3}\big[\big(x_4 + 14.44x_1\cos x_3 + 0.44x_2\cos x_3 - 165.2\sin x_3\cos x_3 \\ & - 4.76x_4\cos^2 x_3 + 7.22\sin x_5\cos x_3 + 0.22x_6\cos x_5\cos x_3 + 22.69x_7\cos x_3 \\ & + 0.65x_8\cos x_3 + 22.69x_9\cos x_3 + 0.65x_{10}\cos x_3 - 29.91x_{11}\cos x_3 \\ & - 0.87x_{12}\cos x_3 - 29.91x_{13}\cos x_3 - 0.87x_{14}\cos x_3\big) + 0.06\varepsilon^{-1}(x_3 + x_4)\big] \\ & + 58.996\big[(x_{14} + 644.07x_1 + 18.64x_2 - 1094.91\sin x_3 - 31.70x_4\cos x_3 \\ & - 644.07\sin x_5 - 18.64x_6\cos x_5 - 3864.41x_{13} - 18.64x_{14}) \\ & + 0.06\varepsilon^{-1}(x_{13} + x_{14})\big] \end{aligned} \tag{112}$$

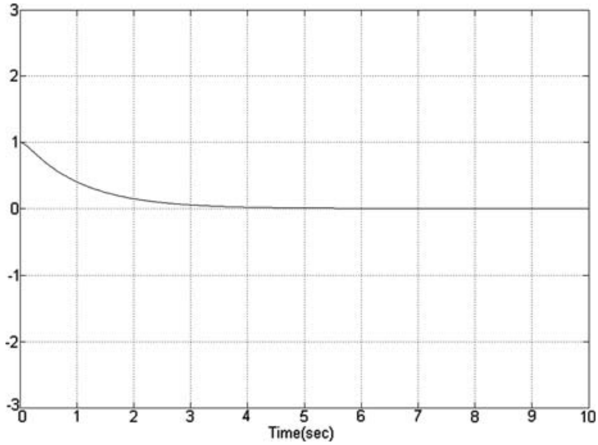
$$\begin{aligned} u_3 = {} & -59.069\big[(x_{12} + 644.07x_1 + 18.64x_2 - 1094.91\sin x_3 - 31.70x_4\cos x_3 \\ & + 1288.14\sin x_5 + 37.29x_6\cos x_5 - 3864.41x_{11} - 18.64x_{12}) \end{aligned}$$

$$+0.06\varepsilon^{-1}(x_{11}+x_{12})\big] \tag{113}$$

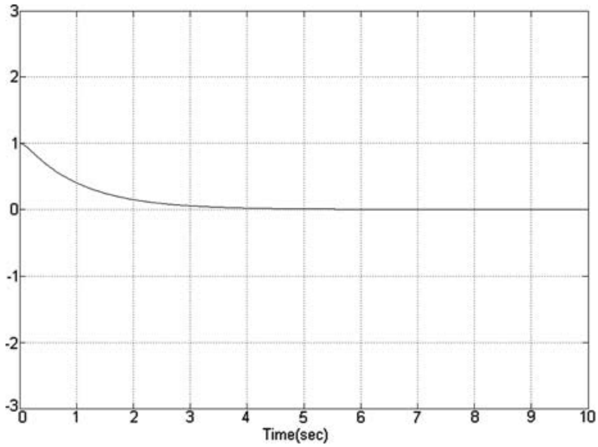
$$\begin{aligned} u_4 = -58.996\big[&(x_{14}+644.07x_1+18.64x_2-1094.91\sin x_3-31.70x_4\cos x_3 \\ &-644.07\sin x_5-18.64x_6\cos x_5-3864.41x_{13}-18.64x_{14}) \\ &+0.06\varepsilon^{-1}(x_{13}+x_{14})\big] \end{aligned} \tag{114}$$

It can be verified that the relative conditions of Theorem 1 are satisfied with $\varepsilon = 0.005$, $B_d^1 = B_d^2 = B_d^3 = B_d^4 = 0$, $k_{11} = 61.02$, $k_{22} = 1.5$, $\omega_1 = \omega_2 = 1$, $\omega_3 = 2$, $\alpha_x = 1$, $M = \sqrt{5}$ and $k = 10\sqrt{\varepsilon}$. Hence the tracking controllers will steer the output tracking errors of the closed-loop system, starting from any initial value, to be asymptotically attenuated to zero by virtue of Theorem 1. The complete trajectories of the outputs are depicted in Fig. 4 and Fig. 5.

Fig. 4 (**a**) The tracking error x_1 driven by u_{fe} for (93). (**b**) The tracking error x_3 driven by u_{fe} for (93). (**c**) The tracking error x_{11} driven by u_{fe} for (93). (**d**) The tracking error x_{13} driven by u_{fe} for (93)

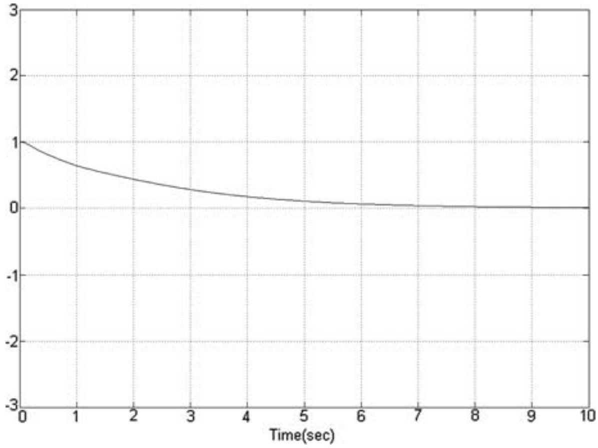


(a)

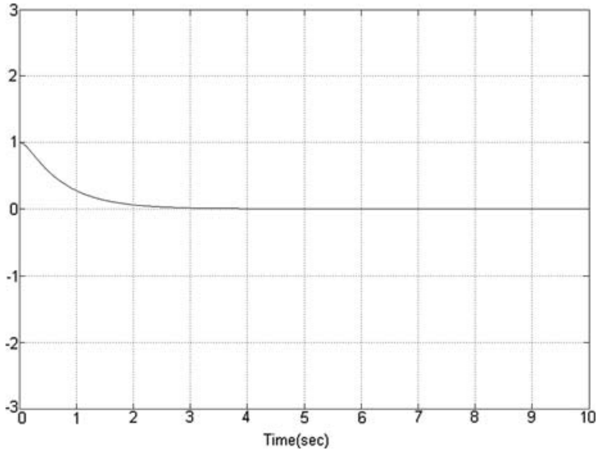


(b)

Fig. 4 (*continued*)



(c)



(d)

4 Comparative Example to Existing Approach

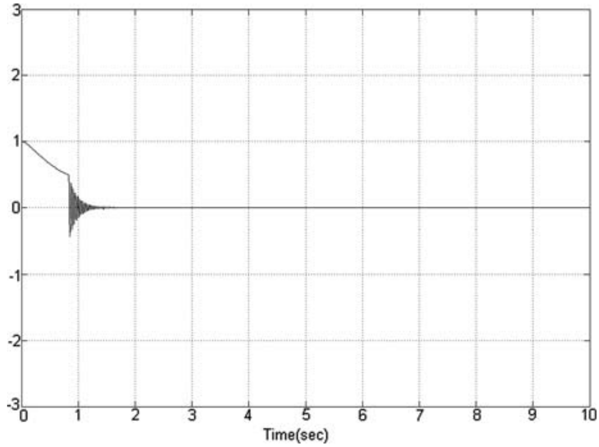
Reference [24] exploited the fact that for nonlinear SISO systems, the almost disturbance decoupling problem could not be solvable, as the following example showed:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \tan^{-1}(x_2) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \theta(t) \tag{115a}$$

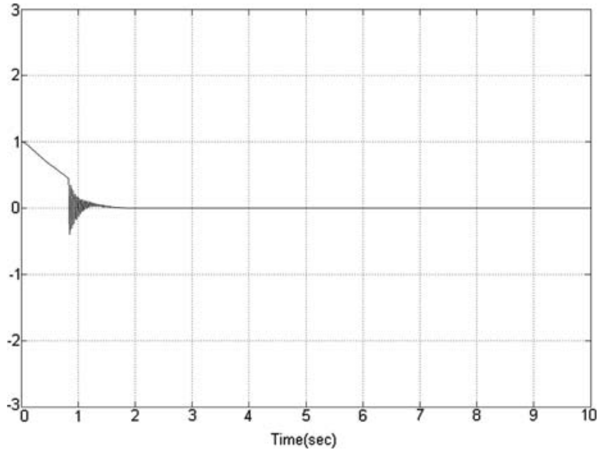
$$y(t) = x_1(t) := h\big(X(t)\big) \tag{115b}$$

where u, y denoted the input and output respectively, $\theta(t) := 2\sin t[u_s(t-1.1) - u_s(t-1.31)]$ was the disturbance shown in Fig. 6 and $u_s(t)$ denotes the unit step function. Assume that the desired tracking signal is equal to $\sin t$. The almost disturbance decoupling problem can be easily solved via the proposed approach in this

Fig. 5 (**a**) The tracking error x_1 driven by u_{fe} for (93). (**b**) The tracking error x_3 driven by $u_{\mathrm{fe+fu}}$ for (93). (**c**) The tracking error x_{11} driven by $u_{\mathrm{fe+fu}}$ for (93). (**d**) The tracking error x_{13} driven by $u_{\mathrm{fe+fu}}$ for (93)



(a)



(b)

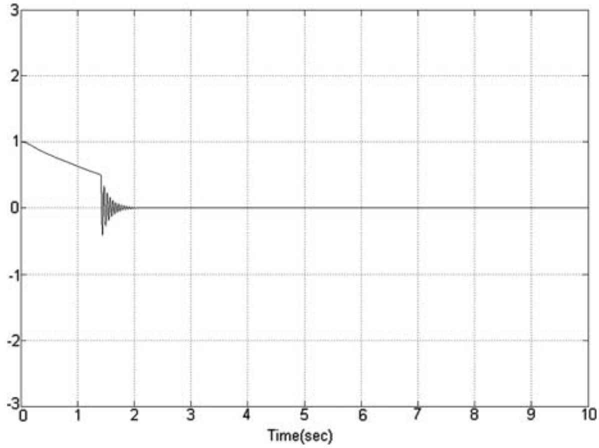
paper. Following the same procedures shown in the demonstrated example, we can solve the tracking problem with almost disturbance decoupling problem by the state feedback controller u defined as

$$u = \left(1 + x_2^2\right)\left[-\sin t - 42(x_1 - \sin t) - 42\left(\tan^{-1} x_2 - \cos t\right)\right]u_s(t) + u_{\mathrm{fuzzy}} u_s(t - t_1) \tag{116}$$

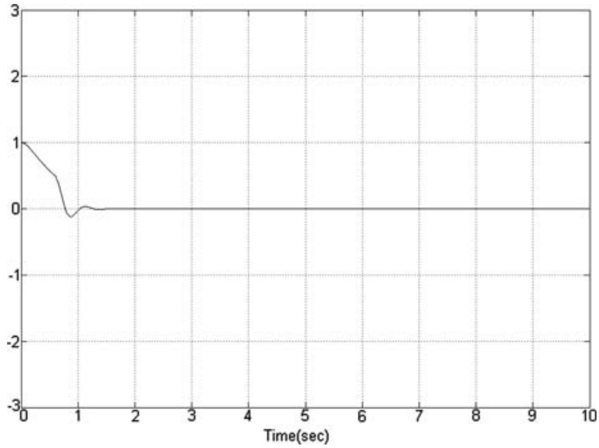
The tracking error dynamics driven by $u_{\mathrm{fe+fu}}$ and u_{feedback} for (115) are depicted in Fig. 7 and Fig. 8, respectively. It is easy to see that the convergence rate driven by both u_{feedback} and u_{fuzzy}, i.e., $u_{\mathrm{fe+fu}}$, is better than that driven only by u_{feedback}.

It is worth noting that the sufficient conditions given in [24] (in particular the structural conditions on nonlinearities multiplying disturbances) are not necessary in this study where a nonlinear state feedback control is explicitly designed which solves the almost disturbance decoupling problem. For instance, the almost disturbance decoupling problem is solvable for the system (115) by a nonlinear state feedback control,

Fig. 5 (*continued*)

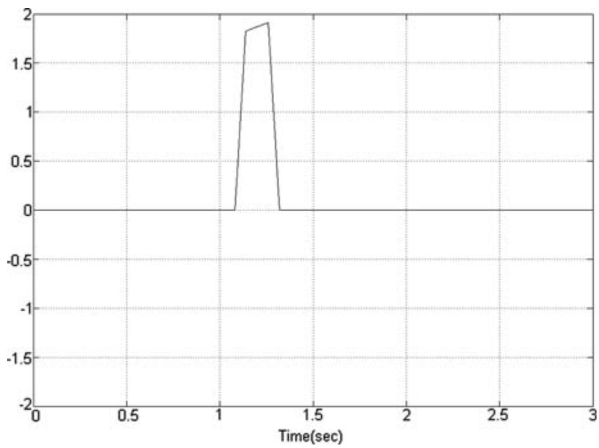


(c)



(d)

Fig. 6 The disturbance of feedback-controlled system for (115)



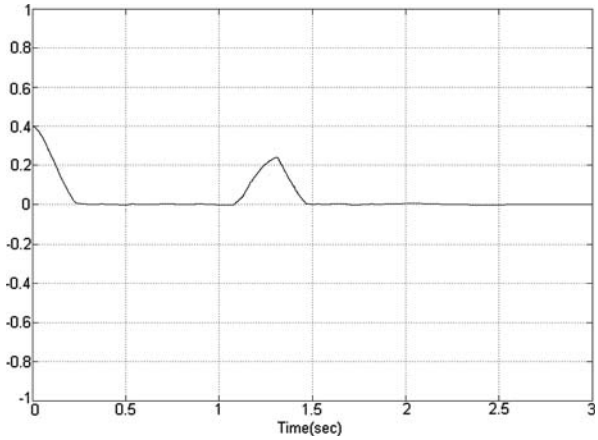


Fig. 7 The tracking error driven by $u_{\mathrm{fe+fu}}$ for (115)

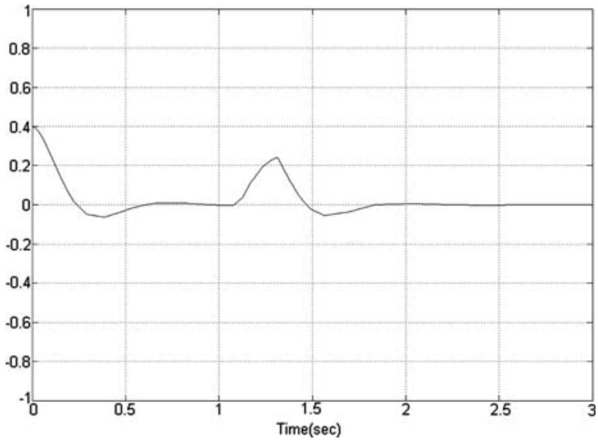


Fig. 8 The tracking error driven by u_{feedback} for (115)

according to our proposed approach, while the sufficient conditions given in [24] fail when applied to the system (115). The design techniques in this study are also entirely different than those in [24] since the singular perturbation tools are not used.

5 Conclusion

In this paper, we propose a novel fuzzy feedback control to solve globally the tracking problem with almost disturbance decoupling for MIMO nonlinear systems. The discussion and practical application of feedback linearization of nonlinear control systems by parameterized co-ordinate transformation have been presented. One comparative example is proposed to show the significant contribution of this paper with respect to some existing approach. Moreover, a practical example of a full-vehicle suspension system demonstrates the applicability of the proposed fuzzy feedback linearization approach and composite Lyapunov approach. Simulation results exploit the fact that the proposed methodology is successfully applied to feedback linearization problems and achieves the desired tracking and almost disturbance decoupling performances of the controlled system.

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